POLYNOMIALS

§§ **Homogeneous expression :** An expression is said to be a homogeneous expression if all its terms are of same degree.

Ex : (i) $ax^2 + hxy + by^2$

(ii) $ax^2 + by^2 + cz^2 + fyz + gzx + hxy$

(iii) $ax^3 + bx^2y$

Note : If a homogeneous expression can be split into product of two algebraic expressions, then each of them are homogeneous and the sum of their degrees is equal to the degree of the original expression.

§§ **Cyclic expression :** An expression $f(x,y,z)$ is said to be cyclic if $f(x,y,z) = f(y,z,x)$

Ex : $f(x,y,z) = x(y+z) + y(z+x) + z(x+y)$

Note : (i) If $f(x,y,z)$ is cyclic, then $f(x,y,z) = f(y,z,x) = f(z,x,y)$

(ii) If $f(x,y,z)$ is cyclic and we know one term, we can write the other two terms.

§§ **Symmetrical function :** A function is said to be symmetrical with respect to two variables if its value is unaltered by interchanging them.

Ex : $f(a,b,c)$ is symmetrical about *a, b* if $f(a,b,c) = f(b,a,c)$

A function is said to be symmetric if it is symmetrical about each pair of its variables.

Ex : (1) $E(a,b,c) = a^2 + b^2 + c^2 + ab + bc + ca$

(2) $E(a,b,c) = a(b-c)^2 + b(c-a)^2 + c(a-b)^2$

¶¶ **Remainder theorem :** If *f*(*x*) is an algebraic expression and *f*(*a*) = 0, then (*x*–*a*) is a factor of *f*(*x*).

Ex : $f(a) = a^2(b-c) + b^2(c-a) + c^2(a-b)$

If we take a = b, then

$f(b) = b^2(b-c) + b^2(c-b) + c^2(b-b) = 0$

$\therefore (a-b)$ is a factor of *f*(*a*).

¶¶ **Alternating function :** A function is said to be alternating with respect to its variables, when its sign but not its value is altered by interchanging any pair of them.

Suppose E (*a, b, c*) is an alternating function.

Then $E(b,a,c) = -E(a,b,c)$

Take *a* = *b*

$E(b,b,c) = -E(b,b,c)$

$\Rightarrow 2E(b,b,c) = 0 \qquad \Rightarrow E(b,b,c) = 0$

This means that $E(a,b,c) = 0$ if *a* = *b*

$\therefore (a-b)$ is a factor of $E(a,b,c)$.

Similarly $(b-c),(c-a)$ are also factors.

Ex : $E(a,b,c) = a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)$

Notice that *E* is an alternating function

$E(b,b,c) = b(b^2 - c^2) + b(c^2 - b^2) + c(b^2 - b^2)$

$\therefore E = 0$ when *a* = *b* so that (*a* –*b*) is a factor of E

Similarly $(b-c),(c-a)$ are also factors

Hence $E(a,b,c) = k(a-b)(b-c)(c-a)$ as E is homogeneous of degree 3.

k can be found by taking a suitable set of values for *a, b, c*.

EXAMPLE

√ **1.** Factorize $x^4 + x^2 - 2ax + 1 - a^2$

Sol : $x^4 + x^2 - 2ax + 1 - a^2$

$= (x^4 + 2x^2 + 1) - (x^2 + 2ax + a^2)$

$= (x^2 + 1)^2 - (x + a)^2$

$= (x^2 + 1 + x + a)(x^2 + 1 - x - a)$

$= (x^2 + x + a + 1)(x^2 - x - a + 1)$

√ **2.** Factorize, $(x^2 + 4x + 8)^2 + 3x(x^2 + 4x + 8) + 2x^2$

Sol : $(x^2 + 4x + 8)^2 + 3x(x^2 + 4x + 8) + 2x^2$

$= k^2 + 3xk + 2x^2$, where $k = x^2 + 4x + 8$

$= k^2 + xk + 2xk + 2x^2$

$= k(k + x) + 2x(k + x)$

$= (k+2x)(k+x)$

$= (x^2+4x+8+2x)(x^2+4x+8+x)$

$= (x^2+6x+8)(x^2+5x+8)$

$= (x+2)(x+4)(x^2+5x+8)$

√ **3.** Factorize $x^{32}+x^{16}y^{16}+y^{32}$

Sol : $x^{32}+x^{16}.y^{16}+y^{32}$

$= x^{32}+2x^{16}.y^{16}+y^{32}-x^{16}.y^{16}$

$= (x^{16}+y^{16})^2-(x^8y^8)^2$

$= (x^{16}+y^{16}+x^8.y^8)\left(x^{16}+y^{16}-x^8y^8\right)$

$= (x^{16}+y^{16}+2x^8y^8-x^8y^8)(x^{16}-x^8y^8+y^{16})$

$= \left\{(x^8+y^8)^2-(x^4.y^4)^2\right\}(x^{16}-x^8y^8+y^{16})$

$= (x^8+y^8+x^4y^4)(x^8+y^8-x^4y^4)(x^{16}-x^8y^8+y^{16})$

$= \left\{(x^4+y^4)^2-(x^2y^2)^2\right\}(x^8-x^4y^4+y^4)(x^{16}-x^8y^8+y^{16})$

$= (x^4+y^4+x^2y^2)(x^4+y^4-x^2y^2)(x^8-x^4y^4+y^8)(x^{16}-x^8y^8+y^{16})$

$= \left\{(x^2+y^2)^2-(xy)^2\right\}(x^4-x^2y^2+y^4)(x^8-x^4y^4+y^8)(x^{16}-x^8y^8+y^{16})$

$= (x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4)(x^8-x^4y^4+y^8)(x^{16}-x^8y^8+y^{16})$

√ **4.** Factorize $x^2-y^2-3z^2-2xz+4yz$

Sol : $x^2-y^2-3z^2-2xz+4yz$

$= (x^2-2xz)-y^2-3z^2+4yz$

$= (x^2-2xz+z^2)-y^2-4z^2+4yz$

$= (x-z)^2-(y^2-4yz+4z^2)$

$= (x-z)^2-(y-2z)^2$

$= \left[(x-z)+(y-2z)\right]\left[(x-z)-(y-2z)\right]$

$= (x+y-3z)(x-y+z)$

√ **5.** Factorize $(x+y+z)^3+(x+y-z)^3+(x-y+z)^3+(x-y-z)^3$

Sol : consider $(a+b)^3+(a-b)^3 = \left[(a+b)+(a-b)\right]\left[(a+b)^2-(a+b)(a-b)+(a-b)^2\right]$

$= 2a\left[a^2 + 2ab + b^2 - (a^2 - b^2) + (a^2 - 2ab + b^2)\right]$

$= 2a\left[a^2 + 3b^2\right]$

$\therefore (x+y+z)^3 + (x-y-z)^3 = \left[x + (y+z)\right]^3 + \left[x - (y+z)\right]^3$

$= 2x\left[x^2 + 3(y+z)^2\right]$ -------------(1)

and $(x+y-z)^3 + (x-y+z)^3 = \left[x + (y-z)\right]^3 + \left[x - (y-z)\right]^3$

$= 2x\left[x^2 + 3(y-z)\right]^2$ -------------(2)

(1)+(2) $\Rightarrow (x+y+z)^2 + (x+y-z)^2 + (x-y+z)^2 + (x-y-z)^2$

$= 2x\left[x^2 + 3(y+z)^2 + x^2 + 3(y-z)^2\right]$

$= 2x\left[x^2 + 6y^2 + 6z^2\right]$

$= 4x\left[x^2 + 3y^2 + 3z^2\right]$

√ **6.** Factorize $a^2(b+c) + b^2(c+a) + c^2(a+b) + 2abc$

Sol : Denote, $E(a,b,c) = a^2(b+c) + b^2(c+a) + c^2(a+b) + 2abc$

Observations :

1. E (*a, b, c*) is a symmetric function and homogeneous of degree 3.
2. If we take a = – b

$E(a,b,c) = E(-b,b,c)$

$= b^2(b+c) + b^2(c-b) + c^2(-b+b) - 2b^2c$

$= b^3 + b^2c + b^2c - b^3 - 2b^2c = 0$

$\therefore (a+b)$ is a factor of *E* (*a, b, c*)

Due to symmetry, $(b+c),(c+a)$ are also factors.

