PERIODICITY

§§ INTRODUCTION:

We saw that each triginometric ratio is defined for all angles whether in radian or other major with the exception of certain quadrant angles. For any such anglee the value of each trigonometric ratio is completely determined and depends only on the angle. we shall define the trigonometric circular functions corresponding to the trigonometric ratios, so that both the domain and the range of the functions will be real numbers.

For any trigonometric function we assume that the domain A consists of real numbers θ for which the corresponding trigonometric ratio of θ radians in standard postion is defined for every $\theta \in$ **A**, we can determine a unique P(x,y) where P is the point of intersection of the terminal ray of θ with the unit circle with centre O.

Thus we can define a function $\theta \longrightarrow$ P(x,y) whose domain is A and whose range is set of all the points (x,y) such that $x^2 + y^2$=1 Unfortunately, it is somewhat inconvinient to deal with functions which associate real numbers with ordered pairs of real numbers. We shall simplify the situation by defining two functions, one which takes θ into x ($=\frac{x}{1}$) and another, which takes θ into y ($=\frac{y}{1}$).

APPLICATIONS IN REAL LIFE:

Example: Construction

In construction, we often need to consider a concept called Static Equilibrium, which is a concept that we use to make sure a stationary structure stays stationary and doesn't move. Torque is a massive consideration in Static Equilibrium, and it is directly related to the angle that a certain weight is acting on the structure.

Using angle measuring tools often introduces another source of error, so we typically use inverse trig functions to find out the angle and determine whether or not the structure will collapse.

Example: Laser Targeting

When we want to use lasers to target something, we need to know an angle at which to point the laser to hit the target with accuracy. If we use inverse trig functions to do so, we can get an exact answer. Additionally, we can use the method for finding the exact angle for programming, so the laser can auto target.

So how do we do this? We use data from different sources (ie, Radar) to find relative distances, which we then use to find an angle via inverse trigonometric functions

§§ **Periodic function:** A real function $f : A \to R$ is said to be a periodic function if there exists a positive real number *P* such that $f(x+p) = f(x),\ \forall\ x \in A$. Least value of *P* is known as the fundamental period of f.

Ø The period of $f(ax+b) = \frac{p}{|a|}$

Ø Let $g(x)=\frac{C_1 f_1(x)\pm C_2 f_2(x)}{C_3 f_3(x)\pm C_4 f_4(x)}$ (where f_1, f_2, f_3, f_4 are trigonometric functions, and C_1, C_2, C_3. C_4 are constants).

Then period of $g(x)$ = L.C.M. of period of f_1, f_2, f_3, f_4

Note: i) Numerator and Denominator can be extended to sum of any number of trignometric functions.

ii) If p and q are rational quantities.Then LCM of p and q exists

iii) If p and q are irrational, then LCM of p and q does not exist unless they have same irrational surd.

iv) LCM of rational and irrational is not possible.

v) L.C. M $\left\{\frac{a}{b},\frac{c}{d},\frac{e}{f}\right\}=\frac{L.C.M\{a,c,e\}}{G.C.D\{b,d,f\}}$

Ø If 'f' is a periodic function with fundamental period '*P*' then '-f' and $\frac{1}{f}$ are also a periodic functions with period *P* .

Ø A constant function is periodic but does not have a fundamental period.

Ø Periods of trignometric functions are given below:

	Function	**Period**
1.	$Sin(ax+b)$	$\frac{2\pi}{\lvert a\rvert}$
2.	$Cos(ax+b)$	$\frac{2\pi}{\lvert a\rvert}$
3.	$Tan(ax+b)$	$\frac{\pi}{\lvert a\rvert}$
4.	$Cosec(ax+b)$	$\frac{2\pi}{\lvert a\rvert}$
5.	$Sec(ax+b)$	$\frac{2\pi}{\lvert a\rvert}$
6.	$Cot(ax+b)$	$\frac{\pi}{\lvert a\rvert}$

Ø Period of $\{x\}$ is 1. where $\{.\}$ denotes the fractional part.

