LOGARITHMS

Logarithms and exponents are closely related to each other. Just as repeated addition of the same ***summand*** evolves a new operation, namely, ***multiplicaton***; the repeated multiplication by the same factor evolves a new operation called ***exponentiation***. The inverse of these operations gives us two distinct inverse operations, i.e, ***extracting roots*** and ***taking logarithms***.

Pre-assessment Questions:

1) What is the 9th power of 2?
2) If $\sqrt{4^x} = 256,$ then find the value of x?
3) If $2^{(x+1)} + 2^{(x-1)} = 10,$ then find the value of x?
4) What is the cuberoot of 9th power of 3?
5) If $a = (2a)^x$, $(2a) = (3a)^y$, $(3a) = (4a)^z$, then find the value of 1+xyz?
6) If $a^x = bc,\ b^y = ca,\ c^z = ab$; then show that $xyz = x + y + z + 2$.

Introduction:

John Napier (1550-1617), a scottish mathematician, invented the logarithms in 1614 with a specific purpose of reducing the amount of work involved in multiplying and dividing large numbers. The word 'logarithm' was coined from two Greek words, logs which means a 'ratio' and arithmos, meaning 'number'. Henry Briggs (1556-1630), a contemporary of Napier, introduced the common (decimal) logarithms. He published 14 figure logarithmic table of numbers from 1 to 2×10^4 *and from* 9×10^4 *to* 10^5, in 1624. The logarithms of missing numbers were calculated by the surveyors Ezechiel De Decker and Adrian Vlacq in 1627. Nowadays, even with the advent of computers and calculators, logarithm have not lost its significance; it is still considered as an important working tool in mathematics.

§§ <u>**Definition:**</u> Let a,b be two positive real numbers and $a \neq 1$. The real number 'x' such that $a^x = b$ is called logarithm of b to the base a. It is denoted by $\log_a b$.

Ex: If $2^3 = 8,$ then $\log_2 8 = 3$

§§ <u>**Properties:**</u>

1) If 'a' is a positive real number and $a \neq 1,$ then $\log_a a = 1$

$a^1 = a \Rightarrow \log_a a = 1$

Ex: $\log_2 2 = 1$

2) If 'a' is a positive real number and $a \neq 1,$ then $\log_a 1 = 0$

$a^0 = 1 \Rightarrow \log_a 1 = 0$

Ex: $\log_2 1 = 0$

3) If a,m are positive real numbers and $a \neq 1$, then $a^{\log_a m} = m$.

$\log_a m = x(say)$

$\Rightarrow m = a^x$

$a^{\log_a^m} = a^x = m$

Ex:- $2^{\log_2 5} = 5$

4) If a,m,n are positive real numbers and $a \neq 1$, then $\log_a(mn) = \log_a m + \log_a n$

$\log_a m = x$ $\log_a n = y$

$\Rightarrow a^x = m$ $a^y = m$

$mn = a^x . a^y = a^{(x+y)}$

$\log_a(mn) = x + y = \log_a m + \log_a n$

Ex: $\log_{10} 2 + \log_{10} 5 = \log_{10}(2 \times 5) = \log_{10} 10 = 1.$

5) If a,m,n are positive real numbers and $a \neq 1$, then $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

$\log_a m = x$ $\log_a n = y$

$\Rightarrow a^x = m$ $\Rightarrow a^y = n$

$\left(\frac{m}{n}\right) = \frac{a^x}{a^y} = a^{(x-y)}$

$\log_a\left(\frac{m}{n}\right) = x - y = \log_a m - \log_a n$

Ex:- $\log_5 15 - \log_5 3 = \log_5\left(\frac{15}{3}\right) = \log_5 5 = 1$

6) If a,m are positive real numbers, $a \neq 1$, n is a real number, then $\log_a(m^n) = n \log_a m$

$\log_a m = x$

$\Rightarrow a^x = m$

$m^n = a^{nx}$

$\log_a(m^n) = nx = n \log_a m$

Ex:- $\log_2 16 = \log_2^{2^4} = 4\log_2^2 = 4.$

7) If a,m are positive real numbers, $a \neq 1,$ n is a real number, then $\log_{a^n} m = \frac{1}{n}\log_a m.$

$\log_a m = x$

$\Rightarrow m = a^x$

$m = a^{\frac{nx}{n}} = \left(a^n\right)^{\frac{x}{n}}$

$\Rightarrow \log_{a^n} m = \frac{x}{n} = \frac{1}{n}\left(\log_a m\right)$

Ex:- $\log_{3^2} 5 = \frac{1}{2}\log_3 5$

Note:- If a,m are positive real numbers, $a \neq 1,$ p,q are real numbers, then $\log_{a^q} m^p = \frac{p}{q}\log_a m.$

Ex: $\log_{2^4} 5^3 = \frac{3}{4}\log_2{}^5$

8) If a,b,m are positive real numbers, $a \neq 1,$ $b \neq 1,$ then $\log_a m = \log_b m \log_a b$

$\log_b m = x \Rightarrow b^x = m$

$\log_a b = y \Rightarrow a^y = b$

$\therefore m = b^x = a^{xy}$

$\Rightarrow \log_a m = xy = \log_b m.\log_a b$

Ex: $\log_5 9 \log_3 5 = \log_3 9 = \log_3\left(3^2\right) = 2\log_3 3 = 2$

NOTE: (1) In $\log_a m = \log_b m \log_a b,$ put $m = a,$ then

$\log_a a = \log_b a \log_a b$

$\Rightarrow 1 = \log_b a \log_a b$

$\Rightarrow \log_b a = \frac{1}{\log_a b}$

Ex: $\log_3 2 = \frac{1}{\log_2 3}$

(2) $\log_a m = \log_b m \log_a b = \frac{\log_b m}{\log_b a}$

$Ex: \frac{\log_{10} 4}{\log_{10} 2} = \log_2 4 = \log_2 \left(2^2\right) = 2\log_2 2 = 2$

9) If $\log_a x = \log_a y \Leftrightarrow x = y \left[\because f(x) = \log_a x \text{ is one-one}\right]$

10) (i) $\log_a x > 0$ iff (x>1,a>1) or (0<x<1, 0<a<1)

(ii) $\log_a x < 0$ iff (x>1, o<a<1) or (0<x<1, a>1)

11) If a>1, then (i) $\log_a x > \log_a y \Leftrightarrow x > y$

$(ii)\ \log_a x < \log_a y \Leftrightarrow x < y$

12) If 0<a<1, then (i) x>y $\Leftrightarrow \log_a x < \log_a y$

$(ii)\ x < y \Leftrightarrow \log_a x > \log_a y$

13) If a>1, then (i) $\log_a x > p \Rightarrow x > a^p$

$(ii)\ 0 < \log_a x < p \Rightarrow 1 < x < a^p$

14) If $0 < a < 1$, then (i) $\log_a x > p \Rightarrow 0 < x < a^p$

(ii) $0 < \log_a x < p \Rightarrow a^p < x < 1$

¶¶ **Logarithmic function :** The mapping $f : R^+ \to R$ defined by $f(x) = \log_a x$, $\forall x \in R^+$, a > 0 and $a \neq 1$ is called logarithmic function.

