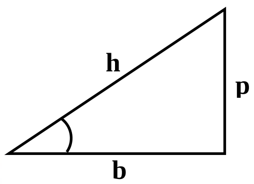
BASIC CONCEPT OF MATHEMATICS

• TRIGONOMETRY

$$\sin\theta = \frac{\text{opposite side}}{\text{Hypotenuse}} = \frac{p}{h}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{Hypotenuse}} = \frac{b}{h}$$

$$\tan\theta = \frac{\text{adjacent side}}{\text{Hypotenuse}} = \frac{p}{b}$$



$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{h}{p},\quad \sec\theta = \frac{1}{\cos\theta} = \frac{h}{b},\quad \cot\theta = \frac{1}{\tan\theta} = \frac{p}{b}$$

TRIGONOMETRIC IDENTITIES

According to Pythagoras's theorem, $p^2 + b^2 = h^2$ …(i)

Dividing (i) by h^2, $\left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = 1 \Rightarrow \sin^2\theta + \cos^2\theta = 1$

Dividing (i) by b^2, $\left(\frac{p}{b}\right)^2 + 1 = \left(\frac{h}{b}\right)^2 \Rightarrow \tan^2\theta + 1 = \sec^2\theta$

Dividing (i) by p^2, $1 + \left(\frac{b}{p}\right)^2 = \left(\frac{h}{p}\right)^2 \Rightarrow 1 + \cot^2\theta = \text{cosec}^2\theta$

SOME VALUES OF TRIGONOMETRIC ANGLES

angle / ratio	0^0	30^0	45^0	60^0	90^0
sin	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

$\sqrt{2} = 1.414$
$\sqrt{3} = 1.732$

- The value of $\sin\theta$ increases form 0 to 1, as θ increases from 0° to 90°.
- The value of $\cos\theta$ decreases from 1 to 0, as θ increase form 0° to 90°.

- The value of tanθ increases from 0 to ∞ (infinity), as θ increases from 0° to 90°

Complementary relations

sin A = cos(90° - A) cosA = sin (90° - A)
tan A = cot(90° - A) cotA = tan (90° - A)
sec A = cosec(90° - A) cosec A = sec (90° - A)

Some relations

$\cos\theta = \sqrt{1-\sin^2\theta}$ $\sin\theta = \sqrt{1-\cos^2\theta}$ $\sec\theta = \sqrt{1+\tan^2\theta}$

$\sec\theta = \dfrac{1}{\sqrt{1-\sin^2\theta}}$ $\operatorname{cosec}\theta = \dfrac{1}{\sqrt{1-\cos^2\theta}}$ $\cos\theta = \dfrac{1}{\sqrt{1+\tan^2\theta}}$

$\tan\theta = \dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$ $\tan\theta = \dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}$ $\sin\theta = \dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$

$\cot\theta = \dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sqrt{1-\cos^2\theta}}$ $\operatorname{cosec}\theta = \dfrac{\sqrt{1+\tan^2\theta}}{\tan\theta}$

$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

Illustrations –1: If 5tanθ = 4, find the value of $\dfrac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta}$

Solution: Given 5tanθ = 4 ⇒ tanθ = $\dfrac{4}{5}$ . . . . . (i)

Now $\dfrac{5\sin\theta-3\cos\theta}{5\sin\theta+2\cos\theta} = \dfrac{5\dfrac{\sin\theta}{\cos\theta}-3\dfrac{\cos\theta}{\cos\theta}}{5\dfrac{\sin\theta}{\cos\theta}+2\dfrac{\cos\theta}{\cos\theta}}$

(Dividing the numerator and denominator by cosθ)

$= \dfrac{5\tan\theta-3}{5\tan\theta+2}$ $\left(\tan\theta = \dfrac{\sin\theta}{\cos\theta}\right)$

$= \dfrac{5(4/5)-3}{5(4/5)+2}$ [using (i)]

$= \dfrac{4-3}{4+2} = \dfrac{1}{6}$

Illustrations –2: If θ is acute and 3sinθ = 4 cos θ, find the value of $4\sin^2\theta - 3\cos^2\theta + 2$

Solution: Given 3sinθ = 4cosθ ⇒ $\dfrac{\sin\theta}{\cos\theta} = \dfrac{4}{3}$ ⇒ tanθ = $\dfrac{4}{3}$

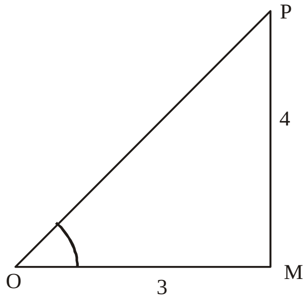
But tanθ = $\dfrac{MP}{OM}$ ⇒ $\dfrac{MP}{OM} = \dfrac{4}{3}$

Draw a triangle OMP right angled at M (Shown in the adjoining figure) such that MP = 4 and OM = 3.

From right-angled ΔOMP, by Pythagoras theorem,

$OP^2 = OM^2 + MP^2$

$\Rightarrow \quad OP^2 = 3^2 + 4^2 = 25$



$\Rightarrow \quad OP = 5$

$\therefore \quad \sin\theta = \dfrac{MP}{OP} = \dfrac{4}{5}$ and $\cos\theta = \dfrac{OM}{OP} = \dfrac{3}{5}$

$\therefore \quad 4\sin^2\theta - 3\cos^2\theta + 2$

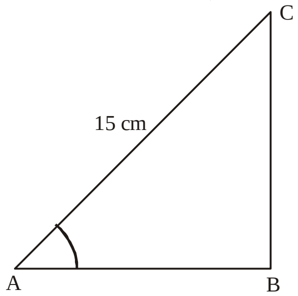
$= 4\left(\dfrac{4}{5}\right)^2 - 3\left(\dfrac{3}{5}\right)^2 + 2 = 4.\dfrac{16}{25} - 3\dfrac{9}{25} + 2$

$= \dfrac{64 - 27 + 50}{25} = \dfrac{87}{25} = 3\dfrac{12}{25}$

Illustrations –3: In the adjoining figure, ΔABC is right-angled at B and $\tan A = \dfrac{4}{3}$. If AC = 15 cm, find the lengths of AB and BC.

Solution: Given $\tan A = \dfrac{4}{3} \Rightarrow \dfrac{BC}{AB} = \dfrac{4}{3}$

Let BC = 4x cm, then AB = 3x cm.



From right-angled ΔABC, by Pythagoras theorem, we get

$AB^2 + BC^2 = AC^2 \Rightarrow (3x^2) + (4x)^2 = 15^2$

$\Rightarrow \quad 9x^2 + 16x^2 = 225 \Rightarrow 25x^2 = 225$

$\Rightarrow \quad x^2 = 9 \Rightarrow x = 3$

$\therefore \quad AB = 3x \text{ cm} \Rightarrow (3 \times 3)\text{cm} = 9 \text{ cm}$ and

BC = 4x cm

$\Rightarrow \quad (4 \times 3) \text{ cm} = 12 \text{ cm}$.

KEY POINTS

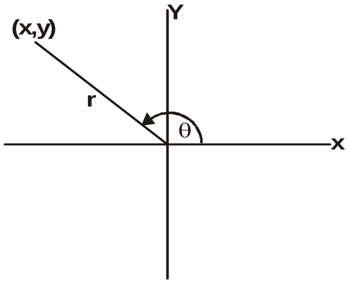
- **Arc length :** An arc "subtends" an angle at the centre of the circle.

- **Analytic Trigonometry :** Analytic Trigonometry takes place on the x-y plane. According to the Pythagorean Theorem

$$\sin\theta = \frac{y}{r}, \operatorname{cosec}\theta = \frac{r}{y}$$

$$\cos\theta = -\frac{x}{r}, \sec\theta = -\frac{r}{x}$$

$$\tan\theta = -\frac{y}{x}, \cot\theta = -\frac{x}{y} \quad r = \sqrt{x^2 + y^2}$$



This is the analytic definition for an angle terminating in any quadrant. It is in terms of the coordinates (x, y) of the endpoint of a radius r.

- **Trigonometric Identities**

Reciprocal Identities

(i) $\cos x = \frac{1}{\sec x}$ and $\sec x = \frac{1}{\cos x}$

(ii) $\sin x = \frac{1}{\operatorname{cosec} x}$ and $\operatorname{cosec} x = \frac{1}{\sin x}$

(iii) $\tan x = \frac{1}{\cot x}$ and $\cot x = \frac{1}{\tan x}$

Quotient Identities

(i) $\tan x = \frac{\sin x}{\cos x}$

(ii) $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities

(i) $\cos^2 x + \sin^2 x = 1$

(ii) $1 + \tan^2 x = \sec^2 x$

(iii) $1 + \cot^2 x = \operatorname{cosec}^2 x$

Negative Angle Identities

(i) $\cos(-x) = \cos x$

(ii) $\sin(-x) = -\sin x$

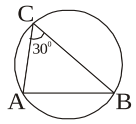
(iii) $\tan(-x) = -\tan x$

ASSIGNMENT – I

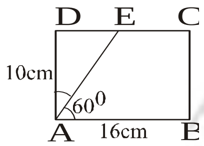
1. If tan A = cot B, prove that A + B = 90^0.

