8th Foundation+

WS-6

Task

①

Given $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ & $\vec{b} = -4\hat{i} - 2\hat{j} + 3\hat{k}$ are taken in order $\therefore \vec{c} = -(\vec{a} + \vec{b})$

$= -[2\hat{i} + \hat{j} + \hat{k} + (-4\hat{i} - 2\hat{j} + 3\hat{k})]$

$= -[-2\hat{i} - \hat{j} + 4\hat{k}]$

②

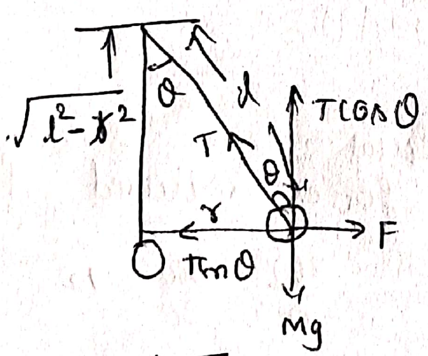
Given $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$; $\vec{b} = 3\hat{i} + 2\hat{j} + \hat{k}$ are represented as the adjacent sides of a triangle taken in order then the closing side $= -(\vec{a} + \vec{b})$

(-) represent closing side is taken in reverse order

$\therefore$ closing side $= 2\hat{i} + \hat{j} - \cancel{\hat{k}} + 3\hat{i} + 2\hat{j} + \cancel{\hat{k}}$

$= 5\hat{i} + 3\hat{j}$

④



$\cos\theta = \dfrac{\sqrt{l^2 - r^2}}{l}$

$T\cos\theta = mg \Rightarrow T = \dfrac{Mg}{\cos\theta}$

$T\sin\theta = F$

$\therefore \dfrac{T\sin\theta}{T\cos\theta} = \dfrac{F}{mg}$

$\Rightarrow \tan\theta = \dfrac{F}{mg}$

$= \dfrac{Mg}{\dfrac{\sqrt{l^2 - r^2}}{l}}$

$T = \dfrac{Mgl}{\sqrt{l^2 - r^2}}$

(5) Given $|\vec{A}| = 12$; $|\vec{B}| = 5$; $|\vec{C}| = 13$.

$\vec{C} = \vec{A} + \vec{B}$

$$\Rightarrow \quad C = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\Rightarrow \quad C^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\Rightarrow \quad 13^2 = 12^2 + 5^2 + 2(12)(5)\cos\theta$$

$$\Rightarrow \quad \cancel{169} = \cancel{144 + 25} + 120\cos\theta \quad \Rightarrow 120\cos\theta = 0$$

$$\Rightarrow \cos\theta = 0$$

$$\Rightarrow \theta = 90^\circ \text{ or } \frac{\pi}{2}$$

(6)

From triangle law of vector addition

The resultant must be zero if the sum of two vectors $\geq$ remaining vector

In all the options except D, the sum of two forces $\geq$ remaining force.

However For option D, $10 + 20 = 30 < 40$

Hence the resultant can not be zero for option D.

(7)

The minimum no. of equal forces required for resultant to be 0 is $\frac{360}{\theta} = \frac{360}{30} = 12$

Here 11 equal forces of magnitude 5N are there.

so remaining 1 more 5N required to make resultant '0'

(8)

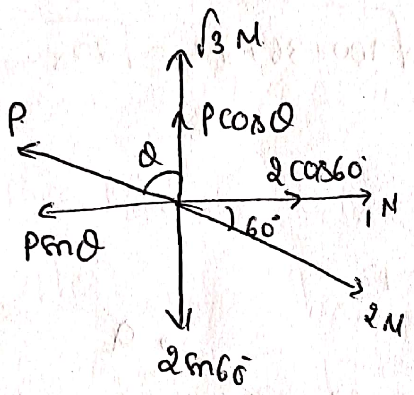
No. of forces are, $N = 6$.

