

Task

① Given $\vec{A}$ and $\vec{B}$ are unit vectors $|\vec{A}|=1$; $|\vec{B}|=1$

$$\begin{aligned}\therefore |\vec{A}-\vec{B}| &= \sqrt{A^2+B^2-2AB\cos\theta} \\ &= \sqrt{1^2+1^2-2 \times 1 \times 1 \cos\theta} \\ &= \sqrt{2-2\cos\theta} = \sqrt{2(1-\cos\theta)} \\ &= \sqrt{2\left(2\sin^2\frac{\theta}{2}\right)} = 2\sin\frac{\theta}{2}\end{aligned}$$

②

Given

$$|\vec{P}-\vec{Q}| = R$$

$$P-Q=R$$

$$\Rightarrow \sqrt{P^2+Q^2-2PQ\cos\theta} = R \quad \& \quad (P-Q)^2 = R^2$$

$$\Rightarrow P^2+Q^2-2PQ\cos\theta = R^2 \quad \& \quad P^2+Q^2-2PQ\cos\theta = R^2$$

$\rightarrow \textcircled{1} \quad \quad \quad \rightarrow \textcircled{2}$

By equating $\textcircled{1}$ & $\textcircled{2}$

$$\Rightarrow P^2+Q^2-2PQ\cos\theta = P^2+Q^2-2PQ$$

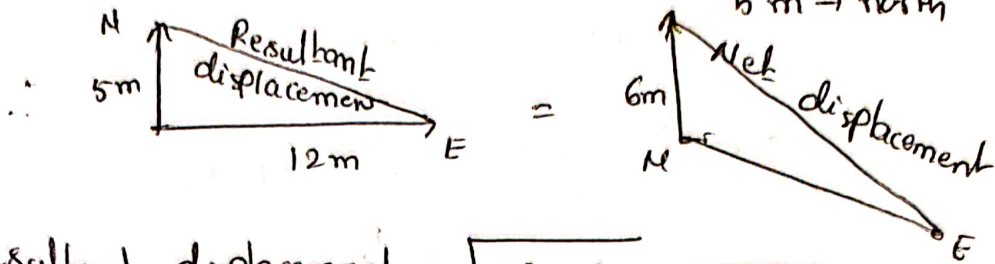
$$\Rightarrow -2PQ\cos\theta = -2PQ$$

$$\Rightarrow \cos\theta = 1 \Rightarrow \theta = 0^\circ$$

③

Given displacement of the particle $12\text{ m} \rightarrow \text{East}$

$5\text{ m} \rightarrow \text{North}$



$$\Rightarrow \text{Resultant displacement} = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = 13\text{ m}$$

$$\text{Net displacement} = \sqrt{13^2 + 6^2} = \sqrt{169 + 36} = 14.32\text{ m}$$

④

Let the two vectors be $\vec{A}$ & $\vec{B}$

Given $|\vec{A}| = |\vec{B}| = P$.

$$\begin{aligned} \therefore |\vec{A} - \vec{B}| &= \sqrt{A^2 + B^2 - 2AB \cos \theta} \\ &= \sqrt{P^2 + P^2 - 2PP \cos \theta} = \sqrt{2P^2 - 2P^2 \cos \theta} \\ &= \sqrt{2P^2 (1 - \cos \theta)} = \sqrt{2P^2 (2\sin^2 \frac{\theta}{2})} = 2P \sin \frac{\theta}{2} \end{aligned}$$

