22 a) $(a+b+c)(b+c-a) = \lambda bc$ (11)

$\Rightarrow (2s)(2s-2a) = \lambda bc$

$\Rightarrow 4\left(\frac{s(s-a)}{bc}\right) = \lambda$

$\Rightarrow 4\cos^2\frac{A}{2} = \lambda \Rightarrow \cos^2\frac{A}{2} = \frac{\lambda}{4}$

We know $0 \le \cos^2\frac{A}{2} \le 1$

$\Rightarrow 0 \le \frac{\lambda}{4} \le 1 \Rightarrow 0 \le \lambda \le 4.$

Greatest value of $\lambda = 4.$

b) In $\triangle ABC$, $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

By Cauch's Inequality.

$(t_1 + t_2 + t_3) \le \sqrt{t_1^2 + t_2^2 + t_3^2}\,(\sqrt{3})$

$\Rightarrow K < 27$

By $GM \le AM \Rightarrow 9.3^{1/3} \le K$

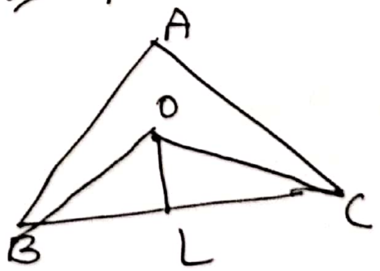
(OR) $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

$AM \ge GM$

$\Rightarrow \frac{\tan^2 A + \tan^2 B + \tan^2 C}{3} \ge (\tan^2 A \cdot \tan^2 B \cdot \tan^2 C)^{1/3}$

$\Rightarrow \frac{K}{3} \ge (9)^{2/3} \Rightarrow K \ge 3.9^{2/3}$

$K \ge 9.3^{1/3}$



c) In $\triangle OBL$, $\cos A = \frac{OL}{OB} = \frac{r}{R}$

$\Rightarrow R\cos A = r$

$\Rightarrow R\cos A = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\Rightarrow \cos A = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\Rightarrow \cos A = \cos A + \cos B + \cos C - 1$

$\Rightarrow \cos A + \cos B = 1$

(12)

d) Given $a=5,\ b=4,\ \cos(A-B)=\frac{31}{32}$

$$\tan\left(\frac{A-B}{2}\right)=\sqrt{\frac{1-\cos(A-B)}{1+\cos(A+B)}}=\sqrt{\frac{1}{63}}$$

$$\Rightarrow \frac{a-b}{a+b}\cot\frac{C}{2}=\frac{1}{\sqrt{63}} \Rightarrow \frac{5-4}{5+4}\cot\frac{C}{2}=\frac{1}{\sqrt{63}}$$

$$\Rightarrow \cancel{\cot}\ \tan\frac{C}{2}=\frac{\sqrt{7}}{3}$$

Now $\cos C=\dfrac{1-\tan^2\frac{C}{2}}{1+\tan^2 C/2}=\dfrac{1-\frac{7}{9}}{1+\frac{7}{9}}=\dfrac{1}{8}$

Now, $c^2=a^2+b^2-2ab\cos C$

$$\Rightarrow c^2=25+16-40\times\frac{1}{8}=36 \Rightarrow c=6.$$

---

23 a) $A=60^\circ \Rightarrow B+C=120^\circ \Rightarrow \frac{B}{2}+\frac{C}{2}=60^\circ$

Now $\dfrac{b}{c+a}+\dfrac{c}{a+b}=\dfrac{\sin B}{\sin A+\sin C}+\dfrac{\sin C}{\sin A+\sin B}$

$$=\frac{\sin B}{2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)}+\frac{\sin C}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$$

$$\cancel{\Rightarrow}\ \frac{\sin\frac{B}{2}}{\cos\left(\frac{A-C}{2}\right)}\ \cancel{+}\ \frac{\sin\frac{C}{2}}{\cos\left(\frac{A-B}{2}\right)}$$

$$=\frac{\sin\frac{B}{2}}{\cos\left(30^\circ-\frac{C}{2}\right)}+\frac{\sin\frac{C}{2}}{\cos\left(30^\circ-\frac{B}{2}\right)}$$

$$=\frac{\sin B/2}{\cos\left(30^\circ-\frac{B}{2}\right)}+\frac{\sin\frac{C}{2}}{\cos\left(30^\circ-\frac{B}{2}\right)}$$

$$=\frac{2\sin30^\circ\cos\left(\frac{B-C}{4}\right)}{\cos\left(30^\circ-\frac{B}{2}\right)}=$$

$$=2\times\frac{1}{2}\times\frac{\cos\left(\frac{B}{2}-30^\circ\right)}{\cos\left(30^\circ-\frac{B}{2}\right)}$$

$$=1$$

b) $c^2 = a^2+b^2-2ab\cos C$, $\angle C = 60^\circ$

$\Rightarrow c^2 = a^2+b^2-2ab\times\frac{1}{2}$

$\Rightarrow c^2 = a^2+b^2-ab \rightarrow (1)$

$\Rightarrow \cancel{\frac{1}{a+c}+\frac{1}{b+c} = \frac{a+b+2c}{a+b+2c(a+c)(b+c)}}$

$\Rightarrow b^2+bc+a^2+ac = ab+ac+bc+c^2$

$\Rightarrow b(b+c)+a(a+c) = (a+c)(b+c)$

÷ by $(a+c)(b+c)$ and add 2 on b/s

$1+\frac{b}{a+c}+1+\frac{a}{b+c} = 3$

$\Rightarrow \frac{1}{a+c}+\frac{1}{b+c} = \frac{3}{a+b+c}$

c) $b^2 = c^2+a^2-2ac\cos B$ since $B=60^\circ$

$\Rightarrow b^2 = c^2+a^2-ac$

Now $\cancel{\frac{1}{a+b}+\frac{1}{a+c} = \frac{a+c+a+b}{(a+b)(a+c)} = \frac{2a+b+c}{a^2+ac+ba+bc} = \frac{2a+b+c}{}}$

$\Rightarrow b^2+ac = c^2+a^2$

$\Rightarrow b^2+ac+bc+ab = c^2+cb+a^2+ab$

$\Rightarrow b(b+c)+a(c+b) = c(c+b)+a(a+b)$

÷ by $(b+c)(a+b)$ and add 2. on b/s

$\Rightarrow \frac{1}{a+b}+\frac{1}{a+c} = \frac{3}{a+b+c}$

d) $C = 60^\circ \Rightarrow c^2 = a^2 + b^2 - 2ab\cos C$

$\Rightarrow c^2 = a^2 + b^2 - ab$

$\Rightarrow ab = a^2 + b^2 - c^2 \longrightarrow (1)$

Also $c^2 - a^2 = b^2 - ab$ $\;\Big|\;$ $\cancel{c^2 - a^2 = b^2 - a^2 - b^2 + c^2}$

