WS-13 - 7th foundation + Energy

Tbasic

(1)

Given $d_{Liquid} = 0.8$ kg per cc ; speed $= 2\,m/s$

we know that $k.E = \frac{1}{2}mv^2$ where $m = mass = d \times vol$

$K.E = \frac{1}{2} \times d \times vol \times v^2$

$\Rightarrow \frac{K.E}{Vol} = \frac{1}{\cancel{2}} 0.8 \times 2^{\cancel{2}} = 0.8 \times 2 = 1.6\,J \rightarrow D$

(2)

we know that $k.E = \frac{1}{2}mv^2$

$m = d \times vol$

$= d \times \frac{4\pi}{3} r^3$

$= \frac{1}{2} d \times vol \times v^2$

$= \frac{1}{2} d \times \frac{4\pi}{3} \times r^3 \times v^2$

$\Rightarrow k.E \propto r^3 d v^2$

$k_1 \propto r^3 d v^2$; $k_2 \propto d \left(\frac{r}{2}\right)^3 \left(\frac{v}{2}\right)^2$; $k_3 \propto \frac{d}{2} (2r)^3 v^2$

$\Rightarrow k_2 \propto \frac{d\, r^3 v^2}{32}$ $\Rightarrow k_3 \propto 4 d r^3 v^2$

In descending order k_3, k_1, k_2

(3)

we know $W = \Delta k.E$; Here body starts from rest (ie) $u = 0$

$\Rightarrow FS = \frac{1}{2}m(v^2 - u^2)$

$\Rightarrow FS = \frac{1}{2}mv^2 \Rightarrow S \propto \frac{v^2}{F}$

$S_1 \propto \frac{v^2}{F}$; $S_2 \propto \frac{(2v)^2}{\frac{F}{2}}$; $S_3 \propto \frac{\left(\frac{v}{2}\right)^2}{2F}$

$\Rightarrow S_2 \propto 8\frac{v^2}{F}$ $S_3 \propto \frac{v^2}{8F}$

In ascending order S_2, S_1, S_3

(4)

Given mass $m' = \frac{m}{2}$; velocity $v' = 2v$.

From $k.E = \frac{1}{2} m v^2$

$k.E' = \frac{1}{2} m' v'^2 \Rightarrow \frac{1}{2} [\frac{m}{2}][2v]^2$

$k.E' = \frac{1}{2} \frac{m}{\cancel{2}} \times \cancel{4}^{2} v^2 = 2 \frac{1}{2} m v^2$

$k.E' = 2 \, kE = 200\% \, k.E = [100 + 100]\% \, k.E$

There is 100% increase in k.E

(5)

Given $m_1 = 1gm$; $m_2 = 4gm$.

we know $k.E = \frac{p^2}{2m} \Rightarrow k.E \propto \frac{1}{m}$

$\Rightarrow \frac{k.E_1}{k.E_2} = \frac{m_2}{m_1} = \frac{4}{1}$

(6)

Given $m = 2 \, kg$ starts from rest means

initial speed $u = 0$

acceleration $a = 2 \, m/s^2$; $t = 2$ sec.

Gain in $k.E = \frac{1}{2} m (v^2 - u^2)$; From $v = u + at$

$= \frac{1}{\cancel{2}} \times \cancel{2} [4^2 - 0^2]$ $\quad v = 0 + 2 \times 2$

$= 16 - 0$ $\quad v = 4 \, m/s$

$= 16 \, J$.

(7)

Given $m = 0.5\ kg$; speed $u = 2\ m/s$; $V = 0$; $S = 2m$

work done $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 = \Delta K.E$

$= \frac{1}{2} \times 0.5 \times (2)^2 - \frac{1}{2}(0.5)(0)^2$

$= \frac{1}{2} \times \frac{1}{2} \times 4 = 1J.$

(8)

let $m_1 = 3\ kg$; $m_2 = 2\ kg$

$K.E_1 + K.E_2 = 600J$

According to LCLM $\Rightarrow P_1 = -P_2$

$m_1 V_1 = -m_2 V_2$

$\Rightarrow V_2 = \frac{-m_1 V_1}{m_2} = -\frac{3}{2} V_1$

$\Rightarrow \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 V_2^2 = 600$

$\Rightarrow \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 \left[\frac{-m_1 V_1}{m_2}\right]^2 = 600$