Note: For $a > 0, a \neq 1$

(i) $\log_a x$ is real if *x* > 0,

(ii) $\log_a x$ is imaginary if *x* < 0

(iii) $\log_a x$ is undefined if *x* = 0

¶¶ **Natural Logarithm:-** The Logarithms which are calculated to the base e(an irrational number which is approximately 2.7182818285........) are called ***Natural Logarithms*** or ***Naperian Logarithms***. $log_e n$ is denoted by *l*n n.

¶¶ **Common Logarithm:-** The Logarithms which are calculated to the base 10 are called ***Common Logarithms*** or ***Briggs Logarithms***. $\log_{10} n$ is simply denoted by $\log n$.

§§ **Characteristic and Mantissa:-** If n is a positive real number, then the integer K such that $K \leq \log n < (K+1)$ is called integral part or charateristic of logn. The non-negative real number (logn)-k is called mantissa of logn.

NOTE: The mantissa of logarithm will be taken from logarithm.

¶¶ Method to find the characteristc of a number:

(1) If n(>1) is a number, whose integral part contains K digits, then the characteristic of log n is (K-1).

Ex: (i) Characteristic of log 729 =2

(ii) Characteristic of log 24537 = 4

(2) If 0<n<1 is a number containing K zeros between the decimal point and first non-zero figure (significant figure), then the charecteristic of logn is $\overline{K+1}$ i.e. -K-1.

Ex: (i) Characteristic of log 0.0526 is $\overline{2}$.

(ii) Characteristic of log 0.0002 is $\overline{4}$.

- If m,n are two positive numbers such that $m = n \times 10^p$ where $p \in Z,$ then log m and log n have the same mantissa.

  Ex:- $\log_{10} 12.23 = 1.0838$

  Here characteristic is 1 and Mantissa is 0.0838

- If the characteristic 'a' of $\log_{10} x$ is positive, then 'x' is a number whose integral part is of (a+1) digits.

  Ex:- If $\log_{10} x = 4.25362,$ then the number of digits in 'x' is (4+1)=5.

- If the characteristic of 'a' of $\log_{10} x$ is negative, say-b, then 'x' is a proper fraction, in the decimal form of which there will be (b-1) zeros immediately after the decimal point before a significant digit appears.

  Ex: If $\log_{10} x = -3.4352$

  $= -3 - 0.4352$

  =-4+1-0.4352

  =-4+0.5648, then 'x' is a proper fraction and the number of zeros between the decimal point and the first significant digit is 4-1=3.

- Mantissa is never negative and lies between 0 & 1. (including 0).

§§ <u>Using logarithm Tables</u>

Let us find the logarithm of 523.7. The number 523.7 has 3 digits in the integral part and hence the characteristic of log 523.7 is 2. From the logarithm tables we refer to the row headed 52 under the column headed 3 and obtain 0.7185. The mean difference for 7 in the same row is 0.0006. The total mantissa is 0.7185+0.0006=0.7191. Thus log523.7=2.7191.

Let us find the logarithm of 0.007253. The number contains 2 zeros between the decimal point and first significant figure and hence characteristic of

log 0.007253 is $\overline{3}$. From the logarithm table we refer to the row headed by 72 under the column headed 5 and obtain 0.8603. The mean difference for 3 in the same row is 0.0002. The total mantissa is $0.8603 + 0.0002 = 0.8605$.

Thus log 0.007253= $\bar{3}.8605$.

§§ Antilogarithm Tables

¶¶ Method of using Antilogarithm Tables

Let us find the antilogarithm of 4.6235. i.e. Antilog 4.6235.

Step 1- The mantissa of the logarithm is 0.6235.

Step-2- From the antilogarithm tables we refer to the row headed by 0.62 under the column headed by 3 and obtain 4198.

Step-3: The mean difference for 5 in the same row (above row) is 5. The total value is 4198+5=4203.

Step-4: Since the characteristic is 4, the no.of digits in the integral part must be 5.

Thus , the number is 42030.

¶¶ Characteristic of $\log_a x$, $a > 0, a \neq 1$

Let $\log_a x = k + f$, where $k \in Z$ and $0 \le f < 1$. Then k is called the characteristic of $\log_a x$ and f is called the mantissa of $\log_a x$.

Ex:- Find the characteristic of $\log_4 71$.

Sol: We have $4^1 = 4,\ 4^2 = 16,\ 4^3 = 64,\ 4^4 = 256$

$\because 4^3 < 71 < 4^4$

$\Rightarrow \log_4 4^3 < \log_4 71 < \log_4 4^4$

$\Rightarrow 3.\log_4 4 < \log_4 71 < 4.\log_4 4$

$\Rightarrow 3 < \log_4 71 < 4$

$\Rightarrow \log_4 71 = 3 + f,\ 0 < f < 1$

$\therefore$ Characteristic of $\log_4 71 = 3$

EXAMPLE

1. Find the value of (i) $\log_{3\sqrt{2}} 324$ (ii) $\log_{3\sqrt{2}} 324$ (iii) $\log_{\sqrt{n}} \sqrt{n\sqrt{n\sqrt{n\sqrt{n}}}}$

Sol: (i) $\log_{3\sqrt{2}} 324 = \log_{3\sqrt{2}} 3^4.2^2 = \log_{(3\sqrt{2})} \left(3\sqrt{2}\right)^4 = 4\log_{3\sqrt{2}} 3\sqrt{2} = 4$

(ii) $\log_{0.11} 0.001331 = \log_{(0.11)} (0.11)^3 = 3\log_{(0.11)} (0.11) = 3$

(iii) $\log_{\sqrt{n}} \sqrt{n\sqrt{n\sqrt{n\sqrt{n}}}} = \log_{\sqrt{n}} n^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}} = 2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}\right)\log_n n$

$$= 2\left(\frac{8+4+2+1}{16}\right) = \frac{15}{8}.$$

2. Find the value of $\log_a x + \log_{a^2} x^2 + \log_{a^3} x^3 + ... + \log_{a^n} x^n$

Sol: $\log_a x + \log_{a^2} x^2 + \log_{a^3} x^3 + ... + \log_{a^n} x^n$

$$= \log_a x + \frac{2}{2}\log_a x + \frac{3}{3}\log_a x + ... + \frac{n}{n}\log_a x$$

$$= \log_a x + \log_a x + ...n\ times = n(\log_a x)$$

3. Show that $\sum_{n=2}^{127} \log_2\left(1+\frac{1}{n}\right) = 6$

Sol: $\sum_{n=2}^{127} \log_2\left(1+\frac{1}{n}\right) = \sum_{n=2}^{127} \log_2\left(\frac{n+1}{n}\right) = \log_2\left(\frac{3}{2}\right) + \log_2\left(\frac{4}{3}\right) + ... + \log_2\left(\frac{128}{127}\right)$

$$= \log_2\left(\frac{3}{4}.\frac{4}{3}.......\frac{128}{127}\right) = \log_2 64 = \log_2 2^6 = 6\log_2 2 = 6.$$