3. Since E (*a, b, c*) is homogeneous of degree 3, it must be in the form,

$E(a,b,c) = k(a+b)(b+c)(c+a)$, where *k* is a constant.

Take $a = 1, b = 1, c = 1$

$E(1,1,1) = k(1+1)(1+1)(1+1)$ = 8*k* = 2 + 2 + 2 + 2

$\Rightarrow k = 1$

$\therefore E(a,b,c) = (a+b)(b+c)(c+a)$

7. Factorize $(x+y+z)^5 - x^5 - y^5 - z^5$

Sol : Denote, $E(x,y,z) = (x+y+z)^5 - x^5 - y^5 - z^5$

Observations :

1. $E(x,y,z)$ is a symmetric function in *x, y, z*
2. $E(x,y,z)$ is homogeneous of degree 5
3. If we take *x* = –*y*,

$E(x,y,z) = E(-y,yz) = (-y+y+z)^5 - (-y)^5 - y^5 - z^5 = z^5 + y^5 - y^5 - z^5 = 0$

$\therefore (x+y)$ is a factor of *E*

Due to symmetry of *E*, $(y+z),(z+x)$ are also factors

4. $E(x,y,z) = (x+y)(y+z)(z+x) \quad F(x,y,z)$

where F is symmetric and homogeneous of degree 2

$\therefore F = A(x^2+y^2+z^2) + B(xy+yz+zx)$

Hence $E(x,y,z) = (x+y)(y+z)(z+x)\left[A(x^2+y^2+z^2) + B(xy+yz+zx)\right]$

$\Rightarrow (x+y+z)^5 - x^5 - y^5 - z^5 = (x+y)(y+z)(z+x)\left[A(x^2+y^2+z^2) + B(xy+yz+zx)\right]$ -------(1)

To find A, B, we take particular values for *x, y, z*

Take $x=0, y=1, z=1$ in (1)

$\Rightarrow 2^5 - 1 - 1 = (1)\,(2)\,(1)\left[A(2)+B\right] \Rightarrow 2(2A+B) = 30$

$\Rightarrow 2A+B = 15$ -------------(2)

Take $x=1, y=1, z=1$ in (1)

$\Rightarrow 3^5 - 1 - 1 - 1 = (2)\,(2)\,(2)\left[3A+3B\right] \Rightarrow 24(A+B) = 240$

$\Rightarrow A+B = 10$ -------------(3)

Solving (2), (3) we get A = 5, B = 5

$\therefore (x+y+z)^5 - x^5 - y^5 - z^5 = 5(x+y)(y+z)(z+x)(x^2+y^2+z^2+xy+yz+zx)$

√ **8.** Without removing the brackets at any stage, factorize $2y(y+z) - (x+y)(x+z)$

Sol : $2y(y+z) - (x+y)(x+z)$

$= \left[(y+x) + (y-x)\right](y+z) - (x+y)(x+z)$

$= (y+x)(y+z) + (y-x)(y+z) - (x+y)(x+z)$

$= (y-x)(y+z) + (x+y)\left[(y+z) - (x+z)\right]$

= $(y-x)(y+z)+(x+y)(y+z-x-z)$

= $(y-x)[(y+z)+(x+y)]$

= $(y-x)(x+2y+z)$

√ **9.** Simplify $(b-c)(b+c-a)^3+(c-a)(c+a-b)^3+(a-b)(a+b-c)^3$.

Sol : Let $E(a,b,c)=(b-c)(b+c-a)^3+(c-a)(c+a-b)^3+(a-b)(a+b-c)^3$

Then E(*a, b, c*) is

(i) Homogenous of degree 4 in *a, b, c*

(ii) Cyclic in *a, b, c*

Take *a* = *b*

$E(b,b,c)=(b-c)(b+c-b)^3+(c-b)(c+b-b)^3+(b-b)(b+b-c)^3$

= $c^3(b-c)+c^3(c-b)+0=0$

$\therefore (a-b)$ is a factor of E.

As E is cyclic, (b – c), (c – a) are also factors. Since $(a-b),(b-c),(c-a)$ are factors and E is homogenous of degree 4,

$E(a,b,c)=(a-b)(b-c)(c-a) \quad k(a+b+c)$, where k is a constant

Take $a=0,\ b=1,\ c=2$

E(0, 1, 2) = (–1) (–1) (2) k(3)

$(-1)(3)^3+(2)(1)^3+(-1)(-1)^3=6k$

–27 + 2 + 1 = 6k

6k = –24 $\Rightarrow k=-4$

$\therefore E(a,b,c)=-4(a-b)(b-c)(c-a)(a+b+c)$

√ **10.** Simplify $(b^2-ca)(c^2-ab)+(c^2-ab)(a^2-bc)+(a^2-bc)(b^2-ca)$.

Sol : Denote the given expression by E.

$E=\sum(b^2-ca)(c^2-ab)$

= $\sum\left[b^2c^2-a(b^3+c^3)+a^2bc\right]$

= $(b^2c^2+c^2a^2+a^2b^2)-\left[a(b^3+c^3)+b(c^3+a^3)+c(a^3+b^3)\right]+abc(a+b+c)$

= $(ab+bc+ca)^2-\left[a(b^3+c^3)+b(c^3+a^3)+c(a^3+b^3)\right]-abc(a+b+c)$

$$= (ab+bc+ca)^2 - \left[a^3b + a^2bc + a^3c\right] - \left[ab^3 + b^3c + ab^2c\right] - \left[abc^2 + bc^3 + ac^3\right]$$

$$= (ab+bc+ca)^2 - a^2(ab+bc+ca) - b^2(ab+bc+ca) - c^2(ab+bc+ca)$$

$$= (ab+bc+ca)^2 - (a^2+b^2+c^2)(ab+bc+ca)$$

$$= (ab+bc+ca)\left[ab+bc+ca-a^2-b^2-c^2\right]$$

$$= -(ab+bc+ca)\left[a^2+b^2+c^2-ab-bc-ca\right]$$

1. Factorisation :

Process of writing the given expression as a product of its factors is called factorisation.

A factor which cannot be further expressed as product of factors is an irreducible factor.

* The form of factorisation where all factors are primes is called product of prime factor form.

Ex : 72

= 1 x 72

= 2 x 36

= 3 x 24

= 4 x 18

= 6 x 12

= 8 x 9

1,2,3,4,6,8,9,12,18,24,36,72 are the factors of 72.

* If the given expression is of the form $x^2 + (a+b)x + ab$, then its factorisation is $(x+a)(x+b)$

2. Polynomial :

An expression is of the form $f(x) = a_0 + a_1x + a_2x^2 + \ldots\ldots\ldots\ldots a_nx^n$, where 'n' is non negative integer and $a_0, a_1, a_2 \ldots\ldots\ldots\ldots\ldots a_n$ are complex numbers such that $a_n \neq 0$ is called a polynomial of degree 'n'.

Ex: 1. $3x^4 - 2x^3 + 4x^2 - 5x + 6$

4 is the degree of polynomial.

2. $6x^6 + 5x^5 - 4x^4 + 3x^3 - 2x^2 + 6x - 7$

6 is the degree of polynomial.

3. Degree of the polynomial :

The highest power of x in the given polynomial is called the degree of that polynomial.

Ex: 1. The degree of $2x^3-6x^2+7x+6$ is 3.

2. The degree of $5x^4+7x^2$ 6x+8 is 4.

4. Value of the polynomial :

Let p(x) is polynomial replacing any value in place of 'x' then the remainder of the polynomial is called value of the polynomial.

If we replace x by "-2" in the polynomial $p(x) = 3x^3 - 2x^2 + x+1$, we have

$$p(-2) = 3(-2)^3-2(-2)^2 + (-2)+1$$
$$= -24 -8-2+1$$
$$=-33$$

Thus, on replacing x by '-2' in the polynomial p(x), we have -33 which is called the value of the polynomial. Hence, if k is any real number, then the value obtained by replacing x by k in p(x), is called the value of the polynomial p(x) at x = k, and it is denoted by p(k).

