8th CLASS
MULTIPLE AND SUB-MULTIPLE ANGLES

TEACHING TASK

1. $Given \dfrac{\sin 3A}{1+2\cos 2A}$

$= \dfrac{3\sin A - 4\sin^3 A}{1+2\left(1-2\sin^2 A\right)}$

$= \dfrac{\sin A\left(3-4\sin^2 A\right)}{\left(3-4\sin^2 A\right)}$

$= \sin A$

2. Given $\sin 2\alpha + \sin 2\beta + \sin 2\gamma - \sin 2(\alpha+\beta+\gamma)$

Let $\alpha = 0^0$, $\beta = 30^0$, $\gamma = 60^0$

Now, $\sin 2\cdot 0^0 + \sin 2\cdot 30^0 + \sin 2\cdot 60^0 - \sin 2\left(0^0+30^0+60^0\right)$

$= \sin 0^0 + \sin 60^0 + \sin 120^0 - \sin 180^0$

$= 0 + \dfrac{\sqrt{3}}{2} + \dfrac{\sqrt{3}}{2}$

$= \sqrt{3}$

Consider option A

$4\sin(\alpha+\beta)\cdot\sin(\beta+\gamma)\cdot\sin(\gamma+\alpha)$

$= 4\sin\left(0^0+30^0\right)\cdot\sin\left(60^0+30^0\right)\cdot\sin\left(60^0+0^0\right)$

$= 4\sin 30^0\cdot\sin 90^0\cdot\sin 60^0$

$= 4\cdot\dfrac{1}{2}\cdot 1\cdot\dfrac{\sqrt{3}}{2}$

$= \sqrt{3}$

Hence option A is correct

ANS: A

3. Given $4\cos 6\theta\cdot\cos 4\theta\cdot\cos 2\theta$

$= 2\cos 6\theta\left(2\cos 4\theta\cdot\cos 2\theta\right)$

$= 2\cos 6\theta\left(\cos 6\theta + \cos 2\theta\right)$

$= 2\cos^2 6\theta + 2\cos 6\theta\cdot\cos 2\theta$

$= \left(2\cos^2 6\theta - 1\right) + \cos 8\theta + \cos 4\theta + 1$

$= \cos 12\theta + \cos 8\theta + \cos 4\theta + 1$

ANS: A

4. Given $\sin^2 2\alpha + \cos^2 2\alpha + \tan^2\alpha$

$= 1 + \tan^2\alpha$

$= \sec^2\alpha$

ANS: A

5. Given $\dfrac{\cos A - \cos 3A}{\cos A} + \dfrac{\sin A + \sin 3A}{\sin A}$

$$= \frac{\cos A - \left(4\cos^3 A - 3\cos A\right)}{\cos A} + \frac{\sin A + 3\sin A - 4\sin^3 A}{\sin A}$$

$$= \frac{\cos A - 4\cos^3 A + 3\cos A}{\cos A} + \frac{4\sin A - 4\sin^3 A}{\sin A}$$

$$= \frac{4\cos A - 4\cos^3 A}{\cos A} + \frac{4\sin A - 4\sin^3 A}{\sin A}$$

$$= 4 - 4\cos^2 A + 4 - 4\sin^2 A$$

$$= 8 - 4\left(\cos^2 A + \sin^2 A\right)$$

$$= 8 - 4(1)$$

$$= 4$$

ANS: D

6. $\left(2\cos^2 3\theta - 1\right)\cos 5\theta$

$$= \cos 2\cdot 3\theta \cos 5\theta$$

$$= \cos 6\theta \cdot \cos 5\theta$$

$$= \frac{1}{2}\left(2\cos 6\theta \cdot \cos 5\theta\right)$$

$$= \frac{1}{2}\left(\cos 11\theta + \cos\theta\right)$$

ANS: A

7. Given $\cos 12^0 + \cos 84^0 + \cos 132^0 + \cos 156^0$

$$= \left(\cos 12^0 + \cos 132^0\right) + \left(\cos 84^0 + \cos 156^0\right)$$

$$= 2\cos\left(\frac{12^0 + 132^0}{2}\right)\cdot\cos\left(\frac{12^0 - 132^0}{2}\right) + 2\cos\left(\frac{84^0 + 156^0}{2}\right)\cdot\cos\left(\frac{84^0 - 156^0}{2}\right)$$

$$= 2\cos 72^0 \cdot \cos 60^0 + 2\cos 120^0 \cdot \cos 36^0$$

$$= 2\cdot\frac{\sqrt{5}-1}{4}\cdot\frac{1}{2} + 2\cdot\left(-\frac{1}{2}\right)\cdot\frac{\sqrt{5}+1}{4}$$

$$= \frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4}$$

$$= \frac{\sqrt{5}-1-\sqrt{5}-1}{4}$$

$$= \frac{-2}{4}$$

$$= \frac{-1}{2}$$

ANS: C

8. Let $\alpha = \sin^2 18^0$, $\beta = \cos^2 36^0$

Now, $\alpha + \beta = \sin^2 18^0 + \cos^2 36^0$

$$=\left(\frac{\sqrt{5}-1}{4}\right)^2+\left(\frac{\sqrt{5}+1}{4}\right)^2$$

$$=\frac{2(5+1)}{16}$$

$$=\frac{12}{16}$$

$$\alpha\cdot\beta=\sin^2 18^0\cdot\cos^2 36^0$$

$$=\left(\frac{\sqrt{5}-1}{4}\right)^2\cdot\left(\frac{\sqrt{5}+1}{4}\right)^2$$

$$=\left(\frac{\sqrt{5}-1}{4}\cdot\frac{\sqrt{5}+1}{4}\right)^2$$

$$=\left(\frac{4}{16}\right)^2$$

$$=\frac{1}{16}$$

The required quadratic equation is

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$\Rightarrow x^2-\frac{12}{16}x+\frac{1}{16}=0$$

$$\Rightarrow 16x^2-12x+1=0$$

ANS: A

9. Given $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=K\cos\theta$

Let $\theta=0^0$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\times O^0}}}=K\cdot\cos 0^0$$