4. Show that $\frac{1}{3} < \log_{20} 3 < \frac{1}{2}$

Sol: $20^{\frac{1}{3}} < 27^{\frac{1}{3}} = 3 = 9^{\frac{1}{2}} < 20^{\frac{1}{2}}$

$$\Rightarrow 20^{\frac{1}{3}} < 3 < 20^{\frac{1}{2}}$$

$$\Rightarrow \log_{20} 20^{\frac{1}{3}} < \log_{20} 3 < \log_{20} 20^{\frac{1}{2}}$$

$$\Rightarrow \frac{1}{3} < \log_{20} 3 < \frac{1}{2}$$

5. If $a^x = b^y = c^z\ and\ y^2 = xz,$ then show that $\log_b a = \log_c b.$

Sol: Let $a^x = b^y = c^z = k$

$$\Rightarrow x\log a = y\log b = z\log c = \log k$$

$$x = \frac{\log k}{\log a}, y = \frac{\log k}{\log b}, z = \frac{\log k}{\log c}$$

$$y^2 = xz$$

$$\left(\frac{\log k}{\log b}\right)^2 = \left(\frac{\log k}{\log a}\right)\left(\frac{\log k}{\log c}\right)$$

$$\frac{\log a}{\log b} = \frac{\log b}{\log c}$$

$$\Rightarrow \log_b a = \log_c b$$

6. If $(3.7)^x = (0.037)^y = 1000$, then find the value of $\left(\frac{1}{x} - \frac{1}{y}\right)$.

Sol: $(3.7)^x = (0.037)^y = 1000$

$$\Rightarrow \log_{10}(3.7)^x = \log_{10}(0.037)^y = \log_{10} 10^3$$

$$\Rightarrow x\log_{10}(3.7) = y\log_{10} 0.037 = 3$$

$$\frac{3}{x} = \log_{10} 3.7,\ \frac{3}{y} = \log_{10} 0.037$$

$$\frac{3}{x} - \frac{3}{y} = \log_{10} 3.7 - \log_{10} 0.037 = \log_{10}\left(\frac{3.7}{0.037}\right) = \log_{10} 10^2 = 2$$

$$3\left(\frac{1}{x} - \frac{1}{y}\right) = 2$$

$$\Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{2}{3}.$$

7. If $\log_{12} 27 = a$, then show that $4\left(\frac{3-a}{3+a}\right) = \log_6 16$.

Sol: $\log_{12} 27 = a$

$$\log_{12} 3^3 = a$$

$$\Rightarrow 3\log_{12} 3 = a$$

$$\frac{3}{a} = \frac{1}{\log_{12} 3} = \log_3 12$$

$$\Rightarrow \frac{(3-a)}{(3+a)} = \frac{(\log_3 12 - 1)}{(\log_3 12 + 1)}$$

$$\Rightarrow \frac{(3-a)}{(3+a)} = \frac{(\log_3 12 - \log_3 3)}{(\log_3 12 + \log_3 3)} = \frac{\log_3\left(\frac{12}{3}\right)}{\log_3(12\times 3)} = \log_{36} 4$$

$$\Rightarrow \frac{(3-a)}{(3+a)} = \log_{(6)^2}(2)^2 = \log_6 2$$

$4\left(\frac{3-a}{3+a}\right)=4\log_6 2=\log_6 2^4=\log_6 16$

8. If $\frac{\log x}{(b-c)}=\frac{\log y}{(c-a)}=\frac{\log z}{(a-b)}$, then show that (i) xyz=1 (ii) $x^a y^b z^c=1$

(iii) $x^{(b+c)}y^{(c+a)}z^{(a+b)}=1$

Sol: $\frac{\log x}{(b-c)}=\frac{\log y}{(c-a)}=\frac{\log z}{(a-b)}=k$

$\log x=k(b-c);\ \log y=k(c-a);\ \log z=k(a-b)$

(i) $\log x+\log y+\log z=k(b-c)+k(c-a)+k(a-b)$

$=k(b-c+c-a+a-b)$

$\log xyz=0$

$\Rightarrow xyz=1$

(ii) $a\log x=ka(b-c);\ b\log y=kb(c-a);c\log x=kc(a-b)$

$\Rightarrow \log x^a=k(ab-ca);\ \log y^b=k(bc-ab);\ \log z^c=k(ca-bc)$

$\Rightarrow \log x^a+\log y^b+\log z^c=k(ab-ca+bc-ab+ca-bc)$

$\log x^a.y^b.z^c=0$

$\Rightarrow x^a.y^b.z^c=1$

(iii) $(b+c)\log x=k\left(b^2-c^2\right);\ (c+a)\log y=k\left(c^2-a^2\right);(a+b)\log z=k\left(a^2-b^2\right)$

$\log x^{(b+c)}+\log y^{(c+a)}+\log z^{(a+b)}=k\left(b^2-c^2+c^2-a^2+a^2-b^2\right)$

$\Rightarrow \log x^{(b+c)}.y^{(c+a)}.z^{(a+b)}=0$

$\Rightarrow x^{(b+c)}.y^{(c+a)}.z^{(a+b)}=1$

9. Find the characteristic of $\log_3 2009$.

Sol: We have $3^1=3,\ 3^2=9,\ 3^3=27,\ 3^4=81,\ 3^5=243, 3^6=729,\ 3^7=2187$

$\therefore 3^6<2009<3^7$

$\Rightarrow \log_3 3^6<\log_3 2009<\log_3 3^7$ as the base is greater than 1.

$\Rightarrow 6<\log_3 2009<7$

$\Rightarrow \log_3 2009=6+f\ ;\ 0<f<1$

$\therefore$ Charateristic of $\log_3 2009 = 6$

10. Prove that $\log_3 5$ is an irrational number.

Sol: Let us assume that $\log_3 5$ is a rational number

Then $\log_3 5 = \frac{p}{q}$ ________ (1) where p,q $\in z$ and $q \neq 0$.

$\because \log_3 5 > 1$, we have p>q>1.

By the definition of logarithm

$5 = 3^{p/q}$

$\Rightarrow 5^q = 3^p$

$\Rightarrow p = q = 0$, which is impossible

This is a contradiction

$\therefore$ Our assumption is wrong.

$\therefore \log_3 5$ is an irrational number.

