SPECIAL PRODUCTS AND EXPANSIONS

Class: IX, Mathematics
BOARD INTEGRATED IIT.

TEACHING TASK	SOLUTIONS

01. $(497)^2 = (500-3)^2$

$= 500^2 - 2\times500\times3 + 9$

$= 250000 - 3000 + 9$

$= 247009$

Ans: B

02 $0.768 \times 0.768 - 2\times0.768 \times 0.568 + 0.568 \times 0.568$

$= (0.768 - 0.568)^2$

$= 0.04$

Ans: A

03 $\dfrac{(5718)^2 - (4135)^2}{5718 + 4135} = \dfrac{(5718+4135)(5718-4135)}{(5718+4135)}$

$= 1583$

Ans: B

04 $(79.01)^2 + 2\times79.01\times20.99 + (20.99)^2$

$= (79.01 + 20.99)^2$

$= (100)^2 = 10000$

Ans C

05 $(687)^2 - (313)^2 = (687+313)(687-313)$

$= 1000 \times 374$

$= 374000$

Ans B

06 $H.C.F\{3a, 6ab^2\} = 3ab$

Ans: C

07 $L.C.M\{6a^2b, 9ab^2, 12abc\} = 36a^2b^2c$

Ans: C

08 $(7\times5-5)(5-5)(3\times7+5) = 0$

Ans: A

09 $a(b+c) = 32,\ b(c+a) = 65,\ c(a+b) = 77$

$ab+ac = 32 \rightarrow ①,\ bc+ba = 65 \rightarrow ②,\ ca+cb = 77 \rightarrow ③$

on adding $2(ab+bc+ca) = 174$

$\Rightarrow ab+bc+ca = 87 \rightarrow ④$

④ − ① ⇒ $bc = 55$

④ − ② ⇒ $ac = 22$

④ − ③ ⇒ $ab = 10$

Now $abc \times bc \times ca = 55\times22\times10$

$(abc)^2 = 55\times22\times10 = 12100$

$= (110)^2$

$\therefore abc = 110$

Ans: B

10 Let $x = a^2+b^2+c^2-ab-bc-ca$

$2x = 2a^2+2b^2+2c^2-2ab-2bc-2ca$

$= (a-b)^2+(b-c)^2+(c-a)^2$

$= (2012-2011)^2+(2011-2010)^2+(2010-2012)^2$

$= 1+1+4$

$= 6$

$\therefore x = 3$

Ans: C

11 $(x+4)^2+(x-2)^2$ (3)

$= x^2+8x+16+x^2-4x+4$

$= 2x^2+4x+20$

$= 2(x^2+2x+10)$

Ans: A, D

12 $x+\frac{1}{x}=2$

S.O.B.S

$x^2+\frac{1}{x^2}+2\cdot x\cdot\frac{1}{x}=4$

$\Rightarrow x^2+\frac{1}{x^2}=2$

$\Rightarrow x^4+\frac{1}{x^4}+2=4 \Rightarrow x^4+\frac{1}{x^4}=2$

Given $x+\frac{1}{x}=2$

By observation $x=1$.

Now $x^8-\frac{1}{x^8}=1-\frac{1}{1}=0$

$\Rightarrow x^{2014}+\frac{1}{x^{2014}}=1+\frac{1}{1}=2$

Also $x^{2014}-\frac{1}{x^{2014}}=1-\frac{1}{1}=0$

Ans: A, B, C, D

13 Statement I: ~~$(a+b)$~~ $(2x+5)(2x-5)=4x^2-25$

Statement II: Conceptual (True)

Ans: A

14 $x^3+y^3 = (x+y)^3 - 3xy(x+y)$ (4)

$= 8^3 - 3.12(8)$

$= 512 - 288$

$= 224$ (False)

Statement II: Conceptual (True)

Ans. D

15 $x^2 + \frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2.x.\frac{1}{x}$

$= (10)^2 - 2$

$= 100 - 2 = 98$

Ans: B

16 $x^4 + \frac{1}{x^4} = \left(x^2+\frac{1}{x^2}\right)^2 - 2.x^2.\frac{1}{x^2}$

$= (98)^2 - 2$

$= 9604 - 2$

$= 9602$

Ans: B

17 $(x^3+y^3)(x^3-y^3)$

$= x^3+y^3-x^3+y^3 = 2y^3$

$= 2x^3$

$= 16$

$x+y=8$

$x-y=4$

$2y=4$

$y=2$

Ans: 17

18 $a^2-b^2=15$

$\Rightarrow (a+b)(a-b)=15$

$\Rightarrow 3x(a-b)=15$

$\Rightarrow a-b=5$

$a+b=3$

$a-b=5$

$a=4 \mid b=-1$

Now $a^2+b^2 = 4^2+(-1)^2$

$=16+1=17$

Ans: 17

19 a) $(a+b)^3 = a^3+b^3+3ab(a+b)$

b) $(a-b)^3 = a^3-b^3-3ab$

(5)

19 a) $(a+b)^3 = a^3+b^3+3ab(a+b)$

b) $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$

c) $(a+b)^3+(a-b)^3 =$

$= a^3+3a^2b+3ab^2+b^3+a^3-3a^2b+3ab^2-b^3$

$= 2a^3+6ab^2$

d) $(a+b)^3-(a-b)^3$

$=(a^3+3a^2b+3ab^2+b^3)-(a^3-3a^2b+3ab^2-b^3)$

$=a^3+3a^2b+3ab^2+b^3-a^3+3a^2b-3ab^2+b^3$

$= 6a^2b+2b^3$

Ans: p, r, s, t

20 a) $a^2+4a = a^2+2\cdot a\cdot 2+(2)^2$ Ans: 4

$=(a+2)^2$

b) $y^2-8yz = y^2-2\cdot y\cdot 4z+(4z)^2$

$=(y-4z)^2$ Ans: 16z

c) $p^2+\_\_\_\_+4q^2 = p^2+2\cdot p\cdot 2q+(2q)^2$

$=(p+2q)^2$ Ans: 4pq

d) $(2l)^2+2\cdot 2l\cdot 3m+(3m)^2 = (2l+3m)^2$ Ans: 12lm

Ans: p, q, r, t

LEARNERS TASK

CUBES

01. $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)=(\sqrt{x})^2-\left(\frac{1}{\sqrt{x}}\right)^2 = x-\frac{1}{x}$