If 'N' forces are acting at point, the resultant is zero if the angle between the forces is $\frac{360}{\theta}$

Here $\theta = \frac{360}{6} = 60^\circ$

So the resultant is zero

(9)



under equilibrium

Net force along x-axis = 0

i.e. $P \sin\theta = 2\cos 60 + 1$

$= 2 \times \frac{1}{2} + 1$

$P \sin\theta = 2$

Net force along y-axis = 0

$\Rightarrow P\cos\theta + \sqrt{3} = 2\sin 60$

$\Rightarrow P\cos\theta + \sqrt{3} = \cancel{2}\frac{\sqrt{3}}{\cancel{2}}$

$\Rightarrow P\cos\theta + \sqrt{3} = \sqrt{3}$

$\Rightarrow P\cos\theta = 0 \Rightarrow \theta = 90^\circ$

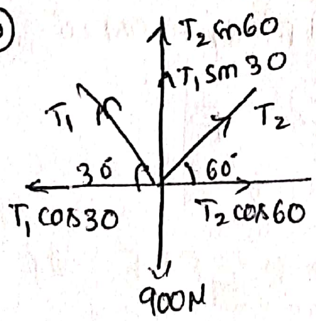
∴ we know that

$P\sin\theta = 2$

$\Rightarrow P\sin 90 = 2$

$\Rightarrow P = 2N$.

(10)



under equilibrium

Net force along x-axis = 0

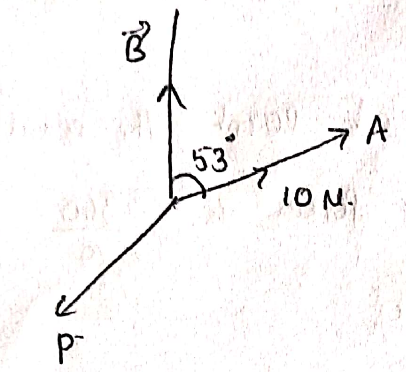
$\Rightarrow T_1 \cos 30 = T_2 \cos 60$

$\Rightarrow T_1 \frac{\sqrt{3}}{\cancel{2}} = T_2 \frac{1}{\cancel{2}}$

$\Rightarrow \frac{T_1}{T_2} = \frac{1}{\sqrt{3}}$

Advanced level

①



we know that when the system is at equilibrium

$$\vec{A}+\vec{B}+\vec{P}=0$$

$$\Rightarrow \vec{P} = -(\vec{A}+\vec{B})$$

$$\Rightarrow |\vec{P}| = \sqrt{A^2+B^2+2AB\cos\theta}$$

Given

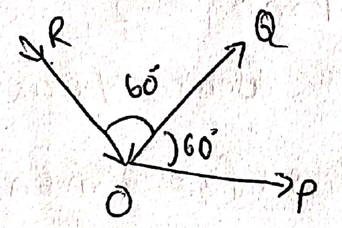
$\cos 53 = \frac{3}{5}$

$$= \sqrt{10^2+6^2+2\times6\times\cancel{10}^{2}\times\frac{3}{\cancel{5}}}$$

$$= \sqrt{100+36+72} = \sqrt{208}$$

②

Given $|\vec{R}| = |\vec{Q}| = |\vec{P}| = F$



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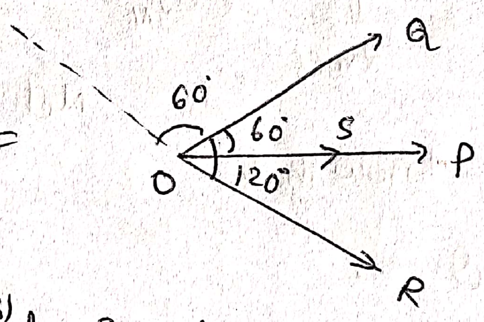