5. Zero of the polynomial:

The value of x for which the polynomial becomes 0.

Ex: 1. The zero of the polynomial ax -b is '$\frac{b}{a}$'.

2. The zero of the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$ is '2'

$$f(2) = (2)^3 -6 (2)^2 + 11(2)-6$$
$$= 8-24+22-6$$
$$= 30-30$$
$$= 0$$

Polynomial	**Zero of the polynomial**
ax + b	$-b/a$
x - a	a
x + a	-a
bx + a	$-a/b$
bx - a	a/b

6. Ploynomials are divided based on its degree and terms:

Polynomials
- Based on terms
  - Monomial
  - Binomial
  - Trinomial
  - Multinomial
- Based on degree
  - Constant polynomial
  - Linear polynomial
  - Quadratic polynomial
  - Cubic polynomial
  - Bi- quadratic polynomial

§§ **Types of polynomials (on basis of number of terms) :**

¶¶ ***Monomial :***

A polynomial is said to be monomial if it contains a single term.

Eg : $4, 4x, 5x^2, 9x^7$ etc..

¶¶ ***Binomial :***

A polynomial is said to be Binomial if it contains only two terms.

Eg : $3x+5$, $9x^2+7$, $5x^9-2x^8$.

¶¶ ***Trinomial :***

A polynomial is said to be Trinomial if it contains only three terms.

Eg : $5x+7x^2-9$, $3x^5-9x^3+7$, $x^5-x^4-x^7$.

¶¶ **Multinomial :**

A polynomials is said to be multinomial if it contains more than two terms (i.e., 3,4,....)

Eg : $x^4+2x^3 -x^2 +2x -8$, $3x^3 +2x^2 -8x +9$.

§§ **Types of polynomials (on the basis of degree) :**

¶¶ ***Constant polynomial :***

A polynomial of degree ‘0’ is called a constant polynomial.

Ex : $f(x) = 7$, $g(x) = \frac{-3}{2}$, $h(x) = 5$.

¶¶ ***Linear polynomial :***

A polynomial of degree ‘1’ is called a linear polynomial.

Eg : $Ax+B$, $5x+7$, $x-101$, x

¶¶ ***Quadratic polynomial :***

A polynomial of degree '2' is called a quadratic polynomial.

Eg : ax^2+bx+c, $2x^2-5x$, x^2-6x+2, x^2

¶¶ ***Cubic polynomial :***

A polynomial of degree '3' is called a cubic polynomial.

Eg : ax^3+bx^2+cx+d, x^3-5x^2+7, x^3-8

¶¶ ***Bi-quadratic polynomial :***

A polynomial of degree '4' is called a Bi-quadratic polynomial.

Eg : $ax^4+bx^3+cx^2+dx+e$, x^4+5x^3+6x-9, x^2y^2+9 .

7. A complex number 'a' is said to be zero of the polynomial if f(a) = 0.

8. If all the terms of an algebraic expression are of the same degree then such expression are called as homogeneous expressions.

9. Standard forms of homogeneous expressions in two (or) more variables.

Variables	Degree	Standard form
x,y	1	$ax+by$
x,y	2	$ax^2+bxy+cy^2$
x,y	3	$ax^3+bx^2y+cxy^2+dy^3$
x,y	4	$ax^4+bx^3y+cx^2y^2+dxy^3+ey^4$
x,y,z	1	$ax+by+cz$
x,y,z	2	$ax^2+by^2+cz^2+dxy+eyz+fzx$
x,y,z	3	$ax^3+by^3+cz^3+dx^2y+exy^2+fy^2z+gyz^2+hz^2x+kzx^2$

10. A homogeneous expression is said to be complete, if it contains all the possible terms in it.

<u>11. Alternating function :</u>

If a function f of x, y, z .... is transformed in to -f by the interchanging any two of the set x,y,z,.... then 'f' is called an alternating function of x,y,z.

<u>12. Symmetric function :</u>

A function which is unaltered by the interchange of any two of the variables which it contains is said to be symmetric with regard these variables i.e., an expression f(x,y) is said to be symmetric if **f(x,y) = f(y,x)**

13. An expression f(x,y,z) is said to be a cyclic expression if **f(x,y,z) = f(y,z,x).**

14. We use the symbols $\sum$ **(read as sigma)** and $\prod$ **(pi)** to write a cyclic expression.

$\sum$ is used for **sum of terms** and $\prod$ is used for **product of terms**. i.e.,

$\sum x(y+z) = x(y+z) + y(z+x) + z(x+y)$

$\prod (a^2 + b^2) = (a^2+b^2)(b^2+c^2)(c^2+a^2)$

15. Division algorithm :

If f(x), g(x) $\neq$ 0 are two polnomials then there exists polynomials q(x), r(x) uniquely such that f(x) = g(x) .q(x) + r(x).

Where r (x) = 0 (or) deg r(x) < deg g(x). The polynomial q (x) is called quotient and the polynomials r (x) is called remainder of f(x) when divided by g(x).

16. *Remainder theorem :*

If f(x) is a polynomial, then the remainder of f(x) when divided by (x-a) is f(a).

17. Let f(x), g(x) be two polynomials. g(x) is said to be factor of f(x), if there exists a polynomials q(x) such that f(x) = q(x) . g(x).

18. *Factor theorem :*

If f(x) is a polynomial and f(a) = 0 then (x - a) is a factor of f(x).

19. a) $x^n - y^n$ is divisible by x - y for every positive integer n.

b) $x^n - y^n$ is divisible by x + y for every even positive integer n.

c) $x^n + y^n$ is divisible by x + y for every odd positive integer n.

20. A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at the most 3 zeroes.

21. If a and b are the zeroes of the quadratic polynomial $ax^2 + bx + c$, a $\neq$ 0, then

$$\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}.$$

22. If a and b are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, a $\neq$ 0, then

$$\alpha + \beta + \gamma = -\frac{b}{a},$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a},$$

$$\text{and} \quad \alpha\beta\gamma = -\frac{d}{a}.$$

♥ ***List of formulae :***

1) $(a+b)^2 = a^2 + 2ab + b^2$

2) $(a-b)^2 = a^2 - 2ab + b^2$

3) $(a+b)(a-b) = a^2 - b^2$

4) $(a+b)^2 - (a-b)^2 = 4ab$

5) $(a+b)^2+(a-b)^2=2(a^2+b^2)$

6) $a^2+b^2=(a+b)^2-2ab=(a-b)^2+2ab$

7) $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

8) $(a-b+c)^2 = a^2+b^2+c^2 +2(-ab-bc+ca)$

9) $(a+b-c)^2=a^2+b^2+c^2+2(ab-bc-ca)$

10) $(a+b)^3 = a^3 +b^3 +3a^2b +3ab^2 = a^3 + b^3 +3ab(a+b)$

11) $(a-b)^3 = a^3 -b^3 -3a^2b +3ab^2 = a^3 - b^3 -3ab(a-b)$

12) $a^3+b^3=(a+b)(a^2-ab+b^2)$ (or) $(a+b)^3-3ab(a+b)$

13) $a^3 -b^3 = (a-b) (a^2 +ab+b^2)$ or $(a-b)^3 +3ab (a-b)$

14) $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

15) If $a+b+c=0$ or $a=b=c$ then $a^3+b^3+c^3=3abc$

16) $(a+b)^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4$

17) $(a+b)^5=a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5$

♣ ***Theroms and its Proof :***

Ψ **Remainder Theorem :**

Let p(x) be any polynomial of degree greater than or equal to one and let 'a' be any real number. If p(x) is divided by the linear polynomial (x-a), then the remainder is p(a).

Proof : Let p(x) be any polynomial with degreee greater than or equal to 1.

Further suppose that when p(x) is divided by a linear polynomial g(x) = (x-a), then quotient is q(x) and the remainder is r (x).

In other words,

p(x) and g(x) are two polynomials such that the degree of p(x) ³ degree of g(x) and g(x)¹0 then we can find polynomials q(x) and r(x)

such that,

where r(x) =0 or degree of r(x) < degree of g(x).

