Dot Product

Thank

(1)

Given line equation $y = x+5$

compare with $y = mx + 5$ $\quad r = 5$

Given $v = 4\sqrt{2}$ m/s ; $m = 8$ kg ; where slope $m = 1$

$\tan\theta = 1$

Angular momentum $\Rightarrow \theta = 45^\circ$

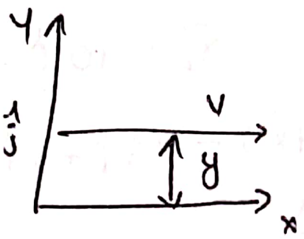
$$\vec{L} = mvr\sin\theta$$

$$= 8\times4\sqrt{2}\times5\times\sin45^\circ$$

$$= 32\sqrt{2}\times5\times\frac{1}{\sqrt{2}}$$

$$= 160 \text{ kg m}^2/\text{s}$$

(2)



$$\vec{p} = mv\,\hat{i}$$

Angular momentum

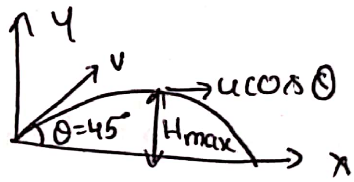
$$\vec{L} = \vec{r}\times\vec{p}$$

$$= y\hat{j}\times mv\hat{i}$$

$$= mvy(\hat{j}\times\hat{i})$$

$$= -mvy\,\hat{k}$$

(3)



$m = \sqrt{2}$ kg

$$\vec{r} = H_{max}\,\hat{j}$$

$$= \frac{u^2\sin^2\theta}{2g}\,\hat{j}$$

$$= \frac{v^2\sin^2 45}{2g}\,\hat{j} \Rightarrow \frac{v^2(\frac{1}{\sqrt{2}})^2}{2g}\hat{j}$$

$$= \frac{v^2}{4g}\,\hat{j}$$

momentum $\vec{p} = mv\cos\theta\,\hat{i}$

$$= \sqrt{2}\,v\cos45\,\hat{i}$$

$$\Rightarrow v\,\hat{i}$$

Angular momentum $\vec{L} = \vec{r}\times\vec{p}$

$$\vec{L} = \frac{v^2}{4g}\hat{j}\times v\hat{i} = \frac{v^3}{4g}(\hat{j}\times\hat{i})$$

$$\vec{L} = \frac{v^3}{4g}\,\hat{k}$$

(4)

$\vec{v} = (3\hat{i} + 2\hat{j}) \times 10^5$ m/s

$\vec{B} = 2\hat{i} + 3\hat{k}$ wb/m2

Force $= q(\vec{v} \times \vec{B})$

$\Rightarrow ma = 1.6 \times 10^{-19} (6\hat{i} - 9\hat{j} - 4\hat{k}) \times 10^5$

$\Rightarrow a = \frac{1.6}{m} \times 10^{-14} (6\hat{i} - 9\hat{j} - 4\hat{k})$

$\Rightarrow a = \frac{1.6 \times 10^{-14}}{\frac{1.6 \times 10^{-19}}{9.6 \times 10^7}} [6\hat{i} - 9\hat{j} - 4\hat{k}]$

$\Rightarrow a = 9.6 \times 10^{12} (6\hat{i} - 9\hat{j} - 4\hat{k}]$

$\vec{V} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 2 & 0 & 3 \end{vmatrix}$

$= (\hat{i}(6-0) - \hat{j}(9-0) + \hat{k}(0-4)] \times 10^5$

$= (6\hat{i} - 9\hat{j} - 4\hat{k}) \times 10^5$

$\frac{q}{m} = 9.6 \times 10^7$ C/kg

$\Rightarrow m = \frac{1.6 \times 10^{-19}}{9.6 \times 10^7}$ kg

(5)

Given $m = 5$ kg ; $\vec{r_1} = 6\hat{i} + 5\hat{j} - 3\hat{k}$ m

$\vec{r_2} = 10\hat{i} - 2\hat{j} + 7\hat{k}$ m.