11. If $\log_{0.5}(x-1) < \log_{0.25}(x-1)$, then show that 2<x<$\infty$.

Sol: Given $\log_{0.5}(x-1) < \log_{0.25}(x-1)$

$\Rightarrow \log_{0.5}(x-1) < \log_{(0.5)^2}(x-1)$

$\Rightarrow \log_{(0.5)}(x-1) < \frac{1}{2}\log_{(0.5)}(x-1)$

$\Rightarrow \frac{1}{2}\log_{0.5}(x-1) < 0$

$\Rightarrow \log_{0.5}(x-1) < 0$

$\because 0 < 0.5 < 1 \Rightarrow (x-1) > 1$

$\Rightarrow x > 2$

$\Rightarrow x \in (2, \infty)$

$\therefore 2 < x < \infty$

12. If $\log_{10} 343 = 2.5353$, then find the least positive integer n such that $7^n > 10^5$.

Sol: $\log_{10} 343 = 2.5353$

$\Rightarrow \log_{10} 7^3 = 2.5353$

$\Rightarrow 3\log_{10} 7 = 2.5353$

$\Rightarrow \log_{10} 7 = 0.8451$

$7^n > 10^5$

$n \log_{10} 7 < 5 \log_{10} 10$

$n(0.8451) > 5$

$\therefore$ The least value of n is 6.

13. If $\log_{10} 2 = 0.3010$, then find the number of digits in 256^{50}.

Sol: Let $x = 256^{50}$

Then $\log x = \log(256)^{50} = \log\left(2^8\right)^{50} = \log 2^{400}$

$\Rightarrow \log x = 400 \log 2 = 400(0.3010) = 120.4$

$\therefore$ No.of digits in x is 121.

14. If log2=0.3010, log3=0.4771, log7=0.8451, then find the number of zeros between the decimal point and the first significant figure in $(0.005332)^{10}$.

Sol: Let $x = (0.005332)^{10}$

$\log x = 10 \log 0.005332 = 10 \log\left(5332 \times 10^{-6}\right)$

$= 10\left(\log 2^2 \times 3^3 \times 7^2 \times 10^{-6}\right)$

$= 10[2\log 2 + 3\log 3 + 2\log 7 - 6\log 10]$

$= 10[2.(0.310) + 3(0.4771) + 2(0.8451) - 6]$

$\log x = 10\left(\bar{3}.7235\right) = \overline{23}.235$

$\therefore$ Required number of zeros = 22.

15. Find the value of $\dfrac{25.62 \times \sqrt{346.5} \times \sqrt[3]{465.7}}{\sqrt{76.42} \times \sqrt[3]{44.44} \times \sqrt[4]{663.5}}$

Sol: Let $x = \dfrac{25.62 \times \sqrt{346.5} \times \sqrt[3]{465.7}}{\sqrt{76.42} \times \sqrt[3]{44.44} \times \sqrt[4]{663.5}}$

$\log x = \log\left[\dfrac{25.62 \times \sqrt{346.5} \times \sqrt[3]{465.7}}{\sqrt{76.42} \times \sqrt[3]{44.44} \times \sqrt[4]{663.5}}\right]$

$= \log 25.62 + \frac{1}{2}\log 346.5 + \frac{1}{3}\log 465.7 - \frac{1}{2}\log 76.42 - \frac{1}{3}\log 44.44 - \frac{1}{4}\log 663.5$

$= 1.4085 + \frac{1}{2}(2.5397) + \frac{1}{3}(2.6682) - \frac{1}{2}(1.8832) - \frac{1}{3}(1.6478) - \frac{1}{4}(2.8218)$

$\log x = 1.3381$

$\therefore x = Anti\log(1.3381) = 21.78$

16. Solve $\log_2(xy) = 5 = \log_{\frac{1}{2}}\left(\frac{x}{y}\right) = 1$

Sol: Here xy>0 & $\left(\frac{x}{y}\right) > 0$ the initial conditions

$\log_2(xy) = 5$ $\log_2\left(\frac{x}{y}\right) = -1$

$\Rightarrow \log_2|x| + \log_2|y| = 5$ ____ (1) $\log_2|x| - \log_2|y| = -1$ ______ (2)

Solving (1) and (2), we get $\log_2|x| = 2$ and $\log_2|y| = 3$

$\Rightarrow |x| = 4, |y| = 8$

$\Rightarrow x = \pm 4, y = \pm 8$

But out of these only two solutions

x=4, y=8 or x=-4, y=-8 satisfy the conditions.

§§ Synopsis points

1. Let a,N be two positive real numbers and $a \neq 1$. The real number x such that $a^x = N$ Then x is called ***logarithm*** of N to the base a. It is denoted by $\log_a N$.
2. If m ($\neq 1$), n ($\neq 1$) are any two coprimes then $x = \log_a N$ is called logarithmic form.
3. If 'a' is even and 'b' is odd (or) 'a' is odd and 'b' is even then $\log_b a$ is an irrational number.
4. The logarithms which are calculated on the base 'e' are called natural logarithm (or) napierian logarithm. $\log_e x$ is denoted by $\ln x$.
5. The logarithm which are calculated on the base '10' are called common logarithm (or) briggs logarithm . $\log_{10} x$ is denoted by $\log x$.
6. If a is positive real number and $a \neq 1$, then $\log_a a$ =1.
7. If 'a' is a positive real number and $a \neq 1$ then $\log_a 1 = 0$
8. If a, m are positive real numbers and $a \neq 1$, then $a^{\log_a m} = m$.
9. If a, m, n are positive real numbers and $a \neq 1$, then $\log_a(mn) = \log_a m + \log_a n$.
10. If a, m, n are positive real numbers and $a \neq 1$, then $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

11. If a, m are positive real numbers $a \neq 1$, n is real number, then $\log_a(m^n) = n\log_a m$.

12. If a, b, m are positive real numbers $a \neq 1$, $b \neq 1$, then $\log_a m = \log_b m \log_a b$.

13. $\log_b a = \frac{1}{\log_a b}$.

14. $\log_a m = \frac{\log_b m}{\log_b a} = \frac{\log m}{\log a}$

15. $\log_{a^b}(m^n) = \frac{n}{b}\log_a m$

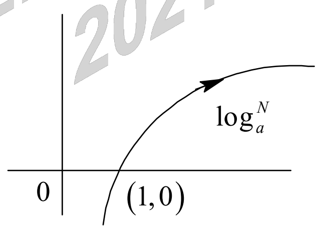
16. $a^{\log_x b} = b^{\log_x a}$

17. Let a be a positive real numbers and $a \neq 1$ the function $f:(0,\infty) \to R$ defined by $f(x) = \log_a x$ is called a logarithmic function.

18. The Graph of $y = \log_a N$ where y is a positive real number and a ($\neq 1$) is also a positive real number , y is any real number is defined as follows if

i) If a >1 then $\log_a N$ is an increasing function.

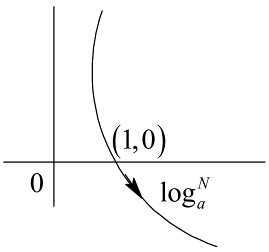
i.e if $N \to \infty$ then $\log_a N \to \infty$



If 0 < N < 1 $\Rightarrow$ y < 0 where a >1.