2. Express $\sin 67^0 + \cos 75^0$ in terms of trigonometric ratios of angles between 0^0 and 45^0.

3. In the figure, the chord AB makes and angle of 30^0 at the point C of the circle. If the radius of the circle is 4 cm, find the length of the chord AB.



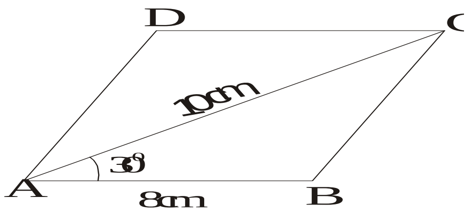
4. In the figure, ABCD is a rectangle with sides 10 cm and 16 cm. AE makes an angle of 60° with AB. Find the perimeter of the trapezium ABCE.



5. A chord of length 10cm makes and angle 60^0 at the center of a circle. Find the radius of the circle.

6. In the figure, ABCD is a rectangle with sides 10 cm and 16 cm. AE makes an angle of 60^0 with AB. Find the perimeter of the trapezium ABCE.

7. In the figure, ABCD is a parallelogram such that AB = 8cm, AC = 10 cm and $\angle BAC = 30^0$.



8. In a right-angled ΔPQR, $\angle R = 90°$, PR = 8 cm and PQ = 17 cm. Find the value of sin P.

9. Evaluate $3\sec^2 30° + 4\tan^2 60° - 6\cot^2 30°$

10. Find the value of $\dfrac{\tan 45^0}{\operatorname{cosec} 30^0} + \dfrac{\sec 60^0}{\cot 45^0} - \dfrac{2\sin 90^0}{\cos 0^0}$

ASSIGNMENT – II

1. If $\sin\theta = \frac{5}{13}$ and θ is less than 90º, find the value of $\tan\theta + \frac{1}{\cos\theta}$.

2. If $\sin\theta = \frac{\sqrt{3}}{2}$ and $\cos\phi = \frac{1}{\sqrt{2}}$, find the value of $\frac{\tan\theta - \tan\phi}{1+\tan\theta\tan\phi}$ (where θ and ϕ are acute).

3. If $\tan\theta = \frac{5}{12}$, find sec θ and sec θ + cosec θ, where θ is acute.

4. From the adjoining figure, find cos y.

5. If 5 tan θ = 4, find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$.

6. If θ is acute and 3 sin θ = 4 cos θ, find the value of 4 $\sin^2\theta$ – 3 $\cos^2\theta$ + 2.

7. In the adjoining figure, ΔABC is right-angled at B and $\tan a = \frac{4}{3}$. If AC = 15 cm, find the lengths of AB and BC.

8. In the adjoining figure, AM is perpendicular to BC. If $\tan B = \frac{3}{4}$, $\tan C = \frac{5}{12}$ and BC = 56 cm, calculate the length of AM.

9. ABCD is a rhombus whose diagonal AC makes an angle α with AB. If $\cos\alpha = \frac{2}{3}$ and OB = 3 cm, then find the side and the diagonals of the rhombus.

10. If tan x + cot x = 2, find the value of $\tan^2 x + \cot^2 x$.

11. Prove that $\tan^2\theta - \frac{1}{\cos^2\theta} + 1 = 0$.

KEY & HINTS

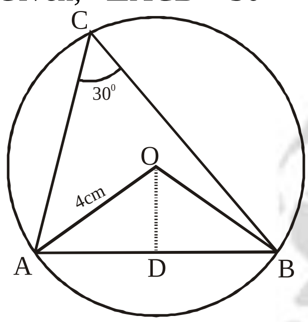
TRIGONOMETRY

ASSIGNMENT – I

1. $\tan A = \cot B \Rightarrow \tan A = \tan(90^0 - B)$
$\Rightarrow A = 90^0 - B \Rightarrow A + B = 90^0$

2. $\sin 67^0 + \cos 75^0 = \cos(90^0 - 67^0) + \sin(90^0 - 75^0) = \cos 23^0 + \sin 15^0$

3. Given, $\angle ACB = 30^0$ OA = 4 cm



$\therefore \angle AOB = 2(30^0) = 60^0$
Draw $OD \perp AB$
Then $\angle AOD = \frac{1}{2}\angle AOB = \frac{1}{2}(60^0) = 30^0$
$\therefore \angle OAD = 90^0 - 30^0 = 60^0$
Now OA = 4 cm and $\angle A = 60^0$
$\therefore \cos A = \cos 60^0 = \frac{AD}{OA} \Rightarrow \frac{1}{2} = \frac{AD}{4} \Rightarrow$ Ad = 2cm
$\therefore$ AB = 2AD = 4cm

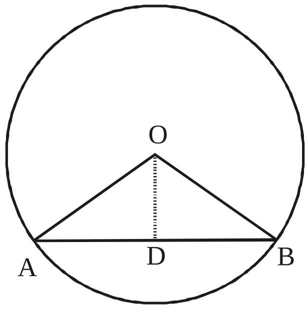
4. Given, $\angle EAB = 60^0$ and AD = 10 cm
Then, $\theta = \angle EAD = 90^0 - 60^0 = 30^0$
$\therefore \cos\theta = \cos 30^0 = \frac{AD}{EA}$
$\Rightarrow \frac{\sqrt{3}}{2} = \frac{10}{EA} \Rightarrow EA = \frac{20}{\sqrt{3}}$ cm
Also, $\tan\theta = \frac{DE}{AD} = \frac{DE}{10} \Rightarrow \frac{1}{\sqrt{3}} = \frac{DE}{10} \Rightarrow DE = \frac{10}{\sqrt{3}}$ cm
$\therefore EC = DC - DE = \left(16 - \frac{10}{\sqrt{3}}\right)$ cm (given, DC = 16 cm)
$\therefore$ perimeter of trap. ABCE

$= AB + BC + CE + EA = 16 + 10 + \left(16 - \frac{10}{\sqrt{3}}\right) + \frac{20}{\sqrt{3}} = 42 + \frac{10}{\sqrt{3}}$

$= 42 + \frac{10\sqrt{3}}{3} = 42 + \frac{17.32}{3} = 42 + 5.77 = 47.77\,cm$

5. Given, AB = 10 cm $\angle AOB = 60^0$

Draw $OD \perp AB$



Then AD = $\frac{1}{2}\angle AB = \frac{1}{2}(10) = 5cm$

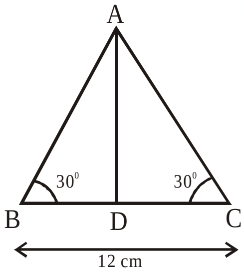
$\angle AOD = \frac{1}{2}\angle AOB = 30^0$

$\therefore \angle OAD = 90^0 - 30^0 = 60^0$

Now AD = 5 cm and $\angle A = 60^0$

$\therefore \cos A = \cos 60^0 = \frac{AD}{OA} \Rightarrow \frac{1}{2} = \frac{5}{OA} \Rightarrow OA = 10$ cm

6. Given, AB = AC BC = 12 cm $\angle ABC = 30^0$



Draw $AD \perp BC$. Then BD = $\frac{1}{2}BC = 6cm$

Also, $\tan B = \frac{AD}{BD} \Rightarrow \tan 30^0 = \frac{AD}{6} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AD}{6} \Rightarrow AD = \frac{6}{\sqrt{3}}$

$\therefore$ area of $\Delta ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2}(12)\left(\frac{6}{\sqrt{3}}\right) = \frac{36}{\sqrt{3}} = \frac{36\sqrt{3}}{3} = 12\sqrt{3}$

$= 12(1.732) = 20.8\ cm^2$

7. In parallelogram ABCD,

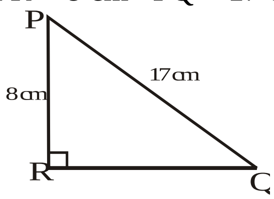
AB = 8 cm AC = 10 cm $\angle BAC = 30^0$

Draw $BE \perp AC$.