$\vec{S} = r_2 - r_1 = (10\hat{i} - 2\hat{j} + 7\hat{k}) - (6\hat{i} + 5\hat{j} - 3\hat{k})$

$\vec{s} = 4\hat{i} - 7\hat{j} + 10\hat{k}$.

and given $\vec{F} = 10\hat{i} - 3\hat{j} + 6\hat{k}$ N

$W = \vec{F} \cdot \vec{s} = (10\hat{i} - 3\hat{j} + 6\hat{k}) \cdot (4\hat{i} - 7\hat{j} + 10\hat{k})$

$= 10 \times 4 + (-3)(-7) + 6 \times 10$

$= 40 + 21 + 60$

$= 121$ J.

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(6)

Given two vectors are

$\vec{A} = \hat{i} + 2\hat{j} + n\hat{k}$; $\vec{B} = 4\hat{i} + 2\hat{j} + 2\hat{k}$

Given $\vec{A}$ and $\vec{B}$ are Perpendicular

$\therefore$ $\vec{A} \cdot \vec{B} = 0$

$(\hat{i} + 2\hat{j} + n\hat{k}) \cdot (4\hat{i} + 2\hat{j} + 2\hat{k}) = 0$

$\Rightarrow 4 + 4 + 2n = 0$

$\Rightarrow 2n = -8 \Rightarrow n = -4.$

(7)

Given $\vec{F} = 8\hat{i} + 4\hat{j}$ N.

$\vec{s} = 3\hat{i} + 3\hat{j}$ m

$P = \frac{w}{t} = \frac{\vec{F} \cdot \vec{s}}{t}$ Given Power = 0.6W

$\Rightarrow 0.6 = \frac{(8\hat{i} + 4\hat{j}) \cdot (3\hat{i} + 3\hat{j})}{t}$

$\Rightarrow 0.6 = \frac{\cancel{24}\ 8 \times 3 + 4 \times 3}{t} = \frac{24 + 12}{t}$

$\Rightarrow t = \frac{36}{0.6} = 60$ sec

(8)

let $\vec{A} = -4\hat{i} + 2\hat{j}$; $\vec{B} = 9\hat{k}$.

$\vec{A} \cdot \vec{B} = (-4\hat{i} + 2\hat{j}) \cdot 9\hat{k} = 0.$

Angle between two vectors $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$

$\cos\theta = \frac{0}{|\vec{A}||\vec{B}|} = 0$

$\cos\theta = 0 \Rightarrow \theta = 90^\circ$

(9)

component of $\vec{B}$ along $\vec{A}$ is

$$A\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|} \quad , \quad B\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|}$$

(17)

Given $\vec{A}\cos\theta = \vec{B}\cos\theta$

$\Rightarrow \vec{A} = \vec{B}$

The angle between $\vec{A}$ and $\vec{B}$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{\vec{A}\cdot\vec{A}}{|\vec{A}||\vec{A}|}$$

$$\cos\theta = \frac{0}{A^2} = 0 \Rightarrow \theta = 90^\circ$$

(10)

Given $\vec{F_1} = 2\hat{i} + 2\hat{j} + 5\hat{k}$

$\vec{F_2} = 3\hat{i} - 5\hat{j} - 4\hat{k}$

$F_{net} = F_1 + F_2 = 2\hat{i} + 2\hat{j} + 5\hat{k} + 3\hat{i} - 5\hat{j} - 4\hat{k}$

$= 5\hat{i} - 3\hat{j} + \hat{k}$

Given $\vec{S} = 18\hat{i} + 17\hat{j} - 7\hat{k}$

$\therefore W = \vec{F}\cdot\vec{S}$

$= (5\hat{i} - 3\hat{j} + \hat{k})\cdot(18\hat{i} + 17\hat{j} - 7\hat{k})$

$= 18\times 5 + (-3)\times 7 + 1\times(-7)$

$= 90 - 51 - 7$

$= 32\,J$

17 (i), (ii), (iii)

Given $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{c} = 3\hat{i} + \hat{j}$

$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

$$\vec{r} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 1 \end{vmatrix}$$

$$\vec{r} = \hat{i}(2-6) - \hat{j}(1+3) + \hat{k}(2-(-2))$$

$$\Rightarrow \vec{r} = -4\hat{i} - 4\hat{j} + 4\hat{k} = -4(\hat{i} + \hat{j} - \hat{k})$$