By division algorithm,

p(x) = g(x). q(x) + r(x)

p(x) = (x-a) .q(x) + r(x) $\because$ g (x) = (x -a)

Since the degree of (x -a) is 1 and the degree of r(x) is less than the degree of (x-a).

Degree of r(x) = 0, implies r(x) is a constant, say K.

so, for every real value of x,

$r(x) = K$.

Therefore,

$p(x) = (x-a)\, q(x) + K$

If x=a, then $p(a) = (a-a)\, q(a) + K$

$= 0+K$

$= K$

$P(a) = K$

Then the remainder is P(a)

Hence proved.

Ψ **Factor Theorem :**

If P(x) is a polynomial of degree n $\geq$ 1 and 'a' is any real number, then (i) x - a is a factor of (x), if p(a) - 0

(ii) and its converse "if (x-a) is a factor of a polynomial p(x) then p(a)=0.

Let us see the simple proof of this theorem.

Proof : By Remainder Theorem,

$p(x) = (x-a)\, q(x) + p(a)$

(i) Consider proposition (i) If p(a) = 0,

then $p(x) = (x-a)\, q(x) + 0$.

$= (x-a)\, q(x)$

Which shows that (x-a) is a factor of p(x).

Hence proved.

(ii) Consider proposition (ii) since (x-a) is a factor of p(x) then

$p(x)=(x-a)q(x)$ for some polynomial q(x).

$p(a) = (a-a)q(a)$

$= 0$

Hence p(a)=0 when (x-a) is a factor of p(x)

§§ **HCF and LCM of polynomials.**

Divisor (Factor)

If a polynomial f(x) is a product of two polynomials g(x) and h(x).

i.e., f(x) = g(x) X h(x) then g(x) and h(x) are called factors of f(x).

Eg: $f(x) = x^2-5x+6$

$f(x) = x^2-5x+6 = (x-2)(x-3)$

then (x-2) and (x-3) are factors of x^2-5x+6.

Note: If g(x) is a factor of f(x) then -g(x) is also a factor of f(x).

§§ **Highest common factor (HCF) or Greatest Common Divisor (GCD)**

The HCF of two polynomials f(x) and g(x) is that common factor which has highest degree among all the factors and in which the co-efficient of highest degree term is positive.

Eg: Find the HCF of the polynomials $150(6x^2+x-1)(x-3)^3$ and $84(x-3)^2(8x^2+14x+5)$

Sol: Lef $f(x) = 150(6x^2+x-1)(x-3)^3$

and $g(x) = 84(x-3)^2(8x^2+14x+5)$

Now $f(x) = 150(6x^2+x-1)(x-3)^3$

$= 2.3.5^2(2x+1)(3x-1)(x-3)^3$

$g(x)=84(x-3)^2(8x^2+14x+5)$

$= 2^2.3.7(x-3)^2(2x+1)(4x+5)$

Common Factor	Least Exponent
2	1
3	1
(2x+1)	1
(x-3)	2

$HCF = 2^1.3^1.(2x+1)^1.(x-3)^2$

$= 6.(2x+1).(x-3)^2$

§§ **LCM of polynomials:**

The LCM of two or more polynomials is the polynomial of the lowest degree having smallest numerical co-efficient which is exactly divisible by the given polynomials and whose co-efficient of highest degree term has the same sign as the sign of the co-efficient of highest degree term in their product.

Eg: LCM of the polynomials $90(x^2-5x+6)(2x+1)^2$ and $140(x-3)^3(2x^2+15x+7)$

Let $f(x) = 90(x^2-5x+6)(2x+1)^2$

$g(x) = 140(x-3)^3(2x^2+15x+7)$

$f(x) = 2.3^2.5.(x-2)(x-3)(2x+1)^2$

$g(x) = 2^2.5.7.(x-3)^3(2x+1)(x+7)$

Factors of f(x) and g(x)	**Greatest exponent**
2	2
3	2
5	1

7	1
(x-2)	1
(x-3)	3
(2x+1)	2
(x+7)	1

LCM $= 2^2.3^2.5^1.7^1.(x-2)^1.(x-3)^3.(2x+1)^2.(x+7)^1$

$= 1260(x-2).(x-3)^3.(2x+1)^2.(x+7)$

¶¶ Relation between LCM and HCF.

LCM X HCF = Product of polynomials.

Eg 1: The HCF of the polynomials $p(x) = (x-3)(x^2+x-2)$ and $q(x) = x^2-5x+6$ is x=3.Find the LCM.

sol: P(x) = (x-3)(x2+x-2) = (X-3)(x-1)(x+2)

Q(x) = (x2-5x+6) = (x-3)(x-2)

HCF = (x-3)

$$\text{LCM} = \frac{P(x).Q(x)}{HCF} = \frac{(x-3)(x-1)(x+2).(x-3)(x-2)}{(x-3)}$$

$$= (x-1)(x+2)(x-3)(x-2)$$

Eg 2 : The LCM and HCF of two polynomials, p(x) and q(x) are $2(x^4-1)$ and $(x+1)(x^2+1)$ respectively. If $p(x) = x^3+x^2+x+1$, Find q(x)

Sol: $p(x) = x^3+x^2+x+1 = (x+1)(x^2+1)$

p(x).q(x) = LCM . HCF

$$q(x) = \frac{LCM.HCF}{p(x)} = \frac{2(x^4-1).(x+1).(x^2+1)}{(x+1)(x^2+1)}$$

$= 2(x^4-1) = (2x^4-2)$.

TEACHING TASK

I) MCQ's with single correct answer :

[Hint :Factorise the following expressions]

1. Factorize $6-x-2x^2$

a) $(x-2)(-2x+3)$
b) $(-2x+3)(x-2)$
c) $(-2x+3)(x+2)$
d) None of these

2. $(2a-b)^2+2(2a-b)-8$ =

a) $(2a-b+4)(2a-b-2)$ b) $(2a+b-4)(2a-b-2)$

c) $(2a+b+4)(2a+b+2)$ d) None of these

3. Factorize $12x^2-23xy+10y^2$ =

a) $(4x-5y)(3x-2y)$ b) $(5x-4y)(3x-2y)$

c) $(5x+4y)(3x+2y)$ d) $(5x-4y)(4x-5y)$

4. Factorize $9-a^6+2a^3b^3-b^6$ =

a) $(a^3+b^3-3)(-a^3+b^3+3)$ b) $(a^3-b^3+3)(-a^3+b^3+3)$

c) $(a^3-b^3-3)(a^3-b^3-3)$ d) None of these

5. Factorize of $x^3+x^2-21x-38$ is

a) 2x b) x^2+x+19 c) $x-2$ d) $x+2$

6. Factor of $x^3+6x^2+11x+6$ is

a) $x-3$ b) x^2-3 c) $x+3$ d) $x+5$

7. The quotient of $a^3+b^3+1-3ab$ by $a+b+1$

a) $a^2+b^2-b-a-ab$ b) $(a+b)(a^2+ab+1)$

c) a^2-b^2+b-a d) None of these

8. The value of $a^3+b^3+c^3$, when $b+c=10$, $c+a=16$ and $a+b=20$

a) 1595 b) 2567 c) 2060 d) 1135

9. The value of $(b+c)(b-c)+(c+a)(c-a)+(a+b)(a-b)$ is

a) 0 b) a^2-b^2 c) c^2+a^2 d) None of these

10. Find the value of $x^{12}-7x^6+2001$ if $x^2=2$

a) 0 b) 2009 c) 2007 d) 2

11. $a^2 + 10a + 25$

A) $(a-5)^2$ B) $(a-5)(a+5)$ C) $(a-5)(a+5)$ D) $(a+5)^2$

12. $25m^2 - 40mn + 16n^2$

A) $(5m-4n)^2$ B) $(5m+4n)^2$ C) $(5m+4n)(5m-4n)$ D) None

13. $81x^2 - 198xy + 121y^2$

A) $(9x+11y)^2$ B) $(9x+11y)(9x-11y)$ C) $(9x-11y)^2$ D) None

14. $(x+y)^2 - 4xy$

A) $(x+y)^2$ B) $(x-y)^2$ C) $(x+y)(x-y)$ D) None

15. $(p^2 - 2pq + q^2) - r^2$

A) $(p+q+r)(p-q-r)$ B) $(p+q+r)(p-q+r)$

C) $(p-q+r)(p-q-r)$ D) $(p+q+r)^2$

16. $(x+y)^2 - (x-y)^2$

A) $(x+y)$ B) $(x-y)$ C) $(x+y)^2$ D) $4xy$

17. $49x^2 - \frac{16}{25}$

A) $\left(7x+\frac{4}{5}\right)^2$ B) $\left(7x+\frac{4}{5}\right)\left(7x-\frac{4}{5}\right)$ C) $\left(7x-\frac{4}{5}\right)^2$ D) None

