6. TRIGONOMETRIC IDENTITIES

Class: 8 : Mathematics

SOLUTIONS.

* TEACHING TASK *

01. $\sec\theta = \frac{m+n}{2\sqrt{mn}}$

$\Rightarrow \cos\theta = \frac{2\sqrt{mn}}{m+n}$

$\therefore \sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-\frac{4mn}{(m+n)^2}} = \sqrt{\frac{(m-n)^2}{(m+n)^2}} = \frac{m-n}{m+n}$

Ans : C

02 $\frac{(1+\tan^2 A)\cdot \cot A}{\operatorname{cosec}^2 A}$

$= \frac{\sec^2 A \cdot \cot A}{\operatorname{cosec}^2 A}$

$= \frac{\frac{1}{\cos^2 A} \times \frac{\cos A}{\sin A}}{\left(\frac{1}{\sin^2 A}\right)}$

$= \frac{\sin A}{\cos A} = \tan A$

Ans : B

03 $\sec^2 A \cdot \operatorname{cosec}^2 A - (\tan^2 A + \cot^2 A)$ (2)

Let $A = 45^\circ$

$= (\sqrt{2})^2 \cdot (\sqrt{2})^2 - (1^2 + 1^2)$

$= 2 \times 2 - 2 = 2$

Ans. A

04 $(\cos^2\theta - 1)(\operatorname{cosec}^2\theta)$

Let $\theta = 45^\circ$

$= \left(\left(\frac{1}{\sqrt{2}}\right)^2 - 1\right)(\sqrt{2})^2$

$= -\frac{1}{2} \times 2 = -1$

Ans: C

05 $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$

$= \sec^2\theta - \cos^2\theta$

Let $\theta = 45^\circ$

$= (\sqrt{2})^2 - \left(\frac{1}{\sqrt{2}}\right)^2$

$= 2 - \frac{1}{2} = \frac{3}{2}$

opt: c $\sin^2\theta \times \tan^2\theta$

$=$

opt: A: $\tan^2\theta + \sin^2\theta$

$= 1 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{3}{2}$ Ans. A

06 $\left(1 + \frac{1}{\tan^2 A}\right)\left(1 + \frac{1}{\cot^2 A}\right)$

Let $A = 45^\circ$

$\left(1 + \frac{1}{1}\right)\left(1 + \frac{1}{1}\right)$

$= 4$

opt: A $\sec^2 A \cdot \operatorname{cosec}^2 A$

$= (\sqrt{2})^2 (\sqrt{2})^2$

$= 2 \times 2$

$= 4$

Ans: A

07. $\sin^2 A \cdot \cos^2 B - \cos^2 A \cdot \sin^2 B$. (3)

Let $A = 30°$, $B = 60°$

$= \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{16} - \frac{9}{16} = \frac{-1}{2}$

opt: D: $\cos^2 B - \cos^2 A$

$= \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{1}{4} - \frac{3}{4} = \frac{-1}{2}$

Ans. D

08 $\frac{\tan\theta}{(1+\tan^2\theta)^2} + \frac{\cot\theta}{(1+\cot^2\theta)^2}$

Let $\theta = 45°$

$= \frac{1}{(1+1^2)^2} + \frac{1}{(1+1^2)^2}$

$= \frac{1}{2}$

opt: c: $\sin\theta \cdot \cos\theta = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$

Ans: C

09 $\sec\theta + \tan\theta = P \rightarrow ①$

$\sec\theta - \tan\theta = \frac{1}{P} \rightarrow ②$

① + ② $2\sec\theta = P + \frac{1}{P} = \frac{P^2+1}{P}$

① − ② $2\tan\theta = P - \frac{1}{P} = \frac{P^2-1}{P}$

$\frac{2\tan\theta}{2\sec\theta} = \frac{P^2-1}{P^2+1}$

$\sin\theta = \frac{P^2-1}{P^2+1}$

Ans: B

10. $\frac{\operatorname{cosec}^2\alpha}{1+\cot^2\alpha} + \frac{\tan^2\alpha}{1+\tan^2\alpha} + \cos^2\alpha$ (4)

Let $\alpha = 45^\circ$

$= \frac{(\sqrt{2})^2}{1+(1)^2} + \frac{(1)^2}{1+(1)^2} + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{2}{2} + \frac{1}{2} + \frac{1}{2}$

$= 2$

Ans: D

---

11. $\sin^6\theta + \cos^6\theta + \sin^2\theta \cdot \cos^2\theta = a\sin^4\theta + b\cos^4\theta$

Let $\theta = 0^\circ$	Let $\theta = 90^\circ$
$1 = b$	$1 = b$

Now $a + 2b = 3$

Ans: C

---

12. $\sin^2 x \cdot \cos^2 y - \cos^2 x \cdot \sin^2 y \longrightarrow (1)$

$(1) = (1-\cos^2 x)\cos^2 y - \cos^2 x (1 - \cos^2 y)$

$= \cos^2 y - \cos^2 x$

$(1) = \sin^2 x (1 - \sin^2 y) - (1 - \sin^2 x) \cdot \sin^2 y$

$= \sin^2 x - \sin^2 y$

Ans: A, C

---

13. $\sin(\alpha+\beta)=1 \Rightarrow \alpha+\beta=90^\circ$

$\sin(\alpha-\beta)=\frac{1}{2} \Rightarrow \alpha-\beta=30^\circ$

$\therefore \alpha=60^\circ, \beta=30^\circ$

Ans: A, C

14. $a\sin^2\theta+b\cos^2\theta=c$

$\Rightarrow a\tan^2\theta+b=c\sec^2\theta$

$\Rightarrow a\tan^2\theta+b=c(1+\tan^2\theta)$

$\Rightarrow a\tan^2\theta+b=c+c\tan^2\theta$

$\Rightarrow (a-c)\tan^2\theta=c-b$

$\Rightarrow \tan^2\theta=\frac{c-b}{a-c}$

Ans: B

15 Statement I:

$(1+\cos\alpha)(1+\cos\beta)(1+\cos\gamma)(1-\cos\alpha)(1-\cos\beta)(1-\cos\gamma)=K$

$\Rightarrow (1-\cos^2\alpha)(1-\cos^2\beta)(1-\cos^2\gamma)=K$

$\Rightarrow \sin^2\alpha\cdot\sin^2\beta\cdot\sin^2\gamma=K$

$\Rightarrow$