Ø Standard results :

i) Period of $|\sin x|, |\cos x|, |\tan x|, |\cot x|, |\sec x|, |\operatorname{cosec} x|$ is π

ii)Period of $|\sin x|+|\cos x|, |\tan x|+|\cot x|$ is $\frac{\pi}{2}$.

iii) Period of $|\sin x+\cos x|, |\sin x-\cos x|$ is π

iv) Period of $|\tan x+\cot x|, |\tan x-\cot x|$ is $\frac{\pi}{2}$

v) If $n \in z^+$, the period of $\sin^{2n} x + \cos^{2n} x$, $\sec^{2n} x + \cos ec^{2n} x, \tan^{2n} x + \cot^{2n} x$ is $\frac{\pi}{2}$

vi) Period of $a\sin^{2n} x + b\cos^{2m} x$, $a\sec^{2n} x + b\cos ec^{2m} x$, $a\tan^{2n} x + b\cot^{2m} x$ is π $\left(n, m \in z^+\right)$

Ø **Maximum and minimum values :**

i) Minimum value of $\sin x$ is -1

ii) Maximum value of $\sin x$ is 1

iii) Range of $\sin x$ is $[-1,1]$

Ø i) Minimum value of $\cos x$ is -1
ii) Maximum value of $\cos x$ = 1

iii) Range of $\cos x$ is $[-1,1]$

Ø Minimum value of $a\cos x + b\sin x + c$ *is* $c - \sqrt{a^2 + b^2}$

Ø Maximum value of $a\cos x + b\sin x + c$ is $c + \sqrt{a^2 + b^2}$

Ø Range of $a\cos x + b\sin x + c$ is $\left[c - \sqrt{a^2 + b^2}, c + \sqrt{a^2 + b^2}\right]$

Ø The minimum value of $a^2 \sin^2 \theta + b^2 \cos ec^2 \theta$, $a^2 \cos^2 \theta + b^2 \sec^2 \theta$, $a^2 \tan\theta + b^2 \cot\theta$ is 2ab.

TEACHING TASK

MCQ's with single correct answer.

1. All trigonometrical functions are
A) non-periodic B) need not be periodic function C) even function D) odd function

2. The period of $\sin\left(\frac{\pi x}{2}\right) + 2\cos\left(\frac{\pi x}{3}\right) - \tan\left(\frac{\pi x}{4}\right)$ is

A) 4 B) 6 C) 12 D) 24

3. The period of tan (4*x*) + sec (4*x*) is

A) 2π B) $\frac{3\pi}{2}$ C) π D) $\frac{\pi}{2}$

4. The period of $\cot\left(\frac{3 - 5x}{7}\right)$ is

A) $-\frac{7\pi}{5}$ B) $\frac{5\pi}{7}$ C) $\frac{7\pi}{5}$ D) $\frac{14\pi}{5}$

5. The period of 5 + 3 $\cos^2 x$ is

A) 2π B) π C) $\frac{\pi}{2}$ D) $\frac{2\pi}{3}$

6. The period of sec (*x* + 2*x* + 4*x*) is

A) 2π B) 14π C) $\frac{2\pi}{7}$ D) $\frac{\pi}{7}$

7. The period of tan (*x* + 8*x* + 27*x* + …+$n^3 x$) is

A) $\frac{8\pi}{n^2(n+1)^2}$ B) $\frac{4\pi}{n^2(n+1)^2}$ C) $\frac{2\pi}{n^2(n+1)^2}$ D) $\frac{4\pi}{n(n+1)}$

