Class X - CBSE Mathematics

Ch-3 Pair of linear Equations in two variables.

Q-1) Form the pair of linear equations in the following problems and find their solutions graphically.

i) 10 students of class X took part in a mathematics quiz. If the number of girls is 4 more than the numb-r of boys, find the number of boys and girls took part in the quiz.

Ans) Assume, No. of girls who took part in the quiz = x

No. of boys who took part in the quiz = y

Given, total no. of students here = 10.

So, $\boxed{x + y = 10} \longrightarrow (1)$

Given, no. of girls is 4 more than no. of boys.

So, $\boxed{x = y + 4} \longrightarrow (2)$.

So, now we formed two linear equations from the information given.

To represent these equations graphically, we need their solutions.

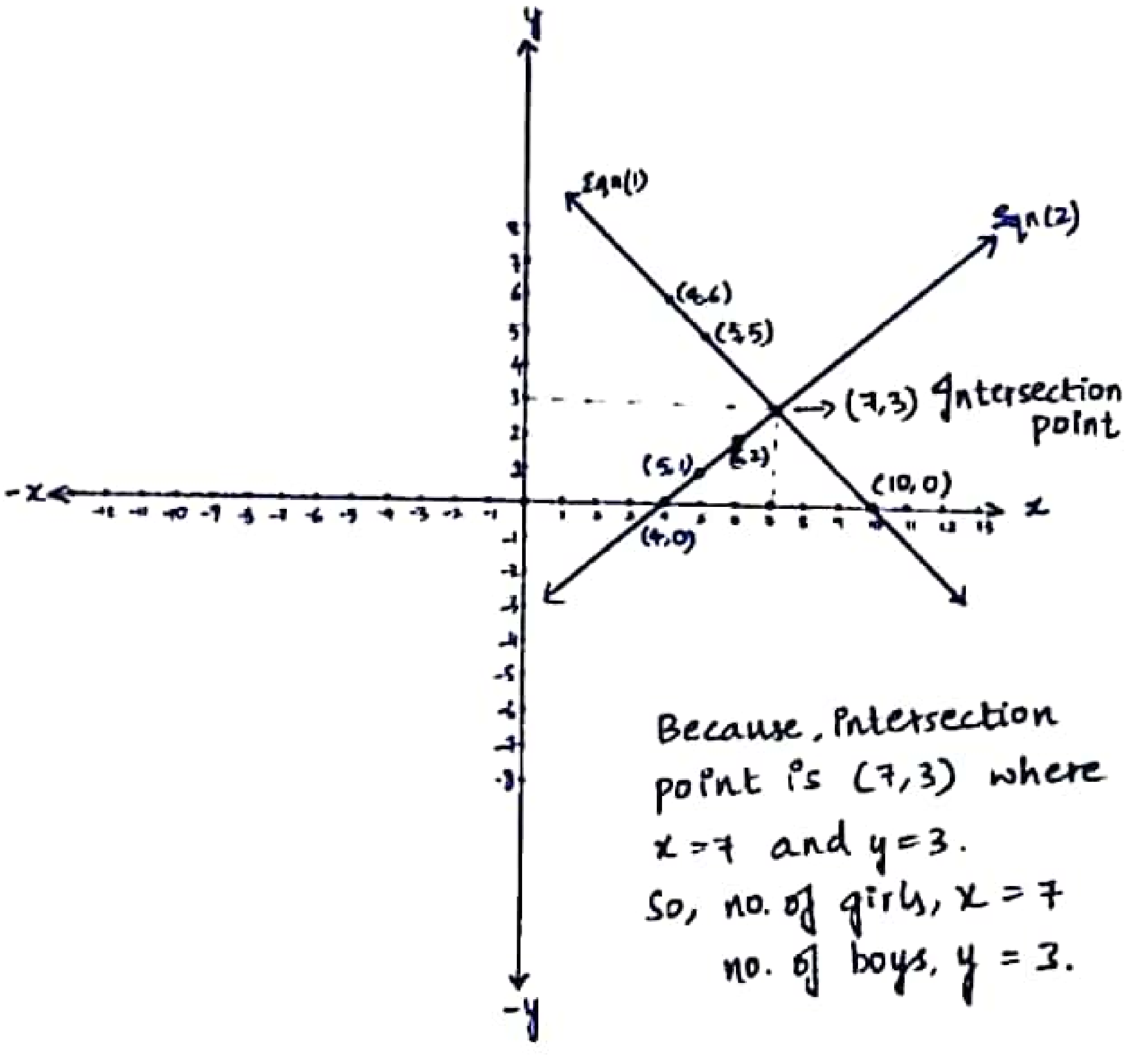
consider equation (1), $x+y=10 \Rightarrow \boxed{x=10-y}$

