COS-13 → 9th foundation + work Thask

(1)

In equilibrium position $F = mg$ also $F = kx$.

$\therefore kx = mg \Rightarrow k = \frac{mg}{x}$

when the object falls freely its Gravitation P.E is converted into elastic P.E of the spring.

$\therefore mg\cancel{e} = \frac{1}{2} k e^{\cancel{2}}$ where 'e' is maximum stretching.

$\Rightarrow mg = \frac{1}{2} \frac{mg}{x} e$

$\Rightarrow e = 2x \rightarrow B$

(2)

Given

Natural length $l = 40\,mm = 40 \times 10^{-3}\,m$.

Force $F = 10N$; elongation $x_1 = 1\,mm = 10^{-3}\,m$.

From $F = kx_1 \Rightarrow 10 = k \times 10^{-3} \Rightarrow k = 10^4\,N/m$.

work done $= \frac{1}{2} k x^2$

$\Rightarrow \frac{1}{2} \times 10^4 \times (40 \times 10^{-3})^2$ [From 1mm upto 40mm]

$\Rightarrow \frac{1}{2} \times 10^4 \times 1600 \times 10^{-6}$

$\Rightarrow \frac{1}{2} \times 16$

$\Rightarrow 8J \rightarrow D$

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3

Under stretching the P.E stored in a spring $U_1 = \frac{1}{2}kx^2$

If it is stretched to nx

Then the P.E $U_2 = \frac{1}{2}k(nx)^2 = n^2 \frac{1}{2}kx^2$

$\Rightarrow U_2 = n^2 U_1$

$\therefore$ The P.E stored in the spring when stretched by n

is $\underline{n^2 U}$ $\rightarrow$ C

4

we know that, the work done by conservative force

is $W_c = -\Delta U \rightarrow (1)$

Total work done $W = W_c + W_{NC}$

$\Rightarrow W = -\Delta U + W_{NC} \rightarrow (2)$

where $W_{NC} \rightarrow$ work done by non conservative force.

According to W-E theorem work $W = \Delta K.E \rightarrow (3)$

$\therefore$ From (2) & (3) $\Delta K.E = -\Delta U + W_{NC}$

$\Rightarrow W_{NC} = \Delta K.E + \Delta U$ [A, C correct]

5

For any type of force work done = change in K.E

$\rightarrow$ A.

WS-13 9th foundation +

Thank

(6)

let length of shorter Piece $= l_1 = x$.

longer Piece $= l_2 = 3x$.

$\therefore$ Total length $L = x + 3x = 4x$

$\Rightarrow x = \frac{L}{4} = l_1$

$l_2 = \frac{3L}{4}$

we know that $F = kx \Rightarrow k \propto \frac{F}{x} \Rightarrow k \propto \frac{1}{x}$

$k_{long} = \frac{k}{l_2} = \frac{k}{\frac{3L}{4}} = \frac{4k}{3L} = \frac{4}{3}k$

where k = original spring constant $= \frac{k}{L}$

$k_{long} = \frac{4}{3}k \rightarrow C$

(7)

we know that elastic P.E of the spring

$U = \frac{1}{2}kx^2 \Rightarrow U \propto x^2$

For $x_1 = 2cm \Rightarrow U_1 = U$ $\therefore \frac{U_2}{U_1} = \left[\frac{x_2}{x_1}\right]^2$

$\Rightarrow \frac{U_2}{U} = \left(\frac{10}{2}\right)^2$

$\Rightarrow \frac{U_2}{U} = 5^2$

$\Rightarrow U_2 = 25U \rightarrow D$

(8)

Given $k_1 = 1500\ N/m$ $k_2 = 3000\ N/m$

$F = \text{constant}$

Elastic P.E $U = \frac{1}{2} k x^2$ (or) $\frac{F^2}{2k}$

$U \propto \frac{1}{k} \Rightarrow \frac{U_1}{U_2} = \frac{k_2}{k_1} = \frac{3000}{1500} = \frac{2}{1}$

(10)

In this force acting on a particle is same and along the path OP and OQP, initial and final positions are same i.e. displacement is same.

w = Force x displacement

$w_1 = w_2$.