8. The period of $\sin\left(\frac{\pi}{3}-x\right)\sin\left(\frac{\pi}{3}+x\right)$ is

A) $\frac{\pi}{2}$ B) π C) $\frac{3\pi}{2}$ D) 2π

9. The period of $\frac{\cot\frac{x}{4}+\tan\frac{x}{4}}{1+\tan\frac{x}{2}-\tan x}$ is

A) $\frac{\pi}{2}$ B) π C) 4π D) 2π

10. The period of $3\sin^6x+5\cos^4x$ is

A) π B) $\frac{\pi}{2}$ C) $\frac{3\pi}{2}$ D) 2π

11. The period of $\sin^6x+\cos^6x$ is

A) $\frac{3\pi}{2}$ B) $\frac{\pi}{2}$ C) π D) 2π

12. The period of $3\sin^5x+\cos^3x$ is

A) π B) 2π C) $\frac{\pi}{2}$ D) $\frac{3\pi}{2}$

13. The function whose period is 5π is

A) cosec (2x) B) sec (5x) C) $\tan\left(\frac{5x}{2}\right)$ D) $\cot\left(\frac{x}{5}\right)$

14. The sine function whose period 3 is

A) $\sin\left(\frac{2\pi}{3}x\right)$ B) $\sin\left(-\frac{2\pi}{3}x\right)$ C) $\sin\left(\pm\frac{2\pi}{3}x\right)$ D) $\sin\left(\frac{x}{3}\right)$

15. The minimum and maximum values of $\cos\theta+2\sqrt{2}\sin\theta$ is

A) -3, 3 B) 3, -3 C) [-3, 3] D) [0,3]

16. The range of $13\cos x+3\sqrt{3}\sin x-4$ is

A) [-18, 10] B) [10,18] C) (-18,10) D) -18, 10

17. The maximum value of $3\cos\theta+4\sin\theta$ is

A) 3 B) 4 C) 5 D) $\sqrt{5}$

18. The range of $f(x)=-3\cos\sqrt{3+x+x^2}$ is

A) [-1, 1] B) [-2, 2] C) [-3, 3] D) [-4, 4]

19. The range of $\cos\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)$ is

A) $\left[-\frac{1}{4},\frac{1}{4}\right]$ B) $\left[-\frac{3}{4},\frac{3}{4}\right]$ C) $\left[-\frac{1}{4},\frac{3}{4}\right]$ D) $\left[-\frac{1}{2},\frac{1}{2}\right]$

20. The greatest (or) least value of $2\sin^2\theta+4\operatorname{cosec}^2\theta$ is

A) $2\sqrt{2}$ B) $4\sqrt{2}$ C) 2 D) 4

21. The period of $\sin^3x+\cos^3x$ is

A) $\frac{\pi}{2}$ B) π C) 2π D) $\frac{3\pi}{2}$

22. The period of the function $f(\theta)=\sin\frac{\theta}{3}+\cos\frac{\theta}{2}$ is

A) 3π B) 6π C) 9π D) 12π

23. If $f(x)=\sin^2\left(\frac{\pi}{8}+\frac{x}{2}\right)-\sin^2\left(\frac{\pi}{8}-\frac{x}{2}\right)$, then the period of f is

A) π B) $\frac{\pi}{2}$ C) $\frac{\pi}{3}$ D) 2π

24. For $x \in R$, $3\cos(4x-5)+4$ lies in the interval

A) [1, 7] B) [4, 7] C) [0,7] D) [2, 7]

25. Statement I: Period of $\cos^3 x+\cos^3(120^0-x)+\cos^3(120^0+x)$ is $\frac{\pi}{3}$

Statement II: Period of $2\cos\frac{\pi x}{3}+\sin\frac{\pi x}{2}-\tan\frac{\pi x}{4}$ is 12

Which of the above statements is correct?

A) only I B) only II C) both I & II D) neither I nor II

26. Arrange the periods for the following in the ascending order

A) $\tan(2x-7)$ B) $\sin x \cos x$ C) $\sin 3x+\cos 3x$ D) $\sin^3 x-\cos^3 x$

A) A, C, B, D B) A, B, C, D C) C, A, D, B D) B, D, A, C

MCQ's with multi correct answer.