If N>1 $\Rightarrow y > 0$ where a >1.

19. If $0 < a < 1$ then $\log_a N$ is decreasing function. That is as $N \to \infty$ then $\log_a N \to -\infty$



If $N > 1 \Rightarrow y < 0$

If $0 < N < 1 \Rightarrow y > 0$ Where $0 < a < 1$.

20. If x_1, x_2 and a ($\neq 1$) be any positive real numbers .

a) If a > 1 then

i) $x_1 > a \Rightarrow \log_a x_1 > 1$; $x_1 < a \Rightarrow \log_a x_1 < 1$

ii) $x_1 > x_2 \Rightarrow \log_a x_1 > \log_a x_2$; $x_1 < x_2 \Rightarrow \log_a x_1 < \log_a x_2$

In this case logarithmic function is increasing function.

b) If $0 < a < 1$ then

i) $x_1 > a \Rightarrow \log_a x_1 < 1$; $x_1 < a \Rightarrow \log_a x_1 > 1$

ii) $x_1 > x_2 \Rightarrow \log_a x_1 < \log_a x_2$; $x_1 < x_2 \Rightarrow \log_a x_1 > \log_a x_2$

Note: a) $\log_e 10 \cong 2.303 \cong 2.3$

b) $\log_{10} e = \frac{1}{\log_e 10}$

$= \frac{1}{2.30258509} = 0.43429448....$

= 0.43

c) $n \in R^+$; $\log_{10} n = (\log_e n) \times (0.43)$

d) Common logarithm is the sum of two parts.
Common logarithm = integral part + decimal part = Characteristic + mantissa

e) Characteristic is the positive or negative integral power of 10

f) Mantissa is a non negative decimal fraction

g) The characteristic of common logarithm of a number greater than unity and with only one digit in its integral part is zero.

h) the characteristic of the logarithm of a number N, with 'n' digits in its integral part is (n-1).

i) The characteristic of the common logarithm of a decimal fraction N with 'n' zeros immediately following the decimal point is - (n+1).

21. If a, b are relatively prime then $\log_b a$ is irrational.

Explanation

If possible suppose that $\log_b^a$ is rational

$\Rightarrow \log_b^a = \frac{p}{q}$ (p,q, $\in z$)

$\Rightarrow a = (b)^{\frac{p}{q}} \Rightarrow a^q = b^p$

which is a contradiction

$\therefore$ if a,b are relatively prime then $\log_b^a$ is irrational

The least value of $\log_b a + \log_a b$ is 2 where $\log_b a > 0$.

TEACHING TASK

1. $\log_2 \log_{25} 5 = ...$

A) 0 B) 1 C) -1 D)1/2

2. $\frac{1}{\log_{ab} abcd} + \frac{1}{\log_{ac} abcd} + \frac{1}{\log_{ad} abcd} + \frac{1}{\log_{bc} abcd} + \frac{1}{\log_{bd} abcd} + \frac{1}{\log_{cd} abcd} =...$

A) 0 B) 1 C) 2 D) 3

3. $\log_{10} \tan 1^0 + \log_{10} \tan 2^0 + ... + \log_{10} \tan 89^0 = ...$

A) 0 B) 1 C) -1 D) None

4. $\frac{1}{\log_{ab} abc} + \frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc} = ...$

A) 1 B) 2 C) -1 D) -2

5. log (1+ 2+ 3) =..........

A) log1.log2.log3 B)log1+log2+log3 C) log1+log2-log3 D)log1+log2.log3

6. $\log_n\left(1-\frac{1}{2}\right) + \log_n\left(1-\frac{1}{3}\right) + ... + \log_n\left(1-\frac{1}{n}\right) =$

A) -1 B) 1 C) 0 D) 2

7. $\frac{\log_a x . \log_b x}{\log_a x + \log_b x} = ...$

A) $\log_b a$ B) $\log_b x$ C) $\log_{ab} x$ D) $\log_x ab$

8. If $\log_{10} 2 = a$ then $\log_{10} 5$ =.........

A) $\frac{1}{a}$ B) 1 - a C) $\frac{1}{1-a}$ D) $\frac{1}{1+a}$

9. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then $f\left(\frac{2a}{1+a^2}\right)$ =.........

A) $f(0)$ B) $-f(a)$ C) $2f(a)$ D) $\frac{1}{2}f(a)$

10. If $\log 2 + \frac{1}{2}\log a + \frac{1}{2}\log b = \log(a+b)$ then......

A) a = b B) a = - b C) a = 2 , b = 0 D) a=1,b=2

11. If $\log\left(\frac{a-b}{2}\right)=\frac{1}{2}(\log a+\log b)$ then $\frac{a}{b}+\frac{b}{a}=$ ......

A) 1 B) 2 C) 4 D) 6

12. If $5^x=(0.5)^y=1000$ then $\frac{1}{x}-\frac{1}{y}$ =

A) 1 B) 1/2 C) 1/2 D) 1/4

13. If $\log_5 x+\log_{25} x+\log_{625} x=\frac{7}{4}$ then x =.........

A) 5 B) 25 C) 125 D) 2

14. $\log_2 x+\log_x 2=\frac{10}{3}=\log_2 y+\log_y 2$ and $x\neq y$ then $x+y=$.......

A) 2 B) 65/8 C) 37/6 D)None

15. $\log_{10} x+\log_{10}\sqrt{x}+\log_{10}\sqrt[4]{x}+\log_{10}\sqrt[8]{x}+$.....= 2 then x =.........

A) 2 B) 10 C) 100 D) 5

16. If $x=\log_7 11$, $y=\log_8 5$ then......

A) $x=y$ B) $x<y$ C) $x>y$ D) None

17. If $\log 2=a$, $\log 3=b$, $\log 7=c$ and $6^x=7^{x+4}$ then x =.........

A) $\frac{4b}{c+a-b}$ B) $\frac{4c}{a+b-c}$ C) $\frac{4b}{c-a-b}$ D) $\frac{4b}{a+b-c}$

18. $0.0001\leq n\leq 0.001\Rightarrow$

A) $-4\leq\log n\leq -3$ B) $-3\leq\log n\leq -2$ C) $-2\leq\log n\leq -1$ D) $-5\leq\log n\leq -4$

