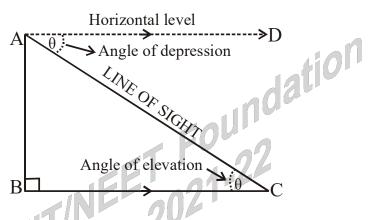
# APPLICATIONS OF TRIGONOMETRY <</p>

# **INTRODUCTION:**

Trigonometry is one of the most ancient subjects studied by scholars all over the world. The astronomers used trigonometry to calculate distance from the Earth to the planets and stars. Trigonometry is also used in geography to construct maps, determine the position of an island in relation to the longitudes and latitudes, etc.

# HEIGHTS AND DISTANCES :

Let A be the top of a tower and C be the eye of a person from where he is observing the top of a tower, then AC is called the line of sight. The angle  $\angle BCA$ , so formed by the line of sight with the horizontal level is called the angle of elevation of the top of tower from the eye of a person.



Hence, the line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer, i.e., the **angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level.

Let C be an object and A be the eye of a person from where he is observing the object C, then AC is called the line of sight. The angle  $\angle CAD$ , so formed by the line of sight with the horizontal level is called the angle of depression of the object fro the eye of a person.

Hence, the line of sight is the line drawn from the eye of angle observer to the point in the object viewed by the observer, i.e., the angle so formed by the line of sight with the horizontal level is called the **angle of depression**. The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal level when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed.

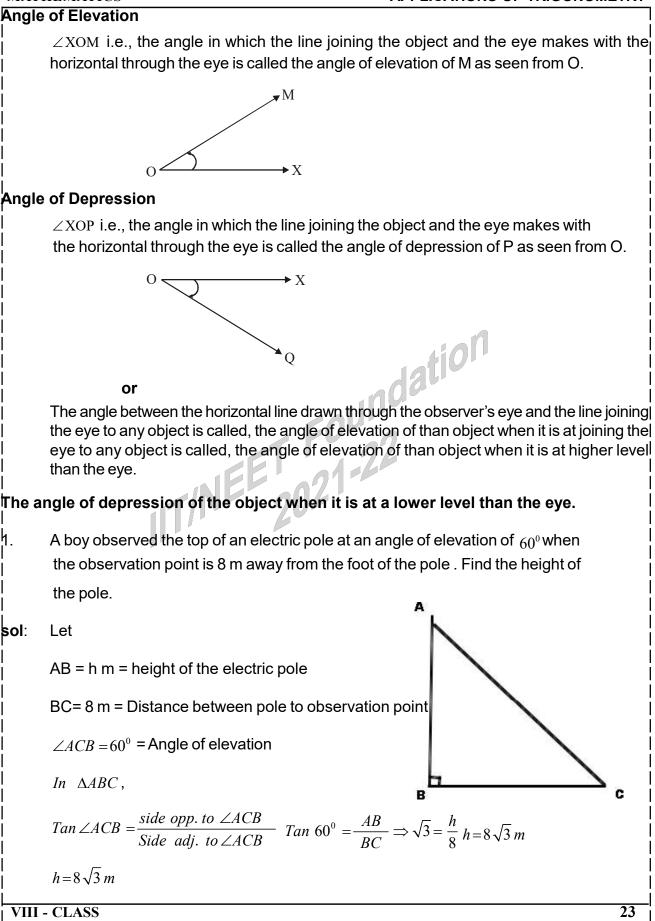
Now, in right  $\Delta ABC$ , we have

BC || AD and AC is a transversal.