18. $4(a+b)^2 - 9(a-b)^2$

A) $(5a-b)^2$ B) $(5b+a)^2$ C) $(5a-b)(5b+a)$

D) $(5a-b)(5b-a)$

19. $a^4 - (b+c)^4$

A) $(a^2+b^2+c^2+2bc)$ B) $(a+b+c)$

C) $(a^2+b^2+c^2+2bc)(a+b+c)(a-b-c)$ D) $(a-b-c)$

20. $l^2 - (m-n)^2$

A) $(l+m-n)(l-m+n)$ B) $(l+m-n)$ C) $(l-m+n)$ D) None

21. If $p(x) = 2 + x + 2x^2 - x^3$ then $P(-2) = ...$

A) 16 B) 15 C) 14 D) 13

22. If '2' is a zero of the polynomial $p(x) = 2x^2 - 3x + 7a$, then value of a=...

A) $\frac{2}{7}$ B) $\frac{7}{2}$ C) $\frac{-2}{7}$ D) $-\frac{7}{2}$

23. The remainder when $9x^3-3x^2+x-5$ is divided by $3x+2$

A) $-\frac{13}{3}$ B) $-\frac{3}{13}$ C) $\frac{3}{13}$ D) $\frac{13}{3}$

24. If $\frac{a}{b}=\frac{b}{c}$ then $(a+b+c)(a-b+c)$ is.....

A) $a^2+b^2-c^2$ B) $a^2-b^2-c^2$ C) $a^2+b^2+c^2$ D) $a^2-b^2+c^2$

25. The remainder when $p(x) = x^3-6x^2+14x-3$ is divided by $g(x) = 1-2x$

A) $\frac{21}{5}$ B) $\frac{8}{21}$ C) $\frac{21}{8}$ D) None

26. If both (x-2) and $\left(x-\frac{1}{2}\right)$ are factors of px^2+5x+r then .....

A) $p=r$ B) $p=2r$ C) $p=3r$ D) $r=2p$

27. If x^2-x-6 and $x^2+3x-18$ have a common factor (x-a) then the value of a......

A) 0 B) 1 C) 2 D) 3

28. The polynomials ax^3+3x^2-13 and $2x^3-5x+a$ are divided by (x-2) the remainder in each case is same then a = ...

A) 0 B) 1 C) 2 D) 3.

29. Let R_1 & R_2 be the remainders when the polynomials $x^3+2x^2-5ax+7$ & x^3+ax^2-12x 6 are divided by (x + 1) and (x- 1) respectively if $R_1 - R_2 = 20$ then a=.........

A) $-\frac{7}{4}$ B) $\frac{4}{7}$ C) $\frac{7}{4}$ D) $-\frac{4}{7}$

30. If x^3+px^2+x+6 leaves the remainder 3 when divided by (x - 3) then P = ...

A) $\frac{11}{3}$ B) $\frac{3}{11}$ C) $-\frac{3}{11}$ D) $\frac{-11}{3}$

31. The polynomial kx^4+3x^3+6 when divided by (x-2) leaves a remainder which is doubled the remainder left by the polynomial $2x^3+17x+k$ when divided by (x-2) then k = ...

A) 5 B) 4 C) 3 D) 1

32. What must be subtracted from $x^3-6x^2-15x+80$ so that the result is exactly divided by $x^2 + x - 12$.

A) $(x-1)$ B) $2(x-1)$ C) $3(x-1)$ D) $4(x-1)$

33. For the expression $f(x)=x^3+ax^2+bx+c$. if f(1) = f(2) =0 & $f(4)=0$ then a = .., b = ..., c = ...

A)7,14,8 B)-7,-14,-8 C)-7,14,-8 D)7,-14,8

34. The quadratic polynomial in x which when divided by (x-1), (x-2), (x-3) leaves the remainder of 11,22 & 37 respectively.

A) $2x^2 5x+4$ B $2x^2+5x-4$ C) $2x^2-5x-4$ D) $2x+5x^2+4$

35. What must be subtracted from $14x^3-2x^2+7x-8$ so that the resulting polynomial is exactly divisible by $x-2$

A)108 B)109 C)110 D)111

36. Factorise $x^3 - 23x^2 + 14x - 120$

A) $(x+1)(x-10)(x-12)$ B) $(x-1)(x-10)(x-12)$

C) $(x-1)(x+10)(x-12)$ D) $(x-1)(x-10)(x+12)$

37. The remainder when x^{100} is divided by x^2-3x+2 **is .....**

A) $(2^{100}-1)x+(2-2^{100})$ B) $(2^{100}-1)+(2-2^{100})x$

C) $(2^{100}-1)x-(2-2^{100})$ D) $(2^{100}-1)-(2-2^{100})x$

38. The H.C.F of the polynomials x^2-3x+2 and x^2+x-6 is ....

A) $x+2$ B) $(x-1)(x-2)(x+3)$ C) $x-2$ D)None

39. The L.C.M of $xy+yz+zx+y^2$ and $x^2+xy+yz+zx$ is....

A) $(x+y)(y+z)$ B) $(x+y)(y+z)(z+x)$

C) $(y+z)(z+x)$ D) $(x+y)(z+x)$

40. The value of $\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$ is.......

A)1 B)1 C)2 D) 3

41. If $a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}}=0$ then the value of $(a+b+c)^3=$

A) 27 abc B) 9 abc C) 3 abc D) 6 abc

42. If a+b+c = 0 then the value of $(a+b-c)^3+(a-b+c)^3+(-a+b+c)^3$ is **......**

A) $a^3+b^3+c^3$ B) $-24abc$ C) $a+b+c$ D) $24abc$

43. The expression $(1+q)(1+q^2)(1+q^4)(1+q^8)(1+q^{16})(1+q^{32})(1+q^{64})$ where $q\neq 1$ is.........

A) $\frac{1-q^{64}}{1-q}$ B) $\frac{1-q^{64}}{1+q}$ C) $\frac{1-q^{128}}{1-q}$ D) $\frac{1-q^{128}}{1+q}$

44. If $(3x-1)^7=a_7x^7+a_6x^6+a_5x^5+........+a_1x+a_0$ then $a_7+a_6+a_5+.......+a_1+a_0=$

A)16 B)32 C)64 D)128

45. $(x-1)$ and $(x+2)$ are the factors of the polynomial (x^3+ax^2+bx-8). then a=.....and b=.....

A)2,5 B)5,-2 C)-5,2 D).5,2

46. The L.C.M of $(16-x^2)$ and (x^2+x-6) is.......

A) $(16-x^2)(x+3)(x-2)$ B) $(x+3)(x-2)$

C) $(16-x^2)(x-3)(x+2)$ D)None

47. G.C.D of x^2-4 and x^2+x-6 is.....

A) $x-4$ B) $x+4$ C) $x-2$ D) $x+2$

48. The H.C.F and L.C.M of the polynomials x^2-5x-6 and $x^2-7x+10$ is.....

A) $(x+2)(x-3)(x-5)(x-2)$ B) $(x-3)(x-5)(x-2)$

C) $(x-3)(x+5)(x+2)$ D) None

49. The G.C.D of the polynomials $(x+3)^2(x-2)(x+1)^2$ and $(x+1)^3(x+3)(x+4)$ is

A) $(x+3)(x+1)^2$ B) $(x+3)^3(x-2)(x+1)^2(x+4)$

C) $(x+3)(x-2)(x+1)(x+4)$ D)None

50. The number of elements in the set

$\{n \in N / n^3 - 8n^2 + 20n - 13 \; is \; a \; prime \; number\}$ is....... (SAT-2010)

A) 1 B) 2 C) 3 D)4

51. Factorise $2y(y+z)-(x+y)(x+z)$=.... (SAT-2006)

A) $(y+x)(x+2y-z)$ B) $(y+x)(x+2y+z)$

C) $(y-x)(x+2y+z)$ D) $(y-x)(x-2y-z)$

52. If ax^2+bx+c is exactly divisible by $x-1$, $x-2$ and leaves remainder 6 when divided by $x+1$ then a = ...b = ...,c = ...