⑤

Let the two forces are F_1 & F_2

When F_1 & F_2 are acting in opp directions $F_1 - F_2 = 10$

When F_1 & F_2 are $\perp$ to each other then $\sqrt{F_1^2 + F_2^2} = 60$

$$(F_1 + F_2)^2 = (F_1 - F_2)^2 + F_1^2 + F_2^2 - 2F_1F_2$$

$$\Rightarrow 10^2 = (60)^2 - 2F_1F_2$$

5th continuation

$$\Rightarrow 100 - 3600 = -2F_1F_2$$

$$\Rightarrow 2F_1F_2 = 3500$$

$$\therefore (F_1 + F_2)^2 = (F_1 - F_2)^2 + 4F_1F_2$$

$$\Rightarrow (10)^2 + 2(3500)$$

$$(F_1 + F_2)^2 = 7100$$

$$\Rightarrow F_1 + F_2 = 10\sqrt{71} = 84.2$$

$$F_1 - F_2 = 10$$

$$2F_1 = 94.2$$

$$\Rightarrow F_1 = 47.1 \text{ and}$$

$$F_1 - F_2 = 10$$

$$\Rightarrow F_2 = F_1 - 10$$

$$= 47.1 - 10$$

$$= 37.1$$

⑥

Let two vectors are $\vec{P}$ & $\vec{Q}$ and their resultant

$$\text{is } \vec{R} = 20 \quad \text{let } \vec{P} = 20\sqrt{3}$$

$$\vec{R} = \vec{P} + \vec{Q}$$

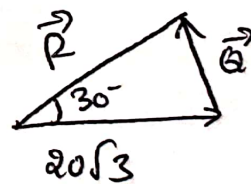
$$\Rightarrow \vec{Q} = \vec{R} - \vec{P}$$

$$\Rightarrow |\vec{Q}| = \sqrt{R^2 + P^2 - 2RP \cos \theta}$$

$$= \sqrt{20^2 + (20\sqrt{3})^2 - 2(20)(20\sqrt{3}) \cos 30}$$

$$= \sqrt{400 + 1200 - 400(2\sqrt{3}) \frac{\sqrt{3}}{2}}$$

$$= \sqrt{1600 - 1200} = \sqrt{400} = 20$$



⑦

let Resultant $\vec{R} = 10\text{m}$; $\vec{P} = 6\text{m}$, $\vec{Q} = 8\text{m}$

$$\therefore \vec{R} = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\Rightarrow 10 = \sqrt{6^2 + 8^2 + 2(6)(8) \cos \theta}$$

$$\Rightarrow 10^2 = 36 + 64 + 96 \cos \theta$$

$$\Rightarrow 100 = 100 + 96 \cos \theta$$

$\Rightarrow \cos \theta = 0 \Rightarrow \theta = 90^\circ$ that two vectors are perpendicular to each other

⑧

let the two forces are $F_1 = 20\text{N}$; $F_2 = 20\text{N}$

$$\theta = 120^\circ$$

$$\text{Magnitude of Resultant} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

$$= \sqrt{(20)^2 + (20)^2 + 2(20)(20) \cos 120^\circ}$$

$$= \sqrt{(20)^2 + (20)^2 + 2(20)^2 \left(-\frac{1}{2}\right)}$$

$$= \sqrt{(20)^2 + (20)^2 - 20^2}$$

$$= \sqrt{20^2} = 20\text{N}$$

$$\text{Direction} \quad \tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta} = \frac{20 \sin 120}{20 + 20 \cos 120}$$

$$= \frac{20 \frac{\sqrt{3}}{2}}{20 \left(1 + \left(-\frac{1}{2}\right)\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$\alpha = 60^\circ$$

⑦

let the two forces are F_1 & F_2

Given that $|\vec{F}_1| = |\vec{F}_2|$ and $\theta = 90^\circ$

Resultant of two forces = 1414 N.

$$\therefore \text{Resultant} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

$$\Rightarrow 1414 = \sqrt{F_1^2 + F_1^2 + 2F_1F_1 \cos 90^\circ}$$

$$\Rightarrow 1414 = \sqrt{2F_1^2 + 2F_1^2(0)}$$

$$\Rightarrow 1414 = \sqrt{2F_1^2} = \sqrt{2} F_1$$

$$\Rightarrow F_1 = \frac{1414}{\sqrt{2}} = \frac{1414}{1.414} = 1000 \text{ N}$$

⑧

let two forces are $F_1 = 1 \text{ N}$ and $F_2 = P \text{ N}$.