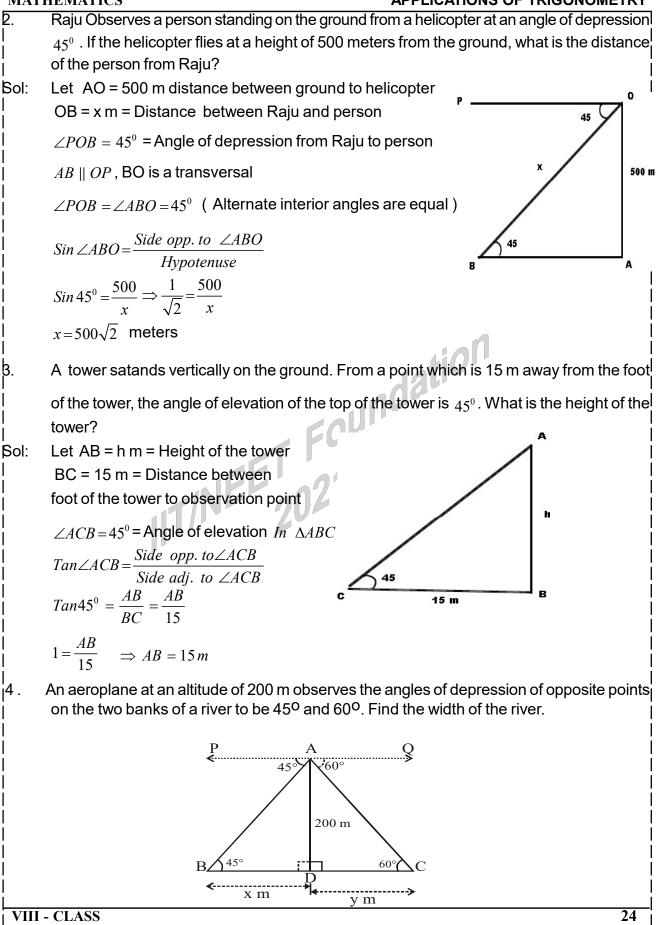
 $\therefore \angle ACB = \angle CAD \qquad [Alternate angles]$ 

 $\Rightarrow \angle \theta = \angle \theta$  Hence, Angle of elevation = Angle of depression.

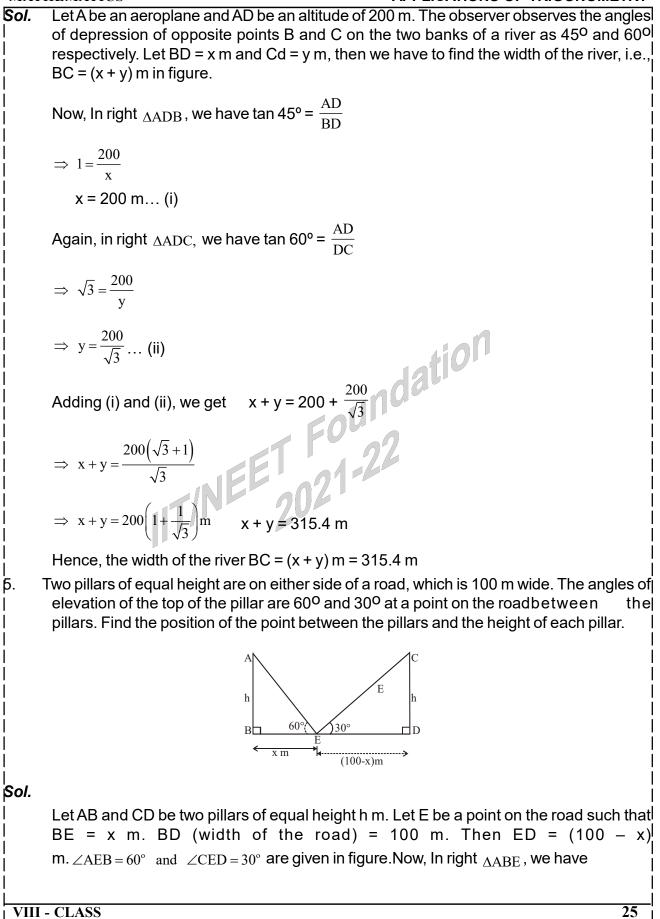




#### APPLICATIONS OF TRIGONOMETRY



## APPLICATIONS OF TRIGONOMETRY



$$\tan 60^{\circ} = \frac{AB}{BE}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \dots (i)$$
Again, in right DCDE, we have
$$\tan 30^{\circ} = \frac{CD}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100 - x}$$

