WS-7 8^{th} foundation +

(1)

Dot Product

Track

(1)

Given $\vec{A} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{B} = -\hat{i} + \hat{j} - 2\hat{k}$

$\therefore \vec{A} \cdot \vec{B} = (\hat{i} + 2\hat{j} - \hat{k}) \cdot (-\hat{i} + \hat{j} - 2\hat{k})$

$= [1 \times (-1)] + 2(1) + (-1)(-2)$

$= -1 + 2 + 2 = 3$

$|\vec{A}| = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6}$; $|\vec{B}| = \sqrt{(-1)^2 + 1^2 + (-2)^2} = \sqrt{6}$

$\therefore \cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \dfrac{3}{\sqrt{6}\sqrt{6}} = \dfrac{3}{6} = \dfrac{1}{2}$

$\theta = 60^\circ$

(2)

We know that $\vec{a} \cdot \vec{b} = ab\cos\theta$ where 'θ' is angle between $\vec{a}$ and $\vec{b}$

From given right angle triangle $\theta = 90^\circ$

$\therefore \vec{a} \cdot \vec{b} = ab\cos 90 = \underline{0}$

③

let the Given vector be $\vec{A} = \hat{i} + 2\hat{j}$.

let $\vec{r}$ is a vector whose magnitude is $3\sqrt{5}$

$\vec{r} = x\hat{i} + y\hat{j}$.

If $\vec{A}$ and $\vec{r}$ are Perpendicular then $\vec{A} \cdot \vec{r} = 0$

$\Rightarrow (\hat{i} + 2\hat{j}) \cdot (x\hat{i} + y\hat{j}) = 0$

$\Rightarrow x + 2y = 0 \rightarrow (1) \Rightarrow$

$\therefore |\vec{r}| = \sqrt{x^2 + y^2} = 3\sqrt{5}$

By doing squaring on both sides

$(\sqrt{x^2 + y^2})^2 = (3\sqrt{5})^2$

$\Rightarrow x^2 + y^2 = 9 \times 5 \Rightarrow x^2 + y^2 = 45 \rightarrow (2)$

From (1) $x = -2y$ $\therefore (-2y)^2 + y^2 = 45$

$\Rightarrow 5y^2 = 45 \Rightarrow y = 3.$

Sub y in $x = -2y \therefore x = -2 \times 3 = -6$

$\therefore \vec{r} = -6\hat{i} + 3\hat{j}$ is the vector Perpendicular to $\vec{A}$

option B correct

(4)

Given $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 2\hat{j} + 3\hat{k}$

$\vec{A} \cdot \vec{B} = (2\hat{i} + 3\hat{j}) \cdot (2\hat{j} + 3\hat{k})$

$\Rightarrow (2 \times 3) = 6.$

$|\vec{A}| = \sqrt{2^2 + 3^2}$

$= \sqrt{4+9} = \sqrt{13}.$

$\therefore$ component of $\vec{B}$ along $\vec{A} = \dfrac{(\vec{A} \cdot \vec{B})}{|\vec{A}|}$

$\Rightarrow \dfrac{6}{\sqrt{13}}$

(5)

$\vec{A} = a\hat{i} + a\hat{j} + 3\hat{k}$; $\vec{B} = a\hat{i} - 2\hat{j} - \hat{k}$.

Given $\vec{A}$ and $\vec{B}$ are perpendicular to each other

$\therefore \vec{A} \cdot \vec{B} = 0 \Rightarrow (a\hat{i} + a\hat{j} + 3\hat{k}) \cdot (a\hat{i} - 2\hat{j} - \hat{k}) = 0$

$\Rightarrow a^2 - 2a - 3 = 0$

$\Rightarrow a^2 - 3a + a - 3 = 0$

$\Rightarrow a(a-3) + 1(a-3) = 0$

$\Rightarrow (a-3)(a+1) = 0$ (ie) $a = 3$ (or) -1

In Question asking only +ve value of a ie 3.