A)1,2,-3 B) 1,-3,2 C) 2,1,-3 D) 2,-3,1

II) **<u>MCQ's with one or more than one correct answer :</u>**

♦ *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options*

1. Which of the following are polynomials

A) $4x^2+5x-2$ B) $\frac{1}{x+1}$ C) $2x^2+\frac{3}{x}-5$ D) $\sqrt{3}x^2+5y$

2. The factors of $\sum ab(a-b)$ ....

A) $a-b$ B) $b-c$ C) $c-a$ D)None

3. Factors of $x^4+3x^3-7x^2-27x-18$

A) $x-2$ B) $x+2$ C) $x-3$ D) $x-1$

4. The zero's of the polynomial $x^3-23x^2+142x-120$

A)1 B)10 C) 12 D)0

III) **<u>Integer type questions :</u>**

1. If $p(x)=4x^4-5x^3-x^2+6$ then $p(1)=$...........

2. If $2x-3$ is a factor of $2x^3-9x^2+x+K$ and $K=2a$ then a = ...

3. If $x^3-23x^2+142x-120 = (x-a)(x-b)(x-c)$ and $a<b<c$ then $c-b+a\cdots$

4. If $(x+4)$ $(x-3)$ and $(x-7)$ are factors of x^3+ax^2+bx+c then $c+3b+3a$

IV) SOLVE THE FOLLOWING

1. Check whether x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

2. Divide $3x^2-x^3-3x+5$ by $x-1-x^2$, and verify the division algorithm.

3. Divide $6x^3+13x^2+x-2$ by 2x+1, and find quotient and remainder.

4. Find other zeroes of the polynomial $x^4+x^3-9x^2-3x+18$, if it is given that two of its zeroes are $\sqrt{3}$ and -$\sqrt{3}$.

5. Find other zeroes of $x^4-7x^3+17x^2-17x+6$, if two of its zeroes are 1 and 2.

6. Divide $(4x^4-8x^3+9x^2+3x-7)$ by $(2x^2-x-2)$ and verify division algorithm.

7. write a quaratic polynomial, the sum and product of whose zeroes are 3 and -2.

8. Form a quaratic polynomial p(y) with sum and product of zeroes are 2 and -3/5 respectively.

9. Find a quadratic polynomial , the sum of whose zeroes is 7 and their product is 12. Hence find the zeroes of the polynomial.

10. If zeroes of the polynomial x^3-3x^2+x+1 are a-b, a and a+b find a and b.

11. Find the zeroes of the polynomial x^2-3 and verify the relationship between the zeroes and the coefficients.

12. Find a quadraic polynomial if the zeroes of it are 2 and $\frac{-1}{3}$ respectively.

13. Verify that $3,-1,-\frac{1}{3}$ are the zeroes of the cubic polynomial $p(x)=3x^3-5x^2-11x-3$ and then verify the relationship between zeroes and the coefficients

14. $\left(\sum_{a,b,c} a^4\right)-\sum_{a,b,c} a^3$

15. $\left(\sum_{a,b,c} a\right)^4-\sum_{a,b.c}(a+b)^4+\sum_{a,b,c} a^4$

16. $\sum_{a,b,c}(a+1)^3\left(b^2-c^2\right)$

17. $\sum_{x,y,z} x^2\left(y^2-z^2\right)$

18. $\sum_{a,b,c} ab(a^2-b^2)$

19. $\sum_{a,b,c} a^2(b^3-c^3)$

20. $\sum_{a,b,c}(a^2+b^2)(a-b)^2$

21. $\prod_{a,b,c}(a^2-b^2)$

LEARNER'S TASK

BEGINNERS (Level - I)

I) MCQ's with Single Answer type:

1. The degree of the polynomial $7-x+3x^2$ is ....
   A) 0 B) 1 C) 2 D) 3

2. The degree of a constant polynomial
   A) 0 B) 1 C) 2 D) not defined

3. Which of the following is cubic polynomial
   A) x-12 B) $3x^3-2x^2+5x+7$ C) 5 D) $2x^2+3x+4$

4. Zero of the polynomial of x^2-3x-4 is
   A) 1 B) 2 C) 3 D) 4

5. Zero's of the polynomial $f(x)=3x^2-1$ are .....
   A) $\pm\frac{1}{\sqrt{3}}$ B) $\pm\frac{2}{\sqrt{3}}$ C) ± 1 D) none

6. The degree of a polynomial $ax^4+bx^3+cx^2+dx+e$, a=0 is ......
   A) 4 B) 3 C) 2 D) 1

7. Which of the following is complet homogeneous expression ?
   A) ax^2+by^2 B) $ax^3+by^3+cx^2y$ C) $ax+by$ D) $ax.by$

8. Which of the following is symmetric expression ?
   A) $2x^2+3xy+2y^2$ B) $x^2+3xy-y^2$ C) $x^2-3xy+2y^2$ D) $2x^2+3xy-y^2$

9. The value of $\sum_{a,b,c} a^2$ if a=0, b=1, c=2 is .....

A) 3 B) 4 C) 5 D) 6

10. The value $\prod_{a,b,c} (a^2 - b^2)$ if a=0, b=1, c=2 is .....

A) -12 B) 3 C) -4 D) 12

11. The remainder of f(x) when divided by ax+b

A) $f\left(\frac{b}{a}\right)$ B) $f\left(-\frac{b}{a}\right)$ C) $f\left(-\frac{a}{b}\right)$ D) $f\left(\frac{a}{b}\right)$

12. The remainder when $x^3 - px^2 + 6x - p$ is divided by (x-p) is.....

A) 6p B) $p^3 + 5p$ C) $p^3 + 6p$ D) 5p

13. $ax^4 + bx^3 + cx^2 + dx + e$ is exactly divisible by $x^2 - 1$, when

A) a+b+c+d+e=0 B) a+c+e=0 C) b+d=0 D) a+c+e=0 (or) b+d=0

14. When a polynomial p(x) is divided by (x-2) the quotient is $3x^2 - x - 5$ and the remainder is -1 then p(-1) is ....

A) 2 B) 3 C) 4 D) 7

15. The factors of $x^3 + 6x^2 + 11x + 6$ are ....

A) (x+1) (x+2) (x+3) B) (x+1) (x-2) (x-3)

C) (x-1) (x-2) (x+3) D) (x-1) (x+2) (x-3)

16. If $x^2 + ax + b$ and $x^2 + bx + a$ have a common factor then

A) a+b=1 B) a+b=-1 C) a=b D) a+b=0

17. $x^n + y^n$ is divisible by x+y when n is .....

A) a positive integer B) an even posiive integer

C) an odd posiive integer D) a real number

18. $x^n - y^n$ is divisible by x-y when n is .....

A) a positive integer B) an even posiive integer

C) an odd posiive integer D) a real number

19. $x^n - y^n$ is divisible by x+y when n is .....

A) a positive integer B) an even posiive integer

C) an odd posiive integer D) a real number

20. $x^4 + 2x^3 + 3x^2 + 2x + 1$=.....