♦ *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D),out of which* ***ONE or MORE*** *is correct. Choose the correct options*

1. The equation $\sin^6 x+\cos^6 x=a^2$ has real solution, if

A) $a \in [-1,1]$ B) $a \in \left[-1,-\frac{1}{2}\right]$ C) $a \in \left[-\frac{1}{2},\frac{1}{2}\right]$ D) $a \in \left[\frac{1}{2},1\right]$

2. In a ΔABC

A) $\sin A \sin B \sin C \le \frac{3\sqrt{3}}{8}$ B) $\sin^2 A+\sin^2 B+\sin^2 C \le \frac{9}{4}$

C) $\sin A \sin B \sin C$ is always positive D) $\sin^2 A+\sin^2 B \le 1+\cos C$

3. If $a=\frac{1}{5\cos x+12\sin x}$ then for all real x

A) the least positive value of a is $\frac{1}{13}$ B) the greatest negative value of a is $-\frac{1}{13}$

C) $a \le \frac{1}{13}$ D) $-\frac{1}{13} \le a \le \frac{1}{13}$

4. Let $y=\sin^2 x+\cos^4 x$. Then for all real x

A) the minimum value of y is 2 B) the minimum value of y is $\frac{3}{4}$ C) $y \le 1$ D) $y \ge \frac{1}{4}$

5. Let $y=\sin x.\sin(60^0+x).\sin(60^0-x)$. Then for all real x

A) the minimum value of y is $-\frac{1}{4}$ B) the maximum value of y is 1 C) $y \le \frac{1}{4}$ D) $y \ge -1$

6. If $7\cos x-24\sin x=\lambda\cos(x+\alpha),\ 0<\alpha<\frac{\pi}{2}$ be true for all $x \in R$ then

A) $\lambda=25$ B) $\alpha=\sin^{-1}\frac{24}{25}$ C) $\lambda=-25$ D) $\alpha=\cos^{-1}\frac{7}{25}$

7. If $A \geq 0, B \geq 0, A+B=\frac{\pi}{3}$ and y = tan A. tanB then

A) the maximum value of y is 3 B) the minimum value of y is $\frac{1}{3}$

C) the maximum value of y is $\frac{1}{3}$ D) the minimum value of y is 0

8. If then $a \leq 5\ \cos\theta + 3\cos(\theta + \pi/3) + 5 \leq b$,

A) a = -2 B) a = 2 C) b = 12 D) b = 7

Match The following

◆ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, shes) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

Column - I	Column - II
A) If maximum and minimum values of $\frac{7+6\tan\theta-\tan^2\theta}{(1+\tan^2\theta)}$ for all real values of $\theta \sim \frac{\pi}{2}$ are λ and μ respectively, then	p) $\lambda+\mu=2$
B) If maximum and minimum values of $5\cos\theta+3\cos\left(\theta+\frac{\pi}{3}\right)+3$ for all real values of θ are λ and μ respectively, then	q) $\lambda-\mu=6$
C maximum and minimum values of $1+\sin\left(\frac{\pi}{4}+\theta\right)+2\cos\left(\frac{\pi}{4}-\theta\right)$ for all real values of θ are λ and μ respectively, then	r) $\lambda+\mu=6$
	s) $\lambda-\mu=10$
	t) $\lambda-\mu=14$

A) a-rs ,b-rt ,c-pq B) a-rt ,b-rs ,c-pq C) a-rs ,b-pq ,c-rt D) a-rp ,b-rs ,c-pq

KEY

ΦΦ TEACHING TASK :

I :	**1. B**	**2.C**	**3. D**	**4. C**	**5. B**	**6.C**	**7. B**	**8. B**	**9. C**	**10. A**	**11. B**	**12. B**
	13. D	**14. C**	**15. A**	**16. A**	**17. C**	**18. C**	**19. C**	**20.B**	**21. C**	**22. D**	**23. D**	**24. A**
	25. B	**26. A**										

II : **1. B,D** **2. A,B,C,D** **3. A,B** **4. B,C** **5. A,C** **6. A,B,D**
7. C,D **8. A, C**

III : **A**

LEARNER'S TASK

BEGINNERS

MCQ's with single correct answer.

