6. TRIGONOMETRIC EQUATIONS

Class: IX . MATHEMATICS

FOUNDATION: SOLUTIONS

TEACHING TASK

JEE MAINS

01. $\operatorname{cosec}\theta + 2 = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}$

B) $\sin 240^\circ = \sin(180^\circ + 60^\circ) = -\sin 60 = -\frac{\sqrt{3}}{2}$ ✗

A) $\sin 210^\circ = \sin(180^\circ + 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$ ✓

$\sin 300^\circ = \sin(360^\circ - 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$ ✗

D) $\sin 330^\circ = \sin(360^\circ - 30^\circ) = -\sin 30^\circ = -\frac{1}{2}$ ✓

∴ Ans. D

02 $7\sin^2 x + 3\cos^2 x = 4$

Short cut method

put $n = 0$ in all the options.

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{6}$

Now D) $\frac{\pi}{6}$

$7\sin^2\frac{\pi}{6} + 3\cos^2\frac{\pi}{6} = 7\left(\frac{1}{2}\right)^2 + 3\left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{7}{4} + \frac{9}{4} = \frac{16}{4} = 4$

Ans. D

03 $\cos 2\theta = 2\sin^2\theta$

Let $\theta = \frac{\pi}{6}$

$\cos 2\theta = \cos 2\times\frac{\pi}{6} = \cos\frac{\pi}{3} = \frac{1}{2}$

$2\sin^2\theta = 2\sin^2\left(\frac{\pi}{6}\right) = 2\times\left(\frac{1}{2}\right)^2 = \frac{1}{2}$

Ans: C

04 $\cot\theta = -\sqrt{3}$, $\operatorname{cosec}\theta = 2$

Let $\theta = \frac{5\pi}{6} = 150^\circ$

$\cot 150^\circ = \cot(180^\circ - 30^\circ) = -\cot 30^\circ = -\sqrt{3}$ ✓

$\operatorname{cosec} 150^\circ = \operatorname{cosec}(180^\circ - 30^\circ) = \operatorname{cosec} 30^\circ = 2$ ✓

Ans: D

05 Short cut method put $n=1$

A) $\pi + \frac{7\pi}{4}$ B) $2\pi + \frac{7\pi}{4}$

C) $\pi + \frac{7\pi}{4}$ D) $\frac{7\pi}{4}$.

Now $\tan\theta = \tan\left(2\pi + \frac{7\pi}{4}\right) = \tan\frac{7\pi}{4}$

$= \tan 315^\circ$

$= \tan(360^\circ - 45^\circ)$

$= -\tan 45^\circ = -1$ ✓

$\cos\theta = \cos\left(2\pi + \frac{7\pi}{4}\right) = \cos\frac{7\pi}{4} = \cos 315^\circ$

$= \cos(360^\circ - 45^\circ)$

$= \cos 45^\circ = \frac{1}{\sqrt{2}}$ ✓

∴ Ans: B

06. Short cut method

put $n=0$, in all the options.

A) $\frac{\pi}{4}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{6} \cup \frac{\pi}{3}$ D) $\frac{\pi}{2} \cup \frac{\pi}{6}$.

Now, $2\sin\alpha + \operatorname{cosec}\alpha = 3$.

Let $\theta = \frac{\pi}{2}$ $2\sin\frac{\pi}{2} + \operatorname{cosec}\frac{\pi}{2} = 2\times1 + 1 = 3$ ✓

$\theta = \frac{\pi}{6}$ $2\sin\frac{\pi}{6} + \operatorname{cosec}\frac{\pi}{6} = 2\times\frac{1}{2} + 2 = 3$ ✓

∴ Ans.. D

07. $\cos 2\alpha = (\sqrt{2}+1)(\cos\alpha - \frac{1}{\sqrt{2}})$

put $\alpha = \frac{\pi}{4}$

LHS $= \cos 2\alpha = \cos 2\times\frac{\pi}{4} = \cos\frac{\pi}{2} = 0$

RHS $= (\sqrt{2}+1)(\cos\alpha - \frac{1}{\sqrt{2}}) = (\sqrt{2}+1)(\cos\frac{\pi}{4} - \frac{1}{\sqrt{2}}) = 0$

Ans: D

08 put $n=1$ in all the options

A) $\pi + \frac{\pi}{2}$ B) $2\pi + \frac{\pi}{2}$ C) $\pi - \frac{\pi}{2}$ D) $\frac{\pi}{2}$

LHS $= \sin^{10}(\pi + \frac{\pi}{2}) - \cos^{10}(\pi + \frac{\pi}{2})$

$= \sin^{10}\frac{\pi}{2} - \cos^{10}\frac{\pi}{2}$

$= 1 - 0$

$= 1$

Ans.. A

NOTE: Here opt: D also satisfies.

But put $n=2$, it becomes π, which fails.

