THEORY OF EQUATIONS - III
TEACHING TASK
Jee Mains Level Questions

1. Let $f\left(x\right)=x^3+3x^2+5x+k=0$

$f\left(x\right)=ax^3+bx^2+cx+d=0$

We have $8a^2d+b^3=4abc$

$\Rightarrow 8(1)^2.K+3^3=4.1.3.5$

$\Rightarrow 8K+27=60$

$\Rightarrow 8K=33$

$\Rightarrow K=\dfrac{33}{8}$

Ans : D

2. Let $f\left(x\right)=32x^3-48x^2+22x-3=0$

Let0 the roots be $\alpha-\beta,\ \alpha,\ \alpha+\beta$

We have $s_1=\left(\alpha-\beta\right)+\alpha+\left(\alpha+\beta\right)=\dfrac{-\left(-48\right)}{32}$

$\Rightarrow 3\alpha=\dfrac{3}{2}$

$\Rightarrow \alpha=\dfrac{1}{2}$

Ans : B

3. Let $f\left(x\right)=4x^3-12x^2+11x+K=0$

Let the roots be $\alpha-\beta,\ \alpha,\ \alpha+\beta$

We have $s_1=\left(\alpha-\beta\right)+\alpha+\left(\alpha+\beta\right)=\dfrac{-\left(-12\right)}{4}$

$\Rightarrow 3\alpha=3$

$\Rightarrow \alpha=1$

We have $4(1)^3-12(1)^2+11(1)+K=0$

$\Rightarrow 4-12+11+K=0$

$\Rightarrow K=-3$

Ans : A

4. Let $f\left(x\right)=6x^3-11x^2+6x-1=0$

Given the roots of $f(x) = 0$ are H.P

We have the roots of $f\left(\frac{1}{x}\right) = 0$ are in A.P

$f\left(\frac{1}{x}\right) = 6\left(\frac{1}{x}\right)^3 - 11\left(\frac{1}{x}\right)^2 + 6\left(\frac{1}{x}\right) - 1 = 0$

$\Rightarrow x^3 - 6x^2 + 11x - 6 = 0$

Let $\alpha - \beta, \alpha, \alpha + \beta$ be the roots.

We have $s_1 = (\alpha - \beta) + \alpha + (\alpha + \beta) = 6$

$\Rightarrow 3\alpha = 6$

$\Rightarrow \alpha = 2$

$\therefore$ One of the root of $f\left(\frac{1}{x}\right) = 0$ is 2

$\therefore$ One of the root of $f(x) = 0$ is $\frac{1}{2}$

Ans : A

5. Given $54x^3 - 39x^2 - 26x + 16 = 0$

Let $\frac{\alpha}{\beta}, \alpha, \alpha\beta$ be the roots

we have, $s_3 = \left(\frac{\alpha}{\beta}\right)(\alpha)(\alpha\beta) = \frac{-16}{54}$

$\Rightarrow \alpha^3 = \left(\frac{-2}{3}\right)^3$

$\Rightarrow \alpha = \frac{-2}{3}$

Ans : A

6. Given $x^3 - 3x^2 + 2x = 0$

$\Rightarrow x(x^2 - 3x + 2) = 0$

$\Rightarrow x(x-1)(x-2) = 0$

$\Rightarrow x = 0$ or $x = 1$ or $x = 2$

Clearly the roots 0,1,2 are in A.P
with first term 0, common difference 1
Required answer = 0+1 = 1

Ans : A

7. Given $x^3 - 13x^2 + Kx - 27 = 0$

Let $\frac{\alpha}{\beta}, \alpha, \alpha\beta$ be the roots

We have $s_3 = \left(\frac{\alpha}{\beta}\right)(\alpha)(\alpha\beta) = 27$

$\Rightarrow \alpha^3 = 27$

$\Rightarrow \alpha = 3$

Now, $3^3 - 13(3)^2 + k(3) - 27 = 0$

$\Rightarrow 27 - 117 + 3k - 27 = 0$

$\Rightarrow k = 39$

Ans : C

8. Given $x^3 - 42x^2 + 336x - 512 = 0$

Let $\frac{\alpha}{\beta}, \alpha, \alpha\beta$ be the roots

We have $s_3 = \frac{\alpha}{\beta}.\alpha.\alpha\beta = 512$

$\Rightarrow \alpha^3 = 512$

$\Rightarrow \alpha^3 = 8^3$

$\Rightarrow \alpha = 8$

We have, $s_1 = \frac{\alpha}{\beta} + \alpha + \alpha\beta = 42$

$\Rightarrow \frac{8}{\beta} + 8 + 8\beta = 42$

$\Rightarrow \frac{4}{\beta} + 4 + 4\beta = 21$

$\Rightarrow 4 + 4\beta + 4\beta^2 = 21\beta$

$\Rightarrow 4\beta^2 - 17\beta + 4 = 0$

Clearly $\beta = 4$ satisfies the above equation.