$\Rightarrow c^2 - a^2 = b(b-a)$ $\;\Big|\;$ $\cancel{c^2 - a^2 = c^2 - a^2}$

Now $$\frac{b}{c^2 - a^2} + \frac{a}{c^2 - b^2} = \frac{b}{b(b-a)} + \frac{a}{a(a-b)}$$

$$= \frac{1}{b-a} + \frac{1}{-(b-a)} = 0$$

---

24 a) $(a-b)^2\cos^2\frac{C}{2} + (a+b)^2\sin^2\frac{C}{2}$

$= (a^2 - 2ab + b^2)\cos^2\frac{C}{2} + (a^2 + 2ab + b^2)\sin^2\frac{C}{2}$

$= (a^2+b^2)\left(\cos^2\frac{C}{2} + \sin^2\frac{C}{2}\right) - 2ab\left(\cos^2\frac{C}{2} - \sin^2\frac{C}{2}\right)$

$= a^2 + b^2 - 2ab\cos C = c^2$

b) $$\frac{b^2-c^2}{a^2}\sin^2 A + \frac{c^2-a^2}{b^2}\sin^2 B$$

$$= \frac{b^2-c^2}{a^2} \times \frac{a^2}{4R^2} + \frac{c^2-a^2}{b^2} \times \frac{b^2}{4R^2}$$

$$= \frac{b^2 - c^2 + c^2 - a^2}{4R^2} = \frac{b^2-a^2}{4R^2}$$

c) Let $a = b = c = 1$ & $A = B = C = 60^\circ$

$$\frac{bc\cos A + ca\cos B + ab\cos C}{\cot A + \cot B + \cot C}$$

$$= \frac{1\cdot 1\cdot \frac{1}{2} + 1\cdot 1\cdot \frac{1}{2} + 1\cdot 1\cdot \frac{1}{2}}{\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{\left(\frac{3}{2}\right)}{\left(\frac{3}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{2}$$

$$= 2 \times \frac{\sqrt{3}}{4}$$

$= 2\Delta$. $\cancel{A}$

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d) Let $a=b=c=1$ & $A=B=C=60^\circ$

$$\frac{\cos C+\cos A}{c+a}+\frac{\cos B}{b}$$

$$=\frac{\frac{1}{2}+\frac{1}{2}}{1+1}+\frac{\frac{1}{2}}{1}\Rightarrow \frac{1}{2}+\frac{1}{2}=1\Rightarrow \frac{1}{b}$$

Ans : p, s, r, q

LEARNERS TASK

01. $a=3 \Rightarrow a=2R\sin A$

$\Rightarrow 1=2R\sin 60^\circ$ | $\Rightarrow R=\frac{1}{\sqrt{3}}$

Ans : A

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02 $a=2R\sin A$

$\Rightarrow 10=2\times5\times\sin A \Rightarrow \sin A=1 \Rightarrow A=90^\circ$

Ans : D

---

03 $A=30^\circ$, $C=90^\circ \Rightarrow B=60^\circ$

$c=2R\sin C \Rightarrow 7\sqrt{3}=2R\sin 90^\circ$

$\Rightarrow R=\frac{7\sqrt{3}}{2}$

$a=2R\sin A$

$\Rightarrow a=2\times\frac{7\sqrt{3}}{2}\times\sin 30^\circ=\cancel{2}\times\frac{7\sqrt{3}}{\cancel{2}}\times\frac{1}{2}=\frac{7\sqrt{3}}{2}$

Ans : C

---

04 $\cos C=\frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \frac{4}{5}=\frac{20^2+21^2-c^2}{2\times20\times21}$

$\Rightarrow c=13$

Ans : B

05 $\cos A = \dfrac{b^2+c^2-a^2}{2bc} = \dfrac{3^2+4^2-2^2}{2\times 3\times 4} = \dfrac{7}{8}$ 16

Ans: A

06 $\tan\dfrac{A}{2} = \dfrac{(s-b)(s-c)}{\Delta}$ OR) $\Delta^2 = s(s-a)(s-b)(s-c)$

$= \cancel{(15)}\ \dfrac{\Delta}{s(s-a)}$

$= \dfrac{30}{15(15-5)} = \dfrac{1}{5}$

Ans: D

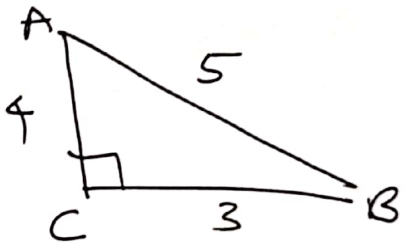
07 $\cot\dfrac{B}{2} = \dfrac{s(s-b)}{\Delta}$

$= \dfrac{66(66-60)}{330} = \dfrac{6}{5}$

Ans: B

08 $C = 90^\circ \Rightarrow \cos\dfrac{C}{2} = \cos 45^\circ$

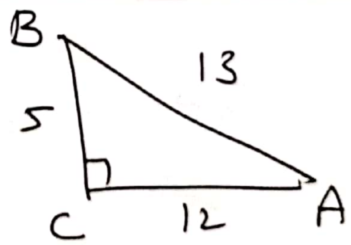
$= \dfrac{1}{\sqrt{2}}$



Ans: D

09 $C = 90^\circ \Rightarrow \sin\dfrac{C}{2} = \sin 45^\circ$

$= \dfrac{1}{\sqrt{2}}$



Ans. A

10 Let $a=b=c=1$, $A=B=C=60^\circ$

$abc \sin\dfrac{A}{2} . \sin\dfrac{B}{2} . \sin\dfrac{C}{2} = 1.1.1. \sin 30^\circ . \sin 30^\circ . \sin 30^\circ = \dfrac{1}{8}$

$\Delta = \dfrac{\sqrt{3}}{4}$

Now $\dfrac{\Delta^2}{s} = \dfrac{\left(\frac{\sqrt{3}}{4}\right)^2}{\left(\frac{3}{2}\right)} = \dfrac{3}{16}\times\dfrac{2}{3} = \dfrac{1}{8}$

$s = 3/2$

Ans : B

JEE MAIN LEVEL

01. Let $A=B=C=60^\circ$ and $a=b=c=1$

$$\sum \frac{\sin(B-C)}{bc} = \sum \frac{\sin(60^\circ-60^\circ)}{1\cdot 1} = \sum 0 = 3\times 0 = 0$$

Ans : C

02 Let $A=B=C=60^\circ$ and $a=b=c=1$

i.e. Consider an equilateral triangle.