1. If $f:R\to R$ is defind by $f(x)=7+\cos(5x+3),\forall x\in R$. Then period of f is

A) 2π B) π C) $\frac{\pi}{5}$ D) $\frac{2\pi}{5}$

2. Period of *cos (x + 2x + 3x + ...+ nx)* is

A) $\frac{2\pi}{n(n+1)}$ B) $\frac{4\pi}{n(n+1)}$ C) $\frac{\pi}{n(n+1)}$ D) $\frac{6\pi}{n(n+1)}$

3. Period of *tan (x + 4x + 9x+...+ n^2x)* is

A) $\frac{2\pi}{n(n+1)(2n+1)}$ B) $\frac{4\pi}{n(n+1)(2n+1)}$ C) $\frac{6\pi}{n(n+1)(2n+1)}$ D) $\frac{8\pi}{n(n+1)(2n+1)}$

4. Period of $cos\left(x+\frac{x}{3}+\frac{x}{3^2}+.....\right)$ is

A) $\frac{\pi}{3}$ B) $\frac{2\pi}{3}$ C) π D) $\frac{4\pi}{3}$

5. Period of *2 $\sin^3$x - 3 $\cos^3$ x* is

A) $\frac{\pi}{2}$ B) π C) 2π D) 4π

6. Period of *$\cos^6$ x + $\sin^6$ x* is

A) $\frac{\pi}{2}$ B) π C) $\frac{3\pi}{2}$ D) 2π

7. Period of $\sin x\sin(120^0-x)\sin(120^0+x)$ is

A) $\frac{\pi}{3}$ B) $\frac{2\pi}{3}$ C) π D) $\frac{\pi}{2}$

8. If $f(x)=sin^2\left(\frac{\pi}{8}+\frac{x}{2}\right)-sin^2\left(\frac{\pi}{8}-\frac{x}{2}\right)$, then the period of f is

A) π B) $\frac{\pi}{2}$ C) $\frac{\pi}{3}$ D) 2π

9. Sine function whose period is 6 is

A) $sin\frac{2\pi x}{3}$ B) $sin\frac{\pi x}{3}$ C) $sin\frac{\pi x}{6}$ D) $sin\frac{3\pi x}{2}$

10. The cosecant function whose period is 4 is

A) $cosec\frac{\pi x}{3}$ B) $cosec\frac{2\pi x}{5}$ C) $\cos ec2\pi x$ D) $cosec\frac{\pi x}{2}$

11. Period of $\frac{2\sin 2x - 5\cos 2x}{7\cos x - 8\sin x}$ is

A) π B) 2π C. $\frac{\pi}{2}$ D. $\frac{\pi}{3}$

12. Period of $\frac{\sin(x+a)}{\cos x}$

A) $\frac{\pi}{2}$ B) π C) 2π D) 3π

13. Period of $\sin\left(e^{\tan x}+e^{\cot x}\right)$ is

A) $\frac{\pi}{2}$ B) π C) $\frac{3\pi}{2}$ D) 2π

Extreme Values:

14. The range of $f(x) = -3\cos\sqrt{3+x+x^2}$

A) [-1,1] B) [-2,2] C) [-3,3] D) [-4,4]

15. The minimum and maximum values of $8\cos 3x - 15\sin 3x$ are

A) -7,7 B) -23,23 C) -17,17 D) -15,8

16. The maximum value of $\frac{3}{5\sin x - 12\cos x + 19}$

A) 1 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$

17. The range of $\sin^6 x + \cos^6 x$

A) $\left[-\frac{1}{4},\frac{1}{4}\right]$ B) $\left[\frac{1}{4},\frac{1}{2}\right]$ C) $\left[\frac{1}{4},1\right]$ D) $\left[-\frac{1}{2},-\frac{1}{4}\right]$

18. The minimum and maximum values of $\sin^2(60^0 - x) + \sin^2(60^0 + x)$ are

A) $-\frac{1}{2},\frac{1}{2}$ B) $\frac{1}{2},1$ C) $\frac{1}{2},\frac{3}{2}$ D) $\frac{3}{2},2$