x	5	10	4
y	5	0	6

consider equation (2), $\boxed{x=y+4}$

x	4	5	6
y	0	1	2

Now plot these points on graph.



Because, intersection point is (7,3) where $x=7$ and $y=3$.

So, no. of girls, $x=7$

no. of boys, $y=3$.

ii) 5 pencils and 7 pens together cost Rs.50, whereas 7 pencils and 5 pens together cost Rs.46. Find the cost of one pencil and that of one pen.

Ans) Here also we need to find the intersection point.

Let pencil cost be x Rs.

and pen cost be y Rs.

from 1st sentence, $\boxed{5x + 7y = 50} \longrightarrow (1)$

from 2nd sentence, $\boxed{7x + 5y = 46} \longrightarrow (2)$

Two linear equations are formed.

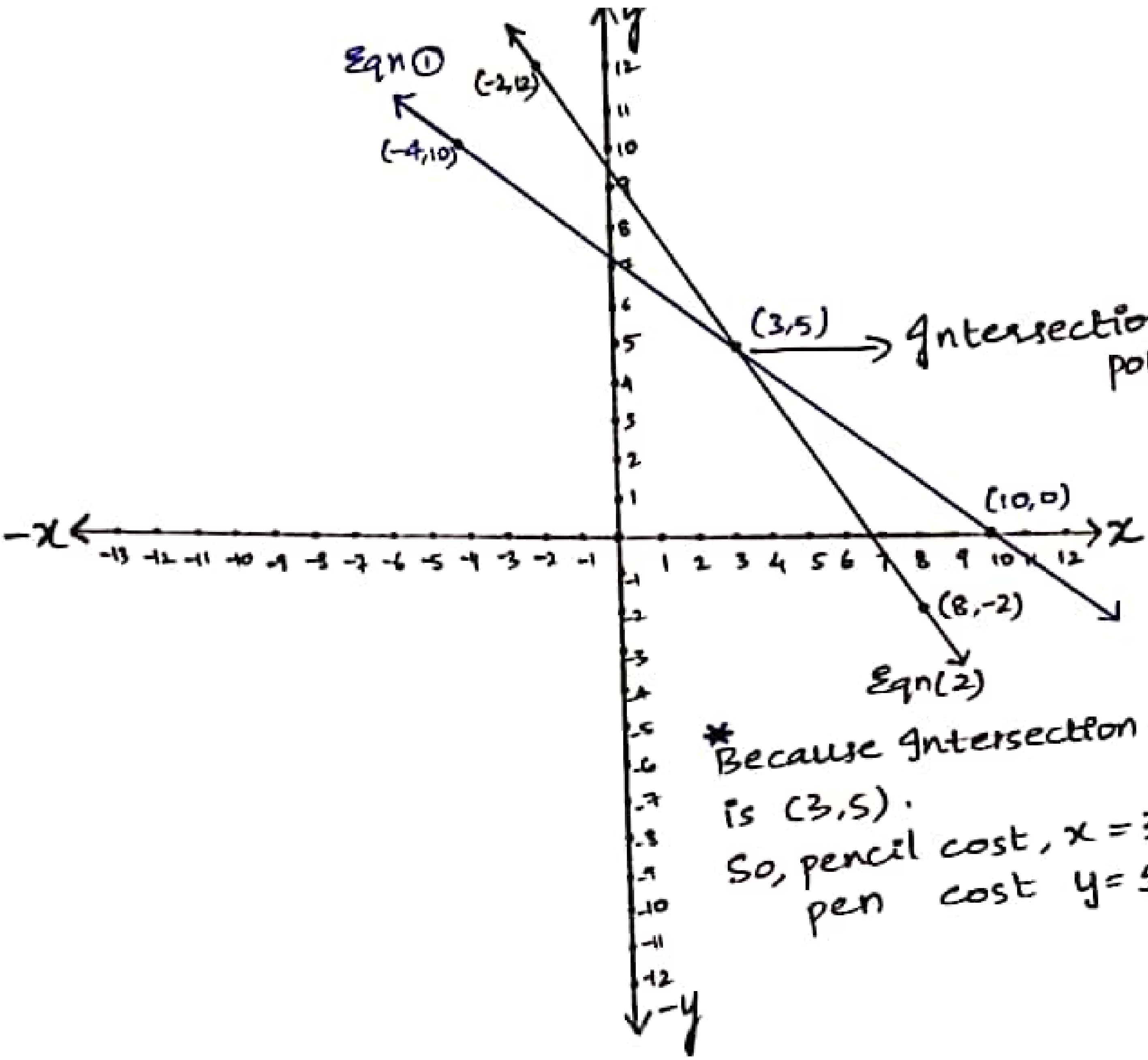
Now find solutions of each of them, to represent them graphically.