$\therefore$ The common ratio $\beta = 4$

Ans : C

9. Given $x^3 - bx^2 + cx - d = 0$

Let the roots be $\frac{\alpha}{\beta}, \alpha, \alpha\beta$

We have $s_3 = \frac{\alpha}{\beta}. \alpha. \alpha\beta = d$

$\Rightarrow \alpha^3 = d$

$\Rightarrow \alpha = (d)^{\frac{1}{3}}$

We have $\left(d^{\frac{1}{3}}\right)^3 - b\left(d^{\frac{1}{3}}\right)^2 + c\left(d^{\frac{1}{3}}\right) - d = 0$

$\Rightarrow d - b.d^{\frac{2}{3}} + cd^{\frac{1}{3}} - d = 0$

$\Rightarrow cd^{\frac{1}{3}} = b.d^{\frac{2}{3}}$

cubing on both sides

$\Rightarrow c^3.d = b^3 d^2$

$\Rightarrow c^3 = b^3 d$

Ans : A

10. Let $f(x) = x^4 + 5x^3 - 30x^2 - 40x + 64 = 0$

$f(-x) = x^4 - 5x^3 - 30x^2 + 40x + 64 = 0$

Let x_1, x_2, x_3, x_4 be the roots of $f(x) = 0$

Which are in G.P

We know, the roots of $f(-x) = 0$ are $-x_1, -x_2, -x_3, -x_4$ Which are also in G.P

Ans : B

Advanced Level Questions

11. Let the roots be $\alpha - \beta, \alpha, \alpha + \beta$

We have $s_1 = (\alpha - \beta) + \alpha + (\alpha + \beta) = 9$

$\Rightarrow \alpha = 3$

$\therefore f(x) = x^3 - 9x^2 + kx + \ell = 0$

$\Rightarrow x^3 - 9.3^2 + k.3 + \ell = 0$

$\Rightarrow 27 - 81 + 3k + \ell = 0$

$\Rightarrow 3k + \ell = 54$

Option : B $\Rightarrow (k, \ell) = (23, -15)$ satisfies the above equation

Ans : B

12. Let $f(x) = x^3 + 3px^2 + 3qx - 8 = 0$

Let the roots be $\frac{\alpha}{\beta}, \alpha, \alpha\beta$

We have $s_3 = \frac{\alpha}{\beta}.\alpha.\alpha\beta = 8$

$\Rightarrow \alpha^3 = 8$

$\Rightarrow \alpha = 2$

We have $2^3 + 3p.2^2 + 3q.2 - 8 = 0$

$\Rightarrow 8 + 12p + 6q - 8 = 0$

$\Rightarrow 2p + q = 0$

$\Rightarrow \frac{q}{p} = -2$

$\Rightarrow \frac{q^3}{p^3} = -8 = \frac{-16}{2}$

Ans : A,D

Statement Type

13. **Statement-I:**

Let $f(x) = x^3 - 7x^2 + 14x - 8 = 0$

Let the roots be $\frac{\alpha}{\beta}, \alpha, \alpha\beta$

We have $s_3 = \frac{\alpha}{\beta}.\ \alpha.\ \alpha\beta = 8$

$\Rightarrow \alpha^3 = 8$

$\Rightarrow \alpha = 2$

$\therefore$ 2 is a root of $f(x) = 0$

Remaining roots can be find by dividing $x^3 - 7x^2 + 14x - 8$ with $x - 2$ using Horner's method of synthetic division (H.M.S.D)

$x = 2$	1	-7	14	-8
	0	2	-10	8
	1	-5	4	0

$\Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow (x-1)(x-4) = 0$

$\Rightarrow x = 1$ or 4

$\therefore$ The roots are 1,2,4

The difference between the largest and the smallest roots = 4-1 = 3

Hence, statement-I is TRUE.