$$\Rightarrow 100 - x = \sqrt{3} h \dots (ii)$$
Substituting the value of x from equation (i) in equation (ii), we get
$$100 - x = \sqrt{3} h$$

$$\Rightarrow 100 - \frac{h}{\sqrt{3}} = \sqrt{3} h \Rightarrow \frac{100\sqrt{3} - h}{\sqrt{3}} = \sqrt{3} h$$

$$\Rightarrow 100\sqrt{3} - h = 3h \Rightarrow 100\sqrt{3} = 4h$$

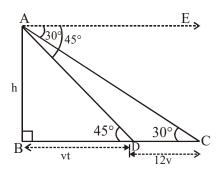
$$\Rightarrow h = 25\sqrt{3}m \qquad \dots (iii)$$

$$\Rightarrow h = 25 \times 1.732m$$

$$h = 43.3 m$$
Again substituting the value of h in equation (i) from (iii), we get
$$x = \frac{h}{\sqrt{3}} \Rightarrow x = \frac{25\sqrt{3}}{\sqrt{3}}m = 25m$$

Hence, the position of the point E from the first pillar is 25 m and 75 m from the second pillar. The height of the pillars = hm = 43.3 m

A man on the top of a vertical tower observes a car having at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to charge from 30<sup>o</sup> to 45<sup>o</sup>, how soon after this, will the car reach the tower? Give your answer to the nearest second.



6.

\_

# Solution:

Let AB be the tower of height h m. Let C be the initial position of the car and after 12 minutes the car be at D. The angles of depression at C and D are 30<sup>o</sup> and 45<sup>o</sup> respectively. Let the speed of the car be v m/s in figure.

 $\therefore$  Distance traveled by the car in 12 minutes = 12 v m

[ $\cdot$  distance = speed × time]

Let the car take t minutes to reach the tower AB from D. Then DB = vt m.Now, in right  $\Lambda ABD$ , we have

$$\tan 45^\circ = \frac{AB}{BD} \Rightarrow 1 = \frac{h}{vt} \Rightarrow h = vt$$
 ... (i)

Again, in right  $\Lambda$  ABC, we have

$$\tan 30^{\circ} = \frac{AB}{BC} \Longrightarrow \frac{1}{\sqrt{3}} = \frac{h}{vt + 12v}$$
$$\Longrightarrow \sqrt{3} h = vt + 12v \qquad \dots ($$

... (ii)

Substituting the value of h from equation (i) in equation (ii), we get undal"

$$\sqrt{3} h = vt + 12v$$
  

$$\Rightarrow \sqrt{3} \times vt = vt + 12v \qquad \Rightarrow \sqrt{3} t = t + 12$$
  

$$\Rightarrow \sqrt{3} t - t = 12 \qquad \Rightarrow t(\sqrt{3} - 1) = 12$$

$$\Rightarrow t = \frac{12}{\sqrt{3}-1} = \frac{12}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{12(\sqrt{3}+1)}{3-1} = 6(\sqrt{3}+1)$$