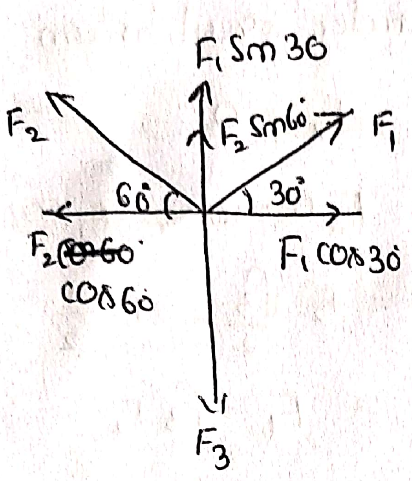
So the resultant(S) of Q and R is along OP, we can find the resultant(S) by using Parallelogram law of vectors

$$S = \sqrt{R^2+Q^2+2RQ\cos\theta} = \sqrt{F^2+F^2+2FF\cos 120^\circ}$$

$$= \sqrt{2F^2+2F^2(-\tfrac{1}{2})} = \sqrt{2F^2-F^2} = F$$

$\therefore$ Net force is along OP $= S+P = F+F = 2F$

4



under equilibrium

$F_x = 0$

$\Rightarrow F_1 \cos 30 = F_2 \cos 60$

$\Rightarrow F_1 \frac{\sqrt{3}}{\cancel{2}} = F_2 \cdot \frac{1}{\cancel{2}}$

$\Rightarrow \sqrt{3} F_1 = F_2 \Rightarrow \sqrt{3} F_1 = 5\sqrt{3}$

$\Rightarrow F_1 = 5N$

$F_y = 0$

$F_1 \sin 30 + F_2 \sin 60 = F_3$

$\Rightarrow F_1 \frac{1}{2} + \sqrt{3} F_1 \frac{\sqrt{3}}{2} = \cancel{8\sqrt{3}}\ F_3$

$\Rightarrow \frac{F_1}{2} + \frac{3F_1}{2} = \cancel{5\sqrt{3}}\ F_3$

$\Rightarrow 2F_1 = \cancel{5\sqrt{3}}\ F_3$

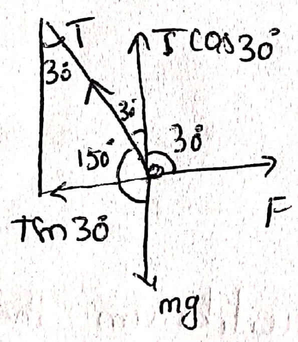
$\Rightarrow \cancel{F_1 = \frac{5\sqrt{3}}{2}}$

$F_3 = 2 \times 5 = 10N.$

Given $F_2 = 5\sqrt{3}$ N

$\therefore \cancel{F_2 = \sqrt{3} \times 5 \times \frac{\sqrt{3}}{2}}$

7, 8, 9



Given $m = 0.05$ kg

$T \sin 30 = F$; $T \cos 30 = mg$

$\Rightarrow \frac{T \sin 30}{T \cos 30} = \frac{F}{mg} \Rightarrow \tan 30 = \frac{F}{mg}$

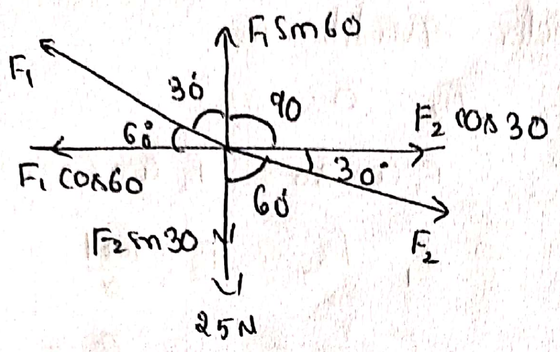
$\Rightarrow F = mg \tan 30 = 0.05 \times 10 \times \frac{1}{\sqrt{3}} = \frac{0.5}{\sqrt{3}}$

$\Rightarrow F = 0.28$

we know $T \sin 30 = F \Rightarrow T\left(\frac{1}{2}\right) = 0.28 \Rightarrow T = 0.56N$

$\frac{F}{T} = \frac{0.28}{0.56} = \frac{1}{2} = 0.5$

(10)



under equilibrium

$F_x = 0$

$F_2 \cos 30 = F_1 \cos 60$

$\Rightarrow F_2 \frac{\sqrt{3}}{2} = F_1 \frac{1}{2}$

$\Rightarrow F_1 = \sqrt{3} F_2$

$F_y = 0$

$F_1 \sin 60 = F_2 \sin 30 + 25$

$\Rightarrow F_1 \frac{\sqrt{3}}{2} = F_2 \left(\frac{1}{2}\right) + 25$

$\Rightarrow (\sqrt{3}) F_2 \frac{\sqrt{3}}{2} = \frac{F_2}{2} + 25$

$\Rightarrow \frac{3F_2}{2} - \frac{F_2}{2} = 25$

$\Rightarrow F_2 = 25N$

$\therefore F_1 = \sqrt{3}(25) = 25\sqrt{3}$ N.