Statement-II:
Clearly statement-II is TRUE
Also, statement-II is the correct explanation of statement-I
Ans : A

14. **Statement-I:**
Let $f(x)=ax^3+bx^2+cx+d=0$

Let $f\left(\frac{x}{k}\right)=a\left(\frac{x}{k}\right)^3+b\left(\frac{x}{k}\right)^2+c\left(\frac{x}{k}\right)+d=0$

$f\left(\frac{x}{k}\right)=ax^3+bkx^2+ck^2x+k^3d=0$

Let x_1, x_2, x_3 are the roots of $f(x)=0$, we know kx_1, kx_2, kx_3 are the roots of $f\left(\frac{x}{k}\right)=0$

since x_1, x_2, x_3 are in G.P.

hence kx_1, kx_2, kx_3 are also in G.P.

Hence $f\left(\frac{x}{k}\right)=ay^3+bky^2+ck^2y+k^3d=0$ roots are in G.P

Hence, Statement-I is TRUE.
Statement-II :
Clearly statement-II is TRUE
Also, Statement-II is the correct explanation of statement-I
Ans : A

Comprehension-I
15. Let $f(x)=18x^3+81x^2+121x+60=0$

Let the roots be $\alpha-\beta, \alpha, \alpha+\beta$.

Since $\alpha=\frac{(\alpha-\beta)+(\alpha+\beta)}{2}$
i.e The roots are in A.P

We have $s_1=(\alpha-\beta)+\alpha+(\alpha+\beta)=\frac{-81}{18}$

$$\Rightarrow 3\alpha=\frac{-81}{18}$$

$$\Rightarrow \alpha=\frac{-3}{2}$$

Ans : A
16. Let $\alpha-\beta, \alpha, \alpha+\beta$ be the roots

We have $\Rightarrow s_1=3\alpha=\frac{-(-9)}{1}$

$\Rightarrow \alpha = 3$

Also, $s_3 = (\alpha - \beta)(\alpha)(\alpha + \beta) = 15$

$\Rightarrow (3-\beta)(3)(3+\beta) = 15$

$\Rightarrow 9 - \beta^2 = 5$

$\Rightarrow \beta^2 = 4 \Rightarrow \beta = \pm 2$

Ans : C

17. **Comprehension-II**

Let $f(x) = x^3 - 3ax^2 + 3bx - c = 0$

Given the roots of $f(x) = 0$ are in H.P.

We have, the roots of $f\left(\frac{1}{x}\right) = 0$ are in A.P.

$\therefore f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{3a}{x^2} + \frac{3b}{x} - c = 0$

$\Rightarrow cx^3 - 3bx^2 + 3ax - 1 = 0$

Let p-q, p, p+q be the roots

We have, $s_1 = 3p = \frac{3b}{c}$

$\Rightarrow p = \frac{b}{c}$

$\Rightarrow \frac{1}{\beta} = \frac{b}{c}$

$\Rightarrow \beta = \frac{c}{b}$

Ans : A

18. $f(x) = kx^3 - 18x^2 - 36x + 8 = 0$

$f\left(\frac{1}{x}\right) = 8x^3 - 36x^2 + 18x + k = 0$

$s_1 = \alpha - \beta + \alpha + \alpha + \beta = \frac{36}{8}$

$\Rightarrow 3\alpha = \frac{36}{8}$

$\Rightarrow \alpha = \frac{3}{2}$

Now $8\left(\frac{3}{2}\right)^3 - 36\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + k = 0$

$\Rightarrow k = 81$

Ans : A

Integer answer type questions

19. $f(x) = x^3 - 14x^2 + 56x - 64 = 0$

Let $\frac{\alpha}{\beta}, \alpha, \alpha\beta$ be the roots

we have $s_3 = \frac{\alpha}{\beta}.\alpha.\alpha\beta = 64$

$\Rightarrow \alpha^3 = 64$

$\Rightarrow \alpha = 4$

Ans : 4

20. Let $\frac{\alpha}{\beta}, \alpha, \alpha\beta$ be the roots of $x^3 - 7x^2 + 14x - 8 = 0$

We have $s_3 = \frac{\alpha}{\beta}.\alpha.\alpha\beta = 8 \Rightarrow \alpha = 2$

2 is a root of $f(x) = 0$

Now, H.M.S.D we have

$x = 2$				
	1	-7	14	-8
	0	2	-10	8
	1	-5	4	0

$\therefore x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0 \Rightarrow x = 1, 4$

$\therefore$ The roots are 1, 2, 4.