$$\Rightarrow t = 691.732 + 1 \qquad \Rightarrow t = 6(2.732)$$

 $\Rightarrow$  t = 16.39 minutes

 $\therefore$  t = 16 minutes 23 seconds.  $[:: 0.39 \times 60 = 23]$ 

Hence, the car will reach the tower from D in 16 minutes and 23 seconds

**TEACHING TASK** 

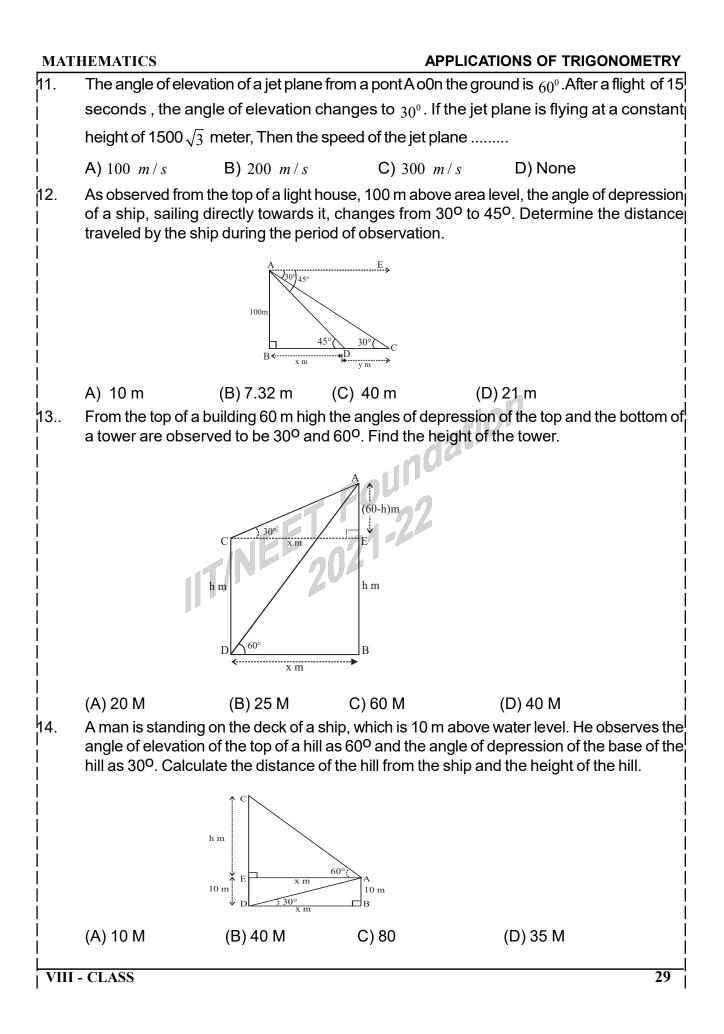
# I. MCQ's with single correct answers

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground by making  $30^{\circ}$  angle with the ground. The distance between the foot of the tree and the top of the tree on the ground is 6m. Then the height of the tree before falling down is .....

C)  $3\sqrt{3} m$ . A)  $6\sqrt{3} m$ B)  $6\sqrt{3} c.m.$ D) $3\sqrt{6}$  m.

You want to erect a pole of height 10 m with the support of three ropes. Each rope has to make an angle  $30^{\circ}$  with the pole. What should be the length of the rope

**MATHEMATICS** APPLICATIONS OF TRIGONOMETRY A) 34.64 m B) 34.46 m C) 31.34 m D) None An electrician wants to repair an electric connection on a pole of height 9 m. He needs to β. reach 1.8 m below the top of the pole to do repair work. What should be the length of the ladder which he should use, when he climbs it at an angle of  $60^{\circ}$  with the ground? A) 5.06 ft B)4.1568 ft C) 4.0068 ft D) 51.96 ft 4. A boat has to cross a river, it crosses the river by making an angle of  $60^{\circ}$  with the bank of the river due to the stream of the river and travels a distance of 600 m to reach the another side of the river. What is the width of the river B)  $200\sqrt{3}$  m. C)  $600\sqrt{3}$  m. A) $100\sqrt{3}$  m. D)  $300\sqrt{3}$  m. AT.V tower stands vertically on the side of a road. From a point on the other side Б. directly opposite to the tower, the angle of elevation of the top of tower is  $60^{\circ}$ . From another point 10 m away from this point, on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is  $30^{\circ}$ , then the height of the tower and the width of the road A)  $5\sqrt{3} m$ . 5m B)  $\sqrt{3} m$ . 5m C)  $5\sqrt{3} m$ .  $5\sqrt{3} m$  D)  $\sqrt{3} m$ .  $\sqrt{3} m$ A 1.5 m tall boy is looking at the top of a temple which is 30 meter in height from a 6. point at certain distance. The angle of elevation from his eye to the top of the crown o the temple increases from  $30^{\circ}$  to  $60^{\circ}$  as he walks towards the temple. Then the distance he walked towards the temple..... C)32.908 m A) 39.208 m B) 31.908 m D) None From the top a building, the angle of elevation of the top of a cell tower is  $60^{\circ}$  and the angle of depression to its foot is 45°. If distance of the building from the tower is 7 m. Then the height of the tower is ..... A) 19.124 m B) 19.114 m C) 18.124 m D) 18.114 m 8. The angle of elevation of the top of a building from the foot of the tower is  $30^{\circ}$  and the angle of elevation of the top of the tower from the foot of the building is  $60^{\circ}$ . If the tower is 30 m high, then the height of the building is..... A) 10 m B) 5 m C) 15 m D) 20 m Two poles of equal heights are standing opposite to each other on either side of the road 9 , which is 120 feet wide. From a point between them on the road, the angles of elevation of the top of the poles are  $60^{\circ}$  and  $30^{\circ}$  respectively. Then the height of the poles and the distances of the point from the poles is ..... A) 50ft, 30ft B)51.96 ft, 30 ft C) 52 ft, 30 ft D) 30 ft, 30 ft The angles of elevation of the top of a tower from two points at a distances of 4 m 110. and 9 m, then the height of the tower from the base of the tower and in the same straght line with it are complementary..... A)2m B) 5 m C) 6 m D) None