(6)

Given $\vec{A} = 5\hat{i} - 2\hat{j} + 3\hat{k}$; $\vec{B} = 2\hat{i} + \hat{j} + 2\hat{k}$

$|\vec{A}| = \sqrt{5^2 + (-2)^2 + 3^2}$

$= \sqrt{25 + 4 + 9}$

$= \sqrt{38}$

$\vec{A} \cdot \vec{B} = (5\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (2\hat{i} + \hat{j} + 2\hat{k})$

$= 5 \times 2 + (-2)(1) + 3 \times 2$

$= 10 - 2 + 6 = 14.$

$\therefore$ component of $\vec{B}$ along $\vec{A} = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}|} = \dfrac{14}{\sqrt{38}}$

(7)

Given $\vec{A} = a\hat{i} + \hat{j} - 2\hat{k}$: $\vec{B} = a\hat{i} - a\hat{j} + \hat{k}$.

$\vec{A}$ and $\vec{B}$ are perpendicular to each other.

$\therefore \vec{A} \cdot \vec{B} = 0 \Rightarrow (a\hat{i} + \hat{j} - 2\hat{k}) \cdot (a\hat{i} - a\hat{j} + \hat{k}) = 0$

$\Rightarrow a^2 - a - 2 = 0$

$\Rightarrow a^2 - 2a + a - 2 = 0$

$\Rightarrow a(a-2) + 1(a-2) = 0$

$\Rightarrow (a-2)(a+1) = 0 \Rightarrow a = 2$ (or) -1.

only +ve value of 'a' is 2.

(8)

Given $F = 8\hat{i} + 4\hat{j}$; $\vec{s} = 3\hat{i} - 3\hat{j}$; $P = 0.6W$; $t = ?$

From def of $P = \dfrac{W}{t}$

$\Rightarrow P = \dfrac{\vec{F} \cdot \vec{S}}{t}$

$\Rightarrow 0.6 = \dfrac{(8\hat{i} + 4\hat{j}) \cdot (3\hat{i} - 3\hat{j})}{t}$

$\Rightarrow 0.6 = \dfrac{24 - 12}{t}$

$\Rightarrow t = \dfrac{12}{0.6}$

$= \dfrac{120}{6}$

$= 20$ sec.

9

Given $\vec{a}$ and $\vec{b}$ are unit vectors

$\therefore |\vec{a}| = |\vec{b}| = 1$; $\theta = 60^\circ$

$$\frac{1+\vec{a}\cdot\vec{b}}{1-\vec{a}\cdot\vec{b}} = \frac{1+ab\cos\theta}{1-ab\cos\theta} = \frac{1+1\times1\times\cos 60}{1-1\times1\times\cos 60} = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$$

10

Given $\vec{A}+\vec{B} = \vec{C}$.

$\therefore$ By doing dot Product with $\vec{B}$ on both sides

$\Rightarrow (\vec{A}+\vec{B})\cdot\vec{B} = \vec{C}\cdot\vec{B}$

$\Rightarrow \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{B} = \vec{C}\cdot\vec{B}$

$\Rightarrow \vec{A}\cdot\vec{B} + B^2 = \vec{C}\cdot\vec{B}$

$\Rightarrow B^2 = \vec{C}\cdot\vec{B} - \vec{A}\cdot\vec{B} \Rightarrow B = \sqrt{\vec{C}\cdot\vec{B} - \vec{A}\cdot\vec{B}}$

Advanced

11

Given $\vec{a} = m\vec{b} + \vec{c}$.

By doing dot Product with $\vec{b}$ on both sides we get

$\Rightarrow \vec{a}\cdot\vec{b} = (m\vec{b}+\vec{c})\cdot\vec{b}$

$\Rightarrow \vec{a}\cdot\vec{b} = m\,\vec{b}\cdot\vec{b} + \vec{c}\cdot\vec{b}$

$\Rightarrow \vec{a}\cdot\vec{b} - \vec{c}\cdot\vec{b} = m\,b^2$

$$\Rightarrow m = \frac{\vec{a}\cdot\vec{b} - \vec{c}\cdot\vec{b}}{b^2}$$

(2)

$\vec{v} = 2\hat{i} + c\hat{j}$; $a = 3\hat{i} + 4\hat{j}$ Given $\vec{v} \perp$ to $\vec{a}$

$\therefore \vec{v} \cdot \vec{a} = 0$

$\Rightarrow (2\hat{i} + c\hat{j}) \cdot (3\hat{i} + 4\hat{j}) = 0$

$\Rightarrow 2\times3 + c\times4 = 0 \Rightarrow 4c = -6 \Rightarrow c = \frac{-6}{4} = -\frac{3}{2}.$

$\Rightarrow c = -1.5$

(3)

(b) Given $\vec{a} = m\vec{b}$ By doing dot Product on both sides

$\vec{a} \cdot \vec{b} = m\,\vec{b} \cdot \vec{b}$

$\Rightarrow \vec{a} \cdot \vec{b} = m\,b^2 \Rightarrow m = \frac{\vec{a} \cdot \vec{b}}{b^2}$

(4)

Given

$\vec{A} = 5\hat{i} + 6\hat{j} + 3\hat{k}$ and $\vec{B} = 6\hat{i} - 2\hat{j} - 6\hat{k}$

If $\vec{A}$ and $\vec{B}$ are Perpendicular.