$$\Rightarrow \sqrt{2+\sqrt{2+\sqrt{2+2}}}=K$$

$$\Rightarrow \sqrt{2+\sqrt{2+2}}=K$$

$$\Rightarrow \sqrt{2+2}=K$$

$$\Rightarrow K=2$$

ANS: A

10. Given $\dfrac{\cos 6x+6\cos 4x+15\cos 2x+10}{\cos 5x+5\cos 3x+10\cos x}$

Let x = 0^0

$$=\frac{\cos 6\cdot 0^0+6\cdot\cos 4\cdot 0^0+15\cdot\cos 2\cdot 0^0+10}{\cos 5\cdot 0^0+5\cdot\cos 3\cdot 0^0+10\cdot\cos 0^0}$$

$$=\frac{1+6+15+10}{1+5+10}=\frac{32}{16}=2$$

Consider option D 2cosx

= 2 cos0^0

= 2

ANS: D

11. $Tan7\frac{1}{2}^{0}=\sqrt{6}-\sqrt{3}+\sqrt{2}-2$

$=\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{2}-1\right)$

$=\left(\sqrt{2}-\sqrt{3}\right)\left(1-\sqrt{2}\right)$

ANS: A, C, D

12. Given xy + yz+ zx = 1

Let x = tanA, y = tanB, z = tanC

$\therefore TanA\cdot TanB+TanB\cdot TanC+TanC\cdot TanA=1$

$\Rightarrow TanB\left(TanA+TanC\right)=1-TanA\cdot TanC$

$\Rightarrow \frac{Tan+TanC}{1-TanA\cdot TanC}=\frac{1}{TanB}$

$\Rightarrow Tan\left(A+C\right)=\cot B$

$\Rightarrow Tan\left(A+C\right)=Tan\left(90^{0}-B\right)$

$\Rightarrow A+C=90^{0}-B$

$\Rightarrow A+B+C=90^{0}$

$\Rightarrow 2A+2B+2C=180^{0}$

$\Rightarrow 2A+2B=180^{0}-2C$

$\Rightarrow Tan\left(2A+2B\right)=Tan\left(180^{0}-2C\right)$

$\Rightarrow Tan\left(2A+2B\right)=-Tan2C$

$\Rightarrow \frac{Tan2A+Tan2B}{1-Tan2A\cdot Tan2B}=-Tan2C$

$\Rightarrow Tan2A+Tan2B=-Tan2C+Tan2A\cdot Tan2B\cdot Tan2C$

$\Rightarrow Tan2A+Tan2B+Tan2C=Tan2A\cdot Tan2B\cdot Tan2C$

$\Rightarrow \sum Tan2A=\pi Tan2A$

$\Rightarrow \sum\frac{2TanA}{1-Tan^{2}A}=\pi\frac{2TanA}{1-Tan^{2}A}$

$\Rightarrow \sum\left(\frac{2x}{1-x^{2}}\right)=\pi\left(\frac{2x}{1-x^{2}}\right)$

$=\pi\left(\frac{2y}{1-y^{2}}\right)$

$=\pi\left(\frac{2z}{1-z^{2}}\right)$

ANS: B, C, D

13. Given $\tan\theta=\frac{p^{2}-q^{2}}{2pq}$

$$\Rightarrow \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{p^2-q^2}{2pq}$$

$$\Rightarrow \frac{2x}{1-x^2} = \frac{p^2-q^2}{2pq} \qquad \text{where } x = \tan\frac{\theta}{2}$$

$$\Rightarrow 4pqx = \left(p^2-q^2\right) - \left(p^2-q^2\right)x^2$$

$$\Rightarrow \left(p^2-q^2\right)x^2 + 4pqx - \left(p^2-q^2\right) = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x = \frac{-4pq \pm \sqrt{16p^2q^2 + 4\left(p^2-q^2\right)^2}}{2\left(p^2-q^2\right)}$$

$$\Rightarrow x = \frac{-4pq \pm 2\sqrt{4p^2q^2 + \left(p^2-q^2\right)^2}}{2\left(p^2-q^2\right)}$$

$$\Rightarrow x = \frac{-4pq \pm 2\sqrt{\left(p^2+q^2\right)^2}}{2\left(p^2-q^2\right)}$$

$$\Rightarrow x = \frac{-2pq \pm \left(p^2+q^2\right)}{p^2-q^2}$$

$$\Rightarrow x = \frac{-2pq + p^2 + q^2}{p^2-q^2} \qquad \text{(Consider + sign)}$$

$$\Rightarrow x = \frac{\left(p-q\right)^2}{\left(p^2-q^2\right)}$$

$$\Rightarrow x = \frac{\left(p-q\right)^2}{\left(p+q\right)\left(p-q\right)}$$

$$\Rightarrow x = \frac{p-q}{p+q}$$

$$\Rightarrow \tan\frac{\theta}{2} = \frac{p-q}{p+q}$$

$$\Rightarrow \cot\frac{\theta}{2} = \frac{p+q}{p-q}$$

ANS: A, D

14. Statement I : Given $\cos 2B = \frac{\cos\left(A+C\right)}{\cos\left(A-C\right)}$

$$= \cos 2B = \frac{\cos A \cos C - \sin A \sin C}{\cos A \cos C + \sin A \sin C}$$

$$\Rightarrow \cos 2B = \frac{1 - \tan A \cdot \tan C}{1 + \tan A \cdot \tan C}$$

$$\Rightarrow \frac{1 - \tan^2 B}{1 + \tan^2 B} = \frac{1 - \tan A \cdot \tan C}{1 + \tan A \cdot \tan C}$$

$$\Rightarrow \left(1 - \tan^2 B\right)\left(1 + \tan A \cdot \tan C\right) = \left(1 + \tan^2 B\right)\left(1 - \tan A \cdot \tan C\right)$$

$$\Rightarrow 1 + \tan A \cdot \tan C - \tan^2 B - \tan^2 B \cdot \tan A \cdot \tan C$$

$$= 1 - \tan A \cdot \tan C + \tan^2 B - \tan^2 B \cdot \tan A \cdot \tan C$$

$$\Rightarrow 2\tan^2 B = 2\tan A \cdot \tan C$$

$$\Rightarrow \tan^2 B = \tan A \cdot \tan C$$

$\therefore \tan A, \tan B$ and $\tan C$ are in G.P

Statement I is TRUE.