Consider equation (1), $5x+7y=50 \Rightarrow \boxed{x = \frac{50-7y}{5}}$

x	10	3	-4
y	0	5	10

Consider equation (2), $7x+5y=46 \Rightarrow \boxed{x = \frac{46-5y}{7}}$

x	8	3	-2
y	-2	5	12



* Because Intersection point is (3,5).
So, pencil cost, $x = 3$ Rs.
pen cost $y = 5$ Rs.

Remember. In case of two linear equations

$a_1x + b_1y + c_1 = 0$ —— ①

$a_2x + b_2y + c_2 = 0$ —— ②,

• If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, both the lines intersect at a point and solution would be unique.

• If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, both the lines coincide and solutions are many.

• If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, both the lines are parallel and no solution will be there.

Question:-2

On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

i, $5x-4y+8=0$ Here, $a_1=5$, $b_1=-4$, $c_1=8$

$7x+6y-9=0$ $a_2=7$. $b_2=6$, $c_2=-9$

$\frac{a_1}{a_2}=5/7$, $\frac{b_1}{b_2}=-4/6=-2/3$. So, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$.

So, these two lines intersect at a point.

ii, $9x+3y+12=0$

$18x+6y+24=0$ Here $a_1=9$, $b_1=3$, $c_1=12$.

$a_2=18$, $b_2=6$, $c_2=24$.

$\frac{a_1}{a_2}=\frac{9}{18}=1/2$, $\frac{b_1}{b_2}=3/6=1/2$, $\frac{c_1}{c_2}=\frac{12}{24}=\frac{1}{2}$.

Therefore, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$. So, this pair of equations are having lines coincident.

iii, $6x-3y+10=0$

$2x-y+9=0$ Here $a_1=6$, $b_1=-3$, $c_1=10$

$a_2=2$, $b_2=-1$, $c_2=9$

$\frac{a_1}{a_2}=6/2=3/1$, $\frac{b_1}{b_2}=-3/-1=3/1$, $\frac{c_1}{c_2}=10/9$.

Here $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$. So, this pair of equations will have parallel lines.

Question-3) On comparing the ratios a_1/a_2, b_1/b_2 and c_1/c_2, find out whether the following pair of linear equations are consistent or in-consistent:

i, $3x+2y=5$; $2x-3y=7$ $\Rightarrow$ $3x+2y-5=0$, $2x-3y-7=0$

$\frac{a_1}{a_2} = \frac{3}{2}$, $\frac{b_1}{b_2} = \frac{2}{-3}$, $\frac{c_1}{c_2} = \frac{-5}{-7} = 5/7$

Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$; So, solutions would be unique and therefore, pair of linear equations are consistent.

ii, $2x-3y=8$, $4x-6y=9$ $\Rightarrow$ $2x-3y-8=0$, $4x-6y-9=0$.