Angle between 25N and F_2 is 60°

Angle between F_1 and $F_2 = 30 + 90 + 30$

$= 150°$

L Task

CUQ's

(5)

To keep a particle in equilibrium the net force acting on the body is zero.

(i.e) $\vec{A} + \vec{B} + \vec{C} = 0$.

$\Rightarrow \vec{C} = -(\vec{A} + \vec{B})$

$= -(\hat{i} + \cancel{\hat{j}} + \hat{k} + 2\hat{i} - \cancel{\hat{j}} + 3\hat{k})$

$= -(3\hat{i} + 4\hat{k})$

(6)

The coordinates of A, B, C, D & E are (0,0) (0,2) (3,1) (2,5) & (1,0)

Displacement = E − A = (1−0, 0−0)

= (1, 0)

(7) If N equal forces are acting at a point in equilibrium, their resultant is zero if the angle between successive forces is $\frac{360}{\theta}$: Given $\theta = 90^\circ$

$$N = \frac{360}{90} = 4.$$

Jee mains level

(11)

Mass = 100 kg

$l = 2m.$

$\theta = 60^\circ$

$$\frac{F}{mg} = \tan\theta$$

$\Rightarrow F = Mg \tan\theta$

$= 100 \times 9.8 \tan 60^\circ$

$= 980\sqrt{3}$ N

(12) Given $F_1 = 2\hat{i} + \hat{j} - \hat{k}$ N

$F_2 = 2\hat{i} + 3\hat{j} - 3\hat{k}$

$F_3 = a(\hat{i} + \hat{j} - \hat{k})$

under equilibrium $\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$

$\Rightarrow 2\hat{i} + \hat{j} - \hat{k} + 2\hat{i} + 3\hat{j} - 3\hat{k} + a(\hat{i} + \hat{j} - \hat{k}) = 0$

$\Rightarrow 4\hat{i} + 4\hat{j} - 4k + a\hat{i} + a\hat{j} - a\hat{k} = 0$

$\Rightarrow (4+a)\hat{i} + (4+a)\hat{j} - (4+a)\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

$a + 4 = 0$

$\Rightarrow a = -4$

(14)

Given $m = 100$ kg

$l = 2m$

$r = 50cm$

we know that $F = mg\tan\theta$

$$F = 100 \times 9.8 \times \frac{r}{\sqrt{l^2 - r^2}}$$

$$F = 980 \times \frac{50 \times 10^{-2}}{\sqrt{4 - \frac{1}{4}}}$$

$$\Rightarrow F = \frac{490}{\frac{\sqrt{15}}{2}} = \frac{2 \times 490}{\sqrt{15}} \simeq 980N.$$

(15)

we know that $T \propto \frac{1}{l}$

$$\Rightarrow \frac{T_1}{T_2} = \frac{l_2}{l_1} = \frac{3}{4}.$$

(18)

No. of forces required to make resultant

zero $N = \frac{360}{\theta} = \frac{360}{90} = 4.$

But Given 3 equal forces of magnitude 5N.

$\therefore$ one more force is required i.e) 5N to make

resultant 0.