Statement II : If a, b, c are in G.P , then $b^2 = ac$

Statement II is TRUE.

ANS: A

15. Statement I : Given $\frac{x}{\cos\theta} = \frac{y}{\cos\left(\theta - \frac{2\pi}{3}\right)} = \frac{z}{\cos\left(\theta + \frac{2\pi}{3}\right)}$

Let

$$\frac{x}{\cos\theta} = \frac{y}{\cos\left(\theta - \frac{2\pi}{3}\right)} = \frac{z}{\cos\left(\theta + \frac{2\pi}{3}\right)} = k$$

$$\therefore x + y + z = k\left(\cos\theta + \cos\left(\theta - \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{2\pi}{3}\right)\right)$$

$$= k\left(\cos\theta + 2\cos\theta\cos\frac{2\pi}{3}\right)$$

$$= k\left(\cos\theta + 2\cos\theta\left(-\frac{1}{2}\right)\right)$$

$$= k\left(\cos\theta - \cos\theta\right) = 0$$

Hence, statement I is FALSE.

Statement II : Given $\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = k \cdot \sec 2\theta$

$$\Rightarrow \frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\cdot\tan\theta}+\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\cdot\tan\theta}=k\cdot\sec 2\theta$$

$$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}=k\cdot\sec 2\theta$$

$$\Rightarrow \frac{(1+\tan\theta)^2+(1-\tan\theta)^2}{(1-\tan\theta)(1+\tan\theta)}=k\cdot\sec 2\theta$$

$$\Rightarrow \frac{2\left(1+\tan^2\theta\right)}{1-\tan^2\theta}=k\cdot\sec 2\theta$$

$$\Rightarrow 2\cdot\sec 2\theta=k\cdot\sec 2\theta$$

$$\Rightarrow k=2$$

Statement II is TRUE.

ANS: D

16. Given $\cos\theta=\cos\alpha\cdot\cos\beta$

$$\Rightarrow \frac{\cos\theta}{\cos\alpha}=\frac{\cos\beta}{1}$$

By componendo and dividend

$$\Rightarrow \frac{\cos\theta+\cos\alpha}{\cos\theta-\cos\alpha}=\frac{\cos\beta+1}{\cos\beta-1}$$

$$\Rightarrow \frac{2\cos\left(\frac{\theta+\alpha}{2}\right)\cdot\cos\left(\frac{\theta-\alpha}{2}\right)}{-2\sin\left(\frac{\theta+\alpha}{2}\right)\cdot\sin\left(\frac{\theta-\alpha}{2}\right)}=\frac{2\cos^2\frac{\beta}{2}}{-2\sin^2\frac{\beta}{2}}$$

$$\Rightarrow \cot\left(\frac{\theta+\alpha}{2}\right)\cdot\cot\left(\frac{\theta-\alpha}{2}\right)=\cot^2\frac{\beta}{2}$$

ANS: A

17. Given $\frac{\cos x}{\cos(x-2y)}=\frac{\lambda}{1}$

$$\Rightarrow \frac{\cos x+\cos(x-2y)}{\cos x-\cos(x-2y)}=\frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{2\cos\left(\frac{x+x-2y}{2}\right)\cdot\cos\left(\frac{x-x+2y}{2}\right)}{-2\sin\left(\frac{x+x-2y}{2}\right)\cdot\sin\left(\frac{x-x+2y}{2}\right)}=\frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{\cos(x-y)\cdot\cos y}{\sin(x-y)\cdot\sin y}=\frac{1+\lambda}{1-\lambda}$$

$$\Rightarrow \tan(x-y)\cdot\tan y=\frac{1-\lambda}{1+\lambda}$$

ANS: B

18. Given $\cos(x-y)=3\cdot\cos(x+y)$

$$\Rightarrow \frac{\cos(x-y)}{\cos(x+y)}=\frac{3}{1}$$

$$\Rightarrow \frac{\cos(x-y)+\cos(x+y)}{\cos(x-y)-\cos(x+y)}=\frac{3+1}{3-1}$$

$$\Rightarrow \frac{2\cos x\cdot\cos y}{2\sin x\cdot\sin y}=\frac{4}{2}$$

$$\Rightarrow \cot x\cdot\cot y=2$$

ANS: C

19. Given $\dfrac{\sin 65^0+\sin 25^0}{\cos 65^0+\cos 25^0}$

$$=\frac{\sin\left(90^0-25^0\right)+\sin\left(90^0-65^0\right)}{\cos 65^0+\cos 25^0}$$

$$=\frac{\cos 25^0+\cos 65^0}{\cos 65^0+\cos 25^0}=1$$

ANS: 1

20. Given $\cos\theta-\sin\theta=\frac{1}{5},\ 0<\theta<\frac{\pi}{2}$

$$\Rightarrow (\cos\theta-\sin\theta)^2=\frac{1}{25}$$

$$\Rightarrow \cos^2\theta+\sin^2\theta-2\cos\theta\cdot\sin\theta=\frac{1}{25}$$

$$\Rightarrow 1-\sin 2\theta=\frac{1}{25}$$

$$\Rightarrow \sin 2\theta=1-\frac{1}{25}$$

$$\Rightarrow \sin 2\theta=\frac{24}{25}$$

a) $(\cos\theta+\sin\theta)^2=(\cos\theta-\sin\theta)^2+4\cos\theta\cdot\sin\theta$

$$=(\cos\theta-\sin\theta)^2+2\cdot\sin 2\theta$$

$$=\left(\frac{1}{5}\right)^2+2\cdot\frac{24}{25}$$

$$=\frac{1}{25}+\frac{48}{25}=\frac{49}{25}$$

$$\therefore \cos\theta+\sin\theta=\frac{7}{5}$$

b) $\sin 2\theta=\frac{24}{25}$

c) $\cos 2\theta=\sqrt{1-\sin^2 2\theta}$

$$=\sqrt{1-\left(\frac{24}{25}\right)^2}$$

$$=\sqrt{1-\frac{576}{625}}$$

$$=\sqrt{\frac{49}{625}}=\frac{7}{25}$$

d) $\cot 2\theta=\dfrac{\cos 2\theta}{\sin 2\theta}$

$$=\frac{\left(\frac{7}{25}\right)}{\left(\frac{24}{25}\right)}=\frac{7}{24}$$

LEARNERS TASK

01.Given $\sin\left(\dfrac{A+B}{2}\right)+\sin\left(\dfrac{A-B}{2}\right)$

$$=\sin\left(\frac{A}{2}+\frac{B}{2}\right)+\sin\left(\frac{A}{2}-\frac{B}{2}\right)$$