09. $r\sin\theta = \sqrt{3}$

$\Rightarrow$ ~~$\sin\theta$~~

$\Rightarrow r = \dfrac{\sqrt{3}}{\sin\theta}$

$r + 4\sin\theta = 2(\sqrt{3}+1)$

$\Rightarrow \dfrac{\sqrt{3}}{\sin\theta} + 4\sin\theta = 2(\sqrt{3}+1)$

$\Rightarrow \sqrt{3} + 4\sin^2\theta = 2\sin\theta(\sqrt{3}+1)$

$\Rightarrow 4\sin^2\theta - 2\sin\theta + \sqrt{3} - 2\sqrt{3}\sin\theta = 0$

$\Rightarrow 2\sin\theta(2\sin\theta - 1) - \sqrt{3}(2\sin\theta - 1) = 0$

$\Rightarrow (2\sin\theta - 1)(2\sin\theta - \sqrt{3}) = 0$

$\Rightarrow \sin\theta = \dfrac{1}{2}$ or $\sin\theta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta = n\pi + (-1)^n \dfrac{\pi}{6}$ or $n\pi + (-1)^n \dfrac{\pi}{3}$

$\therefore \theta \in \left\{\dfrac{\pi}{6}, \dfrac{\pi}{3}, \dfrac{5\pi}{6}, \dfrac{2\pi}{3}\right\}$ Ans.. B

10. $4\cos^2 x \sin x - 2\sin^2 x - 3\sin x = 0$

$\Rightarrow \sin x(4\cos^2 x - 2\sin x - 3) = 0$

$\Rightarrow \sin x = 0$ or $4(1 - \sin^2 x) - 2\sin x - 3 = 0$

$\Rightarrow \boxed{x = n\pi}$ $\Rightarrow 4\sin^2 x + 2\sin x - 1 = 0$

$\Rightarrow \sin x = \dfrac{-1 \pm \sqrt{5}}{4}$

$\sin x = \dfrac{\sqrt{5}-1}{4}$ or $\sin x = -\dfrac{1+\sqrt{5}}{4}$

$\sin x = \sin\dfrac{\pi}{10}$

$\boxed{x = n\pi + (-1)^n \cdot \dfrac{\pi}{10}}$

$\sin x = \sin\left(-\dfrac{3\pi}{10}\right)$

$\boxed{x = n\pi + (-1)^n\left(-\dfrac{3\pi}{10}\right)}$

Ans.. D

JEE ADVANCED LEVEL

01. $a\cos 2\theta + b\sin 2\theta = c$

$\Rightarrow a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = c$

$\Rightarrow a - a\tan^2\theta + 2b\tan\theta - c - c\tan^2\theta = 0$

$\Rightarrow (a+c)\tan^2\theta - 2b\tan\theta + c - a = 0$

product of the roots $= \tan\alpha \cdot \tan\beta = \frac{c-a}{c+a}$

Ans: C

02 put $n=1$ in all the options.

A) $\frac{\pi}{2}$ B) $2\pi + \frac{\pi}{2}$ C) 2π D) π

opt: B: (Ø)

$LHS = \sin^2\theta + \sin\theta\cos\theta + \cos^2\theta$

$= \sin^2\left(2\pi+\frac{\pi}{2}\right) + \sin\left(2\pi+\frac{\pi}{2}\right)\cos\left(2\pi+\frac{\pi}{2}\right) + \cos^2(2\pi+\frac{\pi}{2})$

$= \sin^2\frac{\pi}{2} + \sin\frac{\pi}{2}\cos\frac{\pi}{2} + \cos^2\frac{\pi}{2} = 1$

opt: C $LHS = \sin^2 2\pi + \sin 2\pi\cos 2\pi + \cos^2 2\pi$

$= 0 + 0 + (\pm 1)^2 = 1$ Ans. B, C

03 $\cos\theta = \frac{1}{2} = \cos\frac{\pi}{3}$ False (True)

$\therefore \theta = 2n\pi \pm \frac{\pi}{3}$

Statement II: Conceptual (True) Ans. A

04. Statement I:

$\tan(3x - 2x) = 1 \Rightarrow \tan x = 1$

$\Rightarrow \tan x = \tan\frac{\pi}{4}$

$\Rightarrow x = \frac{\pi}{4}$ (False)

Statement II: Conceptual (True)

Ans: D

05 Assertion: $\sin x = \sin 60$

$\Rightarrow x = n\pi \pm 60°$

put $n = 0 \Rightarrow x = 60°$ ✓

$n = 1 \Rightarrow x = 180° + 60°$ ✓

$= 180° - 60°$ ✓

$n = 2 \Rightarrow x = 360° - 60°$ ✓

(True)

~~Ans. B~~

Reason: Conceptual (True)

~~Ans. A~~

06 Assertion: $\cos x = -1$

$\Rightarrow \cos x = \cos\pi$

$\Rightarrow \boxed{x = 2n\pi \pm \pi}$ $[0, 360]$

put $x = 0 \Rightarrow x = \pi$ ✓ principal solution $[0, \pi]$

True

$x = 1 \Rightarrow x = 2\pi - \pi$ ✗ (~~False~~)

Reason: For infinite values $\cos x = -1$ (False)

Ans: C

07. $\cos^2\theta = \left(\pm\frac{1}{2}\right)^2$

$\Rightarrow \cos^2\theta = \cos^2\frac{\pi}{3}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