Greatest root = 4

Ans : 4

21. **Matrix Matching**

a) Let $f(x) = ax^3 + bx^2 + cx + d = 0$

$f(x+k) = a(x+k)^3 + b(x+k)^2 + c(x+k) + d = 0$

Given the roots of $f(x) = 0$ are in A.P

Let x_1, x_2, x_3 be the roots of $f(x)=0$

We have x_1-k, x_2-k, x_3-k are the roots of $f(x+k)=0$

hence x_1-k, x_2-k, x_3-k are also in A.P.

b) Let $f(x)=ax^3+bx^2+cx+d=0$

$f(kx)=a(kx)^3+b(kx)^2+c(kx)+d=0$

$\Rightarrow ak^3x^3+bk^2x^2+ckx+d=0$

Given the roots of $f(x)=0$ are in G.P

Let x_1, x_2, x_3 be the roots of $f(x)=0$

We have $\frac{x_1}{k}, \frac{x_2}{k}, \frac{x_3}{k}$ are the roots of $f(kx)=0$

hence $\frac{x_1}{k}, \frac{x_2}{k}, \frac{x_3}{k}$ are also in G.P.

c) Let $f(x)=ax^3+bx^2+cx+d=0$

$f\left(-\frac{1}{x}\right)=\frac{-a}{x^3}+\frac{b}{x^2}-\frac{c}{x}+d=0$

$\Rightarrow dx^3-cx^2+bx-a=0$

Given the roots of $f(x)=0$ are in G.P.

Let x_1, x_2, x_3 be the roots of $f(x)=0$

We have $\frac{-1}{x_1}, \frac{-1}{x_2}, \frac{-1}{x_3}$ are the roots of $f\left(-\frac{1}{x}\right)=0$

hence, $\frac{-1}{x_1}, \frac{-1}{x_2}, \frac{-1}{x_3}$ are also in G.P.

d) Clearly the roots are in A.P

Ans : a-t, b-s, c-s, d-t

22. a) Given α, β are the roots of $px^2+qx+r=0$

we have $\alpha+\beta=\frac{-q}{p}, \alpha\beta=\frac{r}{p}$ .... (i)

since p,q,r are in A.P

$\Rightarrow 2q=p+r$ .... (ii)

Also $\frac{1}{\alpha}+\frac{1}{\beta}=4$

$$\Rightarrow \frac{\alpha+\beta}{\alpha\beta} = 4$$

$$\Rightarrow \frac{-q}{r} = 4$$

$\Rightarrow q = -4r$ .... (iii)

From (ii) 2q = p + r

$$\Rightarrow 2(-4r) = p + r$$

$$\Rightarrow p = -9r$$

Now, $\alpha+\beta = \frac{-q}{p} = \frac{-4r}{-9r} = \frac{4}{9}$

$$\alpha\beta = \frac{r}{p} = \frac{r}{-9r} = \frac{-1}{9}$$

$$\alpha-\beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

$$\sqrt{\left(\frac{4}{9}\right)^2 - 4\left(\frac{-1}{9}\right)}$$

$$= \frac{2\sqrt{13}}{9}$$

b) Let $f(x) = ax^2 + bx + c = 0$

Given $\alpha+\beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$$\Rightarrow \alpha+\beta = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$$

$$\Rightarrow \alpha+\beta = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow \frac{-b}{a} = \frac{\left(\frac{-b}{a}\right)^2 - \frac{2c}{a}}{\left(\frac{c}{a}\right)^2}$$

$$\Rightarrow \frac{-b}{a} = \frac{b^2 - 2ac}{c^2}$$

$$\Rightarrow -bc^2 = ab^2 - 2a^2c$$

$$\Rightarrow 2a^2c = ab^2 + bc^2$$

hence bc^2, a^2c, ab^2 are in A.P

c) $f(x) = (a^2+b^2)x^2 + 2(bc+ad)x + (c^2+d^2) = 0$

Given the roots are equal

we have $\Delta = B^2 - 4AC = 0$

$\Rightarrow [2(bc+ad)]^2 - 4(a^2+b^2)(c^2+d^2) = 0$

$\Rightarrow b^2c^2 + a^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2 = 0$