19

$m = 10\ kg$ Horizontal force $F = mg \tan\theta$

$\theta = 60^\circ$

$= 10 \times g \times \tan 60^\circ$

$= 10g \times \sqrt{3}$

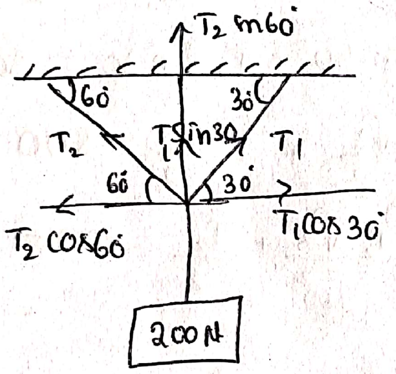
$= 10\sqrt{3}$ kg wt

20

$m = 2\ kg$; $\theta = 60^\circ$ $F = T \sin\theta$

$\Rightarrow \frac{F}{T} = \sin 60^\circ = \frac{\sqrt{3}}{2}$

21



under equilibrium

$F_x = 0 \Rightarrow T_1 \cos 30^\circ = T_2 \cos 60^\circ$

$\Rightarrow T_1 \frac{\sqrt{3}}{\cancel{2}} = T_2 \frac{1}{\cancel{2}}$

$\Rightarrow T_2 = \sqrt{3}\, T_1$

$F_y = 0$

$\Rightarrow T_1 \sin 30 + T_2 \sin 60 = 200$

$\Rightarrow T_1 \frac{1}{2} + T_2 \frac{\sqrt{3}}{2} = 200$

$\Rightarrow \frac{T_1}{2} + \sqrt{3}\, T_1 \frac{\sqrt{3}}{2} = 200$

$\Rightarrow 2T_1 = 200 \Rightarrow T_1 = 100 N$

$T_2 = \sqrt{3}\, T_1$

$= 100\sqrt{3}$ N.

22

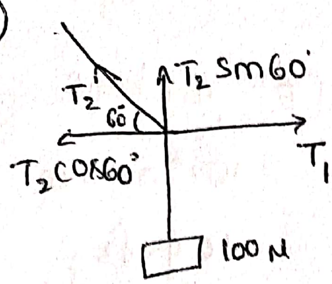
According to triangle law of vector addition

The resultant is zero if sum of two forces $\geq$ Remaining force

From option (A) $3+7 = 10 < 11N$. so the set of

3N, 7N, 11N do not form zero resultant.

(23)



under equilibrium $T_2 \sin 60 = 100$

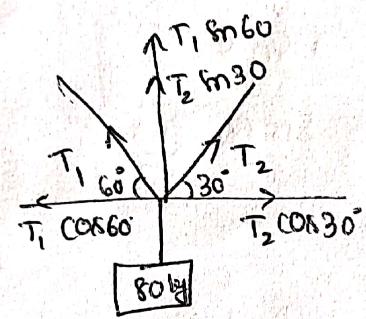
$\Rightarrow T_2 \times \frac{\sqrt{3}}{2} = 100$

$\Rightarrow T_2 = \frac{200}{\sqrt{3}}$ N

$T_1 = T_2 \cos 60$

$= \frac{200}{\sqrt{3}} \frac{1}{2} = \frac{100}{\sqrt{3}}$ N.

(24)



under equilibrium $T_2 \cos 30 = T_1 \cos 60$

$\Rightarrow T_2 \frac{\sqrt{3}}{\cancel{2}} = T_1 \frac{1}{\cancel{2}}$

$\Rightarrow T_1 = \sqrt{3}\, T_2$

$T_1 \sin 60 + T_2 \sin 30 = 80 g$

$\Rightarrow T_1 \frac{\sqrt{3}}{2} + T_2 \frac{1}{2} = 800$

$\Rightarrow \sqrt{3}\, T_2 \frac{\sqrt{3}}{2} + \frac{T_2}{2} = 800 \Rightarrow 2T_2 = 800$

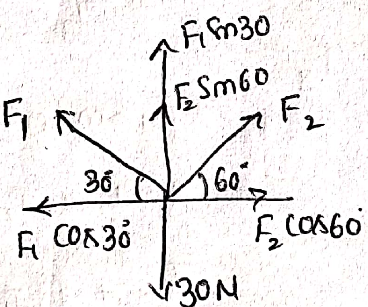
$\Rightarrow T_2 = 400$ N.

$= 40$ kgwt

$T_1 = \sqrt{3} \times 40$

$= 40\sqrt{3}$ kg wt.