$\Rightarrow \theta = 60°$

Ans: A

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08 $\cos^2\theta = \left(\pm\frac{1}{2}\right)^2$

$\cos^2\theta = \cos^2\frac{\pi}{3}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$ Given $(0, 360°)$

$n=0 \Rightarrow \theta = \frac{\pi}{3}$ ✓

$n=1 \Rightarrow \theta = \pi \pm \frac{\pi}{3}$ ✓✓

$n=2 \Rightarrow \theta = 2\pi - \frac{\pi}{3}$ ✓ total 4

Ans: D

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09 $(2\sin x - \cos x)(1+\cos x) = \sin^2 x$

$(2\sin x - \cos x)(1+\cos x) = 1 - \cos^2 x$

$(2\sin x - \cos x)(1+\cos x) = (1+\cos x)(1-\cos x)$

$(2\sin x - \cos x)(1+\cos x) = 0$

$\Rightarrow (1+\cos x)[2\sin x - \cos x - 1 + \cos x] = 0$

$\Rightarrow \cos x = -1$ or $\sin x = \frac{1}{2}$

$\Rightarrow \cos x = \frac{\sqrt{3}}{2}$

$\Rightarrow \boxed{\cos\alpha = \frac{\sqrt{3}}{2}}$

$3\cos^2 x - 10\cos x + 3 = 0$

$\Rightarrow 3\cos^2 x - 9\cos x - \cos x + 3 = 0$

$\Rightarrow 3\cos x(\cos x - 3) - 1(\cos x - 3) = 0$

$\Rightarrow (\cos x - 3)(3\cos x - 1) = 0$

$\Rightarrow \cos x = 3$ (✗) $\qquad \cos x = \frac{1}{3}$

$\boxed{\cos\beta = \frac{1}{3}}$

Again, $1 - \sin 2x = \cos x - \sin x$

$\Rightarrow (\sin x - \cos x)^2 + (\sin x - \cos x) = 0$

$\Rightarrow (\sin x - \cos x)[\sin x - \cos x + 1] = 0$

↓ no need to solve

$\Rightarrow \sin x - \cos x = 0$ or

$\Rightarrow \tan x = 1$

$\Rightarrow \cos x = \frac{1}{\sqrt{2}} \Rightarrow \boxed{\cos\gamma = \frac{1}{\sqrt{2}}}$

Now, $\cos\alpha + \cos\beta + \cos\gamma$

$= \frac{\sqrt{3}}{2} + \frac{1}{3} + \frac{1}{\sqrt{2}}$

$= \frac{2\sqrt{6} + 2\sqrt{2} + 6}{6\sqrt{2}}.$

Ans.. A

10 $\cos\alpha = \frac{\sqrt{3}}{2} \Rightarrow \sin\alpha = \frac{1}{2}$

$\cos\beta = \frac{1}{3} \Rightarrow \sin\beta = \frac{2\sqrt{2}}{3}$

$\cos\gamma = \frac{1}{\sqrt{2}} \Rightarrow \sin\gamma = \frac{1}{\sqrt{2}}$

Now, $\sin\alpha + \sin\beta + \sin\gamma$

$= \frac{1}{2} + \frac{2\sqrt{2}}{3} + \frac{1}{\sqrt{2}}$

$= \frac{14 + 3\sqrt{2}}{6\sqrt{2}}$

Ans: A

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11 $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{1}{2}\cdot\frac{1}{3} - \frac{\sqrt{3}}{2}\cdot\frac{2\sqrt{2}}{3}$

$= \frac{1 - 2\sqrt{6}}{6}$

Ans: C

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12 $|\sin x \cdot \cos x| + \sqrt{2 + \tan^2 x + \cot^2 x} = \sqrt{3} \quad x \in [0, 4\pi]$

$\Rightarrow |\sin x \cdot \cos x| + \sqrt{(\tan x + \cot x)^2} = \sqrt{3}$

$\Rightarrow |\sin x \cdot \cos x| + |\tan x + \cot x| = \sqrt{3}$

$\Rightarrow |\sin x \cdot \cos x| + \left|\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right| = \sqrt{3}$

$\Rightarrow |\sin x \cdot \cos x| + \left|\frac{1}{\sin x \cdot \cos x}\right| = \sqrt{3}$

Let $|\sin x \cdot \cos x| = y$

$\Rightarrow y + \frac{1}{y} = \sqrt{3}$

We know $y + \frac{1}{y} \geq 2$

Given $y + \frac{1}{y} = \sqrt{3} = 1.732$ which is not possible

$\Rightarrow \left(\sqrt{y} - \frac{1}{\sqrt{y}}\right)^2 = \sqrt{3} + 2$

$\Rightarrow \left(\sqrt{y} - \frac{1}{\sqrt{y}}\right)^2$

Hence there is no solution for x

Ans. 0

13. $\cos 4x = \cos(2.2x) = 2\cos^2 2x - 1$ (10)