$\therefore \vec{A} \cdot \vec{B} = (5\hat{i} + 6\hat{j} + 3\hat{k}) \cdot (6\hat{i} - 2\hat{j} - 6\hat{k})$

$= (5\times6 + 6\times(-2) + 3\times(-6))$

$= 30 - 12 - 18 = 0$

$\therefore \vec{A}$ & $\vec{B}$ are perpendicular to each other.

Since dot Product obeys commutative law $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

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(6)

Given $\theta = 60^\circ$

$\vec{a} \cdot \vec{b} = ab \cos\theta = ab \cos 60 = \frac{ab}{2}$

(7)

component of $\vec{b}$ along $\vec{a}$ is $= \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \frac{ab\cos\theta}{|\vec{a}|}$

$= \hat{a}\, b \cos 60^\circ$ $\quad [\hat{a} = \frac{\vec{a}}{|\vec{a}|}$ unit vector$]$

$= \frac{\hat{a}\, b}{2}$

(8)

$F = 1200N$; $S = 10m$; $\theta = 60^\circ$

$\omega = F S \cos\theta$

$= 1200 \times 10 \times \cos 60$

$= 12000 \times \frac{1}{2}$

$= 6000 = 6 \times 10^3 J$

(9)

Let $\vec{A} = 4\hat{i} - 3\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} + 8\hat{j} + 4\hat{k}$

$\therefore$ Dot Product of $\vec{A}$ and $\vec{B} = \vec{A} \cdot \vec{B}$

$= (4\hat{i} - 3\hat{j} + 3\hat{k}) \cdot (3\hat{i} + 8\hat{j} + 4\hat{k})$

$= (4 \times 3 + (-3)(8) + 3 \times 4$

$= 12 - 24 + 12$

$= 0$

(10)

(a) $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

$\Rightarrow \sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{A^2+B^2-2AB\cos\theta}$ S.O.B.S

$\Rightarrow A^2+\cancel{B^2}+2AB\cos\theta = A^2+\cancel{B^2}-2AB\cos\theta$

$\Rightarrow 2AB\cos\theta = -2AB\cos\theta$

$\Rightarrow 4AB\cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$

(b) $\vec{A}\times\vec{B} = \vec{A}\cdot\vec{B}$

$\Rightarrow \cancel{AB}\sin\theta = \cancel{AB}\cos\theta$

$\Rightarrow \sin\theta = \cos\theta \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45^\circ$

(c) $\vec{A}\cdot\vec{B} = \frac{AB}{2}$

$\Rightarrow \cancel{AB}\cos\theta = \frac{\cancel{AB}}{2} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

(D) $\vec{A}\times\vec{B} = \frac{AB}{2}$

$\Rightarrow \cancel{AB}\sin\theta = \frac{\cancel{AB}}{2} \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30$

L Task

CUQ's

1

Given $\vec{A}\cdot\vec{B}=0$ $\Rightarrow AB\cos\theta=0$

$\Rightarrow \theta=90^\circ$

2

let the two vectors be $\vec{A}=\hat{i}+\hat{j}$ and $\vec{B}=\hat{j}+\hat{k}$

$\therefore (\vec{A}\cdot\vec{B})=(\hat{i}+\hat{j})\cdot(\hat{j}+\hat{k})=1\cdot1=1$

$|\vec{A}|=\sqrt{1^2+1^2}=\sqrt{2}$; $|\vec{B}|=\sqrt{1^2+1^2}=\sqrt{2}$.

$\therefore$ If 'θ' is angle between the vectors

$$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}=\frac{1}{\sqrt{2}\sqrt{2}}=\frac{1}{2}$$

$\Rightarrow \theta=60^\circ$

3

let $\vec{A}=4\hat{i}-2\hat{j}+5\hat{k}$; $\vec{B}=3\hat{i}+\hat{j}-2\hat{k}$.