$$=2\sin\left(\frac{A}{2}\right)\cdot\cos\left(\frac{B}{2}\right)$$

ANS: A

02.Given $\cos A-\cos B$

$$=-2\sin\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$$

ANS: B

03.Given $A+B+C=180^0$

$$\Rightarrow 2A+2B+2C=360^0$$

$$\Rightarrow 2A+2B=360^0-2C$$

$$\Rightarrow \sin(2A+2B)=\sin\left(360^0-2C\right)$$

$$\Rightarrow \sin(2A+2B)=-\sin 2C$$

ANS: D

04.Given $\sin 85^0-\sin 35^0-\cos 65^0$

$$= 2\cos\left(\frac{85^0 + 35^0}{2}\right)\cdot\sin\left(\frac{85^0 - 35^0}{2}\right) - \cos 65^0$$

$$= 2\cos 60^0 \cdot \cos 25^0 - \cos 65^0$$

$$= 2\left(\frac{1}{2}\right)\cdot\cos 25^0 - \cos 65^0$$

$$= \cos 25^0 - \cos 65^0$$

$$= -2\sin\left(\frac{25^0 + 65^0}{2}\right)\cdot\sin\left(\frac{25^0 - 65^0}{2}\right)$$

$$= -2\sin 45^0 \cdot \sin\left(-20^0\right)$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)\sin 20^0$$

$$= \sqrt{2}\sin 20^0$$

05.Given $\cos A = \theta$

Now , $\cos 3A = 4\cos^3 A - 3\cos A$

$$= 4\theta^3 - 3\theta$$

ANS: A

06.Given $\sin\theta = 1$

$$\Rightarrow \theta = \frac{\pi}{2}$$

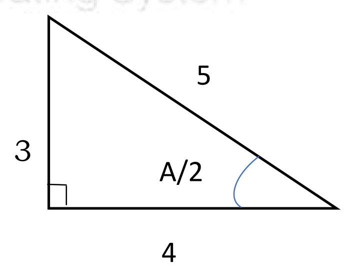
Now, $\cos 2\theta = \cos 2\times\frac{\pi}{2}$

$$= \cos\pi$$

$$= -1$$

ANS: B

07.Given $\cos\frac{A}{2} = \frac{4}{5}$



$$\Rightarrow \sin\frac{A}{2} = \frac{3}{5}$$

Now, $\sin A = 2\sin\frac{A}{2}\cdot\cos\frac{A}{2}$

$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$$

$$= \frac{24}{25}$$

ANS: B

08.Given $\tan 22\frac{1}{2}^0 + \cot 22\frac{1}{2}^0$

$$= \sqrt{2} - 1 + \sqrt{2} + 1$$

$$= 2\sqrt{2}$$

ANS: B

09.Given $\sin 72^0 \cdot \cos 72^0$

$$= \frac{\sqrt{10+2\sqrt{5}}}{4} \cdot \frac{\sqrt{5}-1}{4}$$

ANS: B

10. Given $\tan A = \frac{x}{2}$

Now, $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

$$= \frac{3\left(\frac{x}{2}\right) - \left(\frac{x}{2}\right)^3}{1-3\left(\frac{x}{2}\right)^2}$$

$$= \frac{\frac{3x}{2} - \frac{x^3}{8}}{1-\frac{3x^2}{4}}$$

$$= \frac{12x - x^3}{2\left(4-3x^2\right)}$$

ANS: B

JEE MAINS LEVEL

1. Given $\sin 78^0 - \sin 18^0 + \cos 132^0$

$$= 2\cos\left(\frac{78^0+18^0}{2}\right) \cdot \sin\left(\frac{78^0-18^0}{2}\right) + \cos 132^0$$

$$= 2\cos 48^0 \cdot \sin 30^0 + \cos 132^0$$

$$= 2\cos 48^0 \cdot \left(\frac{1}{2}\right) + \cos 132^o$$

$$= \cos 48^0 + \cos 132^o$$

$$= 2\cos\left(\frac{48^0+132^0}{2}\right) \cdot \cos\left(\frac{48^0-132^0}{2}\right)$$

$$= 2\cos 90^0 \cdot \cos 42^0$$

$$= 2(0) \cdot \cos 42^o = 0$$

ANS: B

2. Given $1+\cos 2\theta + \cos 4\theta + \cos 6\theta$

$$= 2\cos^2\theta + 2\cos\left(\frac{4\theta+6\theta}{2}\right) \cdot \cos\left(\frac{4\theta-6\theta}{2}\right)$$

$$= 2\cos^2\theta + 2\cos 5\theta \cdot \cos\theta$$

$$= 2\cos\theta\left(\cos\theta + \cos 5\theta\right)$$

$$= 2\cos\theta\left(2\cos\left(\frac{\theta+5\theta}{2}\right) \cdot \cos\left(\frac{\theta-5\theta}{2}\right)\right)$$

$$= 4\cos\theta \cdot \cos 2\theta \cdot \cos 3\theta$$

ANS: B

3. $\sqrt{2}\cdot\cos ec20^0\cdot\sec 20^0$

$$=\sqrt{2}\cdot\left(\frac{1}{\sin 20^0}\right)\left(\frac{1}{\cos 20^0}\right)$$

$$=\frac{2\sqrt{2}}{2\sin 20^0\cdot\cos 20^0}$$

$$=\frac{2\sqrt{2}}{\sin 40^0}$$

$$=4\times\left(\frac{1}{\sqrt{2}}\right)\times\cos ec40^0$$

$$=4\sin 45^0\cdot\cos ec40^0$$

ANS: D

4. Given $2\left(1-2\sin^2\theta\right)\cdot\cos 4\theta$

$$=2\cdot\cos 2\theta\cdot\cos 4\theta$$

$$=\cos\left(2\theta+4\theta\right)+\cos\left(2\theta-4\theta\right)$$

$$=\cos 6\theta+\cos 2\theta$$

ANS: C

5. Given $\frac{\sin\alpha}{a}=\frac{\cos\alpha}{b}$

$$\Rightarrow \tan\alpha=\frac{a}{b}$$

$a\sin 2\alpha+b\cos 2\alpha$

$$=a\left(\frac{2\tan\alpha}{1+\tan^2\alpha}\right)+b\left(\frac{1-\tan^2\alpha}{1+\tan^2\alpha}\right)$$