(11)

Forces do not store energy are called as non conservative force. Potential energy is a non-conservative force because the work done by the body depends on the path connecting the starting point and ending point.

(13)

when the particle moves exactly once around any closed path, it's work equals the change in K.E of the particle. it work depends on the end points of motion, not on the path in between.

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(14), (15), (16)

At $x = 5m$ $\quad U = 20 + (x-2)^2$

$= 20 + (5-2)^2$

$= 20 + 3^2 = 29J$

Given at $x = 5m$ $\quad K.E = 20J$

$\therefore$ Total mechanical energy $= K.E + U = 20+29$

$= 49J$

Here the particle has maximum K.E, when P.E is minimum. however Total Energy $E = 49J$

At $x = 2$ $\quad U_{min} = 20 + (2-2)^2 = 20J$

$E = K_{max} + U_{min}$

$\Rightarrow 49 = K.E_{max} + 20 \Rightarrow K.E_{max} = 49 - 20$

$= 29J$

For min P.E condition

From $U = 20 + (x-2)^2$

$= 20 + x^2 - 4x + 4$

$\frac{dU}{dx} = \frac{d}{dx}[24 + x^2 - 4x] = \frac{d}{dx}(20) + \frac{d}{dx}x^2 - 4\frac{dx}{dx}$

$= 0 + 2x - 4$

$\frac{dU}{dx} = 0 \Rightarrow 2x - 4 = 0$

$x = 2.$

limit of x is $\quad$ for $U = 49J$

$20 + (x-2)^2 = 49$

$\Rightarrow (x-2)^2 = 29 \Rightarrow x - 2 = \sqrt{29}$

$\Rightarrow x - 2 = \pm 5.38$

$\therefore x = 7.38$ (or) -3.38 m

(17)

Given $\quad W = 1J \quad ; \quad x = 20\times10^{-2} m$

work done $= \frac{1}{2} kx^2$

$\Rightarrow \quad 1 = \frac{1}{2} \times k \times (20\times10^{-2})^2$

$\Rightarrow \quad 1 = \frac{k}{\cancel{2}} \times \overset{200}{\cancel{400}} \times 10^{-4}$

$\Rightarrow \quad k = \frac{10^4}{200} = 50 \, N/m$

(18)

we know that $\quad U = \frac{1}{2} kx^2$

$U \propto x^2$ $\qquad$ Given $x_1 = 1cm \to U_1 = 4$

$\qquad\qquad\qquad\qquad\qquad\qquad \to U_2 = 250$

$\Rightarrow \quad \frac{U_1}{U_2} = \left(\frac{x_1}{x_2}\right)^2$

$\Rightarrow \quad \frac{4}{250} = \left(\frac{1}{x}\right)^2 \Rightarrow \frac{1}{x} = \sqrt{\frac{1}{25}} \Rightarrow x = 5cm$

(19)

Given $\quad e_1 = 10cm \to U_1 = 4J \qquad k = ?$

$\qquad = 10^{-1} m$

we know that $\quad U = \frac{1}{2} k x^2$

$\Rightarrow \quad 4 = \frac{1}{2} k (10^{-1})^2$

$\Rightarrow \quad 8 = k\times10^{-2} \Rightarrow k = 800 \, N/m$

$\qquad\qquad = 8\times100 \, N/m$

(20)

we know $\quad$ workdone in stretching a string

$W = \frac{1}{2} kx^2 \qquad x = \text{same}$

$W \propto k \qquad W_B < W_A$

$F = kx \quad so \Rightarrow F \propto k \qquad F_A > F_B$

LTask CUQ's

(1) A conservative force is a force with the Property that ~~w~~ the work done in moving a Particle between two points is independent of the Path taken and is equal to zero when path is closed.

So work done by a conservative force is independent of the path resulting in a given displacement

(2) A conservative force is work done move a Particle from one Point to another and is independent of the Path work done over a closed loop is zero for a conservative force.

(3) when the force depends only on initial and final position irrespective of the Path taken. In any closed path, the work done by a conservative force is zero. The work done by a conservative is reversable.