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$, $\frac{c_1}{c_2} = -8/-9 = 8/9$.

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. Therefore, solutions cannot be obtained because these equations give parallel lines.

Hence, these linear equations are inconsistent.

iii, $\frac{3}{2}x + \frac{5}{3}y = 7$, $9x-10y=14$ $\Rightarrow$ $\frac{3}{2}x + \frac{5}{3}y - 7 = 0$, $9x-10y-14=0$

$\frac{a_1}{a_2} = \frac{3/2}{9}$, $\frac{b_1}{b_2} = \frac{5/3}{-10}$, $\frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{1}{6}$, $\frac{b_1}{b_2} = -\frac{1}{6}$. Here, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

So, definitely pair of linear equations given here are consistent.

iv) $5x-3y=11$; $-10x+6y=-22 \Rightarrow 5x-3y-11=0, -10x+6y+22=0$

Here $\frac{a_1}{a_2} = \frac{5}{-10} = -\frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-11}{22} = -\frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, these equations give coincident lines and has many solutions.

This pair of equations are dependent consistent or in general, we say them consistent.

v) $\frac{4}{3}x+2y=8$; $2x+3y=12 \Rightarrow \frac{4}{3}x+2y-8=0$, $2x+3y-12=0$

Here $\frac{a_1}{a_2} = \frac{4/3}{2} = \frac{2}{3}$, $\frac{b_1}{b_2} = \frac{2}{3}$, $\frac{c_1}{c_2} = \frac{2}{3}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, this pair of equations form coincident lines and has many solutions.

This pair of equations are consistent.

Q-4) Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

i, $x+y=5$, $2x+2y=10 \Rightarrow x+y-5=0$, $2x+2y-10=0$

$\frac{a_1}{a_2} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. Here, pair of linear equations are consistent and lines obtained will be coincident

Now, the solution has to be obtained graphically.