Then $\sin 30^0 = \frac{BE}{AB} \Rightarrow \frac{1}{2} = \frac{BE}{8} \Rightarrow BE = 4$ cm

$\therefore$ area of parallelogram = 2(area ΔABC) = $2\left(\frac{1}{2} \times AC \times BE\right) = 2 \times \frac{1}{2} \times 10 \times 4 = 40cm^2$

8. Given, PR = 8 cm PQ = 17 cm $\angle R = 90^0$



$\therefore$ $PR^2 + QR^2 = PQ^2$ or $82 + QR^2 = 17^2$

$\therefore$ QR = $\sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225} = 15cm$

$\therefore$ sin P = $\frac{QR}{PQ} = \frac{15}{17}$

9. $2\sec^2 30^0 + 4\tan^2 60^0 - 6\cot^2 30^0 = 3\left(\frac{2}{\sqrt{3}}\right)^2 + 4(\sqrt{3})^2 - 6(\sqrt{3})^2 = 3\left(\frac{4}{3}\right) + 4(3) - 6(3)$

= 4 + 12 – 18 = -2

10. $\frac{\tan 45^0}{\operatorname{cosec} 30^0} + \frac{\sec 60^0}{\cot 45^0} - \frac{2\sin 90^0}{\cos 0^0}$

$= \frac{1}{2} + \frac{2}{1} - 2 \times \frac{1}{1} = \frac{1}{2} + 2 - 2 = \frac{1}{2}$.

ASSIGNMENT – II

1. We know that $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{5}{13}\right)^2$

$\Rightarrow \quad \cos^2\theta = 1 - \frac{25}{169} = \frac{169 - 25}{169} = \frac{144}{169}$

$\Rightarrow \quad \cos\theta = \frac{12}{13}$

(As θ is acute, cosθ is +ve, so we take +ve value of the square root)

$\therefore \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{13} \div \frac{12}{13} = \frac{5}{13} \times \frac{13}{12} = \frac{5}{12}$

$\therefore \quad \tan\theta + \frac{1}{\cos\theta} = \frac{5}{13} + \frac{1}{\frac{12}{13}} = \frac{5}{12} + \frac{13}{12} = \frac{5 + 13}{12} = \frac{18}{12} = \frac{3}{2}$

2. We know that $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \quad \cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{\sqrt{3}}{2}\right)^2 = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\Rightarrow \quad \cos^2\theta = \frac{1}{2} \qquad (\because \theta \text{ is acute, so } \cos\theta \text{ is +ve})$$

$$\therefore \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$$

Also we know that $\sin^2\phi + \cos^2\phi = 1$

$$\Rightarrow \quad \sin^2\phi = 1 - \cos^2\phi = 1 - \left(\frac{1}{\sqrt{2}}\right)^2 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \quad \sin\phi = \frac{1}{\sqrt{2}} \qquad (\because \phi \text{ is acute, so } \sin\phi \text{ is +ve})$$

$$\therefore \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{1} = 1$$

$$\therefore \quad \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi} = \frac{\sqrt{3}-1}{1+\sqrt{3}\times 1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2 - 1^2} = \frac{3+1-2\sqrt{3}}{3-1} = 2 - \sqrt{3}$$

3. Given $\tan\theta = \frac{5}{12}$ but $\tan\theta = \frac{MP}{OM}$

$$\Rightarrow \quad \frac{MP}{OM} = \frac{5}{12}$$

Draw a triangle OMP right angled at M such that OM = 12 and MP = 5.
From right angled ΔOMP, by Pythagoras theorem,
We get $OP^2 = OM^2 + MP^2 = 12^2 + 5^2 = 144 + 25 = 169$

$\Rightarrow \quad OP = 13$

$$\therefore \quad \sec\theta = \frac{OP}{OM} = \frac{13}{12} \text{ and } \operatorname{cosec}\theta = \frac{OP}{MP} = \frac{13}{5}$$

$$\therefore \quad \sec\theta + \operatorname{cosec}\theta = \frac{13}{12} + \frac{13}{5} = 13\left(\frac{1}{12} + \frac{1}{5}\right)$$

$$= 13.\frac{5+12}{60} = \frac{13\times 17}{60} = \frac{221}{60} = 3\frac{41}{60}$$

4. From right-angled ΔABD, by Pythagoras theorem, we get

$AD^2 = AB^2 + BD^2$

$\Rightarrow \quad AD^2 = (12)^2 + 5^2 = 144 + 25 = 169 \Rightarrow AD = 13$

$$\therefore \quad \cos y = \frac{\text{base}}{\text{hypotenuse}} = \frac{12}{13}$$

5. Given $\tan B = \frac{3}{4} \Rightarrow \frac{AM}{BM} = \frac{3}{4}$

Let AM = 3x cm, then BM = 4x cm.

$\therefore$ MC = BC – BM = (56 – 4x) cm.

Also $\tan C = \frac{5}{12}$ (given) $\Rightarrow \frac{AM}{MC} = \frac{5}{12}$

$\Rightarrow \quad \frac{3x}{56-4x} = \frac{5}{12} \Rightarrow 36x = 280 - 20x$

$\Rightarrow \quad 56x = 280 \Rightarrow x = 5$

$\therefore \quad AM = 3x$ cm $= (3 \times 5)$ cm $= 15$ cm.

6. We know that the diagonals of a rhombus bisects each other at right angles.
From right-angled triangle OAB,

$$\cos\alpha = \frac{OA}{AB} = \frac{2}{3} \text{ (given)}$$

Let OA = 2x cm, then AB = 3x cm.
By Pythagoras theorem, $AB^2 = OA^2 + OB^2$

$\Rightarrow \quad (3x)^2 = (2x)^2 + 3^2 \Rightarrow 9x^2 = 4x^2 + 9$

$\Rightarrow \quad 5x^2 = 9 \Rightarrow x = \frac{3}{\sqrt{5}}$

$\therefore \quad AB = 3 \times \frac{3}{\sqrt{5}}\text{cm} = \frac{9}{\sqrt{5}}\text{cm}$ and $OA = 2 \times \frac{3}{\sqrt{3}}\text{cm} = \frac{6}{\sqrt{5}}\text{cm}$

$\therefore \quad BD = 2 \times OB = (2 \times 3)$ cm $= 6$ cm and $AC = 2 \times OA = \left(2 \times \frac{6}{\sqrt{5}}\right)\text{cm} = \frac{12}{\sqrt{5}}\text{cm}$

$\therefore \quad$ Each side $= \frac{9}{\sqrt{5}}$ cm, diagonal BD = 6 cm and diagonal $AC = \frac{12}{\sqrt{5}}$ cm

7. Given tan x + cot x = 2, on squaring both sides, we get $(\tan x + \cot x)^2 = 2^2$

$\Rightarrow \quad \tan^2 x + \cot^2 x + 2 \tan x \times \cot x = 4$

$\Rightarrow \quad \tan^2 x + \cot^2 x + 2 \times \tan x \times \frac{1}{\tan x} = 4 \qquad \left(\because \cot x = \frac{1}{\tan x}\right)$

$\Rightarrow \quad \tan^2 x + \cot^2 x + 2 = 4$

$\Rightarrow \quad \tan^2 x + \cot^2 x = 4 - 2$

$\Rightarrow \quad \tan^2 x + \cot^2 x = 2$

8. L.H.S. $= \tan^2\theta - \frac{1}{\cos^2\theta} + 1$

$= \tan^2\theta - \sec^2\theta + 1 \qquad \left(\because \frac{1}{\cos\theta} = \sec\theta\right)$

$= \tan^2\theta - (1 + \tan^2\theta) + 1 \qquad (\because \sec^2\theta = 1 + \tan^2\theta)$

$= \tan^2\theta - 1 - \tan^2\theta + 1 = 0 =$ R.H.S.

9. $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right) = \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)$

$= \left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 \qquad [\because (a+b)(a-b) = a^2 - b^2]$

$= \frac{9}{4} - \frac{1}{2} = \frac{9-2}{4} = \frac{7}{4} = 1\frac{3}{4}$

10. $\sin^2 30^o \cos^2 45^o + 4\tan^2 30^o + \frac{1}{2}\sin^2 90^o - 2\cos 90^o + \frac{1}{24}$

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2+4\times\left(\frac{1}{\sqrt{3}}\right)^2+\frac{1}{2}\times(1)^2-2\times 0+\frac{1}{24}$$

$$=\frac{1}{4}\times\frac{1}{2}+4\times\frac{1}{3}+\frac{1}{2}\times 1-0+\frac{1}{24}=\frac{1}{8}+\frac{4}{3}+\frac{1}{2}+\frac{1}{24}=\frac{3+32+12+1}{24}=\frac{48}{24}=2$$

Straight Objective Type

1. $\sin A = \frac{1}{2} \Rightarrow A = 30^0 \Rightarrow \tan A = \tan 30° = \frac{1}{\sqrt{3}}$

2. $3\sin A = 5\cos A \Rightarrow \frac{\sin A}{\cos A}=\frac{5}{3} \Rightarrow \tan A = \frac{5}{3}$