$b^2\sin 2C + c^2\sin 2B$

$= 1^2\cdot\sin 120^\circ + 1^2\cdot\sin 120^\circ \Rightarrow \frac{\sqrt3}{2}+\frac{\sqrt3}{2} = \sqrt3$

opt : $B = 2\Delta = 2\times\frac{\sqrt3}{4} = \frac{\sqrt3}{2}$

$C = 3\Delta = 3\times\frac{\sqrt3}{4} = \frac{3\sqrt3}{4}$

opt. $A = 4\Delta$

$= 4\times\frac{\sqrt3}{4} = \sqrt3$

Ans. A

03 $a\cos A = b\cos B$

$\Rightarrow 2R\sin A\cos A = 2R\sin B\cos B$

$\Rightarrow \sin 2A = \sin 2B$

$\Rightarrow A = B$ (or) $\sin 2A = \sin(180^\circ - 2B)$

$\Rightarrow 2A = 180^\circ - 2B$

$\Rightarrow A+B = 90^\circ \Rightarrow C = 90^\circ$

Ans : C

$\therefore$ Right angled isosceles

04 $\frac{b}{c+a} + \frac{c}{a+b} - 1 = 0$

$\Rightarrow b^2+c^2-a^2-bc = 0$

$\Rightarrow \frac{b^2+c^2-a^2}{bc} = 0$

$\cos A = 0$

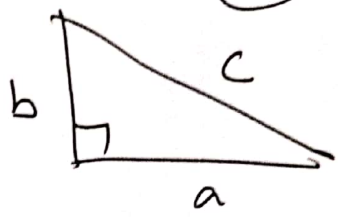
$A = 90^\circ$

Ans : D

05 Let $a = \sqrt{2}$, $b = \sqrt{6}$, $c = \sqrt{8}$ (18)

$a^2 = 2$, $b^2 = 6$, $c^2 = 8$

$a^2 + b^2 = c^2 \Rightarrow \angle C = 90^\circ$



By trial and Error method.

Opt: C, $30^\circ, 60^\circ, 90^\circ$ is Correct Answer

since $45^\circ, 45^\circ, 90^\circ$ Can not be the answer

Since $a \neq b$

Ans: C

06 $aqr + brp + cpq$

$= (2R)^2 [a \cos B \cos C + b \cos A \cos C + c \cos A \cos B]$

$= 4R^2 [\cos C [a \cos B + b \cos C) + c \cos A \cos B]$

$= 4R^2 [c \cos C + c \cos A \cos B]$

$= 4R^2 \cdot c [-\cos(A+B) + \cos A \cos B]$

$= 4R^2 \cdot c [\sin A \cdot \sin B]$

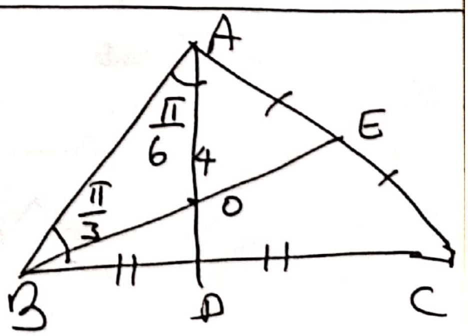
$= 4R^2 \cdot 2R \sin C \cdot \sin A \cdot \sin B$

$= (2R \sin A)(2R \sin B)(2R \sin C) = abc$

Ans: B

07 $AD = 4$, $BD = DC$

Since centroid O divides AD in the ratio 2 : 1



$AO = \frac{8}{3}$, $OD = \frac{4}{3}$

$\Rightarrow BO = AO \cot \frac{\pi}{3}$

$\Rightarrow BO = \frac{8}{3\sqrt{3}}$,

Ar $\triangle ADB = \frac{1}{2} \times 4 \times \frac{8}{3\sqrt{3}} = \frac{16}{3\sqrt{3}}$

Ar $\triangle ABC = 2 \times$ Ar $\triangle ADB$

$= \frac{32}{3\sqrt{3}}$

Ans C

(19)

08 $(a+b+c)(b+c-a) = 3bc$

$\Rightarrow (2s)(2s-2a) = 3bc$

$\Rightarrow \dfrac{s(s-a)}{bc} = \dfrac{3}{4}$

$\Rightarrow \cos^2\dfrac{A}{2} = \dfrac{3}{4} \Rightarrow \cos\dfrac{A}{2} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \dfrac{A}{2} = 30^\circ \Rightarrow A = 60^\circ$

Ans : A

09 $1 - \tan\dfrac{A}{2} \cdot \tan\dfrac{B}{2}$

$= 1 - \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)} \times \dfrac{(s-a)(s-c)}{s(s-b)}}$

$= 1 - \dfrac{s-c}{s} \;\cancel{\Rightarrow}\; \dfrac{c}{s} = \dfrac{2c}{a+b+c}$

Ans : C

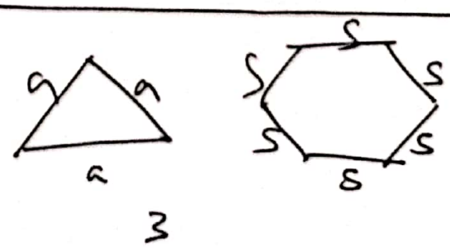
10 a, b, c AP $\Rightarrow 2b = a+c$

$\dfrac{2\sin\frac{A}{2}\sin\frac{C}{2}}{\sin\frac{B}{2}} = \dfrac{2\sqrt{\dfrac{(s-b)(s-c)}{bc}} \cdot \sqrt{\dfrac{(s-a)(s-b)}{ab}}}{\sqrt{\dfrac{(s-a)(s-c)}{ac}}}$

$= 2\left(\dfrac{s-b}{b}\right)$

$= \dfrac{2s-2b}{b} = \dfrac{\cancel{a}+b+\cancel{c}-\cancel{a}-\cancel{c}}{b} = \dfrac{b}{b} = \cancel{\dfrac{1}{2}} = 1$

11. $3a = 6s \Rightarrow a = 2s$



$\dfrac{\sqrt{3}}{4} a^2 : 6 \times \dfrac{\sqrt{3}}{4} s^2$

$\Rightarrow 4s^2 : 6s^2$

$\Rightarrow 2 : 3$

Ans : C

22 a) $(a+b+c)(b+c-a) = \lambda bc$

$\Rightarrow (2s)(2s-2a) = \lambda bc$

$\Rightarrow 4\left(\frac{s(s-a)}{bc}\right) = \lambda$

$\Rightarrow 4\cos^2\frac{A}{2} = \lambda \Rightarrow \cos^2\frac{A}{2} = \frac{\lambda}{4}$

We know $0 \le \cos^2\frac{A}{2} \le 1$

$\Rightarrow 0 \le \frac{\lambda}{4} \le 1 \Rightarrow 0 \le \lambda \le 4.$

Greatest value of $\lambda = 4.$

b) In $\triangle ABC$, $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

By Cauch's Inequality.