$\Rightarrow a^2c^2 + b^2d^2 = 2abcd$

$\Rightarrow (ac)^2 + (bd)^2 - 2(ac)(bd) = 0$

$\Rightarrow (ac-bd)^2 = 0$

$\Rightarrow ac = bd$

$\Rightarrow bd = \sqrt{(ac)^2}$

$\Rightarrow bd = \sqrt{a^2.c^2}$

hence, a^2, bd, c^2 are G.P

d) Given a,b,c are in A.P

$\Rightarrow 2b = a+c$

Now $ax^2 + 2bx + c = 0$

Discriminant $= \Delta = B^2 - 4AC$

$= (2b)^2 - 4ac$

$= (a+c)^2 - 4ac$

$= (a-c)^2 \geq 0$

$\therefore$ The roots are real

Ans : a-t, b-s, c-r, d-q

LEARNER'S TASK

CUQ'S

1. The required condition is $2b^3 + 27a^2d = 9abc$

Ans : D

2. The required condition is $ac^3 = b^3d$

Ans : C

3. The required condition is $2a^3 + 27ad^2 = 9bcd$

Ans : B

4. The required condition is $8a^2d+b^3=4abc$

Ans : A

5. The roots can be taken as $\alpha-\beta,\ \alpha,\ \alpha+\beta$

Ans : D

6. The roots can be taken as $\frac{1}{\alpha-3\beta},\ \frac{1}{\alpha-\beta},\ \frac{1}{\alpha+\beta},\ \frac{1}{\alpha+3\beta}$

Ans : C

7. The required roots can be taken as $\frac{\alpha}{\beta},\ \alpha,\ \alpha\beta$

Ans : B

8. The roots can be taken as $\frac{\alpha}{\beta^3},\ \frac{\alpha}{\beta},\ \alpha\beta,\ \alpha\beta^3$

Ans : C

9. The roots can be taken as $\frac{1}{\alpha-\beta},\ \frac{1}{\alpha},\ \frac{1}{\alpha+\beta}$

Ans : A

10. Let the roots be $\alpha-\beta,\ \alpha,\ \alpha+\beta$.

We have $s_1=\alpha-\beta+\alpha+\alpha+\beta=-1$

$\Rightarrow 3\alpha=-1$

$\Rightarrow \alpha=\frac{-1}{3}$

Now, $\left(\frac{-1}{3}\right)^3+\left(\frac{-1}{3}\right)^2+\left(\frac{-1}{3}\right)+k=0$

$\Rightarrow k=\frac{7}{27}$

Ans : D

JEE MAIN'S LEVEL QUESTIONS

1. Given $x^3-14x^2+56x-64=0$

$\Rightarrow ax^3+bx^2+cx+d=0$

$\Rightarrow a=1,\ b=-14,\ c=56,\ d=-64$

Now $ac^3=1.(56)^3=175616$

$b^3d = (-14)^3.(-64)=175616$

$\therefore ac^3 = b^3d$

The roots are in G.P

Ans : D

2. Given $48x^3 - 44x^2 + 12x - 1 = 0$

$\Rightarrow ax^3 + bx^2 + cx + d = 0$

here a = 48, b = -44, c = 12, d = -1

Now $2c^3+27ad^2$

= $2(12)^3+27(48)(-1)^2$

= 3456+1296

= 4752

Now 9bcd = 9(-44)(12)(-1)

= 4752

hence, the roots are in H.P

Ans : B

3. Given $x^4 - 8x^3 + 14x^2 - 8x - 15 = 0$

from opttion : A, The roots are -1, 1, 3, 5

Now s_1 = -1+1+3+5 = 8

Also $s_1 = \frac{-b}{a} = \frac{-(-8)}{1} = 8$

hence, option A is correct answer

Ans : A

4. Given $x^4 + 5x^3 - 30x^2 - 40x + 64 = 0$

From option : A. The roots are 1, -2, 4, -8

Now s_1= 1- 2 + 4 - 8 = - 5

Also $s_1 = \frac{-b}{a} = \frac{-5}{1} = -5$

Ans : A

5. Given the roots of $f(x) = 24x^3 - 26x^2 + 9x - 1 = 0$ are in H.P

$\therefore$ The roots of $f\left(\frac{1}{x}\right) = 0$ are in A.P

$f\left(\frac{1}{x}\right) = \frac{24}{x^3} - \frac{26}{x^2} + \frac{9}{x} - 1 = 0$

$\Rightarrow x^3 - 9x^2 + 26x - 24 = 0$

from option A : s_1= 2 + 3 + 4

= 9

Also $s_1 = \frac{-b}{a} = \frac{-(-9)}{1} = 9$

$\therefore$ 2, 3, 4 are the roots of $f\left(\frac{1}{x}\right) = 0$

hence $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ are the roots of $f(x) = 0$

Ans : A

6. Given $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$

Option D: $s_1 = -4 - 1 + 2 + 5 = 2$

Also $s_1 = \frac{-b}{a} = \frac{-(-2)}{1} = 2$

Ans : D

7. Given $x^3 - 13x^2 + 39x - 27 = 0$

$ax^3 + bx^2 + cx + d = 0$

here a = 1, b = -13, c = 39, d = -27

Now $ac^3 = 1(39)^3 = 59319$

$b^3d = (-13)^3(-27) = 59319$

hence the roots are in G.P

Ans : B

8. We have $\alpha + \beta = \frac{-b}{a}, \alpha.\beta = \frac{c}{a}$

Given $\alpha + \beta, \alpha^2 + \beta^2, \alpha^3 + \beta^3$ are in G.P

hence $\left(\alpha^2 + \beta^2\right)^2 = (\alpha + \beta)\left(\alpha^3 + \beta^3\right)$