(6) For a completely closed loop work done by conservative force is zero.

SAQ's

(1)

If the spring is stretched by length x, then work done $= \frac{1}{2} k x^2$

so work done by each mass on the spring $= \frac{1}{4} k x^2$

$\therefore$ work done by spring on each mass $= -\frac{1}{4} k x^2$

Since the spring force and the displacement of masses are in opposite direction, work done is -ve.

(2)

we know that

work done = change in energy.

$\Rightarrow W_{NC} = \Delta K.E + \Delta U$

$\Rightarrow W_{NC} = K_F - K_i + U_F - U_i$

$\Rightarrow 0 = \frac{1}{2} m \left(\frac{v}{2}\right)^2 - \frac{1}{2} m v^2 + \frac{1}{2} k x^2 - 0$

$\Rightarrow 0 = \frac{1}{4} \left[\frac{1}{2} m v^2\right] - \left[\frac{1}{2} m v^2\right] + \frac{1}{2} k x^2$

$\Rightarrow 0 = -\frac{3}{8} m v^2 + \frac{1}{2} k x^2$

$\Rightarrow \frac{3}{\cancel{8}\,4} m v^2 = \frac{\cancel{1}}{\cancel{2}} k x^2$

$\Rightarrow \frac{3}{4} m v^2 = k x^2$

$\Rightarrow k = \frac{3}{4} \frac{m v^2}{x^2}$

(3)

we know that workdone by the conservative force = change in P.E

$W_c = -(U_B - U_A)$

$= U_A - U_B$

(6)

let the length of spring be 'l' m

length of shorter path $l_1 = \frac{l}{3}$ m

length of longer part $l_2 = \frac{2l}{3}$ m

work done by the spring $= \frac{1}{2}kx^2$

here $k_2 < k_1$ because spring constant $k \propto \frac{1}{l}$

$\therefore$ The work done is greater for shorter part

(7)

Given $k = 5 \times 10^3$ N/m

stretched position $x_1 = 5$ cm

$\therefore$ $W_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(5\times10^3)(5\times10^{-2})^2$

$= 6.25$ J

For additional ~~a~~ compression $W_2 = \frac{1}{2}kx_2^2$

$W_2 = \frac{1}{2} \times 5\times10^3 \times (10\times10^{-2})^2 = 25$ J

$\therefore$ The additional work done $= W_2 - W_1$

$= 25 - 6.25$

$= 18.75$ J

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(8)

Since $w = \int F \cdot dr$

clearly for forces (A) and (B), the integration do not require any information of path taken.

For (c): $w_c = \int \frac{3(x\hat{i} + y\hat{j})}{(x^2+y^2)^{3/2}} \cdot (dx\hat{i} + dy\hat{j})$

$= 3\int \frac{x dx + y dy}{(x^2+y^2)^{3/2}}$

Taking: $x^2 + y^2 = t$

$\Rightarrow 2x dx + 2y dy = dt$

$\Rightarrow x dx + y dy = \frac{dt}{2}$

$w = 3\int \frac{\frac{dt}{2}}{t^{3/2}} = \frac{3}{2}\int \frac{dt}{t^{3/2}}$ which is solvable.

$\therefore$ A, B, C are conservative force

(9)

From Diagram $(w_F)_{OAC} = \int (xy\,dx + x^2 y\,dy)$

$= \int_O^A (xy\,dx + x^2 y\,dy) + \int_A^C (xy\,dx + x^2 y\,dy)$

on OA Path $y = 0;\ dy = 0$ and on AC Path $x = 1;\ dx = 0$

$(w_F)_{OAC} = \int_O^A (0 \cdot dx + 0 \cdot dy) + \int_{y=0}^{4} (0 + 1 \times y\,dy)$

$= \int_0^4 y\,dy = \left[\frac{y^2}{2}\right]_0^4 = \left[\frac{4^2}{2}\right] = 8\,J$