(25), (26)



under equilibrium

$F_1 \cos 30 = F_2 \cos 60$

$\Rightarrow F_1 \frac{\sqrt{3}}{\cancel{2}} = F_2 \frac{1}{\cancel{2}}$

$\Rightarrow F_2 = \sqrt{3}\, F_1$

$F_1 \sin 30 + F_2 \sin 60 = 30$

$\Rightarrow F_1 \frac{1}{2} + F_2 \frac{\sqrt{3}}{2} = 30$

$\Rightarrow \frac{F_1}{2} + \sqrt{3}\, F_1 \frac{\sqrt{3}}{2} = 30$

$\Rightarrow \frac{F_1}{2} + \frac{3F_1}{2} = 30$

$\Rightarrow 2F_1 = 30 \Rightarrow F_1 = 15$ N

$\therefore F_2 = \sqrt{3} \times 15 = 15\sqrt{3}$ N

(27)

when three forces acting on a body can be represented as sides of triangle taken in order, then the net force acting on the body is zero

$\therefore$ Acceleration $= 0$.

(28)

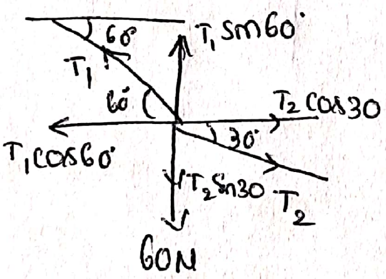
F is the resultant of 20N & 20N here $\theta = 120^\circ$

$$F = \sqrt{(20)^2 + (20)^2 + 2(20)(20)\cos 120^\circ}$$

$$= \sqrt{(20)^2 + (20)^2 + \cancel{2}(20)^2\left(-\frac{1}{\cancel{2}}\right)}$$

$$= \sqrt{(20)^2 + \cancel{(20)^2} - \cancel{(20)^2}} = \sqrt{(20)^2} = 20\,N = 2 \times 10\,N$$

(29)



under equilibrium

$T_1 \cos 60^\circ = T_2 \cos 30$

$\Rightarrow T_1 \frac{1}{\cancel{2}} = T_2 \frac{\sqrt{3}}{\cancel{2}}$

$\Rightarrow T_1 = \sqrt{3}\,T_2$

$T_1 \sin 60 = T_2 \sin 30 + 60$

$\Rightarrow T_1 \frac{\sqrt{3}}{2} = T_2 \left(\frac{1}{2}\right) + 60$

$\Rightarrow \sqrt{3}\,T_2 \frac{\sqrt{3}}{2} = \frac{T_2}{2} + 60$

$\Rightarrow \frac{3T_2}{2} = \frac{T_2}{2} + 60 \Rightarrow T_2 = 60N.$

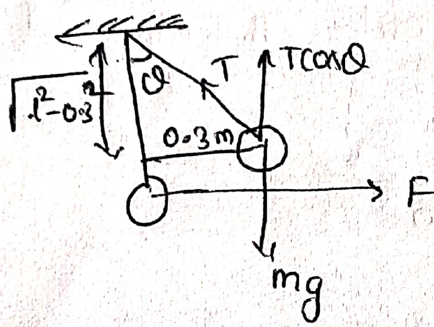
Given $T_2 = 15P$

$\Rightarrow \overset{4}{\cancel{60}} = \cancel{15}P$

$\Rightarrow P = 4$

(30)

$l = 0.5m$



$T\cos\theta = mg$

$\Rightarrow T\frac{\sqrt{l^2-0.3^2}}{l} = 8$

$\Rightarrow T\frac{\sqrt{(0.5)^2-(0.3)^2}}{(0.5)} = 8$

$\Rightarrow T \times \frac{\sqrt{(0.4)^2}}{0.5} = 8$

$\Rightarrow T \times \frac{\cancel{4}}{5} = \cancel{8}\,2$

$\Rightarrow \frac{T}{5} = 2$

$\Rightarrow T = 10\,N$

$\sin\theta = \frac{0.3}{l}$

(b) $F = T\sin\theta$

$= 10 \times \frac{0.3}{0.5}$

$F = 10 \times \frac{3}{5} = 6N$

(c) $T \times F = 10 \times 6 = 60$

(d) $\frac{T}{F} = \frac{10}{6} = 1.66$