Resultant $R = 1 \text{ N}$ and is $\perp$ to 1 N i.e. $\alpha = 90^\circ$

$$\text{Direction of resultant } \tan \alpha = \frac{F_2 \sin \theta}{F_1 + F_2 \cos \theta}$$

$$\Rightarrow \tan 90^\circ = \frac{P \sin \theta}{1 + P \cos \theta}$$

$$\Rightarrow \infty = \frac{P \sin \theta}{1 + P \cos \theta}$$

$$\Rightarrow 1 + P \cos \theta = \frac{P \sin \theta}{\infty} = 0$$

$$\Rightarrow \cos \theta = -\frac{1}{P} \rightarrow \text{①}$$

From ①

$$\cos \theta = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = 135^\circ$$

$$\therefore \text{Magnitude of } R = \sqrt{P^2 + 1^2 + 2P(1) \cos \theta}$$

$$\Rightarrow 1 = \sqrt{1 + P^2 + 2(1)(P) \left(-\frac{1}{P}\right)}$$

$$\Rightarrow 1 \leq 1 + P^2 - 2 \Rightarrow P = \sqrt{2}$$



(11)

Given $\vec{A} = 4\hat{i} - 2\hat{j} + 6\hat{k}$ & $\vec{B} = \hat{i} - 2\hat{j} - 3\hat{k}$

Then $\vec{A} + \vec{B} = 4\hat{i} - 2\hat{j} + 6\hat{k} + \hat{i} - 2\hat{j} - 3\hat{k}$
 $= 5\hat{i} - 4\hat{j} + 3\hat{k}$

$$|\vec{A} + \vec{B}| = \sqrt{5^2 + (-4)^2 + 3^2} = \sqrt{25 + 16 + 9} = 5\sqrt{2}$$

Angle made by the vector $\vec{A} + \vec{B}$ with x-axis

$$\cos \alpha = \frac{\text{x-component}}{|\vec{A} + \vec{B}|} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\alpha = 45^\circ \quad \text{or} \quad \cos \alpha = \frac{5}{5\sqrt{2}} \quad \text{or} \quad \sin \alpha = \frac{5}{5\sqrt{2}}$$

$$\sec \alpha = \sec 45^\circ = \sqrt{2} \Rightarrow \alpha = \sec^{-1}(\sqrt{2})$$

$$\text{or } \cos^{-1}(\frac{1}{\sqrt{2}})$$

(15)

Let $\vec{P} = 5\text{ N}$; $\vec{Q} = 3\text{ N}$ they are acting in same direction ($\theta = 0$) their resultant $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos 0} = \sqrt{P^2 + Q^2 + 2PQ}$$

$$R = \sqrt{(P+Q)^2} = P+Q = 5+3 = 8\text{ N}$$

(16)

Let $\vec{P} = 20\text{ N}$; $\vec{Q} = 25\text{ N}$; $\vec{R} = 39\text{ N}$.

$$\therefore \vec{R} = \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \Rightarrow 39 = \sqrt{20^2 + 25^2 + 2(20)(25) \cos \theta}$$

$$\Rightarrow (39)^2 = 400 + 625 + 1000 \cos \theta$$

$$\Rightarrow 1521 = 1025 + 1000 \cos \theta$$

$$\Rightarrow 1000 \cos \theta = 496$$

$$\Rightarrow \cos \theta = \frac{496}{1000} \approx \frac{1}{2} \Rightarrow \theta = 60^\circ$$

$$\Rightarrow A \approx 60^\circ$$



(17) take solution from Task (9)th question.

(18)

let the two forces are F_1 and F_2 .