$= 2(2\cos^2 x - 1)^2 - 1$

$= 2(4\cos^4 x - 4\cos^2 x + 1) - 1$

$= 8\cos^4 x - 8\cos^2 x + 1$

$= 1 - 8\cos^2 x + 8\cos^4 x$

$= a_0 + a_1 \cos^2 x + a_2 \cos^4 x$

here $a_0 = 1$, $a_1 = -8$, $a_2 = 8$

Now, $5a_0 + a_1 + a_2 = 5(1) - 8 + 8 = 5$ Ans: 5

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14 a) $4\sin^2 x + \sin^2 2x = 3$

$\Rightarrow 4\sin^2 x + 4\sin^2 x \cos^2 x - 3 = 0$

$\Rightarrow 4\sin^2 x + 4\sin^2 x (1 - \sin^2 x) - 3 = 0$

$\Rightarrow 8\sin^2 x - 4\sin^4 x - 3 = 0$

$\Rightarrow 4\sin^4 x - 8\sin^2 x + 3 = 0$

$\Rightarrow 4\sin^4 x - 2\sin^2 x - 6\sin^2 x + 3 = 0$

$\Rightarrow 2\sin^2 x (2\sin^2 x - 1) - 3(2\sin^2 x - 1) = 0$

$\Rightarrow (2\sin^2 a - 1)(2\sin^2 x - 3) = 0$

$\Rightarrow \sin x = \pm \frac{1}{2}$ or $\sin x = \pm \frac{\sqrt{3}}{2}$

$\Rightarrow \sin x = \frac{1}{2}$

$\Rightarrow x = \sin^{-1}\left(\frac{1}{2}\right)$ (s)

b) $4\cos^2 2x + 8\cos^2 x = 7$

$\Rightarrow 4(2\cos^2 x - 1)^2 + 8\cos^2 x = 7$

$\Rightarrow 4(4\cos^4 x - 4\cos^2 x + 1) + 8\cos^2 x - 7 = 0$

$\Rightarrow 16\cos^4 x - 8\cos^2 x - 3 = 0$

$\Rightarrow 16\cos^4 x + 4\cos^2 x - 12\cos^2 x - 3 = 0$

$\Rightarrow 4\cos^2 x(4\cos^2 x + 1) - 3(4\cos^2 x + 1) = 0$

$\Rightarrow (4\cos^2 x + 1)(4\cos^2 x - 3) = 0$

$\Rightarrow \cos^2 x = -\frac{1}{4}$ (x) $\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \sin x = \pm\frac{1}{2} \Rightarrow x = \sin^{-1}(\frac{1}{2})$ (S)

c) $3(1 + \sin x) = 2\cos^2 x$

$\Rightarrow 3(1 + \sin x) = 2(1 - \sin^2 x)$

$\Rightarrow 3 + 3\sin x = 2 - 2\sin^2 x$

$\Rightarrow 2\sin^2 x + 3\sin x + 1 = 0$

$\Rightarrow 2\sin^2 x + 2\sin x + \sin x + 1 = 0$

$\Rightarrow 2\sin x(\sin x + 1) + 1(\sin x + 1) = 0$

$\Rightarrow (\sin x + 1)(2\sin x + 1) = 0$

$\Rightarrow \sin x = -1$ or $\sin x = -\frac{1}{2}$

$\Rightarrow x = \sin^{-1}(-1)$ (Q)

d) short cut method

$4\sin^3\alpha - 2\sin^2\alpha - 2\sin\alpha + 1$

put $\sin\alpha = \frac{1}{2}$

$\therefore 4\left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1$

$= \frac{4}{8} - \frac{2}{4} - 1 + 1$

$= \frac{1}{2} - \frac{1}{2} - 1 + 1 = 0$

$\therefore \sin\alpha = \frac{1}{2} \Rightarrow \alpha = \sin^{-1}\left(\frac{1}{2}\right)$

~~Ans: S~~

Ans: S, S, Q, S

15 a) $\tan^2\alpha + \cot^2\alpha - 2 = 0$

$\Rightarrow (\tan\alpha - \cot\alpha)^2 = 0$

$\Rightarrow \tan\alpha - \cot\alpha = 0$

$\Rightarrow \tan^2\alpha = 1$

$\Rightarrow \tan^2\alpha = \tan^2\frac{\pi}{4}$

$\Rightarrow \alpha = n\pi \pm \frac{\pi}{4}$

$\alpha = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

No. of solutions = 4 (S)

b) $\sin^2\alpha - \cos\alpha = \frac{1}{2}$

$\Rightarrow 2\sin^2\alpha - 2\cos\alpha = 1$

$\Rightarrow 2(1 - \cos^2\alpha) - 2\cos\alpha = 1$

$\Rightarrow 2\cos^2\alpha + 2\cos\alpha - 1 = 0$

$\Rightarrow \cos\alpha = \frac{-1 \pm \sqrt{3}}{2}$ → Two solutions (P)

c) $4\sin^2 x - 6\cos^2 x - 10 = 0$

$\Rightarrow 4\sin^2 x - 6(1 - \sin^2 x) - 10 = 0$

$\Rightarrow \sin^2 x = \frac{8}{5} \Rightarrow \sin x = \frac{2\sqrt{2}}{\sqrt{5}} \approx 1.26$