$$=a\left(\frac{2\frac{a}{b}}{1+\left(\frac{a}{b}\right)^2}\right)+b\left(\frac{1-\left(\frac{a}{b}\right)^2}{1+\left(\frac{a}{b}\right)^2}\right)$$

$$=\frac{2a^2b}{a^2+b^2}+\frac{b\left(b^2-a^2\right)}{b^2+a^2}$$

$$=\frac{2a^2b+b^3-a^2b}{a^2+b^2}$$

$$=\frac{a^2b+b^3}{a^2+b^2}=\frac{b\left(a^2+b^2\right)}{\left(a^2+b^2\right)}=b$$

ANS: B

6. Given $\tan 2\alpha-\tan\alpha\left(1+\sec 2\alpha\right)$

Let $\alpha=30^0$

$= \tan 60^0 - \tan 30^0 \left(1 + \sec 60^0\right)$

$= \sqrt{3} - \frac{1}{\sqrt{3}}(1+2)$

$= \sqrt{3} - \frac{3}{\sqrt{3}}$

$= \sqrt{3} - \sqrt{3} = 0$

ANS: C

7. Given $\frac{\cos A + \sin A}{\cos A - \sin A} - \frac{\cos A - \sin A}{\cos A + \sin A}$

$= \frac{(\cos A + \sin A)^2 - (\cos A - \sin A)^2}{(\cos A - \sin A)(\cos A + \sin A)}$

$= \frac{4\cos A \cdot \sin A}{\cos^2 A - \sin^2 A}$

$= \frac{2(2\sin A \cos A)}{\cos 2A}$

$= 2 \cdot \frac{\sin 2A}{\cos 2A} = 2\tan 2A$

ANS: A

8. $\sin 75^0 + \sin 15^0$

$= 2\sin\left(\frac{75^0 + 15^0}{2}\right) \cdot \cos\left(\frac{75^0 - 15^0}{2}\right)$

$= 2\sin 45^0 \cdot \cos 30^0$

$= 2\left(\frac{1}{\sqrt{2}}\right) \cdot \left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{\sqrt{2}}$

ANS: B

9. Given $\frac{1}{\sin 10^0} - \frac{\sqrt{3}}{\cos 10^0}$

$= \frac{\cos 10^0 - \sqrt{3}\sin 10^0}{\sin 10^0 \cdot \cos 10^0}$

$$=4\frac{\left(\frac{1}{2}\cdot\cos10^0-\frac{\sqrt{3}}{2}\cdot\sin10^0\right)}{2\sin10^0\cdot\cos10^0}$$

$$=4\frac{\left(\sin30^0\cdot\cos10^0-\cos30^0\cdot\sin10^0\right)}{\sin20^0}$$

$$=4\frac{\sin\left(30^0-10^0\right)}{\sin20^0}$$

$$=4\cdot\frac{\sin20^0}{\sin20^0}=4$$

ANS: A

10. Given $\cos^2 76^0+\cos^2 16^0-\cos16^o\cdot\cos76^0$

$$=\cos^2 76^0+1-\sin^2 16^0-\cos16^o\cdot\cos76^0$$

$$=1+\left(\cos^2 76^0-\sin^2 16^o\right)-\frac{1}{2}\left(2\cos16^0\cdot\cos76^0\right)$$

$$=1+\cos72^0\cdot\cos60^0-\frac{1}{2}\left(\cos72^0+\cos60^0\right)$$

$$=1+\cos72^0\cdot\left(\frac{1}{2}\right)-\frac{1}{2}\cos72^0-\frac{1}{2}\cos60^0$$

$$=1-\frac{1}{2}\cdot\frac{1}{2}=1-\frac{1}{4}=\frac{3}{4}$$

ANS: D

11. Given $\frac{\sin70^0+\cos40^0}{\cos70^0+\sin40^0}$

$$=\frac{\cos20^0+\cos40^0}{\sin20^0+\sin40^0}$$

$$=\frac{2\cos\left(\frac{20^0+40^0}{2}\right)\cdot\cos\left(\frac{20^0-40^0}{2}\right)}{2\sin\left(\frac{20^0+40^0}{2}\right)\cdot\cos\left(\frac{20^0-40^0}{2}\right)}$$

$$=\frac{\cos30^0\cdot\cos10^0}{\sin30^0\cdot\cos10^0}=\cot30^0=\sqrt{3}$$

ANS: A

ADVANCED LEVEL QUESTIONS

12. Given $\tan\theta=\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$

$$\Rightarrow \tan\theta = \frac{\tan\alpha - 1}{\tan\alpha + 1}$$

$$= \frac{-(1-\tan\alpha)}{(1+\tan\alpha)}$$

$$= -\tan\left(\frac{\pi}{4} - \alpha\right)$$

$$= \tan\left(\alpha - \frac{\pi}{4}\right)$$

$$\therefore \theta = \alpha - \frac{\pi}{4}$$

$$\Rightarrow \alpha = \theta + \frac{\pi}{4}$$

Option A : $\sin\alpha - \cos\alpha = \pm\sqrt{2}\sin\theta$

Consider $(\sin\alpha - \cos\alpha)^2$

$$= 1 - \sin 2\alpha$$

$$= 1 - \sin 2\left(\theta + \frac{\pi}{4}\right)$$

$$= 1 - \sin\left(\frac{\pi}{2} + 2\theta\right)$$

$$= 1 - \cos 2\theta$$

$$= 2\sin^2\theta$$

$$\therefore \sin\alpha - \cos\alpha = \pm\sqrt{2}\sin\theta$$

Option B: $\sin\alpha + \cos\alpha = \pm\sqrt{2}\cos\theta$

Consider $(\sin\alpha + \cos\alpha)^2 = 1 + \sin 2\alpha$

$$= 1 + \sin\left(\frac{\pi}{2} + 2\theta\right)$$

$$= 1 + \cos 2\theta$$

$$= 2\cos^2\theta$$

$$\therefore \sin\alpha + \cos\alpha = \pm\sqrt{2}\cos\theta$$

13. Given $\sin\alpha = \sin\beta$ and $\cos\alpha = \cos\beta$

$$\Rightarrow \sin\alpha - \sin\beta = 0$$

$$\Rightarrow 2\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right) = 0 \rightarrow \quad 1$$