$(t_1 + t_2 + t_3) \le \sqrt{t_1^2 + t_2^2 + t_3^2}\,(\sqrt{3})$

$\Rightarrow K < 27$

By $GM \le AM \Rightarrow 9 \cdot 3^{1/3} \le K$

(OR) $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$

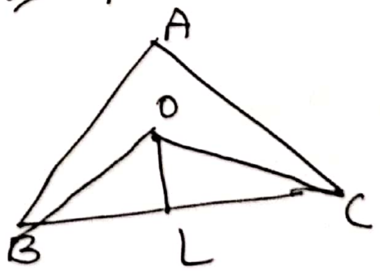
$AM \ge GM$

$\Rightarrow \frac{\tan^2 A + \tan^2 B + \tan^2 C}{3} \ge (\tan^2 A \cdot \tan^2 B \cdot \tan^2 C)^{1/3}$

$\Rightarrow \frac{K}{3} \ge (9)^{2/3} \Rightarrow K \ge 3 \cdot 9^{2/3}$

$K \ge 9 \cdot 3^{1/3}$

c) In $\triangle OBL$, $\cos A = \frac{OL}{OB} = \frac{r}{R}$



$\Rightarrow R\cos A = r$

$\Rightarrow R\cos A = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\Rightarrow \cos A = 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$\Rightarrow \cos A = \cos A + \cos B + \cos C - 1$

$\Rightarrow \cos A + \cos B = 1$

d) Given $a=5,\ b=4,\ \cos(A-B)=\frac{31}{32}$

$$\tan\left(\frac{A-B}{2}\right) = \sqrt{\frac{1-\cos(A-B)}{1+\cos(A+B)}} = \sqrt{\frac{1}{63}}$$

$$\Rightarrow \frac{a-b}{a+b}\cot\frac{C}{2} = \frac{1}{\sqrt{63}} \Rightarrow \frac{5-4}{5+4}\cot\frac{C}{2} = \frac{1}{\sqrt{63}}$$

$$\Rightarrow \cancel{\cot}\ \tan\frac{C}{2} = \frac{\sqrt{7}}{3}$$

Now $\cos C = \dfrac{1-\tan^2\frac{C}{2}}{1+\tan^2 C/2} = \dfrac{1-\frac{7}{9}}{1+\frac{7}{9}} = \dfrac{1}{8}$

Now, $c^2 = a^2+b^2-2ab\cos C$

$$\Rightarrow c^2 = 25+16-40\times\frac{1}{8} = 36 \Rightarrow c=6.$$

23 a) $A=60^\circ \Rightarrow B+C=120^\circ \Rightarrow \frac{B}{2}+\frac{C}{2}=60^\circ$

Now $\dfrac{b}{c+a}+\dfrac{c}{a+b} = \dfrac{\sin B}{\sin A+\sin C} + \dfrac{\sin C}{\sin A+\sin B}$

$$= \frac{\sin B}{2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right)} + \frac{\sin C}{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}$$

$$\cancel{\Rightarrow} \frac{\sin\frac{B}{2}}{\cos\left(\frac{A-C}{2}\right)} + \frac{\sin\frac{C}{2}}{\cos\left(\frac{A-B}{2}\right)}$$

$$= \frac{\sin\frac{B}{2}}{\cos\left(30^\circ-\frac{C}{2}\right)} + \frac{\sin\frac{C}{2}}{\cos\left(30^\circ-\frac{B}{2}\right)}$$

$$= \frac{\sin B/2}{\cos\left(30^\circ-\frac{B}{2}\right)} + \frac{\sin\frac{C}{2}}{\cos\left(30^\circ-\frac{B}{2}\right)}$$

$$= \frac{2\sin 30^\circ\cos\left(\frac{B-C}{4}\right)}{\cos\left(30^\circ-\frac{B}{2}\right)} =$$

$$= 2\times\frac{1}{2}\times\frac{\cos\left(\frac{B}{2}-30^\circ\right)}{\cos\left(30^\circ-\frac{B}{2}\right)}$$

$$= 1$$

(13)

b) $c^2 = a^2 + b^2 - 2ab\cos C$, $\angle C = 60^\circ$

$\Rightarrow c^2 = a^2 + b^2 - 2ab \times \frac{1}{2}$

$\Rightarrow c^2 = a^2 + b^2 - ab \rightarrow (1)$

$\cancel{\Rightarrow \frac{1}{a+c} + \frac{1}{b+c} = \frac{a+b+2c}{(a+c)(b+c)}}$

$\Rightarrow b^2 + bc + a^2 + ac = ab + ac + bc + c^2$

$\Rightarrow b(b+c) + a(a+c) = (a+c)(b+c)$

$\div$ by $(a+c)(b+c)$ and add 2 on b/s

$1 + \frac{b}{a+c} + 1 + \frac{a}{b+c} = 3$

$\Rightarrow \frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$

c) $b^2 = c^2 + a^2 - 2ac\cos B$ since $B = 60^\circ$

$\Rightarrow b^2 = c^2 + a^2 - ac$

$\cancel{\text{Now } \frac{1}{a+b} + \frac{1}{a+c} = \frac{a+c+a+b}{(a+b)(a+c)} = \frac{2a+b+c}{a^2+ac+ba+bc} = \frac{2a+b+c}{}}$

$\Rightarrow b^2 + ac = c^2 + a^2$

$\Rightarrow b^2 + ac + bc + ab = c^2 + cb + a^2 + ab$

$\Rightarrow b(b+c) + a(c+b) = c(c+b) + a(a+b)$

$\div$ by $(b+c)(a+b)$ and add 2. on b/s

$\Rightarrow \frac{1}{a+b} + \frac{1}{a+c} = \frac{3}{a+b+c}$

d) $c = 60^\circ \Rightarrow c^2 = a^2+b^2-2ab\cos C$

$\Rightarrow c^2 = a^2+b^2-ab$

$\Rightarrow ab = a^2+b^2-c^2 \longrightarrow (1)$

Also $c^2-a^2 = b^2-ab$ | $\cancel{c^2-a^2 = b^2-a^2-b^2+c^2}$

$\Rightarrow c^2-a^2 = b(b-a)$ | $\cancel{c^2-a^2 = c^2-a^2}$

Now $\dfrac{b}{c^2-a^2} + \dfrac{a}{c^2-b^2} = \dfrac{b}{b(b-a)} + \dfrac{a}{a(a-b)}$

$= \dfrac{1}{b-a} + \dfrac{1}{-(b-a)} = 0$

---

24 a) $(a-b)^2\cos^2\frac{C}{2} + (a+b)^2\sin^2\frac{C}{2}$

$= (a^2-2ab+b^2)\cos^2\frac{C}{2} + (a^2+2ab+b^2)\sin^2\frac{C}{2}$

$= (a^2+b^2)(\cos^2\frac{C}{2}+\sin^2\frac{C}{2}) - 2ab(\cos^2\frac{C}{2} - \sin^2\frac{C}{2})$

$= a^2+b^2-2ab\cos C = c^2$

b) $\dfrac{b^2-c^2}{a^2}\sin^2 A + \dfrac{c^2-a^2}{b^2}\sin^2 B$

$= \dfrac{b^2-c^2}{a^2} \times \dfrac{a^2}{4R^2} + \dfrac{c^2-a^2}{b^2} \times \dfrac{b^2}{4R^2}$