Given

$$F_1 + F_2 = 7 \text{ N}$$

$$F_1 - F_2 = 1 \text{ N}$$

Given $F_1 = 4x$

$$F_2 = 3x$$

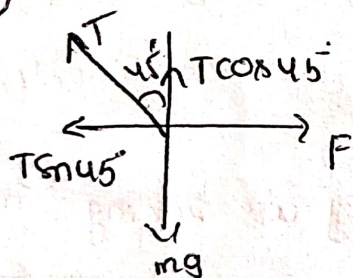
$\therefore$ By substituting F_1 & F_2 value in one of the above we get

$$4x + 3x = 7$$

$$\Rightarrow 7x = 7$$

$$\Rightarrow x = 1$$

(19)



From fig:- under equilibrium

$$F = T \sin 45^\circ$$

$$mg = T \cos 45^\circ$$

$$\Rightarrow \frac{F}{mg} = \frac{T \sin 45^\circ}{T \cos 45^\circ} = \tan 45^\circ = 1$$

(20)

See 9th foundation + ws-3.

CUQA

⑥

Let the two forces are F_1 & F_2

Given

$$F_1 + F_2 = 10 \text{ N}$$

$$F_1 - F_2 = 4 \text{ N} \Rightarrow 7 - F_2 = 4$$

$$2F_1 = 14 \text{ N}$$

$$\Rightarrow F_2 = 3 \text{ N}$$

The large force is $F_1 = 7 \text{ N}$

⑦

$$\text{Given } \vec{P} + \vec{Q} = \vec{R}$$

$$\text{and } \vec{P} - \vec{Q} = \vec{S}$$

$$\Rightarrow P^2 + Q^2 + 2PQ \cos \theta = R^2$$

$$P^2 + Q^2 - 2PQ \cos \theta = S^2$$

$$\begin{aligned} \therefore R^2 + S^2 &= P^2 + Q^2 + 2PQ \cos \theta + P^2 + Q^2 - 2PQ \cos \theta \\ &= 2P^2 + 2Q^2 \\ &= 2(P^2 + Q^2) \end{aligned}$$

⑧

$$\text{Given } F_1 = 3\hat{i} + 4\hat{j} + 2\hat{k}, F_2 = 2\hat{i} + 3\hat{j} - \hat{k}, F_3 = 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\text{their resultant } F_1 + F_2 + F_3 = 3\hat{i} + 4\hat{j} + 2\hat{k} + 2\hat{i} + 3\hat{j} - \hat{k} + 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\Rightarrow 7\hat{i} + 11\hat{j} - 4\hat{k}$$

⑨ For magnitude of $|\vec{A} - \vec{B}|$ is maximum $\theta = 180^\circ$

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB \cos \theta} = \sqrt{A^2 + B^2 - 2AB \cos 180^\circ}$$

$$= \sqrt{A^2 + B^2 - 2AB(-1)} = \sqrt{A^2 + B^2 + 2AB}$$

$$= \sqrt{(A+B)^2} = A+B$$

(10)

Given $|\vec{a} - \vec{b}| = |\vec{a}| + |\vec{b}|$

$$\Rightarrow \sqrt{a^2 + b^2 - 2ab \cos \theta} = a + b$$

$$\Rightarrow a^2 + b^2 - 2ab \cos \theta = (a + b)^2$$

$$\Rightarrow a^2 + b^2 - 2ab \cos \theta = a^2 + b^2 + 2ab$$

$$\Rightarrow -2ab \cos \theta = 2ab$$

$$\Rightarrow \cos \theta = -1 \Rightarrow \theta = 180^\circ$$

Free main level

(1)

Given $|\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$

$$\Rightarrow \sqrt{A^2 + B^2 + 2AB \cos \theta} = A + B$$

$$\Rightarrow A^2 + B^2 + 2AB \cos \theta = (A + B)^2$$

$$\Rightarrow A^2 + B^2 + 2AB \cos \theta = A^2 + B^2 + 2AB$$

$$\Rightarrow 2AB \cos \theta = 2AB$$

$$\Rightarrow \cos \theta = 1 \Rightarrow \theta = 0^\circ$$

(2)

let the forces are F_1 & F_2

Given $\frac{F_1 + F_2}{F_1 - F_2} = \frac{4}{3} \Rightarrow 3(F_1 + F_2) = 4(F_1 - F_2)$

$$\Rightarrow 3F_1 + 3F_2 = 4F_1 - 4F_2$$

$$\Rightarrow 4F_2 + 3F_2 = 4F_1 - 3F_1$$

$$\Rightarrow 7F_2 = F_1$$

$$\Rightarrow \frac{F_1}{F_2} = \frac{7}{1}$$



③

let the given vector is $\vec{A} = \hat{j} + \hat{k}$

magnitude of $\vec{A} = |\vec{A}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