19. If $9a^2+4b^2=18ab$ then $\log(3a+2b)=$..........

A) $\log 5+\log 3+\log a+\log 5b$ B) $\log 5+\log 3+\log 3a+\log b$

C) $\log 5+\log a+\log b$ D) None

20. If $\log(x-y)-\log 5-\frac{1}{2}\log x-\frac{1}{2}\log y=0$ then $\frac{x}{y}+\frac{y}{x}=$ ......

A) 25 B) 26 C) 27 D) 28

21. The Value of $3^{\frac{4}{\log_4 9}}+27^{\frac{1}{\log_{36} 9}}+81^{\frac{1}{\log_5 3}}=$.......

A) 890 B) 860 C) 857 D) None

22. If $\log_a ab=x$ then $\log_b ab=$.....

A) $\frac{x-1}{x}$ B) $\frac{x}{x-1}$ C) $\frac{x}{x+1}$ D) $\frac{x+1}{x}$

23. If $n = 1999!$ then $\sum_{x=1}^{1999} \log_n x$ =

A) 1 B) $\sqrt[1999]{1999}$ C) 0 D) -1

24. If $\log_{10} 3 = 0.4771$ the number of digits in 3^{40} is ....

A) 18 B) 19 C) 20 D) 21

25. $\log_a^n \log_b^n + \log_b^n \log_c^n + \log_c^n \log_a^n$ =

A) $\frac{\log_a^n \log_b^n \log_b^n}{\log_{abc}^n}$ B) $\frac{\log_{abc}^n}{\log_a^n}$ C) $\frac{\log_b^n}{\log_{abc}^n}$ D) None

26. If $\log[1+\log_b\{1+\log_c(1+\log_a x)\}] = 0$ then x =

A) 1 B) 2 C) 3 D) 4

27. If $a^2 = b^2 + c^2$ then $\frac{1}{\log_{a-b}^c} + \frac{1}{\log_{a+b}^c}$ =

A) 0 B) 1 C) 3 D) 2

28. If $3^x = (0.3)^y = (0.03)^z = 10$ then

A) $x = y - 1 = z - 2$ B) $\frac{1}{x} = \frac{1}{y} - 1 = \frac{1}{z} - z$

C) $\frac{1}{x} - 1 = \frac{1}{y} = \frac{1}{z} + 1$ D) $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in G.P

29. $\log_2 7$ is........

A) an integer B) a ratIonal number

C) an irrational number D) a prime number

30. If $\log_2 3 = a$ then the value of $\log_{48} 24$ is terms of a

A) $\frac{3+a}{4+a}$ B) $\frac{3+\log a}{4+\log a}$ C) $\frac{4+a}{3+a}$ D) None

31. If $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \text{..........} \frac{1}{\log_{43} n} = \frac{1}{\log_x n}$ then x =.....

A) 43 B) 43! C) 43^n D) 1/43

32. If $a^x = b^y = c^z = d^w$ then $\log_a bcd$ =

A) $x\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$ B) $\frac{1}{x}\left(\frac{1}{y}+\frac{1}{z}+\frac{1}{w}\right)$

C) $x\left(\frac{1}{y}+\frac{1}{z}-\frac{1}{w}\right)$ D) $\frac{1}{x}\left(\frac{1}{y}+\frac{1}{z}-\frac{1}{w}\right)$

33. If $p^3 + q^3 = r^3$ then $\log_p(r-q) + \log_p(r^2 + rq + q^2) =$

A) 1 B) 4 C) 3 D) None.

34. $\sum_{r=2}^{43} \frac{1}{\log_r n} =$ (**IIT-1975)**

A) $\log_n 43$ B) $\log_{43} n$ C) $\log_{43!} n$ D) $\frac{1}{\log_{43!} n}$

35. Given $\log_{10} 343 = 2.5353$, the least integer n such that $7^n > 10^{10}$ is **(IIT-1976)**

A) 10 B) 11 C) 12 D) 13

36. If a,b,c are distinct positive numbers different from 1 such that

$(\log_b a . \log_c a - \log_a a) + (\log_c b . \log_a b - \log_b b) + (\log_a c . \log_b c - \log_c c) = 0$

then abc = (**RMO-2014**)

A) 0 B) 1 C) 2 D) 33

37. If $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$ then x = (**IIT-1999**)

A) 0 B) 1/2 C) 1 D) 3/2

38. If a>0, $2\log_x a + \log_{ax} a + 3\log_{a^2x} a = 0$, then x = (**IIT-1978)**

A) $a^{1/2}$ B) $a^{-1/2}$ C) $a^{-2/3}$ D) $a^{-4/3}$

NOTE: A.M $\geq$ G.M

i.e., $\left(\frac{a+b}{2} \geq \sqrt{a.b}\right)$ Replace b = $\frac{1}{a}$

$\left(\frac{a+\frac{1}{a}}{2} \geq \sqrt{a.\frac{1}{a}}\right) = \frac{a+\frac{1}{a}}{2} \geq \sqrt{1} = a + \frac{1}{a} \geq 2$.

39. If x>1, the least value of $2\log_{10} x - \log_x 0.01$ is (**IMO-2017**)

A) 10 B) 2 C) 0.1 D) 4

40. For 0 < a < x , the minimum value of $\log_a x + \log_x a$ is (**NIMO-2018)**

A) 0 B) 1 C) 2 D) 3

41. The number of rational roots of the equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ is (**IIT-1989**)

A) 0 B) 1 C) 2 D) 3

42. The number of solutions of the equation $\log_4 (x-1) = \log_2 (x-3)$ is (**IMO-2019**)

A) 0 B) 1 C) 2 D) 3

43. The value of $6+\log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\cdots\cdots}}}\right)$ is (**IIT-2016**)

A) 1 B) 2 C) 3 D) 4

LEARNER'S TASK

BEGINNERS (Level - I)

MCQ's with single answer.

1. The exponential form of $\log_5 125=3$ is ....

A) $5^3=125$ B) $25=3^5$ C) $25=5^5$ D) $5=125^3$

2. $\log_8 128=.....$

A) 7/3 B) 16 C) 3/7 D) 1/16.

3. $9^{\log_3 2}=.....$

A) 27 B) 8 C) 4 D) 2

4. If $x=27$ and $y=\log_3 4$ then $x^y=.....$

A) 16 B) 64 C) 4 D) 1

5. $3^{\log_5 7}-7^{\log_5 3}=......$

A) 0 B) 1 C) -1 D) None

6. $3^{\sqrt{\log_3 7}}-7^{\sqrt{\log_7 3}}=.......$

A) 0 B) 1 C) -1 D) 2

7. $\frac{\log_4 625}{\log_{243} 343}-\frac{\log_8 625}{\log_{243} 49}=...........$

A) 0 B) 1 C) -1 D)None

8. $\log 64=x\log 2$ then x =.....

A) 64 B) 2 C) 6 D) 32

9. $\log_{\sqrt{2}}\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}$ =..........