A) $(x^2 + x - 1)^2$ B) $(x^2 + x + 1)^2$ C) $(x^2 - x + 1)^2$ D) $(x^2 - x - 1)^2$

21. $18a^5 + 8a^3 + 2a + 24a^4 - 12a^3 - 8a^2 = ...$

A) $2a(3a^2+2a+1)^2$ B) $2a(3a^2-2a-1)^2$ C) $2a(3a^2+2a-1)^2$ D) none

22. $(x+y)(1-z)-(y+z)(1-x)=...$

A) $(x-z)(1-y)$ B) $(x-z)(1+y)$ C) $(x+y)(1-y)$ D) $(x-z)(1+y^2)$

23. $(x+y)(a+bz)-(y+z)(a+bx)=...$

A) $(x-z)(a-by)$ B) $(x+z)(a-by)$ C) $(x-z)(a+by)$ D) none

24. $\frac{9}{25}x^2 - \frac{4}{5}xy + \frac{4}{9}y^2$ can be factorised as

A) $\left(\frac{3x}{5}+\frac{2y}{3}\right)^2$ B) $\left(\frac{3x}{5}-\frac{y}{3}\right)^2$ C) $\left(\frac{3x}{5}-\frac{2y}{3}\right)^2$ D) $\left(\frac{x}{5}-\frac{2y}{3}\right)^2$

25. $a^2b + a^2c + ab^2 + b^2c + ac^2 + bc^2 + 3abc = ...$

A) $(ab+ac+bc)(a+b+c)$ B) $(ab-ac-bc)(a+b+c)$

C) $(ab+ac-bc)(a-b+c)$ D) $(ad+ac+bc)(a-b-c)$

26. $a^4 + 4(a-1)^2 - 4(a^3 - a^2) =$

A) $(a^2 + 2a + 2)^2$ B) $(a^2 - 2a + 2)^2$ C) $(a^2 - 2a - 2)^2$ D) None

27. $x^2 - z^2 - 2xy + 2yz =$

A) $(x+z)(x-2y+z)$ B) $(x-z)(x-2y+z)$

C) $(x+z)(x+2y+z)$ D) None

28. $a^4 - 2a^3 + 2a^2 - 2a + 1$ can be factorised as

A) $(a^2+1)(a-1)$ B) $(a^2+1)^2(a-1)$ C) $(a^2+1)(a-1)^2$ D) $(a^2+1)(a-+1)^2$

29. If the value of $a^4 + \frac{1}{a^4} = 119$ the value of $(a^3 - \frac{1}{a^{3)}})$ is .

A) 52 B) 48 C) 36 4) 29

30. If the polynamial $x^{19} + x^{17} + x^{13} + x^{11} + x^7 + x^5 + x^3$ is divided by $(x^2 + 1)$, then the remainder is

A) X B) -X C) 2X D) X^2

ACHIEVERS (Level - II)

Solve the following :

1. Find a quadratic polynomial if the zeroes of it are 2 & -1/3 respectively.

2. Verify that 1, -1, -3 are the zeroes of the cubic polynomial $x^3 + 3x^2 - x - 3$ and check the relationship between zeroes and the coefficients.

3. Give possible values for length and breadth of the rectangle whose area is $2x^2 + 9x - 5$

4. What must be added to $x^4+2x^3-2x^2+x-1$ so that the result is exactly divided by x^2+2x-3

5. Find a if (x-a) is a factor of polynomial. $x^6-ax^5+x^4-ax^3+3x-a+2$

6. If R_1 & R_2 be the remainder when the polynomials. $f(x)=x^3+2ax^2-5x-7$, $g(x)=x^3+x^2-12x+6a$ divided by (x+1) & (x-1) respectively if $2R_1+R_2=12$ then find the value of 'a'?

7. Find all the zeroes of $2x^4-3x^3-3x^2+6x-2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

8. Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

9. On dividing x^3-3x^2+x+2 by a polynomial g(x), the quotient and remainder were x-2 and -2x+4, respectively. Find g(x).

10. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -14 respectively.

EXPLORERS (Level - III)

I) MCQ's with More than one Answer type:

- *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options*

1. Which of the following are symmetric ?

A) $x^2+y^2+z^2$ B) $x^2+y^2+x+y+1$

C) $x^3+y^3+x^2+y^2+x+y$ D) $(a+b)^3+(b+c)^3+(c+a)^3$

2. . Which of the following are homogeneous and complete ?

A) $x^3-3x^2y+3xy^2-y^3$ B) x^2+xy+y^2

C) $ax+by+cz$ D) $3x^2+4y^2+5z^2$

3. Factors of $4x^3+8x^2-6x-12$ is.....

A) $(2x+\sqrt{6})$ B) $(2x-\sqrt{6})$ C) (x+2) D) (x+1)

4. If $ax^2+2a^2x+b^3$ is divisible by x+a then ....

A) a-b=0 B) $a^2+ab+b^2=0$ C) $a^2-ab+b^2=0$ D) $a^2-b^2=0$

5. x-1 is a factor of ....

A) x^n-1 B) $x^3-3ax^2+3ax-1$ C) x^2+2x+1 D) x^2-2x+1

II) Assertion and Reasoning type questions:

- *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.*

a) Both A and R are correct and R is correct explanation of A.
b) Both A and R are correct and R is not correct explanation of A.
c) A is correct and R is incorrect.
d) A is incorrect and R is correct.

1. **A :** $2x^2-3x^{-1}+5$ is a polynomial of degree 2

R : $f(x)=a_0+a_1x+a_2x^2+........+a_nx^n$ where $n\in z^+$ and $a_1,a_2,a_3........a_n\in C$ such that $a_n\neq 0$ is called polynomial os degree n.

2. **A :** $7x^3+4x^2y+3xy^2+y^3$ is a complete homogeneous.

R : A homogeneous expression is said to be complete, if it contains all possible terms in it.

3. **A :** $(a^2-2ab+3b^2)(3a-2b)$ is homogeneous expression.

R : An expression f(x,y) is said to be symmetric if f(x,y)= f(y,x).

4. **A :** f(x,y,z)= $ax^2+ay^2+az^2+2bxyz$ is absolutely symmetric.

R: If the expression is symmetric in x,y; y,z; z,x then it is absolutely symmetric.

5. **A :** $\sum a^4(b-c)=-(a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)$

R : If $(a-b)(b-c)(c-a)$ are factors of 4^{th} degree cyclic expression then other factor is K(a+b+c).

6. **A :** $x^3-3x^2+4x-5=(x+1)(x^2-4x+8)+(-13)$

R : dividend = (divisor) $\times$ (quotient) + remainder.

7. **A:** (x-2) is a factor of $x^3-x^2-8x+12$

R: f(x) is divided by (x-a) then the remainder is f(a)

8. **A:** (x+1) is factor of $x^4+4x^3+3x^2-4x+5$

R: The sum of coefficient of even powers of x= sum of coefficient of odd powers of x if (x+1) is a factor.

9. **A:** (x-1) is a factor of x^3+2x^2-x-2

R: Sum of coefficient is not equal to zero then (x-1) is a factor.

III) **Integer type questions :**

1. The remainder when $x^{2016}+2016x+2016$ is divided by (x+1) is ....

2. $2^{2n}+1$ where n is odd integeris divisible by ....

3. If (x-2) $\left(x-\frac{1}{2}\right)$ are factors of px^2+5x+q then p-q =.......

4. Let p(x) = $x^{2014}+2x^{2013}+..........+2014x+2015$ and q(x)= x+1. If p(x) divided q(x) the sum of the digits in the remainder is ....

5. When a polynomial p(x) is divided by x-2 the quotient is $3x^2-x-5$ and the remainder is 4 then p(-1) = ...

RESEARCHERS (Level - IV)

I) **MCQ's with Single Answer type:**

1. The degree of the polynomial $(1 - 3x + 3x^2)^{2012}$ $(1 - 3x + 2x^3)^{2012}$ is....... (**RMA-2012**)

a) 2012 b) 6026 c) 1060 d) 20060

2. If the polynomials $ax^3 + 3x^2 - 13$ and $2x^3 - 5x + 9$ are divided by (x-2) leaves the same remainder then find the value of a. (**RAM-2013**)

a) 7 b) 0 c)2 d) 1

3. The remainder when a polynomial $x+x^3+x^9+x^{27}+x^{81}+x^{213}$ is divided by x-1 is ... (**RMA-2012**)

a) 4 b)6 c)-4 d) -6

4. When the polynomial $6x^4+8x^3+17x^2+21x+7$ is divided by $3x^2+4x+1$ the remainder is ax+b then (**NTSE-2013**)

a) a=1, b=2 b) a=1, b=-2 c) a=, b= 1 d) a=-1, b=-2

5. If $f(x) = 2x^3+46x^2+229x+6 = (2x+1)\,g(x)$ = ... (**ASRao-2014**)

a) (2x+3) (6x+1) b) (6x+4) (4x+3) c) (3x+6) (2x+1) d) (3x-2) (4x-3)