Angle made by the vector with y-axis is

$$\cos \beta = \frac{\text{y-component}}{|\vec{A}|} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \beta = 45^\circ$$

④

let the two forces are $2P$ and $\sqrt{2}P$

Their resultant $R = \sqrt{10}P$.

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\Rightarrow R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

$$\Rightarrow (\sqrt{10}P)^2 = (2P)^2 + (\sqrt{2}P)^2 + 2(2P)(\sqrt{2}P) \cos \theta$$

$$\Rightarrow 10P^2 = 4P^2 + 2P^2 + 4\sqrt{2}P^2 \cos \theta$$

$$\Rightarrow 10P^2 = 6P^2 + 4\sqrt{2}P^2 \cos \theta$$

$$\Rightarrow 10 - 6 = 4\sqrt{2} \cos \theta \Rightarrow 4\sqrt{2} \cos \theta = 4 \Rightarrow \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

⑤

let the two forces are $Q = 500 \text{ gm wt}$ & $P = 250 \text{ gm wt}$.

Given the resultant is perpendicular to any one of the two forces

let it be P then $\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta} \Rightarrow \tan 90^\circ = \frac{Q \sin \theta}{P + Q \cos \theta}$

$$\Rightarrow \infty = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$\Rightarrow P + Q \cos \theta = 0 \Rightarrow \cos \theta = -\frac{P}{Q} = -\frac{250}{500} = -\frac{1}{2}$$

$$\Rightarrow \theta = 120^\circ$$



⑥

Let the two forces are $\vec{P}$ and $\vec{Q}$

Given $\frac{\vec{P}}{\vec{Q}} = \frac{3}{5} = \frac{3x}{5x}$: Their resultant $R = 35\text{N}$

Angle between them $\theta = 60^\circ$

$\therefore$ the magnitude of ^{Resultant} each force $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$

$$\Rightarrow R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

$$\Rightarrow 35^2 = (3x)^2 + (5x)^2 + 2(3x)(5x) \cos 60^\circ$$

$$\Rightarrow (35)^2 = 9x^2 + 25x^2 + 15x^2 \times \frac{1}{2}$$

$$\Rightarrow (35)^2 = 49x^2$$

$$\Rightarrow x^2 = \frac{(35)^2}{49} \Rightarrow x = \frac{35}{7} = 5$$

$\therefore P = 3x = 15\text{N}$ and $Q = 5 \times 5 = 25\text{N}$.

⑦

Let the two forces are $\vec{P}$ and $\vec{Q}$

The greatest resultant $\vec{P} + \vec{Q} = 29$

least resultant $\vec{P} - \vec{Q} = 5$

$$2P = 34 \Rightarrow P = 17 \text{ kg wt}$$

From $P - Q = 5 \Rightarrow Q = P - 5 = 17 - 5 = 12 \text{ kg wt}$

Given magnitude of each force is increased by 3 kg wt

$$\therefore P' = 17 + 3 = 20 \text{ kg wt} \quad \text{and} \quad Q' = 12 + 3 = 15 \text{ kg wt}$$

$\therefore$ Given P' and Q' are perpendicular each other

Their resultant $R = \sqrt{P'^2 + Q'^2} = \sqrt{(20)^2 + (15)^2}$
 $= \sqrt{400 + 225}$
 $= \sqrt{625} = 25 \text{ kg wt}$

8

let the displacements are $x = 300 \text{ m}$ & $y = 400 \text{ m}$
they are acting at right angles to each other.