$\therefore\ 6\tan A = 6\left(\frac{5}{3}\right)=10$

3. $\tan A = \frac{2}{3}$

$$\therefore\ \frac{3\sin A-\cos A}{3\sin A+\cos A}=\frac{3\frac{\sin A}{\mathrm{coa}A}-\frac{\cos A}{\cos A}}{3\frac{\sin A}{\cos A}+\frac{\cos A}{\cos A}}=\frac{3\tan A-1}{3\tan A+1}=\frac{3(2/3)-1}{3(2/3)+1}=\frac{2-1}{2+1}=\frac{1}{3}$$

4. $\cot\theta = \frac{7}{8}$

$$\therefore\ \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}=\frac{1-\sin^2\theta}{1-\cos^2\theta}=\frac{\cos^2\theta}{\sin^2\theta}=\cot^2\theta=\left(\frac{7}{8}\right)^2=\frac{49}{64}$$

5. $\tan(A + B) = \sqrt{3} \Rightarrow A + B = 60^0$

since $A = 45^0$, $B = 60^0 - A = 60^0 - 45^0 = 15^0$

6. $\tan 2A = \cot(A - 150^0) \Rightarrow \tan 2A = \tan[90^0 - (A- 15^0)]$

$\Rightarrow 2A = 90^0 - (A - 150^0) \Rightarrow 2A = 90^0 - A + 150^0$

$\Rightarrow 3A = 105^0 \Rightarrow A = 35^0$

7. Let the length of ladder be x metes.

$\therefore\ \sin 60^0 = \frac{3}{x} \Rightarrow \frac{\sqrt{3}}{2}=\frac{3}{2}$

$\Rightarrow\ x=\frac{2\times 3}{\sqrt{3}}=2\sqrt{3}$

$\therefore$ length of ladder = $2\sqrt{3}$ meters.

8. Let AB = x meters

$\therefore\ \tan 30^0 = \frac{x}{20} \Rightarrow \frac{1}{\sqrt{3}}=\frac{x}{20}$

$\therefore \quad x = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3} = \frac{1.73 \times 20}{3} = 11.53$

$\therefore$ AB = 11.53 meters

9. Let AB be the tower

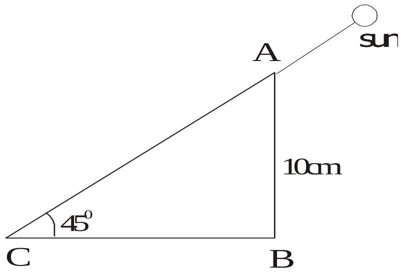
$\therefore$ AB = 11m

The sun is inclined at 45^0

Let BC = x be the length of the shadow

$\therefore \tan 45^0 = \frac{10}{x} \Rightarrow \tan 45^0 = \frac{10}{x} \Rightarrow x = 10$

$\therefore$ length of shadow = 10 metres.



10. Given, $\sin\theta = \frac{12}{13}$

$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{12}{13}\right)^2} = \sqrt{\frac{169-144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$

$\therefore \sec\theta = \frac{13}{5}$ and $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12/13}{1/5} = \frac{12}{5}$

$\therefore \quad \frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta} = \frac{13/5+12/5}{13/5-12/5} = \frac{25/5}{1/5} = 25$

11. In ΔABD,

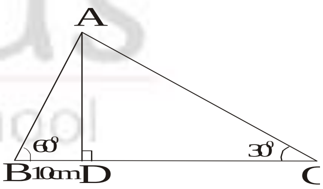
$\tan 60^0 = \frac{AD}{BD} \Rightarrow \sqrt{3} = \frac{AD}{10}$

$\therefore$ AD = $10\sqrt{3}$ cm

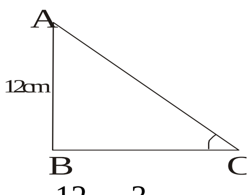
In ΔACD,

$\tan 30^0 = \frac{AD}{DC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{10\sqrt{3}}{DC}$

$\therefore$ DC = $(10\sqrt{3})\sqrt{3} \Rightarrow$ DC = 30cm



12. Given, $\tan\theta = \frac{3}{4} \Rightarrow \frac{AB}{BC} = \frac{3}{4}$



$\therefore \quad \frac{12}{BC} = \frac{3}{4}$ $\quad (\because$ AB = 12 cm)

$\therefore$ 3BC = 48 $\quad \therefore$ BC = 16 cm

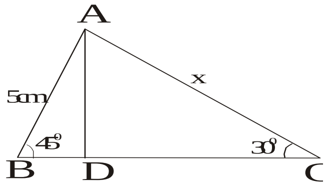
$\therefore \quad AC = \sqrt{AB^2+BC^2} = \sqrt{12^2+16^2}$

$= \sqrt{144+256} = \sqrt{400} = 20cm$

13. Draw AD ⊥ BC

Then, $\sin 45^0 = \frac{AD}{AB}$ or $\frac{1}{\sqrt{2}} = \frac{AD}{5}$

$\therefore$ $AD = \frac{5}{\sqrt{2}}$ cm

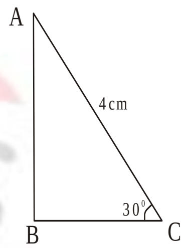


Also, $\sin 30^0 = \frac{AD}{AC}$ or $\frac{1}{2} = \frac{5/\sqrt{2}}{AC}$

$\therefore$ $AC = 2\left(\frac{5}{\sqrt{2}}\right) = 5\sqrt{2}$ cm $= 5 \times 1.4142 = 7.0710$ cm

or $x = 7.1$ cm (approx)

14. From the figure,



$\sin 30^0 = \frac{AB}{AC}$ and AC = 4 cm

$\therefore$ $\frac{1}{2} = \frac{AB}{4}$ or AB = 2 cm

Also, $\cos 30^0 = \frac{BC}{AC}$

$\therefore$ $\frac{\sqrt{3}}{2} = \frac{BC}{4}$ or $BC = 2\sqrt{3}$ cm

$\therefore$ area of $\Delta ABC = \frac{1}{2} \times BC \times AB = \frac{1}{2} \times (2\sqrt{3}) \times 2 = 2\sqrt{3}$

$= 2 \times 1.732 = 3.464$ cm^2

15. (AD)
16. (AC)
17. (ACD)
18. 1. (B)
    2. (A)
    3. (C)
19. (1)

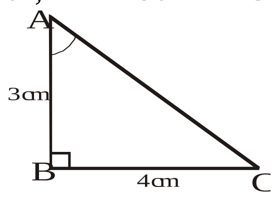
$\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0 = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$

20. $2\sin^2 A + 5\sec^2 A + 3\cos^2 A - 5\tan^2 A$

$= 3(\sin^2 A + \cos^2 A) + 5(\sec^2 A - \tan^2 A)$

$= 3 \times 1 + 5 \times 1 = 8$

21. Given, AB = 3cm BC = 4cm $\angle B = 90^0$



$\therefore$ AC = $\sqrt{AB^2 + BC^2} = \sqrt{3^2 + 4^2} = 5\text{cm}$

$\therefore$ $\sin\theta = \frac{BC}{AC} = \frac{4}{5}$ and $\cos\theta = \frac{AB}{AC} = \frac{3}{5}$

$\therefore$ $2\sin\theta - \cos\theta = 2\left(\frac{4}{5}\right) - \frac{3}{5} = \frac{8}{5} - \frac{3}{5} = \frac{5}{5} = 1$

22. $\tan 20^0 - \cot 70^0 + \tan 20^0 \tan 70^0$

$= \tan(90^0 - 70^0) - \cot 70^0 + \tan(90^0 - 70^0)\tan 70^0$

$= \cot 70^0 - \cot 70^0 + \cot 70^0 \tan 70^0 = 0 + \frac{1}{\tan 70^0}\tan 70^0 = 1$

23. 1

$\tan 10^0 \tan 15^0 \tan 75^0 \tan 80^0$

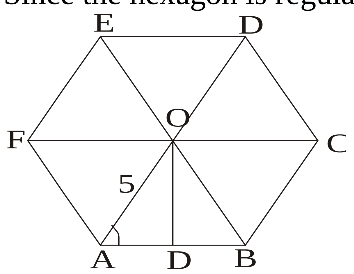
$= \tan 10^0 \tan 15^0 \tan(90^0 - 15^0)\tan(90^0 - 10^0)$

$= \tan 10^0 \tan 15^0 \cot 15^0 \cot 10^0$

$= \tan 10^0 \tan 150^0 \frac{1}{\tan 15^0}\frac{1}{\tan 10^0} = 1$

24. Given, radius of circumcircle = 5cm $\Rightarrow$ OA = 5 cm

Since the hexagon is regular, ΔOAB is an equilateral triangle.



$\therefore$ AB = OA = 5cm

$\therefore$ AD = $\frac{1}{2}(AB) = 2.5\text{cm}$

Also, $\tan\theta = \frac{OD}{AD}$ or $\tan 60^0 = \frac{OD}{2.5}$

$\therefore$ OD = $2.5 \times \sqrt{3}$ = 2.5 × 1.732 = 4.33 cm

$\therefore$ radius of incircle = OD = 4.3 cm and side of hexagon = 5 cm. .