$= \dfrac{b^2-c^2+c^2-a^2}{4R^2} = \dfrac{b^2-a^2}{4R^2}$

c) let $a=b=c=1$ & $A=B=C=60^\circ$

$\dfrac{bc\cos A + ca\cos B + ab\cos C}{\cot A + \cot B + \cot C}$

$= \dfrac{1.1.\frac{1}{2} + 1.1.\frac{1}{2} + 1.1.\frac{1}{2}}{\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \dfrac{\left(\frac{3}{2}\right)}{\left(\frac{3}{\sqrt{3}}\right)} = \cancel{3}\,\dfrac{\sqrt{3}}{2}$

$= 2 \times \dfrac{\sqrt{3}}{4}$

$= 2\Delta.$ $\cancel{A}$

d) Let $a=b=c=1$ & $A=B=C=60^\circ$

$$\frac{\cos C+\cos A}{c+a}+\frac{\cos B}{b}$$

$$=\frac{\frac{1}{2}+\frac{1}{2}}{1+1}+\frac{\frac{1}{2}}{1} \Rightarrow \frac{1}{2}+\frac{1}{2}=1 \Rightarrow \frac{1}{b}$$

Ans: p, s, r, q

LEARNERS TASK

01. $a=3 \Rightarrow a=2R\sin A$

$\Rightarrow 1=2R\sin 60^\circ$

$\Rightarrow R=\frac{1}{\sqrt{3}}$

Ans: A

02 $a=2R\sin A$

$\Rightarrow 10=2\times 5\times \sin A \Rightarrow \sin A=1 \Rightarrow A=90^\circ$

Ans: D

03 $A=30^\circ,\ C=90^\circ \Rightarrow B=60^\circ$

$c=2R\sin C \Rightarrow 7\sqrt{3}=2R\sin 90^\circ$

$\Rightarrow R=\frac{7\sqrt{3}}{2}$

$a=2R\sin A$

$\Rightarrow a=2\times\frac{7\sqrt{3}}{2}\times\sin 30^\circ = \cancel{2}\times\frac{7\sqrt{3}}{\cancel{2}}\times\frac{1}{2}=\frac{7\sqrt{3}}{2}$

Ans: C

04 $\cos C=\frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \frac{4}{5}=\frac{20^2+21^2-c^2}{2\times 20\times 21}$

$\Rightarrow c=13$

Ans: B

05 $\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{3^2+4^2-2^2}{2\times3\times4} = \frac{7}{8}$ 16

Ans: A

06 $\tan\frac{A}{2} = \frac{(s-b)(s-c)}{\Delta}$ (OR) $\Delta^2 = s(s-a)(s-b)(s-c)$

$= \frac{\Delta}{s(s-a)}$

$= \frac{30}{15(15-5)} = \frac{1}{5}$

Ans: D

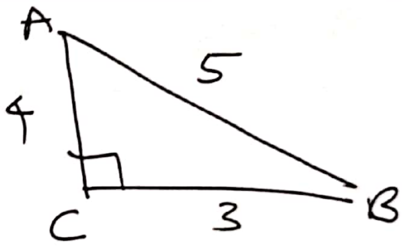
07 $\cot\frac{B}{2} = \frac{s(s-b)}{\Delta}$

$= \frac{66(66-60)}{330} = \frac{6}{5}$

Ans: B

08 $C=90^\circ \Rightarrow \cos\frac{C}{2} = \cos 45^\circ$

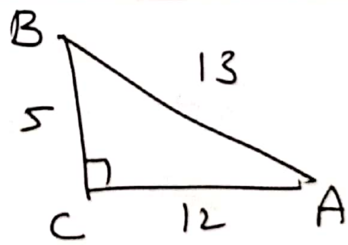
$= \frac{1}{\sqrt{2}}$



Ans: D

09 $C=90^\circ \Rightarrow \sin\frac{C}{2} = \sin 45^\circ$

$= \frac{1}{\sqrt{2}}$



Ans. A

10 let $a=b=c=1$, $A=B=C=60^\circ$

$abc\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2} = 1.1.1.\sin30^\circ.\sin30^\circ.\sin30^\circ = \frac{1}{8}$

Now $\frac{\Delta^2}{s} = \frac{\left(\frac{\sqrt{3}}{4}\right)^2}{\left(\frac{3}{2}\right)} = \frac{3}{16}\times\frac{2}{3} = \frac{1}{8}$

$\Delta = \frac{\sqrt{3}}{4}$

$s = 3/2$

Ans: B

01. Let $A=B=C=60^\circ$ and $a=b=c=1$

$$\sum \frac{\sin(B-C)}{bc} = \sum \frac{\sin(60^\circ-60^\circ)}{1\cdot 1} = \Sigma 0 = 3\times 0$$
$$=0$$

Ans: C

---

02. Let $A=B=C=60^\circ$ and $a=b=c=1$

i.e. Consider an equilateral triangle.

$b^2\sin 2C + c^2 \sin 2B$

$= 1^2 \cdot \sin 120^\circ + 1^2 \cdot \sin 120^\circ \Rightarrow \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}$

opt: $B = 2\Delta = 2\times\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$C = 3\Delta = 3\times\frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$

opt. $A = 4\Delta$
$= 4\times\frac{\sqrt{3}}{4} = \sqrt{3}$

Ans. A

---

03. $a\cos A = b\cos B$

$\Rightarrow 2R\sin A\cos A = 2R\sin B\cos B$

$\Rightarrow \sin 2A = \sin 2B$

$\Rightarrow A = B$ (or) $\sin 2A = \sin(180^\circ - 2B)$

$\Rightarrow 2A = 180^\circ - 2B$

$\Rightarrow A+B = 90^\circ \Rightarrow C = 90^\circ$

Ans: C

$\therefore$ Right angled isosceles

---

04. $\frac{b}{c+a} + \frac{c}{a+b} - 1 = 0$

$\Rightarrow b^2+c^2-a^2-bc=0$

$\Rightarrow \frac{b^2+c^2-a^2}{bc} = 0$

$\cos A = 0$

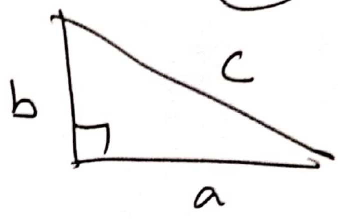
$A = 90^\circ$

Ans: D

05 Let $a = \sqrt{2}$, $b = \sqrt{6}$, $c = \sqrt{8}$ (18)

$a^2 = 2$, $b^2 = 6$, $c^2 = 8$

$a^2 + b^2 = c^2 \Rightarrow \angle C = 90^\circ$



By trial and Error method.