$(w_F)_{OBC} = \int_O^B (xy\,dx + x^2 y\,dy) + \int_B^C (xy\,dx + x^2 y\,dy)$

$\Rightarrow (W_F)_{OBC} = \int_O^B (0 \cdot dx + 0 \cdot dy) + \int_B^C (xy\,dx + x^2 y\,dy)$

$= 0 + \int_{x=0}^{1} (x(4)dx + x^2 \times 4(0)] dx =$

$[y = 4]$

$= \int_0^1 4x\,dx = 4\left[\frac{x^2}{2}\right]_0^1 = 4\left[\frac{1}{2}\right] = 2J$

$(W_F)_{ODC} = \int_O^B [xy\,dx + x^2 y\,dy] + \int_B^D [xy\,dx + x^2 y\,dy]$

$\underline{\underline{y = 4x^2}}$ $\therefore dy = 8x\,dx$ For Path OD $x = 0,\ dy = 0$

$= \int_O^D (0 \cdot dx + 0 \cdot dy) + \int_D^C [xy\,dx + x^2 y\,dy]$

For DC $x = 1$

$= \int_0^1 x\,4x^2\,dx + x^2\,4x^2\,8x\,dx$

$= \int_0^1 4x^3\,dx + 32\int x^5\,dx$

$= 4\left[\frac{x^4}{4}\right]_0^1 + 32\left[\frac{x^6}{6}\right]_0^1$

$= 1 + \frac{32}{6} = \frac{38}{6} = \frac{19}{3} J$

only A-correct

(10)

As the block makes collision with a stationary wall the system rebounds back with same velocity

∴ work done by the spring on the wall = change in K.E

$$W = \frac{1}{2} m (v^2 - u^2) \quad \text{Here } V = -u$$

$$= \frac{1}{2} m ((-u)^2 - u^2)$$

$$= \frac{1}{2} m (u^2 - u^2) = 0 \text{ J}$$

(1.4)

Given $k_1 = 40$ N/m & $k_2 = 60$ N/m.

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{40} + \frac{1}{60} = \frac{3+2}{120}$$

$$k_s = \frac{120}{5} = 24 \rightarrow A$$

(15)

Given $k_1 = 200$ N/m ; $k_2 = 600$ N/m.

$$k_p = k_1 + k_2 = 200 + 600 = 800 \text{ N/m}$$

(16)

Given $k_p = 90$; $k_s = 20$

$\Rightarrow k_1 + k_2 = 90$; $\frac{k_1 k_2}{k_1 + k_2} = 20$

$\Rightarrow k_1 k_2 = 20(k_1 + k_2) = 20 \times 90$

$\Rightarrow k_1 k_2 = 1800$

$\therefore (k_1-k_2)^2 = (k_1+k_2)^2 - 4k_1k_2$

$= (90)^2 - 4(1800)$

$\Rightarrow \cancel{(90)^2}\ 8100 - 7200$

$\Rightarrow 900$

$k_1 - k_2 = 30$

We know $k_1 + k_2 = 90$

$2k_1 = 120$

$\Rightarrow k_1 = 60\ N/m$

$\therefore k_2 = 90 - 60$

$k_2 = 30\ N/m$

(17)

Given if compression $x_1 = 4\ cm = 4 \times 10^{-2}$

$W = 2J$

$\Rightarrow \frac{1}{2}kx_1^2 = 2J$

$\Rightarrow \frac{1}{2}(4\times10^{-2})^2 k = 2$

$\Rightarrow k(4)^2 = 4 \times 10^4$

$\Rightarrow k = \frac{10^4}{4}\ N/cm$

For $x_2 = 8\ cm$

$F = kx_2$

$\Rightarrow F = \frac{10^4}{4} \times 8 \times 10^{-2} =$

$F = 200\ N$

(18)

Given $U = 100\ J$ For $x_1 = x$

$\Rightarrow \frac{1}{2}kx_1^2 = 100\ J$

For $x_2 = 2x$

$U_2 = \frac{1}{2}kx_2^2 = \frac{1}{2}k(2x)^2 = 4\ \frac{1}{2}kx^2$

$= 4(100)$

$= 400\ J$

Additional P.E $= U_2 - U = 400 - 100 = 300\ J$