• SURFACE AREA & VOLUME

- **SOLIDS** The objects having definite shape and size are called solids. A solid occupies a definite space.
- **CUBOID** Solids like matchbox, chalkbox, a tile, a book, an almirah, a room, etc., are in the shape of a cuboid.

FORMULAE: For a cuboid length = l, length = b and height = h, we have

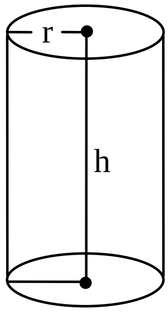
(i) Volume = $(l \times b \times h)$ cubic units
(ii) Total surface area = $2(lb + bh + lh)$ sq units
(iii) Lateral surface area = $[2(l + b) \times h]$ sq units

- **CUBE** Solids like ice cubes, sugar cubes, dice, etc., are in the shape of a cube.

FORMULAE: For cube having each edge = a units, we have

(i) Volume a^3 cubic units
(ii) Total surface area = $6a^2$ sq units
(iii) Lateral surface area = $4a^2$ sq units

- **CYLINDER** Solids like measuring jars, circular pillars, circular pencils, circular pipes, road rollers, gas cylinders etc., are said to have cylindrical shape.



FORMULAE: For a cylinder of base radius = *r* and height (or length) = h, we have

(i) Volume = $(\pi r^2 h)$ cubic units
(ii) Curved surface area = $2\pi rh$ sq units
(iii) Total surface area = $(2\pi rh + 2\pi r^2)$ sq units
$= 2\pi r(h + r)$ sq units

- **HOLLOW CYLINDERS** Solids like iron pipes, rubber tubes are in the shape of hollow cylinders.

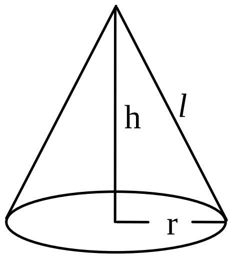
FORMULAE Consider a hollow cylinder having
External radius = R, internal radius = r and height = h.
Then, we have

(i) Volume of material = (external volume) – (internal volume)
$= (\pi R^2 h - \pi r^2 h)$ cubic units
$= \pi h(R^2 - r^2)$ cubic units

(ii) Curved surface area of hollow cylinder
= (external surface area) – (internal surface area)
$= \left(2\pi Rh - 2\pi rh\right)$ sq units
$= 2\pi h(R - r)$ sq units

(iii) Total surface area of hollow cylinder
=(curved surface area) + (area of the base ring)
$= (2\pi Rh + 2\pi rh) + 2(\pi R^2 - \pi r^2)$ sq units
$= 2\pi h\left(R + r\right) + 2\pi\left(R^2 - r^2\right)$ sq units

- **CONE** Solids like ice-cream cones, conical tents, funnels, etc, are having the shape of a cone



FORMULAE Consider a cone in which base radius = r, height = h and slant height,

$l = \sqrt{h^2 + r^2}$.

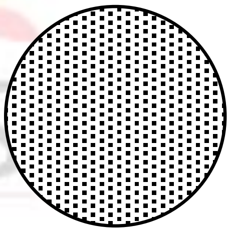
Then, we have

(i) Volume of the cone $= \frac{1}{3}\pi r^2 h$ cubic units

(ii) Curved surface area of the cone $= \pi r l = \pi r \sqrt{r^2 + h^2}$ sq units

(iii) Total surface area of the cone
= (curved surface area) + (area of the base)
$= (\pi r l + \pi r^2) = \pi r(l + r)$ sq units

- **SPHERE** Objects like a football, a cricket ball, etc., are said to have the shape of a sphere.



FORMULAE For a sphere of radius *r*, we have

(i) Volume of sphere $= \left(\frac{4}{3}\pi r^3\right)$ cubic units

(ii) Surface area of the sphere = $\left(4\pi r^2\right)$ sq units

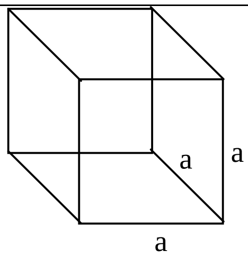
- **HEMISPHERE** A plane through the centre of a sphere cuts it into two equal parts. Each part is called a hemisphere.

FORMULAE For a hemisphere of radius *r*, we have

(i) Volume of the hemisphere $= \frac{2}{3}\pi r^3$ cubic units

(ii) Curved surface area of the hemisphere = $(2\pi r^2)$ sq units

(iii) Total surface area of the hemisphere = $(3\pi r^2)$ sq units.

Name of the solid	Figure	Lateral/ Curved Surface Area	Total Surface Area	Volume
Cube		$4a^2$	$6a^2$	a^3

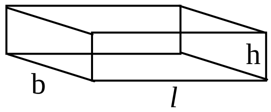
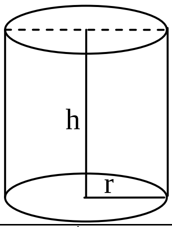
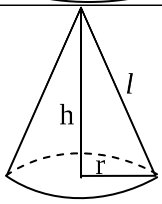
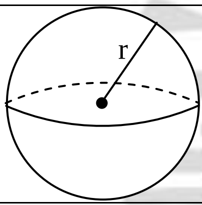
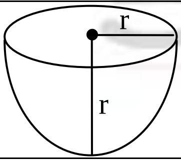
Cuboid		$2(l+b)h$	$2(lb+bh+hl)$	lbh
Right circular cylinder		$2\pi rh$	$2\pi r(r+h)$	πr^2h
Right circular cone		πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2h$
Sphere		$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemisphere		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$

Illustration – 1: Find the volume, the total surface area and the lateral surface area of a cuboid which is 15 m long, 12 m wide and 4.5 m high.

Solution

Here, $l = 15$ m , $b = 12$ m and $h = 4.5$ m

Volume of the cuboid = $(l \times b \times h)$ cubic units

$= (15 \times 12 \times 4.5)\ m^3 = 810\ m^3$

Total surface area of the cuboid

$= 2(lb + bh + lh)$ sq units

$= (15 \times 12 + 12 \times 4.5 + 15 \times 4.5)\ m^2 = 603\ m^2$.

Lateral surface are of the cuboid

$= \left[2(l+b)\times h\right]$ sq units

$= [2(15 + 12) \times 4.5]\ m = 243\ m^2$.

Illustration – 2 How many bricks will be required to construct a wall 13.5 m long, 6 m high and 22.5 cm thick if each brick measures (27 cm × 12.5 cm × 9 cm)?

Solution

Length of the wall = (13.5×100) cm = 1350

Breadth of the wall = 22.5 cm

Height of the wall = (6×100) cm = 600 cm

Volume of the wall = $(1350 \times 22.5 \times 600)\ cm^3$

Volume of each brick = $(27 \times 12.5 \times 9)\ cm^3$

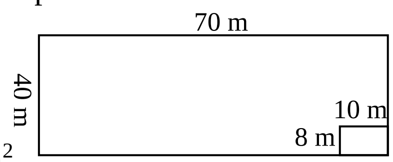
Number of brick required = $\left(\dfrac{\text{volume of wall}}{\text{volume of 1 brick}}\right)$

Illustration – 3 A field is 70 m long and 40 m broad. In one corner of the field, a pit which is 10 m long, 8 m broad and 5 m deep, has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.

Solution : Area of the field = $(70 \times 40)\ m^2 = 2800\ m^2$

Area of the pit = $(10 \times 8)\ m^2 = 80\ m^2$

Area over which the earth is spread over



= $(2800 - 80)\ m^2 = 2720\ m^2$

Volume of the earth dug out = $(10 \times 8 \times 5)\ m^3\ 400\ m^3$

Rise in level of the field = $\left(\dfrac{\text{volume of the earth dug out}}{\text{area on which the earth is spread}}\right)$

$$= \left(\frac{400}{2720}\right) m = \left(\frac{400 \times 100}{2720}\right) cm$$

$$= \frac{250}{17} = 14.70 \text{ cm}$$

Illustration – 4: The total surface area of a cube is 216 cm^2. Find its volume.

Solution Let each side of the cube be *a* cm.

Then, the total surface area of the cube = $(6a)^2\ cm^2$.

$\therefore\ 6a^2 = 216 \Leftrightarrow a^2 = 36 \Leftrightarrow a = \sqrt{36} = 6$

Volume of the cube = a^3 cubic units

= $(6 \times 6 \times 6)\ cm^3 = 216\ cm^3$.

Illustration – 5: Find the (i) volume, (ii) area of the surface and (iii) total surface area of a cylinder having radius of the base 14 cm and height 30 cm.

Solution Here, r = 14 cm and h = 30 cm.