Ans: A

02 $4a^2+\frac{12a}{5}+\frac{9}{25}$

$=(2a)^2+2\times 2a\times\frac{3}{5}+\left(\frac{3}{5}\right)^2$

$=\left(2a+\frac{3}{5}\right)^2$

Ans: A

03 $\left(10+\frac{1}{2}\right)\left(10-\frac{1}{2}\right)$

Ans: B

04 $x^2+2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$

Ans: C

05 $9x^2+12xy+4y^2$

$=(3x)^2+2\times 3x\times 2y+(2y)^2=(3x+2y)^2$

Ans: A

06 $x^2+2\cdot x\cdot\frac{1}{x}+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2$

: 2 should be added

Ans: D

07 $\left(2x+\frac{1}{3y}\right)^2-\left(2x-\frac{1}{3y}\right)^2$

$=2\left((2x)^2+\left(\frac{1}{3y}\right)^2\right)$

$=2\left(4x^2+\frac{1}{9y^2}\right)$

Ans: A

10 $x+y+z=0 \Rightarrow x^3+y^3+z^3=3xyz$ Ans: C

(7)

JEE MAIN LEVEL

01 $x^2-16x+p = x^2-2\times x\times 8+8^2$ $\therefore p=64$

Ans: C

02 $(5x)^2+2\times 5x\times 9y+(9y)^2 = (5x+9y)^2$

And $(5x)^2$

$=25x^2$

Ans: C

03 $a^2+b^2=(a+b)^2-2ab$

$=6^2-2\times 3$

$=30$

Ans: B

04 $9x^2+18xy+$ ——

$=(3x)^2+2\cdot(3x)\cdot(3y)+(3y)^2=(3x+3y)^2$ Ans: C

i.e $9y^2$

05 $a^2+b^2=(a+b)^2-2ab$

$=9^2-2(-20)$

$=81+40$

$=121$

Ans: A

06 $a-b=6,\ ab=135$

$(a+b)^2=(a-b)^2+4ab$

$=(6)^2+4(135)$

$=36+540$

$=576$

$\therefore a+b=\sqrt{576}$

$a+b=24$

Ans: B

07 $(x+2y+4z)^2 = x^2+(2y)^2+4z$ (8)

$= x^2+(2y)^2+(4z)^2+2\cdot x(2y)+2\cdot(2y)(4z)+2\cdot x(4z)$

$= x^2+4y^2+16z^2+4xy+16yz+8xz$

Ans: A

08 $(-2x+5y-3z)^2$

$= (2x-5y+3z)^2$

$= (2x)^2+(5y)^2+(3z)^2-2(2x)(5y)-2(5y)(3z)+2(2x)(3z)$

$= 4x^2+25y^2+9z^2-60xy-30yz+12xz$

Ans: C

09 $(4pq+3q)^2-(4pq-3q)^2$

$= (a+b)^2-(a-b)^2 = 4ab$

$= 4\times 4pq\times 3q$

$= 48pq^2$

Ans: B

10 $x^2+3x+1=0$

$\Rightarrow x^2+1 = -3x$

$\Rightarrow x+\frac{1}{x} = -3$

$x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2-2\cdot x\frac{1}{x}$

$= (-3)^2-2$

$= 7$

Ans: B

11 $\left[\left(x+\frac{1}{x}\right)-\left(x-\frac{1}{x}\right)\right]^2 = \left[x+\frac{1}{x}-x+\frac{1}{x}\right]^2$

$= \left[\frac{2}{x}\right]^2$

$= \frac{4}{x^2} =$

Ans: C, D

12. $a^5 - a^3 = a^3(a^2-1)$

$= a^3(a+1)(a-1)$

(9)

Ans: C, D

13 Statement: I: $6x^2y = 2 \times 3 \times x^2y$

$14xy^2 = 2 \times 7 \times xy^2$

HCF $\{6x^2y, 14xy^2\} = 2xy$ (True)

Statement II: (Conceptual) true

Ans: A

14. $k^2 - 3k + 1 = 0$

$\Rightarrow k^2 + 1 = 3k$

$\Rightarrow k + \frac{1}{k} = 3$

$k^2 + \frac{1}{k^2} + 2 \cdot k \cdot \frac{1}{k} = 9$

$\Rightarrow k^2 + \frac{1}{k^2} = 7$

Ans: A

15 $a^2 - b^2 = (a+b)(a-b)$

$= 9 \times 3$

$= 27$

Ans: 27

16 a) $(3x-4y+z)(3x-4y-z)$

$= (3x-4y)^2 - z^2$

b) $(3x-4y+z)(3x+4y-z)$

$= [3x-(4y-z)][3x+(4y-z)]$

$= (3x)^2 - (4y-z)^2$

c) $(5a+3b+2c)(5a-3b-2c)$

$= [(5a)+(3b+2c)][(5a)-(3b+2c)]$

$= (5a)^2 - (3b+2c)^2$

d) $(2x+3y+5z)(2x+3y-5z)$

$= (2x+3y)^2 - (5z)^2$

$\Rightarrow$ THE END $\Leftarrow$

Ans: S, Q, R, P