$\Rightarrow \alpha^4 + \beta^4 + 2\alpha^2\beta^2 = \alpha^4 + \alpha\beta^3 + \beta\alpha^3 + \beta^4$

$\Rightarrow \alpha\beta\left(\alpha^2 + \beta^2 - 2\alpha\beta\right) = 0$

$\Rightarrow \alpha\beta(\alpha - \beta)^2 = 0$

$\Rightarrow \alpha\beta = 0$ or $\alpha - \beta = 0$

$\Rightarrow \alpha\beta = 0$ or $\alpha = \beta$

$\Rightarrow \frac{c}{a} = 0$ or $\Delta = 0$ (Equal roots)

$\Rightarrow c\Delta = 0$

$\Rightarrow c\,\Delta = 0$ since $a \neq 0$

Ans : D

9. We have $(\alpha+\beta)^2 = \left(\frac{1}{\alpha}+\frac{1}{\beta}\right)(\alpha^2+\beta^2)$

$\Rightarrow (\alpha+\beta)^2 = \left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\left((\alpha+\beta)^2-2\alpha\beta\right)$

$\Rightarrow \left(\frac{-b}{a}\right)^2 = \left(\frac{-b}{c}\right)\left(\left(\frac{-b}{a}\right)^2-\frac{2c}{a}\right)$

$\Rightarrow cb^2+b\left(b^2-2ac\right)=0$

since $b \neq 0$, $bc+b^2-2ac = 0$

a,b,c are in A.P $\therefore b=\frac{a+c}{2}$

We have $\frac{(a+c)c}{2}+\left(\frac{a+c}{2}\right)^2-2ac=0$

$\Rightarrow a^2-4ac+3c^2=0$

$\Rightarrow (a-c)(a-3c)=0$

$\therefore a \neq c \;\; \therefore a=3c \;\Rightarrow \frac{a}{c}=3$

Ans : C

10. Since $\alpha, \beta, \gamma, \delta$ are in H.P, hence $\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma},\frac{1}{\delta}$ are in A.P and they may be taken as a-3d, a-d, a+d, a+3d. Replacing x by $\frac{1}{x}$, we get the equation whose roots are $\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma},\frac{1}{\delta}$. therefore equation $x^2-4x+A=0$ has roots a-3d, a+d and the equation $x^2-6x+B=0$ has roots a-d, a+3d.

sum of the roots is 2(a-d) = 4 and
2(a+d) = 6

solving, we get $a=\frac{5}{2}$ and $d=\frac{1}{2}$

Product of the roots is
(a-3d)(a+d) = A = 3
(a-d)(a+3d) = B = 8

Ans : A

JEE ADVANCE LEVEL QUESTIONS

(i) Multi correct answer type questions

11. Let the roots be $\frac{a}{r}, a, ar$ where a > 0, r > 1.

Now $s_1 = \frac{a}{r} + a + ar = -p$ .... (i)

$s_2 = a\left(\frac{a}{r}\right) + a(ar) + (ar)\left(\frac{a}{r}\right) = q$ .... (ii)

$s_3 = \left(\frac{a}{r}\right)(a)(ar) = 1$ .... (iii)

$\Rightarrow a^3 = 1$

$\Rightarrow a = 1$

Hence (c) is correct.

From (i), putting a = 1, we get

- p - 3 > 0 $\left(\because r + \frac{1}{r} > 2\right)$

$\Rightarrow p < -3$

Hence (b) is not correct.

Also $\frac{1}{r} + 1 + r = -p$ .... (iv)

From (2) putting a = 1, we get

$\therefore \frac{1}{r} + r + 1 = q$ .... (v)

From (iv) and (v) , we have

-p = q $\Rightarrow$ p+q = 0

Hence, (a) is correct.