(i) Volume of the cylinder = $(\pi r^2 h)$ cubic units

$$= \left(\frac{22}{7} \times 14 \times 14 \times 30\right) cm^3 = 18480\ cm^3$$

(ii) Curved surface area of the cylinder = $(2\pi rh)$ sq units.

$$= \left(2 \times \frac{22}{7} \times 14 \times 30\right) cm^2 = 2640\ cm^2$$

(iii) Total surface area of the cylinder

= $(2\pi rh + 2\pi r^2)$ sq units = $2\pi r(r + h)$ sq units

$$= \left\{2 \times \frac{22}{7} \times 14(14+30)\right\} cm^2 = 3872 cm^2$$

Illustration – 6: The curved surface area and the volume of a pillar are 264 m^2 and 396 m^3 respectively. Find the diameter and the height of the pillar.

Solution

Let the radius of the pillar be r meters and its height be h meters. Then,

$2\pi rh = 264$ …… (i)

and $\pi r^2 h = 396$ …… (ii)

On dividing (ii) by (i), we get

$$\frac{\pi r\ h}{2\pi rh} = \frac{396}{294} \Leftrightarrow r = \left(\frac{2 \times 396}{264}\right) = 3 \Leftrightarrow 2r = 6$$

Hence, the diameter of the pillar is 6 m.

Putting r = 3 in (i), we get

$$2 \times \frac{22}{7} \times 3 \times h = 264 \Leftrightarrow h = \left(264 \times \frac{7}{132}\right) = 14$$

Hence, the height of the pillar is 14 m.

Illustration – 7: The diameter of a roller, 120 cm long, is 84 cm. If it takes 500 complete revolutions to level a playground, find the cost of leveling it at 75 paisa per square meter.

Solution

Radius of the roller, r = 42 cm, and its length, h = 120 cm

Area covered by the roller in 1 revolution

= curved surface area of the roller

= $(2\pi rh)$ sq units

$$= \left(2 \times \frac{22}{7} \times 42 \times 120\right) cm^2 = 31680\ cm^2$$

Area covered by the roller in 500 revolutions

$$= (31680 \times 500)\ cm^2 = \left(\frac{31680 \times 500}{100 \times 100}\right) m^2 = 1584\ m^2$$

$\therefore$ area of the playground = 1548 m^2

Cost of leveling the playground = $Rs\left(1584 \times \frac{75}{100}\right)$

= Rs 1188.

Illustration – 8: The height of a cone is 24 cm and the diameter of its base is 14 cm. Find the slant height, volume, area of curved surface and the total surface area of the cone.

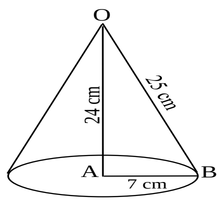
Solution

Here, h = 24 cm and r = 7 cm.

Let the slant height be I cm. Then,

$$l = \sqrt{h^2 + r^2} = \sqrt{(24)^2 + 7^2} = \sqrt{625} = 25$$

$\therefore$ slant height = l cm = 25 cm.



Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 25\right) \text{ cm}^3$$

Curved surface area of the cone $= \pi r l$

$$= \left(\frac{22}{7} \times 7 \times 25\right) \text{ cm}^2 = 550 \text{ cm}^2$$

Total surface area of the cone = $\pi r(l + r)$

$$= \left[\frac{22}{7} \times 7 \times (25 + 7)\right] \text{ cm}^2$$

= 704 cm^2

Illustration – 9: The radius and the height of a right circular cone are in the ratio of 5 : 12 and its volume is 2512 cu cm. Find the curved surface area and the total surface area of the cone. (use π = 3.14.)

Solution

Let radius = 5x cm and height = 12 x cm. Then,

volume $= \left[\frac{1}{3} \times 3.14 \times (5x)^2 \times (12x)\right] \text{cm}^2 = (314x^3) \text{ cm}^3$

$\therefore 314x^3 = 2512 \Leftrightarrow x^3 = \left(\frac{2512}{314}\right) = 8 \Leftrightarrow x = 2$.

$\therefore$ radius = 10 cm and height = 24 cm

$\therefore$ slant height $= \sqrt{r^2 + h^2} = \sqrt{(10)^2 + (24)^2} \text{ cm} = \sqrt{676} \text{ cm}$

= 26 cm

So, area of the curved surface = $\pi r l$

$= (3.14 \times 10 \times 26) \text{ cm}^2$

= 816.4 cm^2

Whole surface area = (curved surface area + base area)

= (816.4 + 3.14 × 10 × 10) cm^2

= (816.4 + 314) cm^2 = 1130.4 cm^2

Illustration – 10: A cone of height 8 m has a curved surface area 188.4 square meters. Find its volume. (Take π = 3.14)

Solution

Let the radius be *r* meters and slant height be *l* meters. Then,

Curved surface area = $\pi r l$ m^2 = $\pi r\sqrt{r^2 + h^2}$ m^2

$$= \left(3.14 \times r \times \sqrt{r^2 + 64}\right) \text{ m}^2$$

$$\therefore\ 3.14 \times r \times \sqrt{r^2 + 64} = 188.4$$

$$\Leftrightarrow r\sqrt{r^2+64} = \frac{188.4}{3.14} = 60$$

$$\Leftrightarrow r^2\left(r^2+64\right) = 3600$$

$$\Leftrightarrow r^4 + 64r^2 - 3600 = 0$$

$$\Leftrightarrow \left(r^2+100\right)\left(r^2-36\right) = 0$$

$$\Leftrightarrow r^2 = 36 \quad \left[\because r^2 = -100 \text{ gives imaginary value of r}\right]$$

$$\Leftrightarrow r = 6$$

So, the radius of the base = 6 m.

$$\text{Volume} = \left(\frac{1}{3}\pi r^2 h\right) \text{ cubic units}$$

$$\left(\frac{1}{3}\times 3.14\times 6\times 6\times 8\right) m^3 = 301.44\ m^3$$

Hence, the volume of the cone is 301.44 m^3

Illustration – 11: Find the volume and surface area of a sphere of radius 21 cm.

Solution

Radius of the sphere = 21 cm

$$\text{Volume of the sphere } = \left(\frac{4}{3}\pi r^3\right) \text{ cubic units}$$

$$= \left(\frac{4}{3}\times\frac{22}{7}\times 21\times 21\times 21\right) cm^3$$

$$= 38808\ cm^3$$

Surface area of the sphere = ($4\pi r^2$) sq units

$$= \left(4\times\frac{22}{7}\times 21\times 21\right) cm^2 = 5544\ cm^2.$$

Illustration – 12: How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter?

Solution

Length of each edge of the cube = 44 cm

Volume of the cube = (44 × 44 × 44) cm^3

Radius of each bullet, r = 2 cm

$$\text{Volume of each bullet } = \left(\frac{4}{3}\pi r^3\right) cm^3$$

$$= \left(\frac{4}{3}\times\frac{22}{7}\times 2\times 2\times 2\right) cm^3 = \frac{704}{21}\ cm^3$$

$$\text{Number of bullets formed } = \frac{\text{volume of the cube in } cm^3}{\text{volume of each bullet in } cm^3}$$

$$= \left(44\times 44\times 44\times\frac{21}{704}\right) = 2541.$$

Hence, the number of bullets formed is 2541.

Illustration – 13: Find the volume, curved surface area and the total surface area of a hemisphere of diameter 7 cm.

Solution

Radius of the hemisphere, r = 3.5 cm

Volume of the hemisphere $= \left(\frac{2}{3}\pi r^3\right)\ \text{cm}^3$

$= \left(\frac{2}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\frac{7}{2}\right)\ \text{cm}^3$

$= \frac{539}{6}\ \text{cm}^3 = 89.83\ \text{cm}^3$.

Curved surface area of the hemisphere = $(2\pi r^2)\ \text{cm}^2$

$= \left(2\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right)\ \text{cm}^2 = 77\ \text{cm}^2$

Total surface area of the hemisphere = $(3\pi r^2)\ \text{cm}^2$

$= \left(3\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right)\ \text{cm}^2 = \frac{231}{2}\ \text{cm}^2 = 115.5\ \text{cm}^2$

KEY POINTS

1. If a cuboid has length *l*, breadth b and height h, then
   (i) Perimeter of the cuboid $= 4(\ell + b + h)$.
   (ii) Surface are of the cuboid $= 2(\ell b + bh + \ell h)$
   (iii) Lateral surface area of the cuboid $= 2(\ell + b) \times h$
   (iv) Length of a diagonal $= \sqrt{\ell^2 + b^2 + h^2}$
   (v) Volume of the cuboid $= \ell \times b \times h$

2. If the length of each edge of a cube is ℓ then
   (i) Perimeter of the cube $= 12\ell = 12(\text{Edge})$
   (ii) Surface area of the cube $= 6\ell^2 = 6(\text{Edge})^2$
   (iii) Lateral surface area of the cube $= 4\ell^2 = 4(\text{Edge})^2$
   (iv) Volume of the cube $= \ell^3 = (\text{Edge})^3$

3. A right circular cone is a solid generated by revolving a line segment which passes thought a fixed point and which makes a constant angle with a fixed line. The fixed point is called the vertex of the cone, the fixed line is called the axis of the cone.