	THEMATICS		APPLICATIONS OF TRIGONOMETRY						
15.	The angle of elevation of the top of a tower from the top and bottom of a building of height 'a' are 30° and 45° respectively. If the tower and the building stand at the same level, the height of the tower is								
	(A) <sub>a√3</sub>	<b>(B)</b> $a(\sqrt{3}-1)$	(C) $a \frac{(3+\sqrt{3})}{2}$	(D) $a(\sqrt{3}+1)$					
16.	from the foot in	•	line and on the sa	hich are at distances a and l me sides of the tower, are					
	(A) ab	(B) √ <u>ab</u>	(C) $\sqrt{a/b}$	D) $\sqrt{b/a}$					
17.	From the top of h metres high cliff, the angles of depression of the top and bottom of a tower are observed to be $30^{\circ}$ and $60^{\circ}$ respectively. The heigth of the tower is-								
	(A) $\frac{2h}{3}$	(B) $\frac{h}{3}$	(C) $\frac{2h}{\sqrt{3}}$	( D) $h\sqrt{3}$					
18.	The angle of ele 30 <sup>0</sup> . Then heigh		ower at a distance c	of 500 metres from its foot i					
	(A) $\frac{500\sqrt{3}}{3}$ m	(B) $\frac{500(\sqrt{3}-1)}{3}$ m	(C) $\frac{500(\sqrt{3}+1)}{3}$ m	(D) 500 m					
19.	tower. At a seco	s an angle of 30° at a p nd point, h metre abov horizontal distance of	e first, the depressio	on of the foot of the					
	(A) h cos 60°	(B) (h/3) cot 30°	(C) (h/3) cot 60°	(D) h cot 30°					
20.	The tops of two poles of height 20 m and 14 m are connected by a wire. If the wire make an angle 30° with the horizontal, then the length of the wire is								
	(A) 12 m	(B) 10 m	(-)	(D) 14 m					
		MULTIPLE COR							
1.	•	evation of the top of a r 30° and 45°. What is th	•	d foot of a 100m. High towe ?					
	(A) $\frac{\sqrt{3}-1}{100\sqrt{3}}$ m	(B) $\frac{100\sqrt{3}}{\sqrt{3}-1}$ m	(C) 246.5 m	(D) 236.5 m					
2. A hedgehog wishes to cross a road without being runover. He observe elevation of a lamp post on the other side of the road to be 45° from the ed and 30° from a point 10m back from the road. how wide is the road?									
	(A) 13.90 m	(B) $\frac{10}{\sqrt{3}}$ m	(C) $\frac{10}{\sqrt{3}-1}$ m	(D) 13.66 m					
ЛАТ	RIX MATCH T	YPE							
l.	elevation and de	-	bottom of another h	use in a street, the angles on nouse on the opposite side of tly					