19. $\sin^2(60^0+x) - \sin^2(60^0-x) \in \left[-\frac{k}{2},\frac{k}{2}\right] \Rightarrow$ k =

A) 3 B) 2 C) $\sqrt{3}$ D) $\sqrt{2}$

20. $\sin\theta\sin(60^0-\theta)\sin(60^0+\theta) \in [-k,k] \Rightarrow$ k =

A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{3}{4}$

21. $16\cos^5 x - 20\cos^3 x + 5\cos x \in$

A) [-1,1] B) $\left[-\frac{1}{4},\frac{1}{4}\right]$ C) $\left[-\frac{3}{4},\frac{3}{4}\right]$ D) [–2, 2]

22. $3\sin^2 x - 4\cos^2 x \in$

A) [0,3] B) [-4,0] C) [-3,4] D) [-4, 3]

23. The maximum value of $\cos x\left(\frac{\cos x}{1-\sin x} + \frac{1-\sin x}{\cos x}\right)$ is

A) 4 B) 3 C) 2 D) 1

24. A + B = $90^0 \Rightarrow$ cos A cos B $\in$

A) $\left[-\frac{1}{3},\frac{1}{3}\right]$ B) $\left[-\frac{1}{2},\frac{1}{2}\right]$ C) $\left[-\frac{1}{4},\frac{1}{4}\right]$ D) $\left[-\frac{3}{4},\frac{3}{4}\right]$

25. A + B = $90^0 \Rightarrow$ sin A + sin B $\in$

A) [-1,1] B) $\left[-\sqrt{2},\sqrt{2}\right]$ C) $\left[-\sqrt{3},\sqrt{3}\right]$ D) $[-2,2]$

26. Minimum value of $9\sec^2\theta + 4\operatorname{cosec}^2\theta + 7$ is

A) 32 B) 25 C) 20 D) 49

RESEARCHERS

1. The period of the function tan(3x+5) is .... **[EAMCET 2011]**

A) $2\pi/3$ B) $\pi/6$ C) $\pi/3$ D) π

2. The period of $\sin^2 x$ is .... **[AIEEE 2013]**

A) π B) 2π C) $\pi/2$ D) 3π

3. The period of the function $f(\theta) = \sin(\theta/3)+\cos(\theta/2)$ is ... **[EAMCET 2013]**

A) 3π B) 6π C) 9π D) 12π

4. If $f(x) = \sin^2(\frac{\pi}{8}+\frac{x}{2}) - \sin^2(\frac{\pi}{8}-\frac{x}{2})$, thwn the period of f is .. **[EAMCET 2009]**

A) π B) $\pi/2$ C) $\pi/3$ D) 2π

5. The period of $\sin^4 x + \cos^4 x$ is .... **[AIEEE 2008]**

A) 2π B) π C) $\pi/2$ D) $\pi/4$

6. The period of $(\tan\theta - \frac{1}{3}\tan^3\theta)(\frac{1}{3} - \tan^2\theta)^{-1}$, where $\tan^2\theta \neq \frac{1}{3}$ is .... **[EAMCET 2010]**

A) $\pi/3$ B) $2\pi/3$ C) π D) 2π

7. If $n \in N$, and the period of $\frac{\cos nx}{\sin(x/n)}$ is 4π then n = ... **[EAMCET 2004]**

A) 4 B) 3 C) 2 D) 1

8. If 5cosx + 12cosy = 13, then the maximum value of 5sinx + 12siny is ... **[EAMCET 2006]**

A) 12 B) $\sqrt{120}$ C) $\sqrt{20}$ D) 13

KEY

ΦΦ LEARNER'STASK :

☐ **BEGINNERS :**

1. D	2. B	3. C	4.D	5.C	6.A	7.B	8.D	9.B
10. D	11. B	12.B	13.B	14.C	15.C	16.B	17.C	18.C
19.C	20.C	21.A	22.D	23.C	24.B	25.B	26.A	

☐ **RESEARCHERS :**

1. C	2. A	3. D	4. D	5. C	6. A	7. C	8. A