$\therefore$ No solution (q)

d) $\sin x = 1$

$\Rightarrow \sin x = \sin\frac{\pi}{2}$

$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{2}$

$\Rightarrow x = \frac{\pi}{2}$ (one solution) (r)

Ans: S, P, Q, R

LEARNERS TASK (CUQ's)

01.	$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$	Ans: C
02	$\sin\theta = 1 \Rightarrow \theta = \frac{\pi}{2}$	Ans: A
03	$\sin\theta = 0 \Rightarrow \theta = n\pi$	Ans: B
04	$\sec\theta = \sqrt{2} \Rightarrow \theta = \frac{\pi}{4}$	Ans: B
05	$-\frac{\pi}{3}$	Ans: D
06	Conceptual	Ans: D
07	Conceptual	Ans: A
08	Conceptual	Ans: E
09	Conceptual	Ans: B

10 Conceptual

Ans. D

JEE MAINS LEVEL

(1h)

01 $\cos\theta = \cos\frac{5\pi}{4}$

$\Rightarrow \theta = 2n\pi \pm \frac{5\pi}{4}$

Ans: C

02 $\sqrt{3}\cos\theta = \sin\theta$

$\Rightarrow \tan\theta = \sqrt{3} = \tan\frac{\pi}{3}$

$\Rightarrow \theta = n\pi + \frac{\pi}{3}$

Ans. A

03 $\sqrt{\sin x} + \cos x = 0$

$\Rightarrow \sqrt{\sin x} = -\cos x$

$\Rightarrow \sin x = \cos^2 x = 1 - \sin^2 x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

$\sin x = \frac{\sqrt{5}-1}{2}$ ✓ $\quad \sin x = \frac{-1-\sqrt{5}}{2}$

$= \frac{-1-2.2}{2}$

$= -\frac{3.2}{2} = -1.6$ ✗

Ans: D

04 put $n=0$ in all the options

A) $\frac{\pi}{6}$ B) $\frac{\pi}{8}$ C) $\frac{-\pi}{6}$ D) $\frac{\pi}{3}$

$LHS = 4\cos\theta - \sec\theta$

$= 4\cos\frac{\pi}{6} - \sec\frac{\pi}{6}$ $\Big| = 4\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{\sqrt{3}}$

$$= \frac{12-4}{2\sqrt{3}} = \frac{8}{2\sqrt{3}} = \frac{4}{\sqrt{3}} = 4\tan\frac{\pi}{6}$$ ✓ (15)

Ans. A

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05 Let $\theta = \frac{\pi}{6}$

$$LHS = 3\cos 2\theta + 2 = 3\cos 2\left(\frac{\pi}{6}\right) + 2$$

$$= 3\cos\frac{\pi}{3} + 2 = 3\left(\frac{1}{2}\right) + 2 = \frac{7}{2}$$

$$RHS = 7\sin\theta = 7\sin\frac{\pi}{6} = 7\left(\frac{1}{2}\right) = \frac{7}{2}$$

Ans: B

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06 $$LHS = \frac{\cos 3\theta}{2\cos 2\theta - 1}$$

$$= \frac{\cos 3\left(2\pi + \frac{\pi}{3}\right)}{2\cos 2\left(2\pi + \frac{\pi}{3}\right) - 1}$$

$$= \frac{\cos(6\pi + \pi)}{2\cos\left(4\pi + \frac{2\pi}{3}\right) - 1}$$

$$= \frac{\cos\pi}{2\cos\frac{2\pi}{3} - 1} = \frac{-1}{2\left(\frac{-1}{2}\right) - 1}$$

$$= \frac{1}{2}$$

NOTE: $\theta = n\pi \pm \frac{\pi}{3}$ also satisfies the given equation.

Ans: B, A

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07 $$\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0$$

$$\Rightarrow \tan^2\theta - \tan\theta - \sqrt{3}\tan\theta + \sqrt{3} = 0$$

$$\Rightarrow (\tan\theta - 1)(\tan\theta - \sqrt{3}) = 0$$

$$\Rightarrow \tan\theta = 1 \quad \text{or} \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \tan\theta = \tan\frac{\pi}{4} \quad \text{or} \quad \tan\theta = \tan\frac{\pi}{3}$$

$$\Rightarrow \theta = n\pi + \frac{\pi}{4} \quad \text{or} \quad \theta = n\pi + \frac{\pi}{3}$$

Ans: C

08. $\sqrt{1-\cos x} = \sin x$

$\Rightarrow 1-\cos x = \sin^2 x$

$\Rightarrow 1-\cos x = 1-\cos^2 x$

$\Rightarrow \cos x(\cos x - 1) = 0$

$\Rightarrow \cos x = 0$ or $\cos x = 1$

$\Rightarrow x = (2n+1)\frac{\pi}{2}$ or $2n\pi \pm 0 = 2n\pi$

Ans: A

09 $\tan\theta - \cot\theta = 1 \Rightarrow \tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \tan^2\frac{\pi}{4}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$

Ans: D

10 $\sin 6\theta = \sin 4\theta - \sin 2\theta$

Clearly, $\theta = 0^\circ$ satisfies the above eqn

put $n=0$ in the options.