$$\begin{aligned}\text{Their resultant} &= \sqrt{x^2 + y^2 + 2xy \cos \theta} \\ &= \sqrt{(300)^2 + (400)^2 + 2 \times 300 \times 400 \cos 90^\circ} \\ &= \sqrt{9 \times 100^2 + 16 \times 100^2 + 2 \times 3 \times 4 \times 100^2 \times 0} \\ &= 100 \sqrt{9 + 16} = 500 \text{ m}\end{aligned}$$

9

The two forces acting on the particle are P and Q

$$\text{Given } P = 3 \text{ N} ; Q = 4 \text{ N}.$$

The net force acting on the particle is in between

$$\begin{aligned}\vec{P} + \vec{Q} \text{ and } \vec{P} - \vec{Q} &\Rightarrow 3 + 4 \text{ to } 3 - 4 \\ &\Rightarrow 7 \text{ N. to } 1 \text{ N}\end{aligned}$$

10

let the two forces are $\vec{P} = 8 \text{ N} ; \vec{Q} = 15 \text{ N}$

Their resultant $\vec{R} = 17 \text{ N}$.

According to parallelogram law of vectors

$$\begin{aligned}\vec{R} &= \sqrt{P^2 + Q^2 + 2PQ \cos \theta} \\ \Rightarrow 17 &= \sqrt{8^2 + 15^2 + 2 \times 8 \times 15 \cos \theta} \\ \Rightarrow 17^2 &= 64 + 225 + 240 \cos \theta \\ \Rightarrow 289 &= 289 + 240 \cos \theta \\ \Rightarrow 240 \cos \theta &= 0 \Rightarrow \cos \theta = 0 \\ &\Rightarrow \theta = 90^\circ\end{aligned}$$

(11)

Let the two forces are $|\vec{P}| = |\vec{Q}| = 10 \text{ N}$ & $\theta = 60^\circ$

The magnitude of Resultant $R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$

$$\begin{aligned} \Rightarrow R &= \sqrt{10^2 + 10^2 + 2(10)(10) \cos 60^\circ} \\ &= \sqrt{100 + 100 + 2 \times 100 \times \frac{1}{2}} \\ &= \sqrt{300} = 10\sqrt{3} \text{ N.} \end{aligned}$$

The direction of resultant $\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$

$$\Rightarrow \tan \alpha = \frac{10 \sin 60^\circ}{10 + 10 \cos 60^\circ} = \frac{10 \frac{\sqrt{3}}{2}}{10(1 + \frac{1}{2})}$$

$$\Rightarrow \tan \alpha = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \alpha = 30^\circ$$

(12)

Let the two forces are $F_1 = x + y$; $F_2 = x - y$.

Resultant $R = \sqrt{2(x^2 + y^2)}$

According to Parallelogram law of vector

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta}$$

$$\Rightarrow \sqrt{2(x^2 + y^2)} = \sqrt{(x+y)^2 + (x-y)^2 + 2(x+y)(x-y) \cos \theta}$$

Squaring on both sides. we get

$$\Rightarrow 2(x^2 + y^2) = x^2 + y^2 + 2xy + x^2 + y^2 - 2xy + 2(x^2 - y^2) \cos \theta$$

$$\Rightarrow 2x^2 + 2y^2 = 2x^2 + 2y^2 + 2(x^2 - y^2) \cos \theta$$

$$\Rightarrow 2(x^2 - y^2) \cos \theta = 0$$

$$\Rightarrow \cos \theta = 0 \Rightarrow \theta = 90^\circ$$

(3)

Let the two vectors are P and Q

Given $|\vec{P}| = 8$, $|\vec{Q}| = 6$; Their resultant $R = 10$.