4. For a right circular cone of based radius r, slant height *l* and height h, we have
   (i) Curved surface area $= \pi r \ell$
   (ii) Total surface area $= \pi r(\ell + r)$
   (iii) Volume $= \frac{1}{3}\pi r^2 h$

   Also, Volume $= \frac{1}{3}(\text{Area of the base}) \times \text{height}.$

ASSIGNMENT – I

1. * Eight identical cuboidal wooden blocks are staked one on top of the other. The total volume of the solid so formed is 128 cm^3. If the height of each block is 1 cm and the base is a square, find the dimensions of each block.

2. A rectangle block of ice measures 40 cm by 25 cm by 15 cm. Calculate its weight in kg, if ice weights $\frac{9}{10}$ of the weight of the same volume of water and 1 cm^3 of water weight 1 gm.

3. What will happen to the volume of a cube if its edge is doubled?

4. A match box measures 4 cm by 2.5 cm by 1.5 cm. What will be the volume of a packet containing 12 such match boxes? How many such packets can be placed in a cardboard box whose size is 60 cm × 30 cm × 24 cm?

5. A water tank built by a municipality of a town to supply water to its 25000 inhabitants at 125 litres per person is 40 m long and 31.25 m broad. The tank, when it is full, can supply water for two days to the inhabitants of the town. Find the depth of the tank.

6. What will be the labour charges for digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs 20 per m^3?

7. A rectangular field is 154 m long and 121 m broad. A well of 14 m length and 11 m breadth is dug inside the field and mud taken out is spread evenly over the remaining part of the field to a thickness of 25 cm. Find the depth of the well.

8. The curved surface area of a cylinder is 1210 cm^2 and its diameter is 20 cm. Find its height and volume.

9. * The curved surface area of a cylinder is 4400 cm^2 and the circumference of its base is 110 cm. Find the height and the volume of the cylinder

10. The sum of the height and radius of the base of a solid cylinder is 37 m. If the total surface area of the cylinder be 1628 m^2, Find its volume.

11. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 ; 3. Calculate the ratio of their volumes and the ratio of their curved surfaces.

12. If 1 cm^3 of cast iron weighs 21 g, find the weight of a cast iron pipe of length 1 m with a bore of 3 cm in which the thickness of the metal is 1 cm.

ASSIGNMENT – II

1. Find the volume, the total surface area and the lateral surface area of a cuboids which is 15 m long, 12 m wide and 4.5 m high.

2. A river 2 m deep and 45 m wide is flowing at the rate of 3 km per hour. find the volume of water that runs into the sea per minute.

3. 2.2 cu dm of brass is to be drawn into a cylindrical wire of diameter 0.50 cm. Find the length of the wire.

4. Find the volume, total surface area, lateral area and the length of diagonal of a cube, each of whose edges measures 20 cm.

5. How many cubic metres of earth must be dug out to sink a well 14 m deep and having a radius of 4 m? If the earth taken out is spread over a plot of dimensions $(24\text{m} \times 16\text{m})$, what is the height of the platform so formed?

6. The external diameter of an iron pipe is 25 cm and its length is 20 cm if the thickness of the pipe is 1 cm, find the total surface area of the pipe.

7. A hollow sphere of external and internal diameters 8 cm and 4 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone.

8. A lead pencil consists of a cylinder of wood with a solid cylinder of graphite fitted into it. The diameter of the pencil is 7 mm, the diameter of the graphite is 1 mm and the length of the pencil is 10 cm. Calculate the weight of the whole pencil, if the specific gravity of the wood is 0.7 g/cm^3 and that of the graphite is 2.1 g/cm^3.

9. The volume of a right circular cone is (100π) cm^3 and its height is 12 cm. Find its slant height and its curved surface area.

10. * The curved surface area of a cone is 4070 cm^2 and its diameter is 70 cm. Find its slant height.

ANSWER KEY

SURFACE AREA & VOLUME

ASSIGNMENT – I

1. $4 \text{ cm} \times 4\text{cm} \times 1 \text{ cm}$.
2. 13.5 kg
3. edge of the4 cube is doubled, then the volume becomes 8 times.
4. 240
5. 0.5 m
6. Rs 2880
7. 30 m
8. Volume = 6050 cm^3, h =19.25   9. Volume = 38500 cm^3, h = 40 cm
10. Volume = 4620 cm^3
11. 10/9

ASSIGNMENT – II

1. 243 sqm
2. 4500 cubic m
3. 112 m
4. 34.64 cm
5. 1.76 m
6. 3168 sq cm
7. 14 cm
8. 0.8925π gm
9. l = 13 cm. Area = 204.4 cm^2
10. l = 37 cm

HINTS & SOLUTION

BASIC CONCEPT OF MATHEMATICS

ASSIGNMENT – I

1. Let the length and breadth are staked one on top of the base of each wooden block be x cm each.
Since eight identical blocks are staked one on top of the other and the height of each block is 1 cm.
So, height of the solid formed = 8 cm
$\therefore$ Volume of the solid formed $=\left(x\times x\times 8\right)\ cm^3 = 8x^2\ cm^3$
But, the volume of solid formed is 128 cm^3 (given)
$\therefore \quad 8x^2 = 128 \Rightarrow x^2 = \dfrac{128}{8} \Rightarrow x^2 = 16x^2 = 4^2 \Rightarrow x = 4\ cm$
Hence, each wooden block is dimension 4 cm × 4cm × 1 cm.

2. We have,
Volume of rectangular block of ice $= 40\times 25\times 15\ cm^2 = 15000\ cm^2$
Now, weight of 1 cm^3 of water = 1 gm
and, weight of 1 cm^3 of ice $=\left(\dfrac{9}{10}\right)^{th}$ of the weight of 1 cm^3 of water
$\therefore \quad$ Weight of 1 cm^3 of ice $=\left(\dfrac{9}{10}\right)$ gm
$\therefore \quad$ Weight of the rectangular block of ice $=\dfrac{9}{10}\times 15000\ gm$
$= 13500\ \ gm = 13.5\ kg$

3. Let the edge of the cube be l cm. Then, its volume V is given by
$V = l^3\ \ cm^3$ .....(i)
Let V_1 be the volume of the cube when its edge is doubled. Then,
$V_1 = \left(2l\right)^3\ \ cm^3$
$\Rightarrow \quad V_1 = 8l^3$
$\Rightarrow \quad V_1 = 8V$
Hence, if each edge of the4 cube is doubled, then the volume becomes 8 times.

4. We have,
Volume of a match box $=\left(4\times 2.5\times 1.5\right)\ cm^3 = 15\ cm^3$
$\therefore$ Volume of a packet containing 12 match boxes $=\left(12\times 15\right)\ cm^3 = 180\ cm^3$
Now, Volume of cardboard box $=\left(60\times 30\times 24\right)\ cm^3 = 43200\ cm^3$
$\therefore$ Number of packet that can be put in a cardboard $=\dfrac{43200}{180} = 240$

5. Water consumed by the inhabitants in one day
= Number of inhabitants × Water consumed by an inhabitant in one day

$= (25000 \times 125)$ litres $= 3125000$ litres

Water consumed by the inhabitants in to days

$= (3125000 \times 2)$ litres

$= 6250000$ litres $= \dfrac{6250000}{1000} \text{m}^3$ $\quad$ [$\therefore 1000$ litres $= 1 \text{ m}^3$]

$= 6250 \text{ m}^3$

Volume of the tank $= 6250$ litres

Depth of the tank $= \dfrac{\text{Volume}}{\text{Lenght} \times \text{Breadth}} = \dfrac{6250 \text{ m}^3}{40\text{m} \times 31.25 \text{ m}} = \dfrac{625}{1250} \text{m} = 0.5 \text{ m}$

6. We have,

Volume of the pit $= (\text{Length} \times \text{Breadth} \times \text{Height}) = (8 \times 6 \times 3) \text{ m}^3 = 144 \text{ m}^3$

Since labour charges are at the rate of Rs 20 per m^3

Total labour charges $= \text{Rs } (144 \text{ m}^3 \times 20) = \text{Rs } 2880$

7. We have,

Length of the well = 14 m, Breadth of the well = 11 m

Area of the base of the well $= (14 \times 11) \text{m}^2 = 154 \text{ m}^2$

Also, area of the field $= (154 \times 121) \text{m}^2 = 18634 \text{ m}^2$

Area of the field in which mud is spread $= (18634 - 154) \text{m}^2 = 18480 \text{ m}^2$

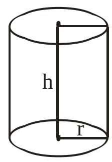
Thickness of the mud $= 25 \text{cm} = \dfrac{25}{100} \text{m} = \dfrac{1}{4} \text{m}$

Volume of the mud = $18480 \times 1/4 \text{ m}^3$

= 4620 m^3

Depth of the well = $4620/14 \times 11 = 30$ m.