option: B) $\theta = \frac{n\pi}{4} \Rightarrow \theta = 0$

Ans. B

JEE ADVANCED LEVEL

01 $a\cos x + b\sin x = c$

$\Rightarrow a\cos x + c = b\sin x$

S.O.B.S

$\Rightarrow a^2\cos^2 x + 2ac\cos x = b^2\sin^2 x$

$= b^2(1-\cos^2 x)$

$= b^2 - b^2\cos^2 x$

$\Rightarrow (a^2+b^2)\cos^2\alpha + 2ac\cos\alpha - b^2 = 0$

Sum of the roots $= \cos\alpha + \cos\beta = \dfrac{-2ac}{a^2+b^2}$ Ans: B

02 $a\tan\theta + b\sec\theta = c$

$\Rightarrow a\tan\theta - c = -b\sec\theta$

$\Rightarrow a^2\tan^2\theta - 2ac\tan\theta + c^2 = b^2(1+\tan^2\theta)$

$= b^2 + b^2\tan^2\theta$

$\Rightarrow (a^2-b^2)\tan^2\theta - 2ac\tan\theta + c^2 - b^2 = 0$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} =$

$\Rightarrow \dfrac{\left(\dfrac{2ac}{a^2-b^2}\right)}{1-\left(\dfrac{c^2-b^2}{a^2-b^2}\right)} = \dfrac{2ac}{a^2-c^2}$ Ans. A

03 Statement I: $\sin x = 1$

$\Rightarrow \sin x = \sin\dfrac{\pi}{2}$

$\Rightarrow x = n\pi + (-1)^n \cdot \dfrac{\pi}{2}$ (False)

Statement II: Conceptual (True) Ans: D

04 Statement I: $3\sin x + 4\cos x = 7$

$|c| = |7| = 7$

$\sqrt{a^2+b^2} = \sqrt{3^2+4^2} = 5$

$7 > 5$ (True)

Statement II: Conceptual (True) Ans: A (18)

05 Assertion: Conceptual (True) Ans: A

Reason: Conceptual (True)

06 Assertion: Conceptual (True)

Reason: Conceptual (False) Ans: C

07 Given $\cos\theta \cdot \cos 2\theta \cdot \cos 3\theta = \frac{1}{4} \quad (0 \le \theta < \pi)$

$= 2(2\cos 3\theta \cdot \cos\theta)\cdot \cos 2\theta = 1$

$\Rightarrow 2(\cos 4\theta + \cos 2\theta)\cdot \cos 2\theta = 1$

$\Rightarrow 2\cos 4\theta \cos 2\theta + 2\cos^2 2\theta - 1 = 0$

$\Rightarrow 2\cos 4\theta \cos 2\theta + \cos 4\theta = 0$

$\Rightarrow \cos 4\theta\,[2\cos 2\theta + 1] = 0$

$\Rightarrow \cos 4\theta = 0$ or $\cos 2\theta = -\frac{1}{2}$

$\Rightarrow 4\theta = (2n+1)\frac{\pi}{2}$ | $\cos 2\theta = \cos\left(\frac{2\pi}{3}\right)$

$\Rightarrow \theta = (2n+1)\frac{\pi}{8}$ | $2\theta = 2n\pi \pm \left(\frac{2\pi}{3}\right)$

$\theta = n\pi \pm \frac{\pi}{3}$

$n=0 \Rightarrow \theta = \frac{\pi}{8}$ ✓

$n=1 \Rightarrow \theta = \frac{3\pi}{8}$ ✓

$n=2 \Rightarrow \theta = \frac{5\pi}{8}$ ✓

$n=3 \Rightarrow \theta = \frac{7\pi}{8}$ ✓

$n=4 \Rightarrow \theta = \frac{9\pi}{8}$ ×

Now $\theta = n\pi \pm \frac{\pi}{3}$

$n=0 \Rightarrow \theta = \frac{\pi}{3}$ ✓

$n=1 \Rightarrow \theta = \pi - \frac{\pi}{3}$ ✓ $= \frac{2\pi}{3}$ ✓

$n=2 \Rightarrow \theta = 2\pi \pm \frac{\pi}{6}$ ✗.

∴ Sum of the solutions

$= \frac{\pi}{8} + \frac{3\pi}{8} + \frac{5\pi}{8} + \frac{7\pi}{8} + \frac{\pi}{3} + \frac{2\pi}{3}$

$= \frac{\pi + 3\pi + 5\pi + 7\pi}{8} + \frac{\pi + 2\pi}{3}$

$= \frac{16\pi}{8} + \frac{3\pi}{3} = 2\pi + \pi = 3\pi$

Ans: C

68 from the above discussion

Ans: D

09. $2\sin^2\theta = \sin\theta$

$\Rightarrow 2\sin^2\theta - \sin\theta = 0$

$\Rightarrow \sin\theta(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 0$ or $\sin\theta = \frac{1}{2}$