Now as r > 1,

$\frac{a}{r} = \frac{1}{r} < 1$

and ar = r > 1

Hence (d) is correct

Ans : A,C,D

12. We have $\alpha + \beta = \frac{-b}{a}, \alpha.\beta = \frac{c}{a}, \alpha^2 + \beta^2 = \frac{-b^2}{a^2}$ and $\alpha^2.\beta^2 = \frac{c^2}{a^2}$

Now $(\alpha + \beta)^2 - 2\alpha\beta = \frac{-b^2}{a^2}$

$\Rightarrow \left(\frac{-b}{a}\right)^2 - \frac{2c}{a} = \frac{-b^2}{a^2}$

$\Rightarrow b^2 = ac \Rightarrow a,b,c$ are in G.P

Ans : A

Statement Type

13. Statement-I : Let $\alpha-\beta,\ \alpha,\ \alpha+\beta$ be the roots.

We have $s_1 = \alpha-\beta+\alpha+\alpha+\beta = -2$

$$\Rightarrow 3\alpha = -2$$

$$\Rightarrow \alpha = \frac{-2}{3}$$

Now $\left(\frac{-2}{3}\right)^3 + 2\left(-\frac{2}{3}\right)^2 + k\left(\frac{-2}{3}\right) + 3 = 0$

$$\Rightarrow k = \frac{97}{18}$$

Hence, Statement-I is TRUE

Statement-II :

Clearly, stetement-II is also TRUE, but satement -II is not the correct explanation of Statement-I

Ans : B

14. **Statement-I:**

Let the roots be $\frac{p}{\beta},\ p,\ p\beta$

We have $s_3 = \frac{p}{\beta}.p.p\beta = -27$

$\Rightarrow p^3 = -27$

$\Rightarrow p = -3$

Now $(-3)^3 - \alpha + 27 = 0$

$\Rightarrow \alpha = 0$

hence, statement-I is TRUE

Statement-II:

Clearly statement-II is not true (Taking the roots like $\alpha, \alpha\beta, \alpha\beta^2$ may be theoretically true but practically they are not suggestable)

Ans : C

Comprehension-I

15. Given $ax^3+bx^2+cx+d=0$, roots are A.P

Condition is $2b^3+27a^2d = 9abc$

Given equation is $px^3+qx^2+rx+s=0$

comparing the coefficients of like terms, we get a = p, b = q, c = r, d = s condition is $2q^3+27p^2s = 9pqr$

Ans : D

16. Clearly, If the roots of $mx^3+nx^2+\ell x+p=0$ are in A.P, We have $2n^3+27m^2p = 9mnl$. Compare the coefficients of like terms with given statement.

Ans : C

Comprehension-II

17. Let $f(x)=24x^4-50x^3+35x^2-10x+1=0$

$$f\left(\frac{1}{x}\right)=x^4-10x^3+35x^2-50x+24=0$$

If the roots of $f(x)=0$ are in H.P

There roots of $f\left(\frac{1}{x}\right)=0$ are in A.P.

Let $\alpha-3\beta,\ \alpha-\beta,\ \alpha+\beta, \alpha+3\beta$ be the roots.

We have $s_1=\alpha-3\beta+\alpha-\beta+\alpha+\beta+\alpha+3\beta$

$= 4\alpha$

We have $s_1=-\frac{(-10)}{1}=10$

$\therefore 4\alpha=10$

$\Rightarrow \alpha=\frac{5}{2}$

$\therefore \frac{5}{2}$ is a root of $f\left(\frac{1}{x}\right)=0$

$\frac{2}{5}$ is one root of $f(x)=0$

Ans : D

18. Let the roots be $\alpha-\beta,\ \alpha,\ \alpha+\beta$

We have $s_1=3\alpha=-1$

$\Rightarrow \alpha=\frac{-1}{3}$

We have

$$\Rightarrow \left(\frac{-1}{3}\right)^3+\left(\frac{-1}{3}\right)^2+\left(\frac{-1}{3}\right)+k=0$$

$$\Rightarrow \frac{-1}{27}+\frac{1}{9}-\frac{1}{3}+k=0$$

$\Rightarrow k = \frac{-7}{27}$

Ans : C

19. Let $f(x) = x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$

Clearly $f(1) = 1^4 - 8(1)^3 + 14(1)^2 + 8(1) - 15$
= 1 - 8 + 14 + 8 - 15
= 0

By H.M.S.D, we have

$x = 1$	1	-8	14	8	-15
	0	1	-7	7	15
	1	-7	7	15	0

Now $g(x) = x^3 - 7x^2 + 7x + 15$

$g(-1) = (-1)^3 - 7(-1)^2 + 7(-1) + 15$
= -1 - 7 - 7 + 15
= 0

-1 is a root of $g(x) = 0$

By H.M.S.D, we have

$x = -1$	1	-7	7	15
	0	-1	8	-15
	1	-8	15	0

We have $x^2 - 8x + 15 = 0$

$\Rightarrow (x-3)(x-5) = 0$

$\Rightarrow x = 3, 5$

$\therefore$ The roots are -1, 1, 3, 5
Least root = -1
Greatest root = 5
$\therefore$ sum = 4