# **APPLICATIONS OF TRIGONOMETRY**

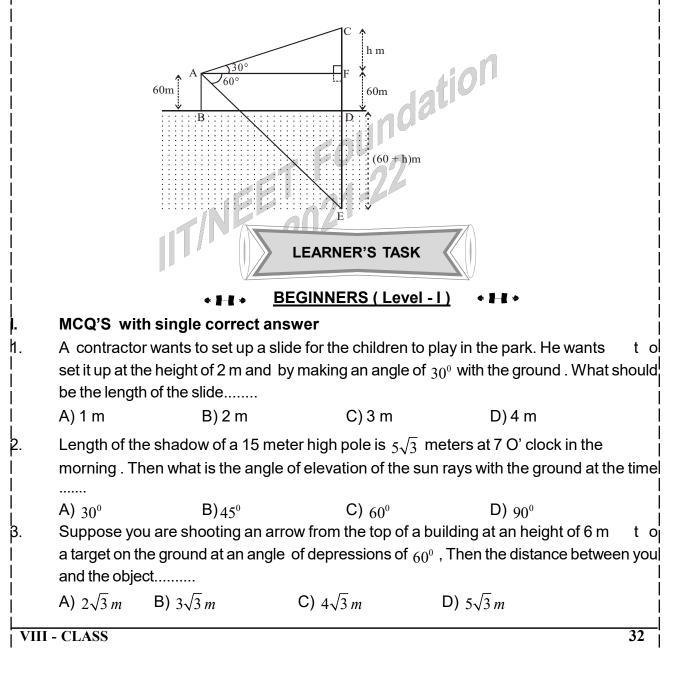
MATHEMA						APP	LICATIONS OF TRIGONOMETRY			
Colu	ımn l		Column II							
(A) D	B	[	]	(p)	h(1-	$-\tan\alpha\cos$	$\operatorname{ot}\beta)$			
(B) C	E	[	]	(q)	h sir	ιβ				
(C) (	CE	[	]	(r)	h tar	n cotβ				
(D) A	٨D	[	]	(s)	h c	ot ß				
away	A ladder (length = <i>l</i> ) rests against a wall at an angle $\alpha$ to the horizontal. Its foot is pulled away from the wall through a distance d, so that it slides a distance h down the wall making an angle $\beta$ with the horizontal, then match the column									
	Column I			Column II						
	(A) d			[	]	(p)	$l(\sin \alpha - \sin \beta)$			
	(B) h			[	]	(q)	$l(\cos\alpha - \cos\beta)$			
	(C) d/h			[	]	(r)	$\frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$			
	(D) dh/	j2		[	]	(s) s	$in(\alpha + \beta) - cos(\alpha - \beta)$			
NTEGER	ТҮРЕ					nu				
bridg 15m. 2. A sta eleva the to 3. The a	A bridge across the river makes an angle of 30° with the river bank. The length of the bridge across the river is 60m. If width of the river is k meter, then what is the value of k 15m. A statue 1.46 m tall, stands on top of a pedestal. From a point on the ground the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. What is the height of the pedestal? The angle of depression of point C, when observed from point A is 45°. If BC = $1 \text{m} \text{ determine } \frac{\text{AB}}{\text{Im}}$ .									
. The t	ops of two p	oles of ł	neight	16m and	10m a	are conr	nected by a wire. If the wire			
make	makes an angle of 30° with the horizontal and x is the length of wire then find $rac{x}{4m}$									
5. In the	In the given figure QR = 20cm, PS = 13 cm, $\angle PQR = 45^{\circ}$ , then value of $\frac{SR}{7cm}$									
$ \begin{array}{c}                                     $										
VIII - CLAS	88						31			

# SOLVE THE FOLLOWING

- A tower subtends an angle of 30<sup>0</sup> at a point on the same level as the foot of the tower. At a second point h metres above the first, the depression of the foot of the tower is 60<sup>0</sup>.
   Find The horizontal distance of the tower from the point.
- 2. A man is standing on the deck of a ship, which is 8m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.
- β. An aeroplane flying horizontally at a height of 1.5 km above the ground is observed
   at a certain point on earth to subtend an angle of 60°. After 15 seconds, its angle of