According to parallelogram law of vectors

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\Rightarrow 10 = \sqrt{8^2 + 6^2 + 2 \times 8 \times 6 \cos \theta}$$

$$\Rightarrow 10^2 = 64 + 36 + 96 \cos \theta$$

$$\Rightarrow 100 = 100 + 96 \cos \theta \Rightarrow 96 \cos \theta = 0$$

$$\Rightarrow \cos \theta = 0 \Rightarrow \theta = 90^\circ$$

(14)

Let the two vectors are P and Q

Their resultant is R . Given $|\vec{P}| = |\vec{Q}| = |\vec{R}|$

According to parallelogram law of vectors

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

$$\Rightarrow R^2 = P^2 + P^2 + 2PP \cos \theta$$

$$\Rightarrow 0 = P^2 + 2P^2 \cos \theta \Rightarrow 2P^2 \cos \theta = -P^2$$

$$\Rightarrow 2 \cos \theta = -1$$

$$\Rightarrow \cos \theta = -\frac{1}{2} \Rightarrow \theta = 120^\circ$$

(15)

Given $|\vec{F}_1| = |\vec{F}_2| = f$

According to parallelogram law of vector

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta$$

$$\Rightarrow R^2 = f^2 + f^2 + 2ff \cos \theta$$

$$\Rightarrow R^2 = 2f^2 + 2f^2 \cos \theta$$

$$\Rightarrow R^2 = 2f^2 (1 + \cos \theta) = 2f^2 (2 \cos^2 \frac{\theta}{2})$$

$$\Rightarrow R^2 = 4f^2 \cos^2 \frac{\theta}{2}$$

$$\Rightarrow R = 2f \cos \frac{\theta}{2}$$

(16)

Given $|\vec{A}| = |\vec{B}|$ and

$$|\vec{A} + \vec{B}| = n |\vec{A} - \vec{B}|$$

$$\Rightarrow \sqrt{A^2 + B^2 + 2AB \cos \theta} = n \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

on squaring both sides

$$\Rightarrow (\sqrt{A^2 + B^2 + 2AB \cos \theta})^2 = n^2 (\sqrt{A^2 + B^2 - 2AB \cos \theta})^2$$

$$\Rightarrow A^2 + B^2 + 2AB \cos \theta = n^2 (A^2 + B^2 - 2AB \cos \theta)$$

Since $|\vec{A}| = |\vec{B}|$

$$\therefore A^2 + A^2 + 2AA \cos \theta = n^2 (A^2 + A^2 - 2AA \cos \theta)$$

$$\Rightarrow 2A^2 + 2A^2 \cos \theta = n^2 (2A^2 - 2A^2 \cos \theta)$$

$$\Rightarrow 2A^2 (1 + \cos \theta) = n^2 2A^2 (1 - \cos \theta)$$

$$\Rightarrow 2 \cos^2 \frac{\theta}{2} = n^2 2 \sin^2 \frac{\theta}{2}$$

$$\Rightarrow \cos \frac{\theta}{2} = n \sin \frac{\theta}{2} \Rightarrow \frac{1}{n} = \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}}$$

$$\Rightarrow \tan \frac{\theta}{2} = \frac{1}{n} \Rightarrow \frac{\theta}{2} = \tan^{-1} \left(\frac{1}{n} \right)$$

$$\Rightarrow \theta = 2 \tan^{-1} \left(\frac{1}{n} \right)$$

(19)

let the two forces are $\vec{P} = 7\text{N}$ & $\vec{Q} = 5\text{N}$.

The resultant of two forces (or) vectors lies in between

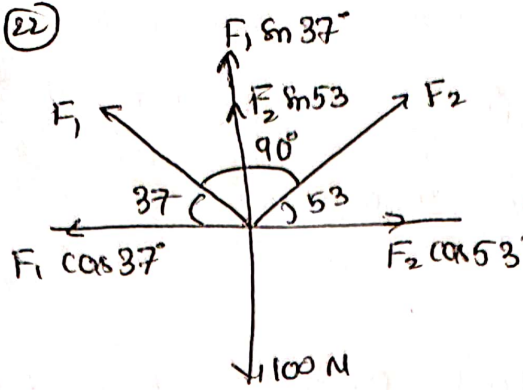
$$|\vec{P} - \vec{Q}| \text{ and } \vec{P} + \vec{Q}$$

$$\Rightarrow 7 - 5 \text{ and } 7 + 5$$

$$\Rightarrow 2\text{N and } 12\text{N, so } 3\text{N lies in between } 2\text{N and } 12\text{N.}$$