Ans : 4

20. Let $f(x) = x^4 + 5x^3 - 30x^2 - 40x + 64 = 0$

f(1) = 1 + 5 - 30 - 40 + 64
= 0

$\therefore x = 1$ is a root of $f(x) = 0$

by H.M.S.D we have

x = 1	1	5	-30	-40	64
	0	1	6	-24	-64
x = -2	1	6	-24	-64	0
	0	-2	-8	64	
x = 4	1	4	-32	0	
	0	4	32		
x = -8	1	8	0		
	0	-8			
	1	0			

$\therefore$ The roots are 1, -2, 4, -8

The sum of positive roots = 5

Ans : 5

21.

a) Given $f(x) = x^3 - 6x^2 + 8x = 0$

$\Rightarrow x\left(x^2 - 6x + 8\right) = 0$

$\Rightarrow x\left(x-2\right)\left(x-4\right) = 0$

$\Rightarrow x = 0, 2, 4$

$\therefore$ The roots are in A.P

b) Given $f(x) = x^3 - 7x^2 + 14x - 8 = 0$

$f(x) = ax^3 + bx^2 + cx + d = 0$

Now $ac^3 = 1.(14)^3 = 2744$

$b^3d = (-7)^3(-8) = 2744$

$\therefore ac^3 = b^3d$

Hence, the roots are in G.P

c) Given $f(x) = 6x^3 - 11x^2 + 6x - 1 = 0$

$f(x) = ax^3 + bx^2 + cx + d = 0$

Now, $2c^3+27ad^2 = 2(6)^3+27(6)(-1)^2$

$= 432 + 16^2$

$= 594$

$9bcd = 9(-11)(6)(-1)$

$= 594$

$\therefore 2c^3+27ad^2 = 9bcd$

Hence, the roots are in H.P

d) Given $f(x) = x^3 - 2x^2 - x + 2 = 0$

$f(1) = 1^3 - 2(1)^2 - (1) + 2$

$= 1 - 2 - 1 + 2$
$= 0$

$\therefore\ x = 1$ is a root of $f(x) = 0$

By H.M.S.D, we have

$x = 1$	1	-2	-1	2
	0	1	-1	2
	1	-1	-2	0

$\therefore\ x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow\ x = 2, -1$

$\therefore$ The roots are -1, 1, 2

$\therefore$ They are neither A.P, nor G.P, nor H.P

Ans : a-p, b-q, c-r, d-s

22

a) If the roots of $f(x) = 0$ are in AP, then the roots of $f\left(\frac{1}{x}\right) = 0$ are in H.P.

b) The sum of the roots of $x^3 - 2x + 1 = 0$ are

$$s_1 = \frac{-b}{a} = \frac{-0}{1} = 0$$

c) Given $x^3 - 1 = 0$

$\Rightarrow (x-1)(x^2 + x + 1) = 0$

$\Rightarrow x - 1 = 0$ or $x^2 + x + 1 = 0$

$\Rightarrow x = 1$

d) Given $f(x) = x^3 - (\sqrt{2} + 3)x^2 + (3\sqrt{2} + 2)x - 2\sqrt{2} = 0$

$f(1) = 1^3 - (\sqrt{2} + 3)1^2 + (3\sqrt{2} + 2)1 - 2\sqrt{2}$

$= 1 - \sqrt{2} - 3 + 3\sqrt{2} + 2 - 2\sqrt{2}$

$= 0$

$x = 1$ is a root of $f(x) = 0$

By H.M.S.D we have

$x = 1$	1	$-(\sqrt{2}+3)$	$3\sqrt{2}+2$	$-2\sqrt{2}$
	0	1	$-(\sqrt{2}+2)$	$2\sqrt{2}$
	1	$-(\sqrt{2}+2)$	$2\sqrt{2}$	0

$\therefore x^2 - \left(\sqrt{2}+2\right)x + 2\sqrt{2} = 0$

$\Rightarrow x^2 - \sqrt{2}x - 2x + 2\sqrt{2} = 0$

$\Rightarrow x\left(x-\sqrt{2}\right) - 2\left(x-\sqrt{2}\right) = 0$

$\Rightarrow \left(x-\sqrt{2}\right)\left(x-2\right) = 0$

$\Rightarrow x = \sqrt{2}$ or 2

$\therefore$ The roots are $1, \sqrt{2}, 2$.

Clearly the roots are in G.P

Ans : a-t, b-p, c-s, d-q