(ii)

$$|\vec{r}| = 4\sqrt{1^2+1^2+1^2}$$

$$= 4\sqrt{3}$$

Angle between $\vec{r}$ and $\vec{c}$ $\quad \cos\theta = \dfrac{\vec{r}\cdot\vec{c}}{|\vec{r}||\vec{c}|}$

$$|\vec{c}| = \sqrt{3^2+1^2} = \sqrt{10}$$

$$\cos\theta = \frac{-\cancel{4}(\hat{i}+\hat{j}-\hat{k})\cdot(3\hat{i}+\hat{j})}{\cancel{4}\sqrt{3}\sqrt{10}}$$

$$= -\frac{[1\times 3 + 1\times 1]}{\sqrt{30}} = \frac{-4}{\sqrt{30}}$$

$$\theta = \cos^{-1}\left(-\frac{4}{\sqrt{30}}\right)$$

19

Given Force $\vec{F} = 3\hat{i} + 4\hat{k}$ N.

Displacement $\vec{s} = 2\hat{i} + 4\hat{j}$ m ; $t = 3$ sec

$$\text{Power} = \frac{\vec{F}\cdot\vec{s}}{t} = \frac{(3\hat{i}+4\hat{k})\cdot(2\hat{i}+4\hat{j})}{3} = \frac{3\times 2 + 4\times 4(\hat{k}\cdot\hat{j})}{3}$$

$$= \frac{6+16(0)}{3} = \frac{6}{3} = 2\,W$$

(18)

Given Angular velocity $\vec{\omega} = 4\hat{i} + \hat{j} - 2\hat{k}$

Linear velocity $\vec{r} = 2\hat{i} - 3\hat{j} + \hat{k}$

$$\vec{V} = \vec{\omega} \times \vec{r} = \begin{vmatrix} i & j & k \\ 4 & 1 & -2 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= \hat{i}(1-6) - \hat{j}(4+4) + \hat{k}(-12-2)$$

$$\Rightarrow -5\hat{i} - 8\hat{j} - 14\hat{k}$$

L Task

Jee mains level

(1) let the two vectors are.

$$\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{B} = \hat{i} + 4\hat{j} - 5\hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & -3 & 4 \\ 1 & 4 & -5 \end{vmatrix}$$

$$\Rightarrow i[(-3)(-5) - 4\times4] - j[2(-5) - 1\times4] + k[2\times8 + 1\times(-3)]$$

$$\Rightarrow i[15-16] - j(-10-4) + k(8+3)$$

$$\Rightarrow -\hat{i} + 14\hat{j} + 11\hat{k}.$$

(2)

Given $m = 0.2\,kg$

let $\vec{v} = \hat{i} - \hat{j} + 2\hat{k}$

$\vec{r} = 4\hat{i} + \hat{j} - \hat{k}$

$$\vec{r} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 4 & 1 & -1 \end{vmatrix}$$

$\Rightarrow \hat{i}[(1\times 2 - (-1)(-1)] - \hat{j}[4\times 2 - 1(-1)] + \hat{k}[1\times 1 - 4(-1)]$

$\Rightarrow \hat{i}(2-1) - \hat{j}(8+1) + \hat{k}(-4-1)$

$\Rightarrow +\hat{i} - 9\hat{j} - 5\hat{k} = \sqrt{1^2 + (-9)^2 + (-5)^2}$

$= \sqrt{107}$.

Angular momentum $\vec{L} = m(\vec{r} \times \vec{v})$

$= 0.2 \times \sqrt{107} = \sqrt{4.28}$ units.

(3)

let two vectors are $\vec{A}$ and $\vec{B}$

Given $\vec{A} \cdot \vec{B} = AB\cos\theta = 48\sqrt{3}$

and $\vec{A} \times \vec{B} = AB\sin\theta = 144$

$$\therefore \frac{\vec{A} \cdot \vec{B}}{\vec{A} \times \vec{B}} = \frac{\cancel{AB}\cos\theta}{\cancel{AB}\sin\theta} = \frac{\cancel{48}\sqrt{3}}{\cancel{144}\,3} = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \cot\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \theta = 60^\circ$

(4) ~~(5) let~~ the vectors are $\vec{A} = \underset{A_x}{2}\hat{i} + \underset{A_y}{3}\hat{j} - \underset{A_z}{4}\hat{k}$.