$\therefore \vec{A}\cdot\vec{B}=(4\hat{i}-2\hat{j}+5\hat{k})\cdot(3\hat{i}+\hat{j}-2\hat{k})$

$=4\times3+(-2)(1)+(5)(-2)$

$=12-2-10=0$

Angle between $\vec{A}$ and $\vec{B}$ is $\cos\theta=\dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

$\Rightarrow \cos\theta=\dfrac{0}{|\vec{A}||\vec{B}|}=0$

$\Rightarrow \theta=90^\circ$

(4)

let $\vec{A} = 4\hat{i} - 2\hat{j}$, $\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore \vec{A} \cdot \vec{B} = (4\hat{i} - 2\hat{j}) \cdot (\hat{i} + 2\hat{j} + 3\hat{k})$

$= (4 \cdot 1) + (-2)(2) + 0 \times 3$

$= 4 - 4 + 0 = 0$

(5)

Given $\vec{A} = \hat{i} + \hat{j}$.

Angle made by the vector with x-axis is

$$\tan\theta = \frac{y\text{-component}}{x\text{-component}}$$

$$\Rightarrow \tan\theta = \frac{1}{1} = 1 \Rightarrow \theta = 45°$$

(6)

Given $\vec{A} = \underset{x}{2\hat{i}} + \underset{y}{3\hat{j}}$.

Angle made by the vector with y-axis

$$\tan\theta = \frac{x\text{-component}}{y\text{-component}} = \frac{2}{3}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2}{3}\right)$$

(7)

Given $\vec{A} = a\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} - 4\hat{k}$

If $\vec{A}$ is $\perp^{er}$ to $\vec{B}$ $\therefore \vec{A} \cdot \vec{B} = 0$

$\Rightarrow (a\hat{i} + 2\hat{j} - 5\hat{k}) \cdot (2\hat{i} - \hat{j} - 4\hat{k}) = 0$

$\Rightarrow 2a - 2 + (-5)(-4) = 0 \Rightarrow 2a - 2 + 10 = 0$

$\Rightarrow a = -4.$

(8)

Given $\vec{F} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{s} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

$\therefore$ $\omega = \vec{F} \cdot \vec{s} = (2\hat{i} + 4\hat{j} + \hat{k}) \cdot (3\hat{i} + 2\hat{j} + 5\hat{k})$

$= (2 \cdot 3) + (4 \cdot 2) + (1)(5)$

$= 6 + 8 + 5 = 19$

(9)

let the vector be $\vec{A} = \sqrt{3}\,\hat{i} + \hat{j}$.

Angle made by the vector with x-axis

$\tan\theta = \dfrac{y\text{-component}}{x\text{-component}} \rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$

$\rightarrow \theta = 30^\circ$ (or) $\dfrac{\pi}{6}$

(10)

(a) $\vec{A} = 3\hat{i} + 4\hat{j}$

Angle made by $\vec{A}$ with x-axis $\tan\theta = \frac{4}{3} = \theta = 53^\circ$

(b) $\vec{B} = 4\hat{i} + 3\hat{j}$

Angle made by $\vec{B}$ with x-axis ; $\tan\theta = \frac{3}{4} \Rightarrow \theta = 37^\circ$

(c) $\vec{C} = \hat{i} + \hat{j}$

Angle made by $\vec{C}$ with x-axis $\rightarrow \tan\theta = \frac{1}{1} \Rightarrow \theta = 45^\circ$

$\Rightarrow$ ans : b, c, a

(11)

If (a) $\vec{A}$ and $\vec{B}$ are parallel $\theta = 0^\circ$

$\vec{A}\cdot\vec{B} = AB\cos\theta = AB$

(b) $\theta = 60^\circ$ $\vec{A}\cdot\vec{B} = AB\cos 60 = \frac{AB}{2}$

(c) $\theta = 180^\circ$ $\vec{A}\cdot\vec{B} = AB\cos 180 = -AB$

c, b, a.

Jee mains level

(1)

$\vec{F} = 2\hat{i} - 0.75\hat{j} + 5\hat{k}$ N. $t = 4$ sec.