Opt: C, $30^\circ, 60^\circ, 90^\circ$ is Correct Answer

since $45^\circ, 45^\circ, 90^\circ$ Can not be the answer

Since $a \neq b$

Ans: C

06 $aqr + brp + cpq$

$= (2R)^2 [\cancel{a}\cos B\cos C + b\cos A\cos C + c\cos A\cos B]$

$= 4R^2 [\cos C\,[a\cos B + b\cos C) + c\cos A\cos B]$

$= 4R^2 (c\cos C + c\cos A\cos B]$

$= 4R^2 \cdot c\,[-\cos(A+B) + \cos A\cos B]$

$= 4R^2 \cdot c\,[\sin A \cdot \sin B]$

$= 4R^2 \cdot 2R\sin C \cdot \sin A \cdot \sin B$

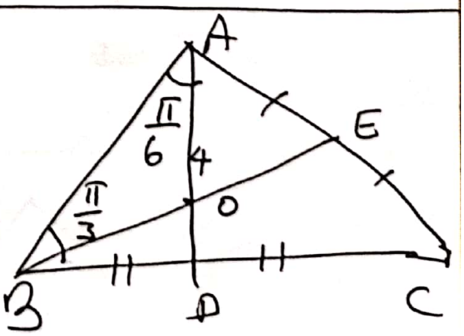
$= (2R\sin A)\,(2R\sin B)(2R\sin C) = abc$

Ans: B

07 $AD = 4$, $BD = DC$

Since centroid O divides AD in the ratio 2 : 1

$AO = \frac{8}{3}$, $OD = \frac{4}{3}$



$\Rightarrow BO = AO \cot\frac{\pi}{3}$

$\Rightarrow BO = \frac{8}{3\sqrt{3}}$,

$Ar\,\triangle ADB = \frac{1}{2} \times 4 \times \frac{8}{3\sqrt{3}} = \frac{16}{3\sqrt{3}}$

$Ar\,\triangle ABC = 2 \times Ar\,\triangle ADB$

$= \frac{32}{3\sqrt{3}}$

Ans C

08 $(a+b+c)(b+c-a) = 3bc$

$\Rightarrow (2s)(2s-2a) = 3bc$

$\Rightarrow \dfrac{s(s-a)}{bc} = \dfrac{3}{4}$

$\Rightarrow \cos^2\dfrac{A}{2} = \dfrac{3}{4} \Rightarrow \cos\dfrac{A}{2} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \dfrac{A}{2} = 30^\circ \Rightarrow A = 60^\circ$

Ans: A

---

09 $1 - \tan\dfrac{A}{2} \cdot \tan\dfrac{B}{2}$

$= 1 - \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)} \times \dfrac{(s-a)(s-c)}{s(s-b)}}$

$= 1 - \dfrac{s-c}{s} \Rightarrow \dfrac{c}{s} = \dfrac{2c}{a+b+c}$

Ans: C

---

10 a, b, c AP $\Rightarrow 2b = a+c$

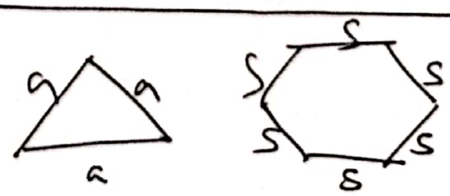
$$\frac{2\sin\frac{A}{2}\sin\frac{C}{2}}{\sin\frac{B}{2}} = \frac{2\sqrt{\frac{(s-b)(s-c)}{bc}} \cdot \sqrt{\frac{(s-a)(s-b)}{ab}}}{\sqrt{\frac{(s-a)(s-c)}{ac}}}$$

$= 2\left(\dfrac{s-b}{b}\right)$

$= \dfrac{2s-2b}{b} = \dfrac{\cancel{a}+b+\cancel{c}-\cancel{a}-\cancel{c}}{b} = \dfrac{b}{b} = \cancel{\dfrac{1}{2}} = 1$

---

11. $3a = 6s \Rightarrow a = 2s$



3

$\dfrac{\sqrt{3}}{4}a^2 : 6 \times \dfrac{\sqrt{3}}{4}s^2$

$\Rightarrow 4s^2 : 6s^2$

$\Rightarrow 2:3$

Ans: C

08 $(a+b+c)(b+c-a) = 3bc$

$\Rightarrow (2s)(2s-2a) = 3bc$

$\Rightarrow \dfrac{s(s-a)}{bc} = \dfrac{3}{4}$

$\Rightarrow \cos^2\dfrac{A}{2} = \dfrac{3}{4} \Rightarrow \cos\dfrac{A}{2} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \dfrac{A}{2} = 30^\circ \Rightarrow A = 60^\circ$ Ans: A

09 $1 - \tan\dfrac{A}{2} \cdot \tan\dfrac{B}{2}$

$= 1 - \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)} \times \dfrac{(s-a)(s-c)}{s(s-b)}}$

$= 1 - \dfrac{s-c}{s} \Rightarrow \dfrac{c}{s} = \dfrac{2c}{a+b+c}$ Ans: C

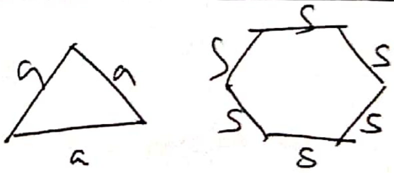
10 a, b, c AP $\Rightarrow 2b = a+c$

$\dfrac{2\sin\frac{A}{2}\sin\frac{C}{2}}{\sin\frac{B}{2}} = \dfrac{2\sqrt{\dfrac{(s-b)(s-c)}{bc}} \cdot \sqrt{\dfrac{(s-a)(s-b)}{ab}}}{\sqrt{\dfrac{(s-a)(s-c)}{ac}}}$

$= 2\left(\dfrac{s-b}{b}\right)$

$= \dfrac{2s-2b}{b} = \dfrac{\cancel{a}+b+\cancel{c}-\cancel{a}-\cancel{c}}{b} = \dfrac{b}{b} = \dfrac{1}{1} = 1$

11. $3a = 6s \Rightarrow a = 2s$



3

$\dfrac{\sqrt{3}}{4}a^2 : 6 \times \dfrac{\sqrt{3}}{4}s^2$

$\Rightarrow 4s^2 : 6s^2$

$\Rightarrow 2 : 3$ Ans: C

12 $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

possible only when $A=B=C$ and $a=b=c$

i.e. $\triangle ABC$ is an equilateral triangle.

If $a = 2\text{ cm}$, Area $= \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4}(2)^2 = \sqrt{3}$ Ans: A, B

---

13 This problem can be done in the following way also.

Consider $\triangle ABC$ is an equilateral $\triangle$le.

We have $a=b=c=1$, $R = \frac{1}{\sqrt{3}}$, $s = \frac{3}{2}$, $\Delta = \frac{\sqrt{3}}{4}$.