8. Surface area = 1210 cm^2



Diameter = 20 cm

R = 10 cm

Surface area = $2\pi rh$

$1210 = 2 \times \dfrac{22}{7} \times 10 \times h$

h = 19.25 cm

Volume = $\pi r^2 h$

$= \dfrac{22}{7} \times 100 \times 19.25$

= 6050 cm^3

9. Curved surface area = 4400 cm^2

$2\pi rh = 4400$

$2\pi r = 110$

$h = \dfrac{4400}{2\pi r} = \dfrac{4400}{110} = 40 \text{ cm}$

Volume = $\pi r^2 h$

$$= \pi\left(\frac{110}{2\pi}\right)^2.40$$

$= 38500 \text{ cm}^3$

10. $\Rightarrow$ $h + r = 37$

$\Rightarrow$ $2\pi rh + 2\pi r^2 = 1628$

$2\pi r(h + r) = 1628$

$$2\pi r = \frac{1628}{37}$$

$r = 7$ m

$\Rightarrow$ $h = 37 - 7 = 30$ m

$\Rightarrow$ Volume $= \pi r^2 h$

$$= \frac{22}{7} \times 7^2 \times 30$$

$= 4620 \text{ m}^3$

11. $$\frac{r_1}{r_2} = \frac{2}{3}, \ \frac{h_1}{h_2} = \frac{5}{3}$$

$\Rightarrow$ $V_1 = \pi r_1^2 h_1$

$\Rightarrow$ $V_2 = \pi r_2^2 h_2$

$$\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 . \left(\frac{h_1}{h_2}\right)$$

$$= \frac{4}{9}.\frac{5}{3}$$

$$= \frac{20}{27}$$

$\Rightarrow$ $$\frac{S_1}{S_2} = \frac{2\pi r_1 h_1}{2\pi r_2 h_2}$$

$$= \frac{r_1}{r_2}.\frac{h_1}{h_2}$$

$$= \frac{2}{3}.\frac{5}{3} = \frac{10}{9}$$

ASSIGNMENT – II

1. Ans: 243 m^2

Solution: Here, l= 15 , b = 12 m and h = 4.5m

Volume of the cuboid $= (l \times b \times h)$ cubic units

$$= (15 \times 12 \times 4.5)\text{m}^3 = 810\text{m}^3$$

Total surface area of the cuboid

$= 2(lb \times bh \times lh)$ sq units

$= 2(15 \times 12 + 12 \times 4.5 + 15 \times 4.5)\mathrm{m}^2 = 603\mathrm{m}^2$

Lateral surface area of the cuboid

$= [2(l + b) \times \mathrm{h}]$ sq units

$= [2(15 + 12) \times 4.5]\mathrm{m} = 243\mathrm{m}^2$

2. Ans: 4500 m^3

3 km /hr $= \left(\frac{3 \times 1000}{60}\right)\mathrm{m/min} = 50\mathrm{m/min}$.

Volume of water running into the sea per minute

$= (50 \times 45 \times 2)\mathrm{m}^3 = 4500\mathrm{m}^3$.

3. Ans: 112 m

Volume of brass $= (2.2 \times 10 \times 10 \times 10)\mathrm{cm}^3 = 2200\mathrm{cm}^3$.

Radius of the wire = 0.25 cm.

Let the required length of wire be x cm.

Then, volume of the wire = ($\pi r^2 x$) cu units.

$= \left(\frac{22}{7} \times 0.25 \times 0.25 \times \mathrm{x}\right)\mathrm{cm}^3$

$\therefore \frac{22}{7} \times 0.25 \times 0.25 \times \mathrm{x} = 2200$

$\Leftrightarrow \mathrm{x} = \left(\frac{2200 \times 7}{22 \times 0.25 \times 0.25}\right) = 11200 \text{ cm} = 112 \text{ m}$

Hence, the length of the wire is 112 m.

4. Here, a = 20 cm.

Volume of cube = a^3 cubic units

$= (20 \times 20 \times 20)\mathrm{cm}^3 = 8000 \text{ cm}^3$

Total surface area of the cube $= (6a^2)$ sq units

$= (6 \times 20 \times 20)\mathrm{cm}^2 = 2400 \text{ cm}^2$

Lateral surface are of the cube $= (4a^2)$ sq units

$= (4 \times 20 \times 20)\mathrm{cm}^2 = 1600 \text{ cm}^2$

Diagonal of the cube $= (\sqrt{3}\,a)$ units

$= (\sqrt{3} \times 20)$ cm $= (1.732 \times 20)$ cm

= 34.64 cm.

5. Clearly, we have r = 4 m and h = 14 m.

Volume of the earth dug out of the well

$= (\pi r^2 h)$ cubic units $= \left(\frac{22}{7} \times 4 \times 4 \times 14\right)\mathrm{m}^3 = 704\mathrm{m}^3$

Area of the given plot $= (25 \times 16)\mathrm{m}^2 = 400\mathrm{m}^2$

Volume of the platform formed = volume of the earth dug out = 704 m^3

Height of the platform $=\left(\dfrac{\text{volume in m}^3}{\text{area in m}^2}\right)$

$$=\left(\frac{704}{400}\right)\text{m}=\frac{176}{100}=1.76\text{m}$$

Hence, the height of the platform so formed = 1.76 m

6. Total surface area of the pipe =
= (external curved surface area) + (internal curved surface area) + 2 (area of the base ring)

$$=\left[2\pi Rh+2\pi rh+2(\pi R^2-\pi r^2)\right]\text{sq units}$$

$$=2\pi(R+r)\left[h+(R-r)\right]\text{sq units}$$

$$=\left\{2\times\frac{22}{7}\times(12.5+11.5)(20+12.5-11.5)\right\}\text{cm}^2$$

$$=\left(2\times\frac{22}{7}\times 24\times 21\right)\text{cm}^2=3168\text{ cm}^2.$$

Hence, the total surface area of the pipe is 3168 cm^2.

7. Volume of the material of the sphere $=\dfrac{4}{3}\pi\left(4^3-2^3\right)\text{cm}^3=\left(\dfrac{224\pi}{3}\right)\text{cm}^3$

Volume of the cone $=\left(\dfrac{1}{3}\pi r^2h\right)\text{cm}^3$

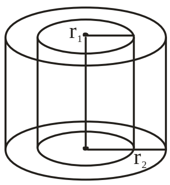
$$=\left(\frac{1}{3}\pi\times 4\times 4\times h\right)\text{cm}^3=\left(\frac{16\pi h}{3}\right)\text{cm}^3$$

$$\therefore\ \frac{16\pi h}{3}=\frac{224\pi}{3}\Leftrightarrow h=\left(\frac{224}{3}\times\frac{3}{16}\right)=14.$$

Hence, the height of cone is 14 cm.

8. $r_2=\dfrac{7}{2}\text{ mm}=\dfrac{7}{20}\text{cm}$

$r_1=\dfrac{1}{2}\text{ mm}=\dfrac{1}{20}\text{cm}$



Volume of graphite (V_1) = $\pi r_1^2 h$

$$=\pi\left(\frac{1}{20}\right)^2\times 10$$

Volume of wood $=\pi h(r_2^2-r_1^2)$

$$=\pi.10\left(\left(\frac{7}{20}\right)^2-\left(\frac{1}{20}\right)^2\right)$$

Weight of graphite $=\dfrac{\pi\times 10}{400}\times 2.1$

= 0.0525 πg

Weight of wood $=10\pi\left(\dfrac{49}{400}-\dfrac{1}{400}\right)\times 0.7$

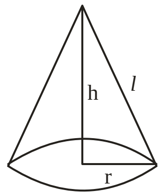
$= 7\pi\left(\frac{48}{400}\right)$

$= 0.84\ \pi$ g

Weight of the whole pencil = $(0.0525 + 0.84)\pi$ g

$= 0.8925\ \pi$ g

9. Volume = $100\ \pi\ cm^3$



h = 12 cm

$\frac{1}{3}\pi r^2 h = 100\pi$

$r^2 h = 300$

$r^2 = \frac{100}{4}$

$r = \frac{10}{2} = 5$ cm

$l = \sqrt{h^2 + r^2}$

$= \sqrt{144 + 25}$

= 13 cm

Curved surface area

$= \pi r l$

$= 204.3\ cm^2$

10. $\pi r l = 4070\ cm^2$

r = 35 cm

$\Rightarrow \quad l = \frac{4070}{\pi r} = 37$ cm