(4)

Given adjacent sides

$$\vec{P} = 2\hat{i} - 3\hat{j} + \hat{k} ; \quad \vec{Q} = -2\hat{i} + 4\hat{j} - \hat{k}$$

$$\text{Area} = \vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ -2 & 4 & -1 \end{vmatrix}$$

$$\Rightarrow \hat{i}\left[(-3)(-1) - (1 \times 4)\right] - \hat{j}\left[2(-1) - (-2)(1)\right] + \hat{k}\left(2 \times 4 - (-2)(-3)\right)$$

$$\Rightarrow \hat{i}\,(3-4) - \hat{j}\,[-2+2] + k[8-6]$$

$$\Rightarrow -\hat{i} + 2\hat{k}$$

$$|\vec{P} \times \vec{Q}| = \sqrt{(-1)^2 + 2^2} = \sqrt{5}\ m^2$$

(5)

Let the vectors are $\vec{A} = \underset{A_x}{2\hat{i}} + \underset{A_y}{3\hat{j}} - \underset{A_z}{4\hat{k}}$

$$\vec{B} = \underset{B_x}{3\hat{i}} - \underset{B_y}{a\hat{j}} + \underset{B_z}{b\hat{k}}$$

If two vectors are parallel

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

$$\Rightarrow \frac{2}{3} = \frac{3}{-a} = \frac{-4}{b}$$

$$\Rightarrow \frac{2}{3} = \frac{3}{-a} \qquad \frac{2}{3} = \frac{-4}{b}$$

$$\Rightarrow a = -\frac{9}{2} \qquad \Rightarrow b = -6$$

$$= -4.5$$

7

Given $\vec{A} = -4N.$; $\vec{B} = 3N.$

$$|\vec{A} \times \vec{B}|^2 + |\vec{A} \cdot \vec{B}|^2 = (AB\sin\theta)^2 + (AB\cos\theta)^2$$

$$= A^2B^2\sin^2\theta + A^2B^2\cos^2\theta$$

$$= A^2B^2[\sin^2\theta + \cos^2\theta]$$

$$= (-4)^2(3)^2[1]$$

$$= 16 \times 9 \times 1 = 144\,N.$$

8

Given $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ & $\vec{C} = [6\hat{i} + 2\hat{j} - 3\hat{k}]k.$

$\vec{B} = 3\hat{i} - 6\hat{j} + 2\hat{k}$

Given $\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 3 & -6 & 2 \end{vmatrix}$

$$= \hat{i}[3\times2 - (6\times(-6))] - \hat{j}(2\times2 - 6\times3) + k[2\times(-6) - 3\times3]$$

$$= \hat{i}[6+36] - \hat{j}(4-18] + k(-12-9]$$

$$= 42\hat{i} + 14\hat{j} - 21\hat{k}$$

$$\vec{C} = 7(6\hat{i} + 2\hat{j} - 3\hat{k})$$

Given $\vec{C} = k[6\hat{i} + 2\hat{j} - 3\hat{k})$

$k = 7.$

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(9)

let $\vec{a} = \hat{i} + 3\hat{j} + 2\hat{k}$; $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{c} = -\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{AB} = \vec{B} - \vec{A} = (2\hat{i} - \hat{j} + \hat{k}) - (\hat{i} + 3\hat{j} + 2\hat{k})$

$= 2\hat{i} - \hat{j} + \hat{k} - \hat{i} - 3\hat{j} - 2\hat{k}$

$\Rightarrow \hat{i} - 4\hat{j} - \hat{k}$.