$\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2}$

$= \cot 30^\circ + \cot 30^\circ + \cot 30^\circ = 3 \times \sqrt{3} = 3\sqrt{3}$

opt: A: $\frac{s^2}{\Delta} = \frac{\left(\frac{3}{2}\right)^2}{\left(\frac{\sqrt{3}}{4}\right)} = \frac{9}{\sqrt{3}} = 3\sqrt{3}$

opt C: $\frac{(a+b+c)^2}{abc}R = \frac{(1+1+1)^2}{1\cdot1\cdot1} \times \frac{1}{\sqrt{3}} = \frac{9}{\sqrt{3}} = 3\sqrt{3}$

Ans: A, C

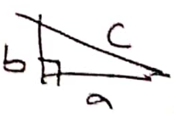
---

14. Statement I:

$4s(s-a)(s-b)(s-c) = a^2b^2$

$4\Delta^2 = a^2b^2 \Rightarrow 2\Delta = ab \Rightarrow \Delta = \frac{1}{2}ab$

$\Rightarrow \triangle ABC$ is a right angled $\triangle$le (True)



Statement II: $\sin A + \sin B + \sin C$ is maximum

$= \frac{3\sqrt{3}}{2}$. when $A = B = C = 60^\circ$

Which is the condition for equilateral $\triangle$le

Ans: A

5 Statement I: Given the angles of $\triangle ABC$ are in A.P. i.e. $2B = A+C$

$$\Rightarrow 3B = A+B+C$$
$$\Rightarrow 3B = 180$$
$$\Rightarrow B = 60^\circ.$$

Now $30^\circ, 60^\circ, 90^\circ$ are in A.P.

Now $\cos 60^\circ + \cos 60^\circ + \cos 60^\circ = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$.

(Given statement is false)

Statement II: In $\triangle ABC$, A, B, C are in A.P.

$30^\circ, 60^\circ, 90^\circ$ are in A.P.

Now ~~cos~~ $\cos 30^\circ + \cos 60^\circ + \cos 90^\circ$
$$= \frac{\sqrt{3}}{2} + \frac{1}{2} + 0 = \frac{\sqrt{3}+1}{2}.$$

Ans : B

(Given statement is false)

---

16 $c^2 = a^2 + b^2 - 2ab\cos C$
$$\Rightarrow c^2 = 2^2 + 4^2 - 2.2.4.\cos 60^\circ$$
$$= 20 - 16 \times \frac{1}{2}$$
$$= 20 - 8$$
$$= 12$$
$\therefore c = \sqrt{12}$

Ans: C

---

17 $b^2 = c^2 + a^2 - 2ac\cos B$
$$\Rightarrow 5^2 = 4^2 + 3^2 - 2.3.4.\cos B$$
$$\Rightarrow 25 = 16 + 9 - 24\cos B$$
$$\Rightarrow \cos B = 0 \Rightarrow B = 90^\circ$$

Ans: C

---

18 Consider an equilateral $\triangle^{le}$

Let $a=b=c=1$ and $A=B=C=60^\circ$, $\Delta=\frac{\sqrt{3}}{4}$.

Now $a^2+b^2+c^2 = 1^2+1^2+1^2 = 3$

opt A: $4\Delta(\cot A+\cot B+\cot C)$

$= \cancel{4}\times\frac{\sqrt{3}}{\cancel{4}}(\cot 60^\circ+\cot 60^\circ+\cot 60^\circ)$

$= \sqrt{3}(3\times\cot 60^\circ) = 3\sqrt{3}\times\frac{1}{\sqrt{3}} = 3$

$=$

Ans: A

---

19 Given that $a=6$, $b=3$, $\cos(A-B)=\frac{4}{5}$

$\tan\left(\frac{A-B}{2}\right) = \sqrt{\frac{1-\cos(A-B)}{1+\cos(A-B)}} = \sqrt{\frac{1-4/5}{1+4/5}} = \frac{1}{3}$

Now $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$

$\Rightarrow \frac{1}{3} = \frac{6-3}{6+3}\cot\frac{C}{2}$

$\Rightarrow C=\frac{\pi}{2}$ (right angle $\triangle^{le}$)

Ans: B

Ans. B

---

20 Area of $\Delta = \frac{1}{2}\times a\times b$

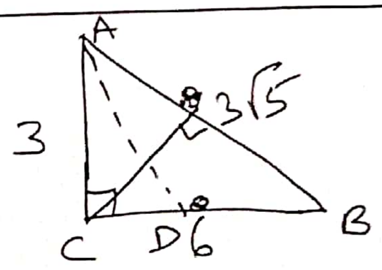
$= \frac{1}{2}\times 6\times 3$

$= 9.$

Ans: A

---

21) $\cancel{AB^2 = 3^2+6^2 = 3\sqrt{5}}$



$AC=3$, $CD=3$

$AD^2 = 3^2+3^2 = 18$

$AD = \sqrt{18} = \sqrt{9\times 2} = 3\sqrt{2}$

Ans: C

22 ~~Cos A + Cos 2B~~ $\cos A + \sin A - \dfrac{2}{\cos B + \sin B} = 0$ 23

$\Rightarrow (\cos A + \sin A)(\cos B + \sin B) = 2$

$\Rightarrow \cos(A-B) + \sin(A+B) = 2$

$\Rightarrow A = B = \dfrac{\pi}{4}$ and $C = \dfrac{\pi}{2}$

Now $\left(\dfrac{a+b}{c}\right)^4 = \left(\dfrac{\sin A + \sin B}{\sin C}\right)^4 = 4$. Ans: 4

---

23 $a+b-c=2$ and $2ab-c^2=4$

Now $2ab - (a+b-2)^2 = 4$

$\Rightarrow 2ab - a^2 - b^2 - 4 - 2ab + 4b + 4a = 4$

$\Rightarrow (a-2)^2 + (b-2)^2 = 0$

$\Rightarrow a = 2$ & $b = 2 \Rightarrow c = 2$

Thus $\triangle ABC$ is an equilateral $\triangle$le

$\Delta = \left[\dfrac{\sqrt{3}}{4}(2)^2\right]^2 = 3$ Ans: 3

24

05 $A=30^\circ,\ B=45^\circ \Rightarrow C=105^\circ$

$\cancel{\frac{a}{\sin A}}$ $\quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \frac{\sin 30^\circ}{\sqrt{3}+1} = \sin$

$\Rightarrow \frac{\sin 30^\circ}{a} = \frac{\sin 45^\circ}{b} = \frac{\sin 105^\circ}{\sqrt{3}+1}$

$\Rightarrow \frac{\left(\frac{\sqrt{3}}{2}\right)}{a} = \frac{1}{1}$

---

05 Let $A=30^\circ,\ B=45^\circ \Rightarrow C=105^\circ$

Now $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \frac{\sin 30^\circ}{a} = \frac{\sin 45^\circ}{b} = \frac{\sin 105^\circ}{\sqrt{3}+1}$

$\Rightarrow \frac{\left(\frac{1}{2}\right)}{a} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{b} = \frac{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\sqrt{3}+1} = \frac{1}{2\sqrt{2}}$

$\therefore\ a=\sqrt{2},\quad b=2$

Area $\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{\cancel{2}}\times\sqrt{2}\times\cancel{2}\times\sin 105^\circ$

$= \cancel{\sqrt{2}}\times\left(\frac{\sqrt{3}+1}{2\cancel{\sqrt{2}}}\right) = \frac{\sqrt{3}+1}{2}$ Ans: A