Now, $\cos\alpha = \cos\beta$

$$\Rightarrow \cos\alpha - \cos\beta = 0$$

$$\Rightarrow -\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)=0 \rightarrow \quad 2$$

$$\frac{2}{1} \Rightarrow \frac{-2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)}=\frac{0}{0}$$

$$\Rightarrow \tan\left(\frac{\alpha+\beta}{2}\right)=0$$

$$\Rightarrow \sin\left(\frac{\alpha+\beta}{2}\right)=0$$

$$\Rightarrow \alpha+\beta=0$$

ANS: A, B

14. Let $A=11\frac{1}{4}^{0}$

$$\Rightarrow 2A=22\frac{1}{2}^{0}$$

$$\Rightarrow Tan2A=Tan22\frac{1}{2}^{0}$$

$$\Rightarrow \frac{2\tan A}{1-Tan^2 A}=\sqrt{2}-1$$

$$\Rightarrow \frac{2x}{1-x^2}=\sqrt{2}-1 \qquad (\because Let\ \tan A = x)$$

$$\Rightarrow 2x=\left(\sqrt{2}-1\right)-\left(\sqrt{2}-1\right)x^2$$

$$\Rightarrow \left(\sqrt{2}-1\right)x^2+2x-\left(\sqrt{2}-1\right)=0$$

$$\Rightarrow x=\frac{-2\pm\sqrt{4+4\left(\sqrt{2}-1\right)\left(\sqrt{2}-1\right)}}{2\left(\sqrt{2}-1\right)}$$

$$\Rightarrow x=\frac{-2\pm 2\sqrt{1+2+1-2\sqrt{2}}}{2\left(\sqrt{2}-1\right)}$$

$$\Rightarrow x=\frac{-2\pm 2\sqrt{4-2\sqrt{2}}}{2\left(\sqrt{2}-1\right)}$$

$$\Rightarrow x=\frac{-1\pm\sqrt{4-2\sqrt{2}}}{\sqrt{2}-1}$$

$$\Rightarrow x=\frac{-1\pm\sqrt{4-2\sqrt{2}}}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$\Rightarrow x=\frac{-\left(\sqrt{2}+1\right)+\sqrt{\left(4-2\sqrt{2}\right)\left(\sqrt{2}+1\right)^2}}{2-1}$$

$$\Rightarrow x=-\left(\sqrt{2}+1\right)+\sqrt{\left(4-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)}$$

$$\Rightarrow x=-\left(\sqrt{2}+1\right)+\sqrt{4+2\sqrt{2}}$$

$$\therefore Tan11\frac{1}{4}^0=\sqrt{4+2\sqrt{2}}-\left(\sqrt{2}+1\right)$$

ANS: C

15. Given $\tan142\frac{1}{2}^0=\tan\left(90^0+52\frac{1}{2}^0\right)$

$$=-\cot52\frac{1}{2}^0$$

$$=\frac{-1}{\tan52\frac{1}{2}^0}$$

$$=\frac{-1}{\tan\left(45+7\frac{1}{2}^0\right)}$$

$$=-\frac{1-\tan7\frac{1}{2}^0}{1+\tan7\frac{1}{2}^0}$$

$$=-\frac{\left(\cos7\frac{1}{2}^0-\sin7\frac{1}{2}^0\right)}{\left(\cos7\frac{1}{2}^0+\sin7\frac{1}{2}^0\right)}\times\frac{\left(\cos7\frac{1}{2}^0-\sin7\frac{1}{2}^0\right)}{\left(\cos7\frac{1}{2}^o-\sin7\frac{1}{2}^0\right)}$$

$$=-\frac{1-2\sin7\frac{1}{2}^0\cdot\cos7\frac{1}{2}^0}{\cos^2 7\frac{1}{2}^0-\sin^2 7\frac{1}{2}^0}$$

$$=-\frac{1-\sin15^0}{\cos15^0}$$

$$=\frac{1-\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}=2+\sqrt{2}-\sqrt{3}-\sqrt{6}$$

ANS: D

16. Let $A = 7\frac{1}{2}^0$

$\Rightarrow 2A = 15^0$

$\Rightarrow \cot 2A = \cot 15^0$

$\Rightarrow \cot 2A = \frac{\cos 15^0}{\sin 15^0}$

$\Rightarrow \cot 2A = \frac{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$

$\Rightarrow \frac{\cot^2 A - 1}{2\cot A} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$\Rightarrow \frac{\cot^2 A - 1}{2\cot A} = \frac{3+1+2\sqrt{3}}{2}$

$\Rightarrow \frac{x^2 - 1}{x} = 4 + 2\sqrt{3}$ (Let cotA = x)

$\Rightarrow x^2 - 1 = \left(4+2\sqrt{3}\right)x$

$\Rightarrow x^2 - \left(4+2\sqrt{3}\right)x - 1 = 0$

$\Rightarrow x = \frac{\left(4+2\sqrt{3}\right) \pm \sqrt{\left(4+2\sqrt{3}\right)^2 + 4}}{2 \cdot 1}$

$\Rightarrow x = \frac{\left(4+2\sqrt{3}\right) \pm \sqrt{32+16\sqrt{3}}}{2}$

$\Rightarrow x = \frac{\left(4+2\sqrt{3}\right) + \sqrt{8\left(4+2\sqrt{3}\right)}}{2}$

$\Rightarrow x = \frac{\left(4+2\sqrt{3}\right) + 2\sqrt{2}\sqrt{\left(\sqrt{3}+1\right)^2}}{2}$