A) $\frac{15}{16}$ B) $\frac{7}{16}$ C) $\frac{15}{8}$ D) $\frac{31}{32}$

10. If $(4\cdot 2)^x=(0\cdot 42)^y=1000$ then $\frac{1}{x}-\frac{1}{y}$ =.........

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$

11. If $\log_{10} 3=0.4771$ then the number of digits in 81^{25} is

A) 4 B) 47 C) 48 D) 46

12. The solution of $\log_{\pi}\left(\log_2(\log_7 x)\right)=0$ is

A) 7^2 B) π^2 C) 2^2 D) None

13. $\frac{1}{\log_{a^5} abc}+\frac{1}{\log_{b^5} abc}+\frac{1}{\log_{c^5} abc}$ =

A) 0 B) 1 C) 5 D) $\frac{1}{5}$

14. $\frac{1}{x\log_{a^x} abc}+\frac{1}{y\log_{b^y} abc}+\frac{1}{z\log_{c^z} abc}$ =

A) 0 B) 1 C) abc D) xyz

15. The value of $\log_{y^2} x^3.\log_{z^2} y^3.\log_{x^2} z^3 = ...$

A)1 B)3/2 C)9/2 D)27/8

16. If $\log_2 3 = a$ then the value of $\log_{48} 12$ is terms of a is.........

A) $\frac{3+a}{4+a}$ B) $\frac{3+\log a}{4+\log a}$ C) $\frac{4+a}{3+a}$ D) $\frac{2+a}{4+a}$

17. If $a=log_{24}12, b=log_{36}24, c=log_{48}36$ then $\frac{1+abc}{bc}$ =.......

A) 0 B) 1 C) 2 D) 4

18. The value of $\log\left(Sin1^0\right).\log\left(Sin2^0\right)...........\log\left(Sin179^0\right) = ...$

A) 0 B)1 C) 2 D) Not Define

19. The Smallest among the following $\log_{\frac{1}{3}}\left(\frac{2}{5}\right), \log_{\frac{1}{3}}\left(\frac{5}{2}\right), \log_{\frac{1}{3}}\left(\frac{1}{4}\right), \log_{\frac{1}{3}}\left(\frac{7}{3}\right)$

A) $\log_{\frac{1}{3}}\left(\frac{2}{5}\right)$ B) $\log_{\frac{1}{3}}\left(\frac{5}{2}\right)$ C) $\log_{\frac{1}{3}}\left(\frac{1}{4}\right)$ D) $\log_{\frac{1}{3}}\left(\frac{7}{3}\right)$

20. The value of $\frac{\log(1+3x+3x^2+x^3)}{\log(1+4x+6x^2+4x^3+x^4)}$ =...........

A) $\frac{3}{2}$ B) $\frac{2}{3}$ C) $\frac{3}{4}$ D) $\frac{4}{3}$

21. If $x=\frac{25^{37}-25^{-37}}{25^{37}+25^{-37}}$ then $\log_{25}\left(\frac{1+x}{1-x}\right)$ =........

A) 148 B) 74 C) 37 D) 34

22. If $f(a)=\log\left(\frac{2+a}{2-a}\right)$ then $\frac{1}{2}f\left(\frac{8a}{4+a^2}\right)$ =

A) $f(a)$ B) $2f(a)$ C) $\frac{1}{2}f(a)$ D) $-f(a)$

23. If $(3.7)^x = (0.037)^y = 10000$ Then $\frac{1}{x} - \frac{1}{y} = ...$

A)1/2 B)1/3 C)1/4 D)1/5

24. If $\log_4(\log_3 x) = \frac{1}{2}$ then x =

A) 3 B) 4 C) 9 D) 12

25. $\log_a 10$ Exists for

A) $a \neq 1, a > 0$ B) $a > 0$ C) $a \neq 1$ D) $a \in R$

ACHIEVERS (Level - II)

Descriptive type questions :

1. Prove that $\log abc = \log a + \log b + \log c$.

2. Show that log 10800 = 4 log 2 + 3 log 3 + 2 log 5

3. Show that $7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80} = \log 2$.

4. Show that $\log_3\left(1+\frac{1}{3}\right) + \log_3\left(1+\frac{1}{4}\right) + \log_3\left(1+\frac{1}{5}\right) + ... + \log_3\left(1+\frac{1}{242}\right) = 4$.

5. Show that $\sum_{n=2}^{127} \log_2\left(1+\frac{1}{n}\right) = 6$

6. If $a^2 + b^2 = 6ab$ then show that

i) $2\log(a+b) = \log a + \log b + 3\log 2$

ii) $2\log(a-b) = \log a + \log b + 2\log 2$.

7. If $a^4 + b^4 = c^4$ then show that $\log_a(c+b) + \log_a(c-b) + \log_a(c^2+b^2) = 4$.

8. If $\log\frac{x+y}{3} = \frac{1}{2}(\log x + \log y)$ then show that $\frac{x}{y} + \frac{y}{x} = 7$.

9. If $\log(a+c) + \log(a-2b+c) = 2\log(a-c)$ then show that $\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$.

10. If $\frac{1}{\log_a x} + \frac{1}{\log_c x} = \frac{2}{\log_b x}$ then show that $b^2 = ac$ also prove conversely.

11. If $\frac{\log a}{1} = \frac{\log b}{2} = \frac{\log c}{5}$ then show that $a^4 b^3 c^{-2} = 1$.

12. Solve $5^{\log x} + x^{\log 5} = 50$.

13. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ then show that $f(x_1) + f(x_2) = f\left(\frac{x_1 + x_2}{1 + x_1 - x_2}\right)$.

14. If $x = 1 + \log_a bc$, $y = 1 + \log_b ca$, $z = 1 + \log_c ab$ then show that $xyz = xy + yz + zx$

15. Solve $\log_{10}\left(98+\sqrt{x^2-12x+36}\right)=2$.

EXPLORERS (Level - III)

MCQ with one or more than one correct answer

♦ *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which* ***ONE or MORE*** *is correct. Choose the correct options*

1. The value of $\log\{\log_b a \log_c b \log_d c \log_a d\}$ is....

A) 0 B) $\log abcd$ C) $\log 1$ D) 1

2. If $\frac{\log_2 x}{4}=\frac{\log_2 y}{6}=\frac{\log_2 z}{3k}$ and $x^3y^2z=1$ then k = ...

A) -8 B) -4 C) 0 D) $\log_2\left(\frac{1}{256}\right)$

3. If $\log_{0.3}(x-1)<\log_{0.09}(x-1)$ then the satisfying value of x is......

A) 5 B) 6 C) 3 D) >2.

4. If $\frac{\log a}{y-z}=\frac{\log b}{z-x}=\frac{\log c}{x-y}$ then ....

A) $a^x.b^y.c^z=1$ B) $a^{y^2+yz+z^2}.b^{z^2+xz+x^2}.c^{x^2+xy+y^2}=1$

C) $a^{y+z}.b^{z+x}.c^{x+y}=1$ D) abc = 1

5. If x = 9 is the solution of $\log(x^2+15a^2)-\log(a-2)=\log\left(\frac{8ax}{a-2}\right)$ then ....

A) a = 3 B) a = $\frac{3}{5}$ C) a = -$\frac{3}{5}$ D) a = $\frac{9}{5}$

6. The solution of the equation $3^{\log_a x}+3x^{\log_a 3}=2$ is....

A) $a^{\log_3 a}$ B) $\left(\frac{2}{a}\right)^{\log_3 2}$ C) $a^{-\log_3 2}$ D) $2^{-\log_3 2}$

7. If $a^x=b,\ b^y=c,\ c^z=a,\ x=\log_b a^{k_1},\ y=\log_c b^{k_2},\ z=\log_a c^{k_3}$ then $k_1\,k_2\,k_3$ is...

A) 1 B) abc C) $x^2y^2z^2$ D) 0

8. $\log_p \log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\sqrt[p]{\cdots\cdots\sqrt[p]{p}}}}}}_{n\ times}$, p > 0 and p $\neq$ 1 is......