---

06 $B=120^\circ,\ C=30^\circ \Rightarrow A=30^\circ$

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \Rightarrow \frac{\sin 30^\circ}{2} = \frac{\sin 120^\circ}{b} = \frac{\sin 30^\circ}{c}$

$\Rightarrow \frac{1}{4} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{b} = \frac{1}{2(c)} \Rightarrow b=2\sqrt{3},\quad c=2$

$\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}\times 2\sqrt{3}\times 2\times\sin 30^\circ = \sqrt{3}$ Ans: B

24 a) Let us consider equilateral triangle (24)

$a=b=c=1$, $A=B=C=60^\circ$, $\Delta=\frac{\sqrt{3}}{4}$, $s=\frac{3}{2}$, $R=\frac{1}{\sqrt{3}}$

$a^2\sin 2C + c^2\sin 2A$

$= (1)^2\sin 2\times 60^\circ + (1)^2 \sin 2\times 60^\circ$

$= \sin 120^\circ + \sin 120^\circ = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$.

option t $\Rightarrow 2s = 2\times\frac{3}{2} = 3$

b) $R^2(\sin 2A + \sin 2B + \sin 2C)$

$=\left(\frac{1}{\sqrt{3}}\right)^2(\sin 2\times 60^\circ + \sin 2\times 60^\circ + \sin 2\times 60^\circ)$

$= \frac{1}{3}(3\times\sin 120^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

option: $p \Rightarrow 2\Delta = 2\times\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

c) $c\cos^2\frac{A}{2} + a\cos^2\frac{C}{2}$

$= 1\cos^2 30^\circ + 1\cos^2 30^\circ = 2\left(\frac{\sqrt{3}}{2}\right)^2 = 2\times\frac{3}{4} = \frac{3}{2}$

option: $\Rightarrow s = \frac{3}{2}$

d) $a\sin^2\frac{C}{2} + c\sin^2\frac{A}{2} = 1.\sin^2 30^\circ + 1.\sin^2 30^\circ$

$= 2\left(\frac{1}{2}\right)^2 = 2\times\frac{1}{4} = \frac{1}{2}$

option: $S \Rightarrow s-b = \frac{3}{2} - 1 = \frac{1}{2}$

Ans: p, q, r, s

ADDITIONAL PRACTICE QUESTIONS

01. Sum of two sides must be greater than third side

$1+2004 = 2005$. Let the third side be x

$\therefore$ $1+2004 = 2005$.

opt A $\Rightarrow x+1+2004 = 3006 \Rightarrow x = 1000 < 2005$ (True)

opt B $\Rightarrow x+1+2004 = 4009 \Rightarrow x = 2005 < 2005$ (False)

opt C $\Rightarrow x+1+2004 = 5008 \Rightarrow x = 3003 < 2005$ (False)

opt D $\Rightarrow x+1+2004 = 6001 \Rightarrow x = 3996 < 2005$ (False)

Ans: opt: A

02 $a\cos^2\frac{C}{2} + c\cdot\cos^2\frac{A}{2}$

$\Rightarrow \cancel{a}\cdot\frac{s(s-c)}{\cancel{a}b} + \cancel{c}\cdot\frac{s(s-a)}{b\cancel{c}} = \frac{3b}{2}$

$\Rightarrow s(s-c)+s(s-a) = \frac{3b^2}{2}$

$\Rightarrow 2s^2 - s(c+a) = \frac{3b^2}{2}$

$\Rightarrow 2s^2 - s(2s-b) = \frac{3b^2}{2}$

$\Rightarrow sb = \frac{3b^2}{2}$

$\Rightarrow 2sb = 3b^2$

$\Rightarrow (a+b+c)b = 3b^2$

$\Rightarrow ab+b^2+bc = 3b^2 \Rightarrow ab+bc = 2b^2$

$\Rightarrow a+c = 2b$.

$\therefore a, b, c$ are in A.P.

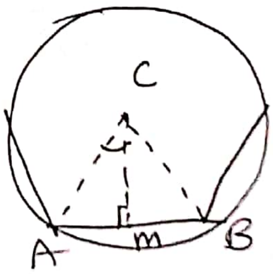
Ans: A

03 Let $AB = 2$

Central Angle $= \frac{360^\circ}{9} = 40^\circ$

$\sin 20^\circ = \frac{AM}{AC} = \frac{1}{r}$

$\Rightarrow r = \operatorname{cosec} 20^\circ = \operatorname{cosec}\frac{\pi}{9}$



Ans: A

04 Consider an equilateral triangle

Let $a=b=c=1$, $A=B=C=60^\circ$

$\therefore (b-c)\cot\frac{A}{2} + (c-a)\cot\frac{B}{2} + (a-b)\cot\frac{C}{2}$

$= (1-1)\cot\frac{60^\circ}{2} + (1-1)\cot\frac{60^\circ}{2} + (1-1)\cot\frac{60^\circ}{2} = 0$

Ans: D

07 $\sin\frac{A}{2}\cdot\cancel{\cos}\cos\frac{B}{2} = \left(\frac{a+c-b}{2c}\right)K$

$\Rightarrow \sqrt{\frac{(s-b)(s-c)}{bc}} \times \sqrt{\frac{s(s-b)}{ac}} = \left(\frac{a+c-b}{2c}\right)K$

$\Rightarrow \frac{(s-b)}{c}\sqrt{\frac{s(s-c)}{ab}} = \left(\frac{2s-2b}{2c}\right)K$

$\Rightarrow K = \sqrt{\frac{s(s-c)}{ab}} = \cos\frac{C}{2}$

Ans: D

---

08 $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$

$\Rightarrow \frac{2(b^2+c^2-a^2)}{2abc} + \frac{c^2+a^2-b^2}{2abc} + \frac{2(a^2+b^2-c^2)}{2abc} = \frac{a^2+b^2}{abc}$

$\Rightarrow$ upon simplifying we get

$\Rightarrow b^2+c^2 = a^2 \Rightarrow \angle A = \frac{\pi}{2}$

Ans: D

---

09 $a^2 = b^2+c^2-2bc\cos A$

$\Rightarrow c^2 - 2b\cos A\, c + b^2 - a^2 = 0$. This is a Q.E. let c_1 & c_2 be the roots.

$c_1 + c_2 = 2b\cos A$

Now $A_1 + A_2 = \frac{1}{2}bc_1\sin A + \frac{1}{2}bc_2\sin A$

$= \frac{1}{2}b\sin A\,(c_1+c_2) = \frac{1}{2}b\sin A\cdot 2b\cos A$

$= \frac{1}{2}b^2\sin 2A$

Ans: B

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10 Statement I: (Conceptual) (True)

Statement II: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ (Conceptual)

$\therefore \frac{a}{\sin A} = 2R$ (which is constant) (True)

Ans: A

$\Longrightarrow$ THE END $\Longleftarrow$