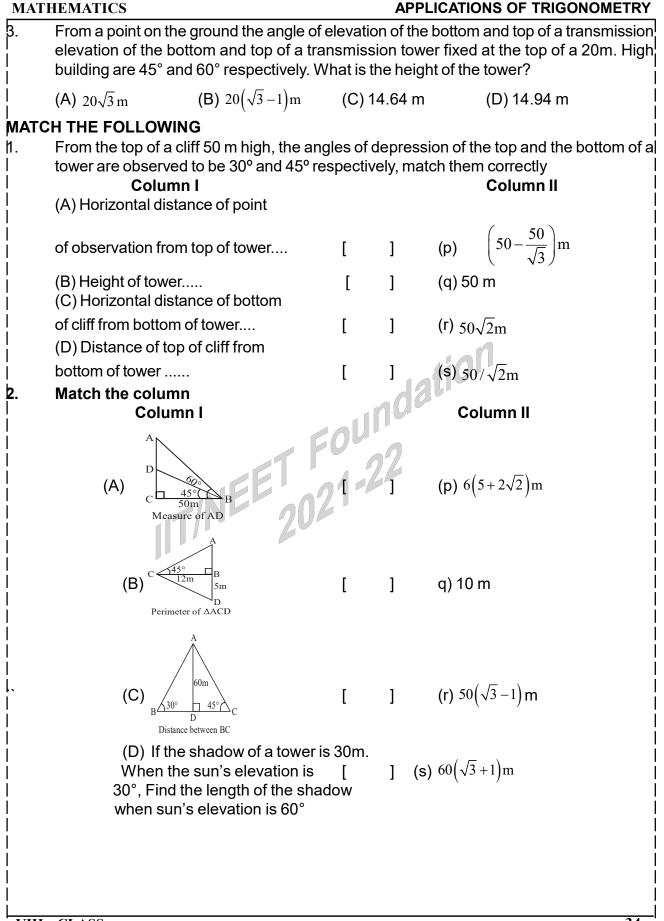
elevation is observed to be 30°. Calculate the speed of the aeroplane in km/h  $\sqrt{3} = 1.732$ .

4. The angle of elevation of a cloud from a point 60 m above a lake is 30<sup>o</sup> and the angle of depression of the reflection of cloud in the lake is 60<sup>o</sup>. Find the height of the cloud.



# APPLICATIONS OF TRIGONOMETRY

MA	HEMATICS		AFFLICAI	IONS OF TRIGONOMETRI						
4.	An Observer of height 1.8 m is 13.2 m away from a palm tree. The angle of elevat									
	the top of the tree from his eyes is $45^{\circ}$ Then the height of the palm tree is									
	, , ,									
5.	Two men on eith	Two men on either side of a temple of 30 meter height observe its top at the angles of								
	elevation $_{30^{\circ}}$ and $_{60^{\circ}}$ respectively . Then the distance between the two men is									
	A) $10\sqrt{3} m$	B) $40\sqrt{3} m$	C) $60\sqrt{3} m$	$D$ )80 $\sqrt{3}$ m						
<b>)</b> .		ay leads to the foot of a	tower. Ramaiah stand	dding at the top of the towe						
				paching the foot of the towe						
	with a uniform sp	beed. Six seconds later	, the angle of depress	sion of the car is found to be						
	-	e taken by the car to re								
	A) 6 Seconds	B) 3 second		•						
-	A statue stands of	on the top of a 2 m tall p	edestal . From a point	t on the ground, the angle o						
	elevation of the t	op of the statue is $60^{\circ}$ is	and from the same po	pint, the angle of elevation						
	of the top of the	pedestal is $45^{\circ}$ , then th	e height of the statue	is						
	A) 1.464 M	B) 1.646 M	C) 14.64 M	D)16.46 M						
	A wire of length 18 m had been tied with electric pole at an angle of elevation $30^{\circ}$ with the									
	ground .Because it was covering a long distance, it was ut and tied at an angle of elevation									
	$_{60^{\circ}}$ with the ground . then the length of the wire was cut									
	A) 6. 708 m	B) 8.706 m	C) 7.608 m	D) None						
	The angle of elevation of the top of a tower from the foot of the building is $30^{\circ}$ and the									
	angle of elevation of the top of the building from the foot of the tower is $60^{\circ}$ , then the ratio									
	of heights of tower and building is									
	A) 1 : 2	B)2:1	C) 1 :3	D) 3 :1						
		◆ <b>I</b> II → <u>ACHIEVE</u>	<u>RS ( Level - II )</u> •	}-1 >						
	N'S WITH MIII	TIPLE CORRECT	ANSWERS							
				° with the ground. How higl						
-	up the wall will th	-								
	-	-								
	(A) $\frac{15\sqrt{3}}{2}$ m	<b>(B)</b> 8√3 m	(C) 12.99 m	(D) 14.38 m						
	A man in a boat rowing away from light house 100 m. He takes 2 minute to									
	change the angle of elevation of the top of light house from 60° to 45°. What is the									
	speed of the bo	at?								
	$(\Lambda)^{-5} (3 \sqrt{3}) m/$	s (B) $\frac{18}{5} (\sqrt{3} - 1) \text{m/s}$	(C) 2.264 m/br	(D) 1.269  km/b						
	$(A) \frac{1}{18} (3 - \sqrt{3}) m/$	$(D) - \frac{1}{5} (\sqrt{5} - 1) \frac{1}{5} \sqrt{5}$	(0) 2.304 11/111.	(U) 1.200 KIII/II						