Consider Equation (1) $x=5-y$

x	3	2	4
y	2	3	1

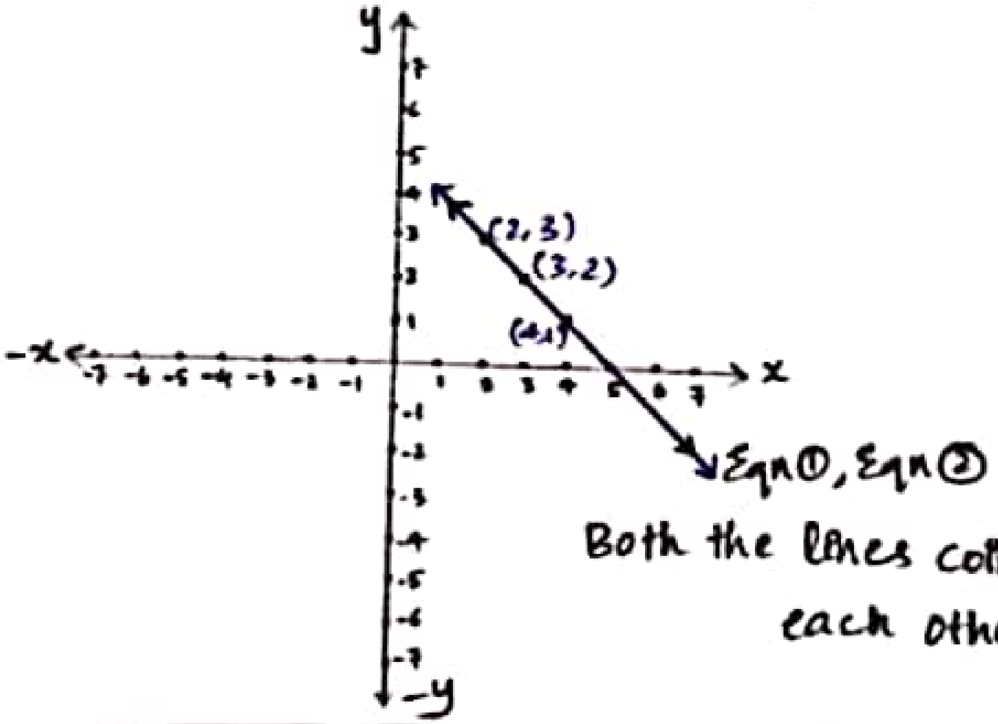
solutions for eqn-①

Consider equation (2)

$x = \frac{10-2y}{2}$ [you can simplify this or leave it as it is]

x	4	3	2
y	1	2	3

solutions for eqn-②



Both the lines coincide with each other.

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ii, $x-y=8$; $3x-3y=16$

$x-y-8=0$ —①

$3x-3y-16=0$ —②

$\frac{a_1}{a_2}=1/3$, $\frac{b_1}{b_2}=-1/-3=1/3$, $\frac{c_1}{c_2}=\frac{-8}{-16}=1/2$

Here, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$.

So, the pair of linear equations here are inconsistent and will have no solution. (lines obtained will be parallel)

iii, $2x+y-6=0$; $4x-2y-4=0$
L① L②

$\frac{a_1}{a_2}=2/4=1/2$, $\frac{b_1}{b_2}=\frac{1}{-2}$, $\frac{c_1}{c_2}=\frac{-6}{-4}=3/2$

$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$; So, the pair of linear equations here are consistent and will have unique solution, as the lines obtained will be intersecting at one point.

Graph is required here,

Consider eqn①,

$2x=6-y$

$$\boxed{x=\frac{6-y}{2}}$$

x	3	2	1
y	0	2	4

consider eqn②,

$4x-2y=4$

$2x-y=2$

$$\boxed{x=\frac{2+y}{2}}$$

x	1	2	3
y	0	2	4

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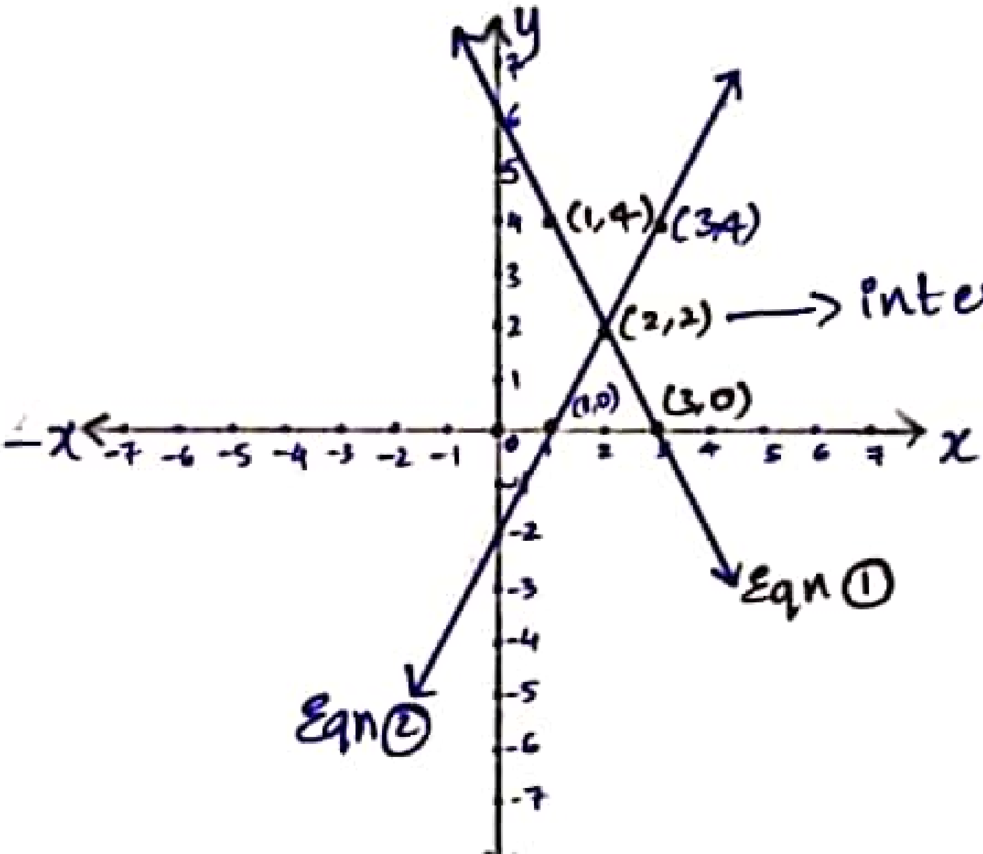