$\Rightarrow \theta = n\pi$ or $\theta = n\pi + (-1)^n \frac{\pi}{6}$

$\downarrow$

✗

$n=0 \Rightarrow \theta = \frac{\pi}{6}$ ✓

$n=1 \Rightarrow \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ ✓

Ans: C

10. (From above problem)

$n=2 \Rightarrow \theta = 2\pi + \frac{\pi}{6}$ ✗

$n=3 \Rightarrow \theta = 3\pi - \frac{\pi}{6}$

10. From the above solution $[0, 2\pi]$ (20)

$\theta = n\pi$

$n=0 \Rightarrow \theta = 0$ ✓

$n=1 \Rightarrow \theta = \pi$ ✓

$n=2 \Rightarrow \theta = 2\pi$ ✓

$\theta = n\pi + (-1)^n \frac{\pi}{6}$

$n=0 \Rightarrow \theta = \frac{\pi}{6}$ ✓

$n=1 \Rightarrow \theta = \pi - \frac{\pi}{6}$ ✓

$n=2 \Rightarrow \theta = 2\pi + \frac{\pi}{6}$ ×

∴ Total No. of solutions. 5

Ans. (C)

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11. Given $3\sec\theta - 5 = 4\tan\theta$ $[0, 4\pi]$

$\Rightarrow \frac{3}{\cos\theta} - 5 = \frac{4\sin\theta}{\cos\theta}$

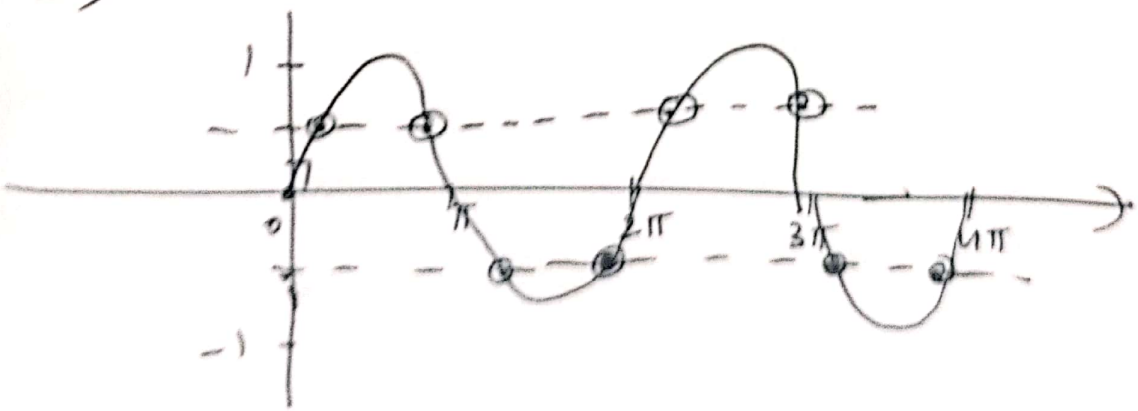
$\Rightarrow 3 - 5\cos\theta = 4\sin\theta$

$\Rightarrow 3 - 4\sin\theta = 5\cos\theta$

$\Rightarrow 9 - 24\sin\theta + 16\sin^2\theta = 25\cos^2\theta = 25 - 25\sin^2\theta$

$\Rightarrow 41\sin^2\theta - 24\sin\theta - 16 = 0$

$\Rightarrow \sin\theta = 0.982$ or -0.397



Total No. of solutions = 8

Ans. 8

12 $(\sqrt{3}+1)^{2x} + (\sqrt{3}-1)^{2x} = 2^{3x} = (2\sqrt{2})^{x}$

$\Rightarrow \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^{x} + \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^{x} = 1$

$\Rightarrow (\sin 75^\circ)^{2x} + (\cos 75^\circ)^{2x} = 1$

This is possible only when $x = 1$

$\therefore$ No. of solutions $= 1$

Ans. 1

13 a) $\tan\theta = 1 \Rightarrow \tan\theta = \tan\frac{\pi}{4} \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$ (q)

b) $\cos\theta = \frac{1}{4} \Rightarrow \cos\theta = \cos\left(\frac{\pi}{3}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$ (r)

c) $\sin\theta = \frac{1}{4} \Rightarrow \sin\theta = \sin\left(\frac{\pi}{6}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{6}$ (p)

d) $\operatorname{cosec}\theta = 1 \Rightarrow \sin\theta = 1$

$\Rightarrow \sin\theta = \sin\frac{\pi}{2} \Rightarrow \theta = n\pi \pm \frac{\pi}{2}$ (s)

Ans. q, r, p, s

14 a) $1 + \sin 2x = \sin x + \cos x$

$\Rightarrow (\sin x + \cos x)^2 - (\sin x + \cos x) = 0$

$\Rightarrow (\sin x + \cos x)\left[(\sin x + \cos x) - 1\right] = 0$

$\Rightarrow \sin x + \cos x = 0$ (or) $\sin x + \cos x = 1$

$\Rightarrow \sin x = -\cos x$

$\Rightarrow \tan x = -1$

$\Rightarrow x = -\frac{\pi}{4}$

put $x = 0$

$\sin 0 + \cos 0 = 1$ ✓

$\therefore \cos x = 0$ ✓ (q)

b) put $x = 30°$

LHS $= (2\sin 3i - \cos 3i)(1 + \cos 3i)$

$\Rightarrow \left(2\left(\frac{1}{2}\right) - \frac{\sqrt{3}}{2}\right)\left(1 + \frac{\sqrt{3}}{2}\right)$