# COMPREHENSION TYPE

## The following question is based on the paragraph given below

- A guard observes an enemy boat, from an observation tower at a height of 200m abovel sea level to be an angle of depression of 30°.
  - (i) What is the distance of the boat from the foot of the observation tower.
  - (ii) If the boat is 200m from the observation tower, then what is the angle of depression.

# SOLVE THE FOLLOWING

- A vertical to stands on a horizontal plane and is surrounded by a vertical flag staff of height 6 meters. At point on the plane, the angle of elevation of the bottom and the top of the flag staff are respectively 30° and 60°. Find the height of tower.
- 2. The angles of depressions of the top and bottom of 8 m tall building from the top of a multistoried building are 30° and 45° respectively. Find the height of multistoried building and the distance between the two buildings.
- β. The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight of
   15 sec, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 metres,
   find the speed of the aeroplane.
- From the top of a light house 60 m high with its base at the sea level the angle of depression of a boat is 15°. Then find the distance of the boat from the foot of light house.
- At a distance 2h meter from the foot of a tower of height h metre the top of the tower and
   a pole at the top of the tower subtend equal angles. Find the Height of the pole
  - A house subtends a right angle at the window of an opposite house and the angle of elevation of the window from the bottom of the first house is 60°. If the distance between the two houses be 6 m, then find the height of the first house.
  - The base of a cliff is circular. From the extremities of a diameter of the base of angles of elevation of the top of the cliff are 30° and 60°. If the height of the cliff be 500 m, then find the diameter of the base of the cliff .
- 8. A boat is being rowed away from a cliff 150 m high. At the top of the cliff the angle of depression of boat changes from 60° to 45° in 2 min. Then find the speed of the boad (in m/h)
- An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°l
   and after 10s the elevation is observed to be 30°. Then find uniform speed of the
   aeroplane(in km/h)

6

APPLICATIONS OF TRIGONOMETRY

**KEY** 

ΦΦ <u>TEACHING TASK</u> : MCQ'S with single correct answer 1. A 2. A3. B 4. D 5. A 6. C 7. A 8. A 9. B 10. C 11.B 12. B 13. D 14. B 15. C 16. B 17. A 18. A 19. B 20. D 21. B 22. B 23. A 24. D Multi correct answers: 1) B, D 2) B , D Matching : 1) s, r, p, q 2) q, p, r, s Integer type: 1)22)2 3)1 4)3 5)1  $\Phi\Phi$  LEARNER'STASK : BEGINNERS : MCQ'S with single correct answer 2.A3.C 4.C 5.B 6.B 7.A 8.C 9.C 1. D ACHIEVERS : 2, A, D 2)r, p, s, q 1 (i) 346.4 (ii) 45<sup>0</sup> 2) A , D 3) B, C Multi correct answers: 1) A, C Matching : Comprehension : 1 VIII - CLASS