intersection point

(2,2)

$\boxed{x = 2, y = 2}$

↓

unique solution.

iv) $2x-2y-2=0$;
$4x-4y-5=0$

$2x-2y-2=0$ — ① $4x-4y-5=0$ — ②

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$. This pair of linear equations are inconsistent.

Lines obtained would be parallel and hence, no solution.

Q-5) Half the perimeter of a rectangular region garden, whose length is 4 m more than its width, is 36 m. find the dimensions of the garden.

Ans) Here dimensions of the rectangular garden means its length and breadth/width.

Perimenter of a rectangle = 2(length + breadth).

Here, length of the garden be x m, width be y m
↓
breadth.

Perimeter = $2(x+y)$

Half of the perimeter = $\frac{1}{2}(2(x+y)) = 36$ (Given)

$\Rightarrow x+y=36$ —— ①

Given, length is 4 m, more than its width

$\rightarrow x = y+4$ —— ②

Now obtain solutions for these equations

consider eqn ① :-

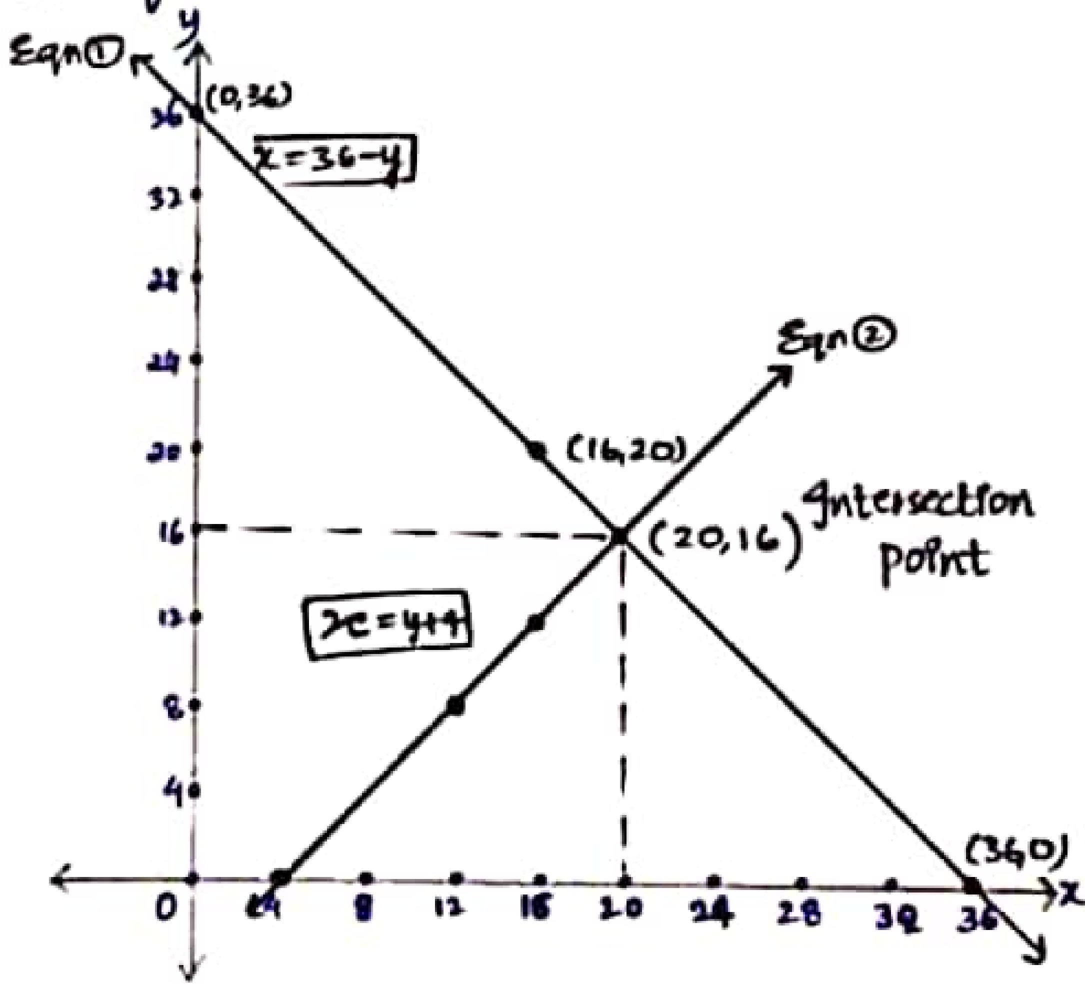
$x = 36-y$

x	36	0	16
y	0	36	20

consider eqn ② :-

$x = y+4$

x	4	12	16
y	0	8	12



solution ⇒ (20,16)

x, length = 20

y, breadth = 16

Q-6)

Given the linear equation $2x+3y-8=0$, write another linear equation in ~~a~~ two variables such that the geometrical representation of the pair so formed is:-

i, Intersecting lines-

For two equations to be intersecting lines,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ $\quad 2x+3y-8=0$ —① $\quad a_1=2, b_1=3, c_1=-8$

let another equation be $2x+2y+4=0$ —②

$a_2=2, b_2=2, c_2=4$