$\vec{BC} = \vec{c} - \vec{B} = -\hat{i} + 2\hat{j} + 3\hat{k} - (2\hat{i} - \hat{j} + \hat{k})$

$= -\hat{i} + 2\hat{j} + 3\hat{k} - 2\hat{i} + \hat{j} - \hat{k}$

$= -3\hat{i} + 3\hat{j} + 2\hat{k}$

Area $= \frac{1}{2} | \vec{AB} \times \vec{BC} |$

$$= \frac{1}{2} \begin{vmatrix} i & j & k \\ 1 & -4 & -1 \\ -3 & 3 & 2 \end{vmatrix}$$

$$= \frac{1}{2} \Big| i(-4\times2 - 3\times(-1)] - j[1\times2 - (-3)(-1)] + \hat{k}[1\times3 - (-3)(-4)]$$

$$= \frac{1}{2} [i(-8+3) - j(2-3) + k(3-12)]$$

$$= \frac{1}{2} [-5\hat{i} + \hat{j} - 9\hat{k}]$$

$$= \frac{1}{2} \sqrt{5^2 + 1^2 + 9^2}$$

$$= \frac{1}{2} \sqrt{107}.$$

(6)

$\vec{a} \times \vec{b} = 0$

$ab \sin\theta = 0$

$\Rightarrow \theta = 0$

$\vec{a} \cdot \vec{b} = ab\cos\theta$

$= ab \cos 0$

$= ab$.

$+2\hat{k})$

let $\vec{A} = 2\hat{i} - 3\hat{j} + 6\hat{k}$; $\vec{B} = 6\hat{i} + 3\hat{j} - 2\hat{k}$

Angle between $\vec{A}$ and $\vec{B}$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{(2\hat{i}-3\hat{j}+6\hat{k})\cdot(6\hat{i}+3\hat{j}-2\hat{k})}{\sqrt{2^2+(-3)^2+6^2}\ \sqrt{6^2+3^2+(-2)^2}}$$

$$\cos\theta = \frac{2\cdot 6 + (-3)(3) + 6(-2)}{\sqrt{49}\ \sqrt{49}}$$

$$\cos\theta = \frac{12-9-12}{49} = -\frac{9}{49}$$

(ii)

unit vector $\perp^{a}$ to $\vec{A}$ and $\vec{B}$

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} \ ; \quad \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 6 \\ 6 & 3 & -2 \end{vmatrix}$$

$$= \hat{i}\,[-3\times -2 - 6\times 3] - \hat{j}\,[2\times -2 - 6\times 6] + \hat{k}\,[2\times 3 - 6\times 6]$$

$$= \hat{i}\,[6 - 18] - \hat{j}\,[-4-36] + \hat{k}\,[6+18]$$

$$= -12\hat{i} + 40\hat{j} + 24\hat{k} = 4(-3\hat{i} + 10\hat{j} + 6\hat{k}$$

$$|\vec{A}\times\vec{B}| = 4\sqrt{(-3)^2+10^2+6^2} = 4\sqrt{9+100+36} = 4\sqrt{145}$$

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{\cancel{4}\,(-3\hat{i}+10\hat{j}+6\hat{k})}{\cancel{4}\sqrt{145}}$$

$$\hat{n} = \frac{-3\hat{i}+10\hat{j}+6\hat{k}}{\sqrt{145}}$$

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(17)

let $\vec{A} = \hat{i} + 4\hat{j}$; $\vec{B} = 2\hat{i} + 3\hat{j}$

Area $\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 1 & 4 & 0 \\ 2 & 3 & 0 \end{vmatrix}$

$\Rightarrow i[4\times0 - 3\times0] - j[1\times0 - 2\times0] + k[3-8]$

$\Rightarrow i(0) - j(0) - 5\hat{k} = -5\hat{k}$

$|\vec{A} \times \vec{B}| = 5$ units

(18)

let $\vec{A} = 2\hat{i} - 2\hat{j} + c\hat{k}$; $\vec{B} = 3\hat{i} + 6\hat{j} + 4\hat{k}$

Given $\vec{A}$ and $\vec{B}$ are perpendicular to each other $\vec{A} \cdot \vec{B} = 0$

$(2\hat{i} - 2\hat{j} + c\hat{k}) \cdot (3\hat{i} + 6\hat{j} + 4\hat{k}) = 0$

$\Rightarrow 2\times 3 + (-2)(6) + c\times 1 = 0$

$\Rightarrow 6 - 12 + c = 0$

$\Rightarrow c = 6$