6. The remainder when x^{2015} is divided with $x^2 - 1$ is .... (**RMA-2013**)

a) 2x b) x\2 c) 2+1 d) x

7. If x^2+x-6 is a factor of $2x^4+x^3-ax^2+bx+a+b-1$ then the value of a+b is ......

a) 22 b) 18 c) 19 d) 17

8. $x^{n+1} - x^n - x + 1$ is exactly divisible by $(x-1)^2$ if n is .... (**AMTI-2011**)

a) an odd positive integer b) an even positive integer
c) an ood prime d) any positive integer

9. If f(x) is a quadratic polynomial with f(0) = 6, f(1) = 1 and f(2) = 0 then f(3) = ... (**AMTI-2011**)

a) 1 b) 3 c) 5 d) 0

10. The remainder $x^5 + kx^2$ is divided by (x-1) (x-2) (x-3) contains no term in x^2 find the value of k. (**NTSC-2013**)

a) - 50 b) -60 c) -80 d) -90

II) ADDITIONAL PRACTISE PROBLEMS.

MCQ's with Single Answer type:

1. $98a^4 - 16\ 2a^2b^2c^2 =$

A)$2a^2(7a + 9bc)(7a - 9bc)$ A)$a^2(7a + 9bc)(7a - 9bc)$
C)$3a^2(7a + 9bc)(7a - 9bc)$ C)$4a^2(7a + 9bc)(7a - 9bc)$

2. $4a^2 - 4ab + b^2 - 9c^2 + 12cd - 4d^2 =$

A) (2a -b -3c -2d) (2a +b + 3c +2d) B) (2a -b -3c -2d) (2a +b -3c -2d)
C) (2a -b +3c -2d) (2a -b - 3c +2d) D) (2a +b +3c -2d) (2a -b + 3c -2d)

3. $a^3 - 8b^3 - 27c^3 - 18abc =$

A)$(a + 2b + 3c)(a^2 + 4b^2 + 9c^2 - 2ab - 3ac + 6bc)$
B)$(a - 2b - 3c)(a^2 + 4b^2 + 9c^2 + 2ab + 3ac - 6bc)$
C)$(a - 2b - 3c)(a^2 - 4b^2 - 9c^2 + 2ab + 3ac + 6bc)$
D) None

4. $x^6 - 1$ can be factorized as

A)$(x+1)(x^2 + x + 1)(x - 1)(x^2 + x - 1)$
B)$(x+1)(x^2 - x + 1)(x - 1)(x^2 + x + 1)$
C)$(x+1)(x^2 - x + 1)(x - 1)(x^2 + x + 1)$
D)$(x+1)(x^2 - x - 1)(x - 1)(x^2 + x + 1)$

5. $a^2 - b^2 - c^2 + 2bc + a + b - c$ can be resolved as factors.

A)(a+ b + c) (a + b + c + 1) B)(a+ b + c) (a - b - c - 1)
C)(a+ b - c) (a- b + c + 1) D)(a- b - c) (a + b - c + 1)

6. $a^2x^3 + a^5 - 2ab.\ x^3 + b^2\ x^3 + a^3b^2 - 2a^4b$ can be resolve as factors .

A)$(x+a)(x^2 - xa + a^2)(a - b)^2$
A)$(x-a)(x^2 + xa - a^2)(a - b)$
A)$(x-a)(x^2 - xa + a^2)$ D) None

7. If $a^3 = 117 + b^3$ and a = 3+ b than the value of (a + b) is

A) ± 2 B) ± 3 C) ± 5 D) ± 7

8. If $x^4 + x^3$ is divided by (x + 9), then find the degree of the remainder ?

A) 1 B) 0 C) 2 D)3

9. If the degree of the expression $(x^4 - \frac{3}{8})(x^n + \frac{16}{17})$, is 12 then $n =$

A) 2 B) 3 C) 8 D)None

10. which of the following expression is a polynomial ?

A) $3\sqrt{z} + 4z + 5z^2$ B) $\sqrt{ax} + x^2 - x^3$

C) $\sqrt{a}\, x^{\frac{1}{2}} + ax + 9x^2 + 5$ D) $3z^3 - \sqrt{5}\, Z + 9$

11. If (2x + 3) and (x-1) are two factors of $(2x^2 + x)^2 - 4(2x^2 + x) + 3$: then remaining two factors are

A)(2x- 1) (x - 1) B)(2x- 1) (x + 1) C)(2x- 1) (x - 1) D) None

12. The factors of $x^8 - x^4 - 30$ are

A) $(x^4 + 6)$ and $(x^4 + 5)$ B) $(x^4 + 6)$ and $(x^4 - 5)$

C) $(x^4 - 6)$ and $(x^4 + 5)$ D) None

13. $x^2 - y^2 - z^2 + 2yz + x + y - z$ can be expreesed as

A) (x- y + z) (x + y + z -1) B) (x + y - z) (x - y + z -1)

C)) (x- y + z) (x + y + z -1) D) None

14. If (x + a) is the H . C. F of $x^2 + px + q$ and . $x^2 + lx + m$, then the value of 'a' is given by.

A) $\frac{P-l}{Q-m}$ B) $\frac{q-m}{p-l}$ C) $\frac{q+m}{p+l}$ D) $\frac{l+p}{Q+m}$

15. $a^6 - 6a^4 + 12a^2 - 8$ is equel to

A) $(a^2 + 2)^3$ B) $(a^2 - 2)^3$ C) $(a^2 + 3)^2$ D) $(a^2 + 3)^2$

KEY

ΦΦ TEACHING TASK :

I.	1) C	2) A	3) A	4) B	5) D	6) C	7) A	8) B	9) A	10) B
	11D.	12.A	13.C	14.B	15.C	16.D	17.B	18.D	19.C	20.A
	21.A	22.C	23.D	24.C	25.C	26.A	27.D	28.B	29.C	30.D
	31.A	32.D	33.C	34.A	35.C	36.B	37.A	38.C	39.B	40.D
	41.A	42.B	43.C	44.D	45.D	46.A	47.C	48.B	49.A	50.C
	51.C	52.B								
II.	1. A,D		2. A,B,C		3. B,C		4. A,B,C			
III.	1.4		2.6		3.3		4.9			

IV. 1. Yes. It's a factor 3. Q= $3x^2+5x-2$ R=0 4. 2, - 3 5. 1, 3

7. x^2-3x-2 8. $5y^2-10y-3$ 9. $x^2-7x+12$

10. 1, $\pm\sqrt{2}$ 11. 0, -3 12. $3x^2-5x-2$

13. $\frac{5}{3},-\frac{11}{3},1$

ΦΦ LEARNER'STASK :

☐ **BEGINNERS :**

1.C 2.A 3.B 4.D 5.A 6.B 7.C 8.A 9.C 10.D
11.B 12.D 13.A 14.A 15.A 16.B 17.C 18.A 19.B
21) B, 22) B 23) A 24) C 25) A 26) B 27)B 28) C 29) C 30)B

☐ **ACHIEVERS :**

1. $3x^2-5x-2$ 3. l=x+5, b= 2x-1 4. (x-2) 5. -1 6. $a=\frac{14}{5}$

7. 1, $\frac{1}{2}$ 8. -1, -1 9. x^2-x+1 10. $x^3-2x^2-7x+14$

☐ **EXPLORERS:**

I. 1. A,B,C,D 2. A,B,C,D 3. A,B,C 4. A,B 5. A,B,D

II. 1. D 2. A 3. B 4. A 5. B 6. A 7. B 8. D 9. C

III. 1. 1 2. 5 3. 0 4. 9 5.7

☐ **RESEARCHERS :**

I. 1. B 2. C 3.B 4. A 5. B 6. D 7. C 8. D 9. B 10. D
13) B 14)B 15)B

ADDITIONAL PRACTICE PROBLEMS

II. 1)A 2) C 3)B 4)C 5)C 6)A 7)D 8)B 9)C 10)D 11)B 12)C