Here $\frac{a_1}{a_2}=\frac{2}{2}=1$, $\frac{b_1}{b_2}=3/2$. So, $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$.

• ~~Required~~ possible linear equation to make the pair of equations having intersecting lines :- $\boxed{2x+2y+4=0}$

• like this any number of equations can be formed, so that ~~this pair~~ the pair formed has intersecting lines.

ii, parallel lines-

for two equations to be parallel lines,

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$. $\quad 2x+3y-8=0$ —① $\quad a_1=2, b_1=3, c_1=-8$

let another equation be $4x+6y-12=0$ —②

$a_2=4, b_2=6, c_2=-12$

$\frac{a_1}{a_2}=2/4=1/2$, $b_1/b_2=3/6=1/2$, $c_1/c_2=-8/-12=2/3$

$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$. So, possible eqn to make the pair of equations have ~~||el~~ parallel lines

$\Rightarrow \boxed{4x+6y-12=0}$.

• Many possible equations like this will exist.

iii) coincident lines -

for two equations to be coincident lines,

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$. $2x+3y-8=0$ —①

$a_1=2, b_1=3, c_1=-8$

let another equation be, $4x+6y-16=0$ —②

$a_2=4, b_2=6, c_2=-16$.

$\frac{a_1}{a_2} = 2/4 = 1/2$, $\frac{b_1}{b_2} = 3/6 = 1/2$, $\frac{c_1}{c_2} = \frac{-8}{-16} = 1/2$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

So, this would be the possible equation, for which the pair obtained will have coincident lines.

$$\boxed{4x+6y-16=0}$$

• Many possible equations will exist which would ~~give the~~ make a pair with the given equation, whether both equations have coincident lines.