A) n B) -n C) $\frac{1}{n}$ D) $\log_{1/p}\left(p^{n}\right)$

II. Comprehension questions :

♦ *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

i). If n is a positive real number then the integer k such that $k \le \log n < k+1$ is called integral part (or) characteristic of log n the non negative real number $[\log n - k]$ is called mantissa.

If the characteristic of logarithm is -k and is denoted by $\bar{k}$, mantissa is always non negative and less than 1.

1. Characteristic of $\log 293 = 2.4669$ is...
   A) 3 B) 2 C) 1 D) 0
2. Characteristic of $\log_{10}(0.00602) = -2.2204$ is.....
   A) 3 B) 2 C) $\bar{1}$ D)None
3. Mantissa of $\log_{10} 15 = 1.1761$ is....
   A) 1761 B) 1 C) 0.1761 D)None
4. Mantissa of $\log_{10}(463.1)$ is 0.6657 then mantissa of $\log_{10}(4.631)$ is .....
   A)0.6657 B) 0.06657 C) 0.006657 D) 1.6657
5. If $\log_{10} x$ = $\bar{3}.7961$ then, number of zeroes between decimal point and first significant digit is.....
   A) 3 B) 1 C) 2 D) 0

III. Matrix Matching

♦ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

a)

1. If a > 1, 0 < x < 1 then.......... [ ] a) $\log_a x$ > 0
2. If 0 < a < 1, 0 < x <1 then.... [ ] b) $\log_a x$ < 0
3. If a > 1 , x > 1 then .... [ ] c) $\log_a x$ is decreasing function
4. If 0 < a < 1, x > 1 then .... [ ] d) $\log_a x$ is increasing function

b)

5. $\frac{\log_8 17}{\log_9 23} - \frac{\log_{2\sqrt{2}} 17}{\log_3 23} =$......... [ ] a) $\frac{1}{5}$
6. $\log_{\frac{1}{x}} 4 = 2$ then x = ... [ ] b) 0

7. $\log_5 2.\log_{11} 5.\log_{32} 11=$......... [ ] c) $\frac{1}{2}$

8. $7^{\log_7(x+1)}=6$ then x = ... [ ] d) 5

IV. Integer type questions:

1. $\log_3(4^x-3)+\log_3(4^x-1)=1$ then x = ...

2. $\log_5\left(\frac{2+x}{10}\right)=\log_5\left(\frac{2}{x+1}\right)$ then x = ...

3. $\log_2(4.3^x-6)-\log_2(9^x-6)=1$ then x = ...

4. $\frac{2\log x}{\log(6x-5)}=1$ then x = ...

5. $7\log_2 x=\log_2 128$ then x = ...

6. If $\log_3\left(1+\log_3(2^x-7)\right)=1$ then x = ...

7. If $\log_3(3^x-8)=2-x$ then x = ...

V. HOTS

1. The value of $\log_3\sqrt[4]{729\sqrt[3]{9^{-1}\cdot 27^{-4/3}}}$ is.......

A) 1 B) -1 C) 2 D) 0

2. If $\frac{\log 2}{b-c}=\frac{\log 3}{c-a}=\frac{\log 5}{a-b}$ then $2^a.3^b.5^c$ =......

A) 5 B) 7 C) 8 D) 1

3. If $(4\cdot 2)^x=(0\cdot 42)^y=100$ then $\frac{1}{x}-\frac{1}{y}$ =......

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{5}$

4. $\log_2\left(1+\frac{1}{2}\right)+\log_2\left(1+\frac{1}{3}\right)+.........+\log_2\left(1+\frac{1}{31}\right)$ =

A) 0 B) 1 C) 5 D) 4

5. If $x=\frac{a^n-a^{-n}}{a^n+a^{-n}}$ then $\frac{1}{2}\log_a\left(\frac{1+x}{1-x}\right)$ =.......

A) 4n B) 3n C) 2n D) n

6. If $4^{\log_9 3}+9^{\log_2 4}=10^{\log_x 83}$, then x =

A) 2 B) 3 C) 5 D) 10

7. The number of solutions of $\log_7\log_5(\sqrt{x+5}+\sqrt{x})=0$ is

A) 0 B) 1 C) 2 D) >2

8. If $3^x = 4^{x-1}$, then x is equal to

A) $\frac{2\log_3 2}{2\log_3 2-1}$ B) $\frac{2}{2-\log_2 3}$ C) $\frac{1}{1-\log_4 3}$ D) $\frac{2\log_2 3}{2\log_2 3-1}$

9. If $16^{2x+1} = 2X\ 4^{x-3}$ Then x = ...

A) $\frac{2}{3}$ B) - $\frac{3}{2}$ C) 1 D) 0

10. If $x = Log_{2a}a$, $y = Log_{3a}2a$, $z = Log_{4a}3a$ then $1 + xyz =$.....

A) 0 B) yz C)2yz D)3xyz

KEY

ΦΦ TEACHING TASK :

1.C 2.D 3.A 4.B 5.B 6.A 7.C 8.B 9.C 10.A
11.D 12.C 13.A 14.B 15.B 16.C 17.B 18.A 19.D 20.C
21.C 22.B 23.A 24.A 25.A 26A 27.D 28.C 29.C 30.A
31.B 32.C 33.C 34. D 35.C 36.B 37.D 38.B,D 39.D
40. C 41.C 42.B 43.D

ΦΦ LEARNER'STASK :

☐ BEGINNERS :

1.A 2.A 3.C 4.B 5.A 6.A 7.A 8.C 9.C 10.B
11.C 12.A 13.C 14.B 15.D 16.D 17.C 18.A 19.B 20.C
21.C 22.A 23.A 24.C 25.A

☐ ACHIEVERS :

12). 100 15). 4 , 8

☐ EXPLORERS :

I. 1) A, C 2) A, D 3) A,B,C,D 4).A,B,C,D 5)A ,D
6) C, D 7) C 8)B, D

II. 1). b 2)a 3).c 4).b 5). c

III. 1).b,c 2). a,d 3)a,d 4).b,c 5). b 6).c 7). a 8).d

IV. 1) 1 2) 3 3) 1 4) 5 5) 2 6) 4 7) 2

V. 1.A 2.D 3.B 4.D 5.D 6.D 7.B 8.A,B 9.B 10.C