$\vec{s} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ m.

$$\text{Power} = \frac{W}{t} = \frac{\vec{F}\cdot\vec{S}}{t} = \frac{(2\hat{i} - 0.75\hat{j} + 5\hat{k})\cdot(3\hat{i} + 4\hat{j} + 5\hat{k})}{4}$$

$$P = \frac{2\times 3 + (-0.75)(4) + 5\times 5}{4} = \frac{6 - 3 + 25}{4} = \frac{28}{4} = 7W$$

(2)

Given $\frac{1 - \vec{A}\cdot\vec{B}}{1 + \vec{A}\cdot\vec{B}} = \frac{1 - AB\cos\theta}{1 - AB\cos\theta}$ Given A, B are unit vectors $|\vec{A}| = |\vec{B}| = 1$

$$= \frac{1 - 1\times 1\cos\theta}{1 + 1\times 1\cos\theta} = \frac{1 - \cos\theta}{1 + \cos\theta} = \frac{2\sin^2\theta/2}{2\cos^2\theta/2}$$

$$= \tan^2\theta/2$$

(3)

$\vec{V} = 4\hat{i} - 2\hat{j} + \hat{k}$ m/s ; $\vec{F} = 5\hat{i} - 10\hat{j} + 6\hat{k}$ N

Power $= \vec{F} \cdot \vec{V}$

$= (5\hat{i} - 10\hat{j} + 6\hat{k}) \cdot (4\hat{i} - 2\hat{j} + \hat{k})$

$\Rightarrow (5 \cdot 4) + (-10)(-2) + 6 \times 1$

$\Rightarrow 20 + 20 + 6 = 46W$

(4)

let $\vec{A} = -2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = \hat{i} + 2\hat{j} + 4\hat{k}$

$\therefore \vec{A} \cdot \vec{B} = (-2\hat{i} + 3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} + 4\hat{k})$

$= (-2)(1) + 3 \times 2 + (-1)(4)$

$\Rightarrow -2 + 6 - 4 = 0$

The angle b/w two vectors is $\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|}$

$\Rightarrow \cos\theta = \dfrac{0}{|\vec{A}|\,|\vec{B}|} = 0 \Rightarrow \theta = 90^\circ$

(5)

Given $\vec{A} = 2\hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{B} = 3\hat{i} + 6\hat{j} + m\hat{k}$

If $\vec{A}$ and $\vec{B}$ are perpendicular to each other

$\therefore \vec{A} \cdot \vec{B} = 0$

$\Rightarrow (2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} + 6\hat{j} + n\hat{k}) = 0$

$\Rightarrow 2 \times 3 + 2 \times 6 + 3n = 0$

$\Rightarrow 6 + 12 + 3n = 0 \Rightarrow 3n = -18$

$\Rightarrow n = -6$

⑥

Given $\vec{v} = 3\hat{i} + 4\hat{j}$

$|c\,u| = 705$

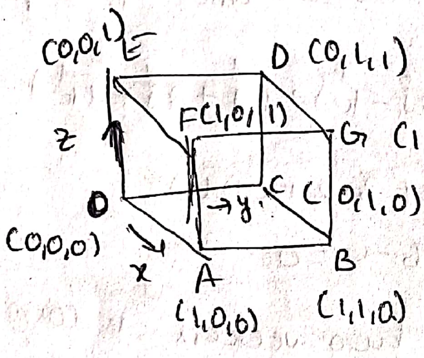
$\Rightarrow |c|\,|\vec{v}| = 705$

$\Rightarrow c\sqrt{3^2+4^2} = 705$

$\Rightarrow c \times 5 = 705$

$\Rightarrow c = 105.$

⑦



Diagonals AD, OG, CF, BE

$\vec{AD} = (0-1)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k}$

$= -\hat{i} + \hat{j} + \hat{k}$

$\vec{OG} = (1-0)\hat{i} + (1-0)\hat{j} + (1-0)\hat{k}$

$= \hat{i} + \hat{j} + \hat{k}$

$|\vec{AD}| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}$

$|\vec{OG}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\vec{OG} \cdot \vec{AD} = (\hat{i} + \hat{j} + \hat{k}) \cdot (-\hat{i} + \hat{j} + \hat{k})$

$= -1 + 1 + 1 = 1$

$$\cos\theta = \frac{\vec{OG} \cdot \vec{AD}}{|\vec{AD}|\,|\vec{OG}|} = \frac{1}{\sqrt{3} \times \sqrt{3}} = \frac{1}{3}$$

$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{3}\right)$ [Between two diagonals]