$\Rightarrow x = \frac{4+2\sqrt{3}+2\sqrt{2}\left(\sqrt{3}+1\right)}{2}$

$\Rightarrow x = \frac{4+2\sqrt{3}+2\sqrt{6}+2\sqrt{2}}{2}$

$\Rightarrow x = 2+\sqrt{3}+\sqrt{6}+\sqrt{2}$

$\Rightarrow x = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

ANS: A

17. Given $\cos^2 A\left(3-4\cos^2 A\right)^2 + \sin^2 A\left(3-4\sin^2 A\right)^2$

$$=\left[\cos A\left(3-4\cos^2 A\right)\right]^2+\left[\sin A\left(3-4\sin^2 A\right)\right]^2$$
$$=\left[3\cos A-4\cos^3 A\right]^2+\left[3\sin A-4\sin^3 A\right]^2$$
$$=\left(-\cos 3A\right)^2+\left(\sin 3A\right)^2$$
$$=\sin^2 3A+\cos^2 3A=1$$

18. Given $\dfrac{\cos\left(\theta_1-\theta_2\right)}{\cos\left(\theta_1+\theta_2\right)}+\dfrac{\cos(\theta_3+\theta_4)}{\cos\left(\theta_3-\theta_4\right)}=0$

$$\Rightarrow\frac{\cos\theta_1\cdot\cos\theta_2+\sin\theta_1\cdot\sin\theta_2}{cos\theta_1\cdot\cos\theta_2-\sin\theta_1\cdot\sin\theta_2}+\frac{\cos\theta_3\cdot\cos\theta_4-\sin\theta_3\cdot\sin\theta_4}{\cos\theta_3\cdot\cos\theta_4+\sin\theta_3\cdot\sin\theta_4}=0$$
$$\Rightarrow\frac{1+\tan\theta_1\cdot\tan\theta_2}{1-\tan\theta_1\cdot\tan\theta_2}+\frac{1-\tan\theta_3\cdot\tan\theta_4}{1+\tan\theta_3\cdot\tan\theta_4}=0$$

$$=\left(1+\tan\theta_1\cdot\tan\theta_2\right)\left(1+\tan\theta_3\cdot\tan\theta_4\right)+\left(1-\tan\theta_3\cdot\tan\theta_4\right)\left(1-\tan\theta_1\cdot\tan\theta_2\right)=0$$
$$=1+\tan\theta_3\cdot\tan\theta_4+\tan\theta_1\cdot\tan\theta_2+\tan\theta_1\cdot\tan\theta_2\cdot\tan\theta_3\cdot\tan\theta_4+1-\tan\theta_1\cdot\tan\theta_2-\tan\theta_3\cdot\tan\theta_4+\tan\theta_1\cdot\tan\theta_2\cdot\tan\theta_3\cdot\tan\theta_4=0$$
$$\Rightarrow 2+2\tan\theta_1\cdot\tan\theta_2\cdot\tan\theta_3\cdot\tan\theta_4=0$$
$$\Rightarrow\tan\theta_1\cdot\tan\theta_2\cdot\tan\theta_3\cdot\tan\theta_4=-1$$

19. a) Given $\dfrac{1}{\cos 290^0}+\dfrac{1}{\sqrt{3}\sin 250^0}$

$$=\frac{1}{\cos\left(270^0+20^0\right)}+\frac{1}{\sqrt{3}\sin\left(270^0-20^0\right)}$$
$$=\frac{1}{\sin 20^0}-\frac{1}{\sqrt{3}\cos 20^0}$$
$$=\left(\frac{1}{\sin 20^0}-\frac{1}{\sqrt{3}\cos 20^0}\right)$$
$$=\left(\frac{\sqrt{3}\cos 20^0-\sin 20^0}{\sqrt{3}\sin 20^0\cdot\cos 20^0}\right)$$
$$=\left(4\frac{\frac{\sqrt{3}}{2}\cos 20^0-\frac{1}{2}\sin 20^0}{\sqrt{3}\left(2\sin 20^0\cdot\cos 20^0\right)}\right)$$
$$=\frac{4}{\sqrt{3}}\frac{\sin 60^0\cdot\cos 20^0-\cos 60^0\cdot\sin 20^0}{\sin 2\times 20^0}$$
$$=\frac{4}{\sqrt{3}}\cdot\frac{\sin\left(60^0-20^0\right)}{\sin 40^0}$$
$$=\frac{4}{\sqrt{3}}\cdot\frac{\sin 40^0}{\sin 40^0}$$
$$=\frac{4}{\sqrt{3}}$$

b) $\dfrac{1}{\sin 10^0}-\dfrac{\sqrt{3}}{\cos 10^0}$

$$= \frac{\cos 10^0 - \sqrt{3}\sin 10^0}{\sin 10^0 \cdot \cos 10^0}$$

$$= \frac{4\left(\frac{1}{2}\cdot\cos 10^0 - \frac{\sqrt{3}}{2}\cdot\sin 10^0\right)}{\left(2\sin 10^0\cdot\cos 10^0\right)}$$

$$= 4\frac{\sin\left(30^0 - 10^0\right)}{\sin 20^0}$$

$$= 4\frac{\sin 20^0}{\sin 20^0} = 4$$

c) $\sqrt{3}\cdot\cos ec 20^0 - \sec 20^0$

$$= \sqrt{3}\frac{1}{\sin 20^0} - \frac{1}{\cos 20^0}$$

$$= \frac{\sqrt{3}\cos 20^0 - \sin 20^0}{\sin 20^0\cdot\cos 20^0}$$

$$= 4\frac{\left(\frac{\sqrt{3}}{2}\cos 20^0 - \frac{1}{2}\sin 20^0\right)}{2\sin 20^0\cdot\cos 20^0}$$

$$= 4\frac{\left(\sin 60^0\cdot\cos 20^0 - \cos 60^0\cdot\sin 20^0\right)}{\sin 40^0}$$

$$= 4\cdot\frac{\sin 40^0}{\sin 40^0} = 4$$

d) $6\cdot\sin 20^0 - 8\sin^3 20^0$

$$= 2\left(3\sin 20^0 - 4\sin^3 20\right)$$

$$= 2\cdot\sin 3\times 20^0$$

$$= 2\cdot\sin 60^0$$

$$= 2\cdot\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

ANS: $a - s, b - p, c - p, d - q$

20. $\cos 5\theta = \cos\left(2\theta + 3\theta\right)$

$$= \cos 2\theta\cdot\cos 3\theta - \sin 2\theta\cdot\sin 3\theta$$

$$= \cos 2\theta\cdot\cos 3\theta - 2\sin\theta\cos\theta\left(3\sin\theta - 4\sin^3\theta\right)$$