$= \left(1 - \frac{\sqrt{3}}{2}\right)\left(1 + \frac{\sqrt{3}}{2}\right) = 1 - \frac{3}{4} = \frac{1}{4}$

$RHS = \sin^2 x = \sin^2 30 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\therefore \sin x = \frac{1}{2}$ (P)

c) $\tan\theta + \cot\theta = 2$

$\Rightarrow \theta = \frac{\pi}{4}$ ✓ (S)

d) $\sin 7\theta = \sin 4\theta - \sin\theta$

$\Rightarrow \sin 7\theta + \sin\theta = \sin 4\theta$

$\Rightarrow 2\sin 4\theta \cdot \cos 3\theta - \sin 4\theta = 0$

$\Rightarrow \sin 4\theta = 0$ or $\cos 3\theta = \frac{1}{2} = \sin\frac{\pi}{3}$

$4\theta = n\pi$

$\Rightarrow \theta = \frac{n\pi}{4}$

$\Rightarrow 3\theta = \frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{9}$ (t)

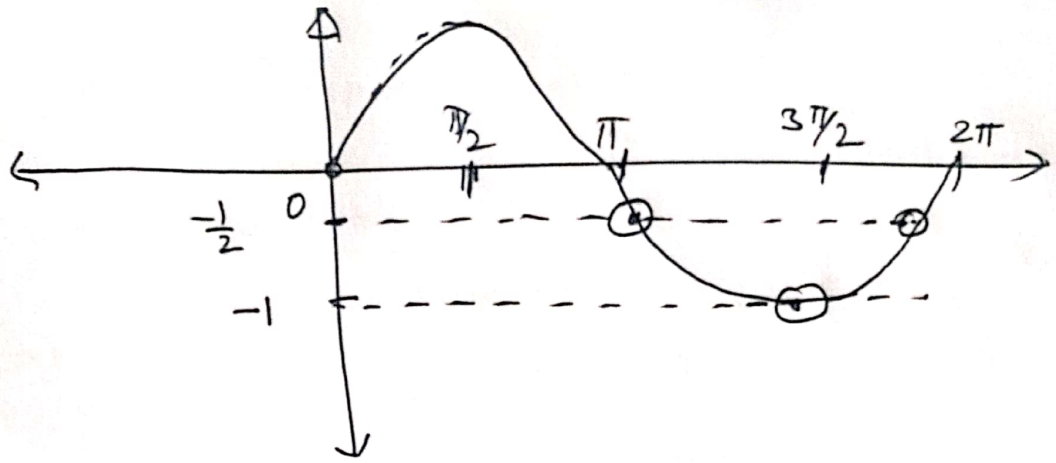
Ans: Q, P, S, t

ADDITIONAL PRACTICE

Q1. $2\sin^2\theta + 3\sin\theta + 1 = 0 \quad (0, 2\pi)$

$\Rightarrow (\sin\theta + 1)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = -1$ or $\sin\theta = -\frac{1}{2}$



: No. of Solution = 3

Ans. C

02 $\sin(x+28^\circ) = \cos(3x-7^\circ)$

$\Rightarrow x+28^\circ+3x-7^\circ = 90^\circ$

$\Rightarrow x = 35^\circ$

Also, $x=8^\circ \Rightarrow \sin 36^\circ = \cos 54^\circ$ (Proved)

Ans. B

03 $3\tan(\theta-15^\circ) = \tan(\theta+15^\circ)$

Let $\theta = 45^\circ$

LHS $= 3\tan(45^\circ-15^\circ)$

$= 3\tan 30^\circ$

$= 3\times\frac{1}{\sqrt{3}}$

$= \sqrt{3}$

RHS $= \tan(45^\circ+15^\circ)$

$= \tan 60^\circ$

$= \sqrt{3}$

Ans. B

04. $1+\sin x+\sin^2 x+\cdots = 4+2\sqrt{3}$

Sum $= \frac{1}{1-\sin x} = 4+2\sqrt{3}$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$

NOTE:

Infinite G.P.

$S_\infty = \frac{a}{1-r}$

05 $(1+\tan\alpha)(1+\tan 4\alpha) = 2$

$\Rightarrow 1+\tan 4\alpha + \tan\alpha + \tan\alpha \cdot \tan 4\alpha = 2$

$\Rightarrow \dfrac{\tan\alpha + \tan 4\alpha}{1 - \tan\alpha \cdot \tan 4\alpha} = 1$

$\Rightarrow \tan(\alpha + 4\alpha) = \tan\dfrac{\pi}{4}$

$\Rightarrow 5\alpha = \dfrac{\pi}{4}$

$\Rightarrow \alpha = \dfrac{\pi}{20}$ ✓

Ans: A

$\Rightarrow$ THE END ∈