Their resultant 3N is possible

(20), (21), (22)



under equilibrium
Along x-axis
 $F_2 \cos 53 = F_1 \cos 37$

$$\Rightarrow F_2 \times \frac{3}{5} = F_1 \times \frac{4}{5}$$

$$\Rightarrow F_2 = \frac{4}{3} F_1$$

Along y-axis

$$F_1 \sin 37 + F_2 \sin 53 = 100$$

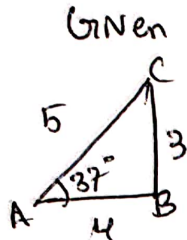
$$\Rightarrow F_1 \times \frac{3}{5} + \frac{4F_1}{3} \times \frac{4}{5} = 100$$

$$\Rightarrow \frac{9F_1 + 16F_1}{15} = 100$$

$$\Rightarrow \frac{25F_1}{15} = 100$$

$$\Rightarrow F_1 = 60 \text{ N}$$

$$\therefore F_2 = \frac{4}{3} \times 60 = 80 \text{ N}$$



Given $\tan 37 = \frac{3}{4}$; $\tan 53 = \frac{4}{3}$

$$AC^2 = AB^2 + BC^2$$

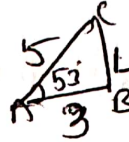
$$= 4^2 + 3^2$$

$$= 16 + 9$$

$$\Rightarrow AC^2 = 25$$

$$\Rightarrow AC = 5 \text{ N}$$

$$\cos 37 = \frac{4}{5} ; \sin 37 = \frac{3}{5}$$



$$\cos 53 = \frac{3}{5}$$

$$\sin 53 = \frac{4}{5}$$

Angle between F_1 & F_2 is 90°

(33)

let two forces are F_1 & F_2

Given $F_1 = F_2 = 15 \text{ N}$ and $\theta = 120^\circ$; Resultant = $5x$

According to Parallelogram law of vectors

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta}$$

$$\Rightarrow 5x = \sqrt{(15)^2 + (15)^2 + 2(15)(15) \cos 120^\circ}$$

$$\Rightarrow 5x = \sqrt{(15)^2 + (15)^2 + 2(15)^2 \left(-\frac{1}{2}\right)}$$

$$\Rightarrow 5x = \sqrt{(15)^2 + (15)^2 - (15)^2}$$

$$\Rightarrow 5x = \sqrt{15^2}$$

$$\Rightarrow 5x = 15$$

$$\Rightarrow x = 3$$

(24)

Let the two forces are $F_1 = 10\text{ N}$; $F_2 = 5\text{ N}$.

Here the force is minimum when angle between them is 180°

Then the resultant $R = F_1 - F_2 = 10 - 5 = 5\text{ N}$

(25)

Let the two forces are $\vec{P}$ and $\vec{Q}$

Given $\vec{Q} = 2\vec{P}$

Given resultant is $\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$

Perpendicular to smaller

force i.e. $\alpha = 90^\circ \Rightarrow \tan 90^\circ = \frac{Q \sin \theta}{P + Q \cos \theta}$

$$\Rightarrow P + Q \cos \theta = \frac{Q \sin \theta}{\tan 90^\circ}$$

$$\Rightarrow P + Q \cos \theta = 0$$

$$\Rightarrow \cos \theta = -\frac{P}{Q} = -\frac{P}{2P} = -\frac{1}{2}$$

$$\Rightarrow \theta = 120^\circ$$

$$\Rightarrow \frac{2\pi}{n} = \frac{2\pi}{3}$$

$$\Rightarrow \underline{n=3}$$

$$\text{Given } \theta = \frac{2\pi}{n}$$