$$= \cos 2\theta\cdot\cos 3\theta - 6\sin^2\theta\cos\theta + 8\sin^4\theta\cos\theta$$

$$= \cos 2\theta\cdot\cos 3\theta - 6\left(1-\cos^2\theta\right)\cdot\cos\theta + 8\left(1-\cos^2\theta\right)^2\cdot\cos\theta$$

$$= \left(2\cos^2\theta - 1\right)\left(4\cos^3\theta - 3\cos\theta\right) - \left(6\cos\theta - 6\cos^3\theta\right) + 8\left(1 - 2\cos^2\theta + \cos^4\theta\right)\cos\theta$$

$$= 8\cos^5\theta - 6\cos^3\theta - 4\cos^3\theta + 3\cos\theta - 6\cos\theta + 6\cos^3\theta + 8\cos\theta - 16\cos^3\theta + 8\cos^5\theta$$

$$= 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$$

$$= p_0 + p_1\cos\theta + p_2\cos^2\theta + p_3\cos^3\theta + p_4\cos^4\theta + p_5\cos^5\theta$$

Comparing we get

$p_0 = 1, p_1 = 5, p_3 = -20, p_5 = 16$

ADDITIONAL PRACTICE QUESTIONS FOR STUDENTS

1. $\cos 2A + \cos 2B + \cos 2C$

$$= 2\cos(A+B)\cdot\cos(A-B) + \cos 2C$$
$$= 2\cos\left(180^0 - C\right)\cdot\cos(A-B) + 2\cos^2 C - 1$$
$$= -2\cos C\cdot\cos(A-B) + 2\cos^2 C - 1$$
$$= -1 - 2\cos C\left[\cos(A-B) - \cos C\right]$$
$$= -1 - 2\cos C\left[\cos(A-B) - \cos\left(180^0 - (A+B)\right)\right]$$
$$= -1 - 2\cos C\left[\cos(A-B) + \cos(A+B)\right]$$
$$= -1 - 2\cos C\left[2\cos A\cos B\right]$$
$$= -1 - 4\cos A\cos B\cos C$$

ANS: D

2. $\sin 47^0 - \sin 25^0 + \sin 61^0 - \sin 11^0$

$$= 2\cos\left(\frac{47^0 + 25^0}{2}\right)\sin\left(\frac{47^0 - 25^0}{2}\right) + 2\cos\left(\frac{61^0 + 11^0}{2}\right)\sin\left(\frac{61^0 - 11^0}{2}\right)$$
$$= 2\cos 36^0\cdot\sin 11^0 + 2\cos 36^o\cdot\sin 25^0$$
$$= 2\cos 36^o\left(\sin 11^0 + \sin 25^0\right)$$
$$= 2\cos 36^o\left(2\sin\left(\frac{11^0 + 25^0}{2}\right)\cdot\cos\left(\frac{11^0 - 25^0}{2}\right)\right)$$
$$= 4\cos 36^o\cdot\sin 18^0\cos 7^0$$
$$= 4\times\frac{\sqrt{5}+1}{4}\cdot\frac{\sqrt{5}-1}{4}\cdot\cos 7^0$$
$$= \cos 7^0$$

ANS: A

3. We know, $\cot 60^0 = \cot\left(16^o + 44^0\right)$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\cot 16^0\cdot\cot 44^0 - 1}{\cot 16^0 + \cot 44^0}$$
$$\Rightarrow 1 + \frac{\cot 16^0 + \cot 44^0}{\sqrt{3}} = \cot 16^o\cdot\cot 44^0 \rightarrow \textcircled{1}$$

Similarly,

$$1 + \frac{\cot 76^0 + \cot 44^0}{-\sqrt{3}} = \cot 44^0\cdot\cot 76^0 \rightarrow \textcircled{2}$$
$$-1 + \frac{\cot 16^0 - \cot 76^0}{\sqrt{3}} = \cot 76^0\cdot\cot 16^o \rightarrow \textcircled{3}$$

(1) + (2) - (3) gives

$$=\cot 16^0\cdot\cot 44^0+\cot 44^o\cdot\cot 76^0-\cot 76^0\cdot\cot 16^0$$

$$=1+1+1=3$$

ANS: C

4. $\sec(\theta-\alpha),\sec\theta,\sec(\theta-\alpha)$ are in A.P

$$2\sec\theta=\sec(\theta-\alpha)+\sec(\theta+\alpha)$$

$$\Rightarrow 2\sec\theta=\frac{1}{\cos(\theta-\alpha)}+\frac{1}{\cos(\theta+\alpha)}$$

$$\Rightarrow 2\sec\theta=\frac{\cos(\theta+\alpha)+\cos(\theta-\alpha)}{\cos(\theta-\alpha)\cdot\cos(\theta+\alpha)}$$

$$\Rightarrow \frac{2}{\cos\theta}=\frac{2\cos\theta\cdot\cos\alpha}{\cos^2\theta-\sin^2\alpha}$$

$$\Rightarrow \cos^2\theta-\sin^2\alpha=\cos^2\theta\cdot\cos\alpha$$

$$\Rightarrow \cos^2\theta-\cos^2\theta\cdot\cos\alpha=\sin^2\alpha$$

$$\Rightarrow \cos^2\theta(1-\cos\alpha)=1-\cos^2\alpha$$

$$\Rightarrow \cos^2\theta(1-\cos\alpha)=(1+\cos\alpha)(1-\cos\alpha)$$

$$\Rightarrow \cos^2\theta=1+\cos\alpha$$

$$\Rightarrow \cos^2\theta=2\cos^2\frac{\alpha}{2}$$

$$\Rightarrow \cos^2\theta\cdot\sec^2\frac{\alpha}{2}=2$$

$$\Rightarrow \cos\theta\cdot\sec\frac{\alpha}{2}=\pm\sqrt{2}$$

ANS: C