$\vec{OA}$ = side length $= (1-0)\hat{i} + (0-0)\hat{j} + (0-0)\hat{k} = \hat{i}$

$|\vec{OA}| = 1$

7th continuation

$$\vec{OA}\cdot\vec{OG} = \hat{i}\cdot(\hat{i}+\hat{j}+\hat{k})$$

$$= \hat{i}\cdot\hat{i} = 1$$

$$\therefore \cos\theta = \frac{\vec{OA}\cdot\vec{OG}}{|\vec{OA}|\,|\vec{OG}|} = \frac{1\cdot 1}{1\times\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is the angle made by the diagonals with unit length of cube.

(8)

Given $\vec{A} = 2\hat{i} + 3\hat{j}$

Angle made by the vector with y-axis

$$\tan\theta = \frac{x\text{-component}}{y\text{-component}} = \frac{2}{3}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2}{3}\right)$$

(10)

let $\vec{A} = 6\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} - \hat{k}$

$$\therefore \vec{A}\cdot\vec{B} = (6\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}-2\hat{j}-\hat{k})$$

$$= 6\times 2 + (3)(-2) + 2(-1) = 12-6-2 = 4.$$

$|\vec{A}| = \sqrt{6^2+3^2+2^2} = \sqrt{49} = 7$; $|\vec{B}| = \sqrt{2^2+(-2)^2+(-1)^2} = \sqrt{9} = 3$

$$\therefore \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|} = \frac{4}{7\times 3} = \frac{4}{21}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

(11)

(I) $\vec{P}\cdot\vec{Q} = PQ\cos\theta$ if $\theta = 0^\circ$

$= PQ\cos 0^\circ$

$= PQ.$

(II) $\vec{P}\cdot\vec{Q} = PQ\cos 30$

$= \frac{\sqrt{3}}{2} PQ \quad \underline{\theta = 30^\circ}$

statement II is wrong, I is correct

(12) , (13)

$F_1 = 5\hat{i} - 3\hat{j} + 2\hat{k}$; $r_1 = 2\hat{i} + 7\hat{j} + 4\hat{k}$

$r_2 = 5\hat{i} + 2\hat{j} + 8\hat{k}$

Displacement $= \vec{r_2} - \vec{r_1} = (5\hat{i} + 2\hat{j} + 8\hat{k}) - (2\hat{i} + 7\hat{j} + 4\hat{k})$

$= 3\hat{i} - 5\hat{j} + 4\hat{k}$

(13) work done $= \vec{F}\cdot\vec{S}$

$= (5\hat{i} - 3\hat{j} + 2\hat{k})\cdot(3\hat{i} - 5\hat{j} + 4\hat{k})$

$\Rightarrow 5\times 3 + (-3)(-5) + 2(4)$

$\Rightarrow 15 + 15 + 8$

$= 38\,J$

(4)

Given Scalar Product of $(3\hat{i}+4\hat{j}-7\hat{k})$ &

$(2\hat{i}+4\hat{j}+p\hat{k})$ is 8.

$\Rightarrow (3\hat{i}+4\hat{j}-7\hat{k})\cdot(2\hat{i}+4\hat{j}+p\hat{k}) = 8$

$\Rightarrow 3\times2+4\times4-7p=8$

$\Rightarrow 6+16-7p=8$

$\Rightarrow -7p=-14 \Rightarrow p=2.$

(15)

let $\vec{A} = \sqrt{3}\,\hat{i}+\hat{j}.$

Angle made by the vector with x-axis

$\tan\theta = \dfrac{y\text{-comp}}{x\text{-comp}}$

$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ = \dfrac{\pi}{6}.$

$\Rightarrow \dfrac{\pi}{k} = \dfrac{\pi}{6} \Rightarrow k=6$

(6)

let $\vec{A} = \hat{i}-\hat{j}-3\hat{k}$; $\vec{B} = -2\hat{i}-c\hat{j}+\hat{k}$

$\theta=90^\circ$ ie) $\vec{A}\cdot\vec{B}=0$

$\Rightarrow (\hat{i}-\hat{j}-3\hat{k})\cdot(-2\hat{i}-c\hat{j}+\hat{k})=0$

$\Rightarrow 1\times(-2)+(-1)(-c)+(-3)(1)=0$

$\Rightarrow -2+c-3=0$

$\Rightarrow c=5$

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