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Q-7)

Draw the graphs of the equations $x-y+1=0$ and $3x+2y-12=0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Ans)

Consider eqn ①:-

$x-y+1=0 \Rightarrow$

$\boxed{x=y-1}$

x	0	-1	1
y	1	0	2

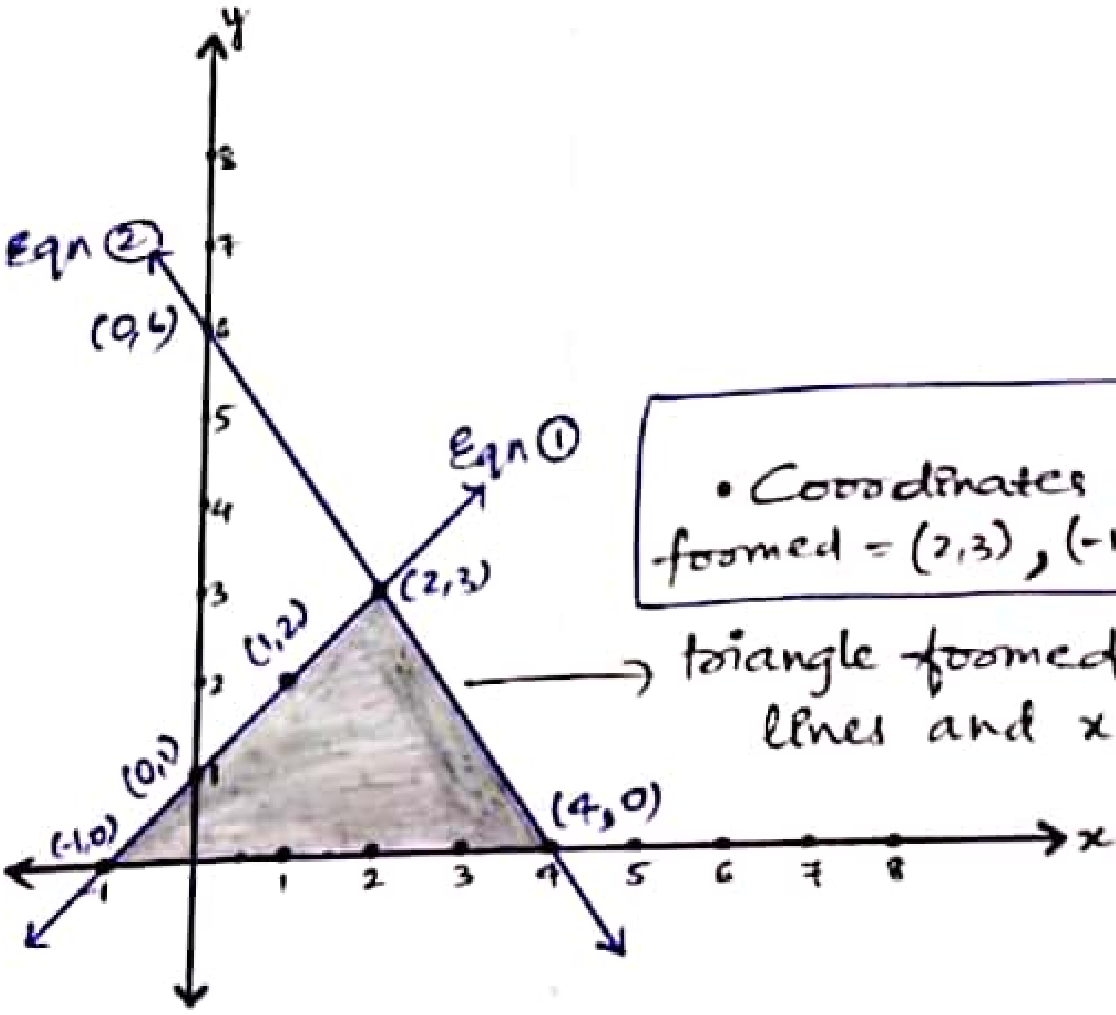
Consider eqn ②:-

$3x+2y-12=0$

$3x = 12-2y$

$\boxed{x = \frac{12-2y}{3}}$

x	4	2	0
y	0	3	6



• Coordinates of Δ^{le} formed = (2,3), (-1,0), (4,0)

→ triangle formed by the lines and x-axis.