$\Rightarrow \frac{1}{2}m_1 V_1^2 + \frac{1}{2}\cancel{m_2} \frac{m_1^2 V_1^2}{m_2^{\cancel{2}}} = 600$

$\Rightarrow \frac{1}{2}m_1 V_1^2 \left[1 + \frac{m_1}{m_2}\right] = 600 \Rightarrow K.E_1 \left[1 + \frac{3}{2}\right] = 600$

$\Rightarrow K.E_1 \left[\frac{5}{2}\right] = 600 \Rightarrow K.E_1 = 240J$

(9)

Given $m = 200\ gm = 200 \times 10^{-3}\ kg$; $l = 1m$; $\theta = 60°$

$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\sqrt{2gl(1-\cos\theta)}\right)^2$

$= \frac{1}{2} \times 200 \times 10^{-3} (2 \times 10 \times 1 (1 - \cos 60))$

$\Rightarrow 2\left(1 - \frac{1}{2}\right)$

$= 2\left(\frac{1}{2}\right)$

$= 1J$

(10)

According to law of conservation of linear momentum

$P_{gun} = - P_{bullet}$ $m_{bullet} = m$

$\Rightarrow M_{gun} V_{gun} = - m_{bullet} u_{bullet}$ $M_{gun} = M$

$\Rightarrow V_{gun} = \frac{-mu}{M}$ $u_{bullet} = u$

$\therefore \frac{K.E_{Bullet}}{K.E_{gun}} = \frac{\frac{1}{2} m u^2}{\frac{1}{2} M V_{gun}^2} = \frac{m u^2}{M \left[-\frac{mu}{M}\right]^2} = \frac{m u^2}{M \frac{m^2 u^2}{M^2}}$

$= \frac{M}{m}.$

(14) (15)

we know that $P = mv$; $K.E = \frac{P^2}{2m}$

when $P =$ constant $K.E \propto \frac{1}{m}$.

so lighter body has large K.E

(17)

Given $m = 12$ kg ; S = displacement = h = 0.30 m.

Maximum height = 1.5 m.

At highest point $P.E = mgh = 12 \times 10 \times 1.5 = 180 J$

At maximum height velocity = 0

(18)

when momentum doubled i.e. $P' = 2P \rightarrow$ 4t's $K.E = K.E'$

we know $K.E \propto \frac{P^2}{2m}$ $\Rightarrow K.E' = 4 K.E$

$\frac{K.E'}{K.E} = \left[\frac{P'}{P}\right]^2 = \left(\frac{2P}{P}\right)^2 = 4$ $= 400\%$ K.E

$= [100 + 300]\%$ K.E

There is 300% increase in K.E

(17)

Given mass $m = 1.5\,kg$ | speed $= 3\hat{i} + 4\hat{j} + 5\hat{k}$ m/s

$|velocity| = \sqrt{3^2+4^2+5^2} = \sqrt{9+16+25} = \sqrt{50}$

$K.E = \frac{1}{2} m v^2 = \frac{1}{2} \times 1.5 \times (\sqrt{50})^2 = \frac{1.5 \times 50}{2} = \frac{75}{2}$

$= 37.5\,J$

LTask

CUQ's

(1)

we know $K.E = \frac{p^2}{2m}$ when momentum is doubled

ie $p' = 2p$

$K.E \propto p^2$

$\Rightarrow \frac{K.E'}{K.E} = \left(\frac{p'}{p}\right)^2 = \left(\frac{2p}{p}\right)^2 = 2^2 = 4$

$K.E' = 4\,K.E$

(2), (3)

let m_1, m_2 are masses of two bodies

Given $K.E_1 = \cancel{K.E_2}$ $P_1 = P_2$

$\Rightarrow \frac{\cancel{P_1^2}}{2m_1} \neq \frac{\cancel{P_2^2}}{2\cancel{m_2}}.$ $K.E \propto \frac{P^2}{2m}$

when $P = $ constant $K.E \propto \frac{1}{m}$

so lighter body have more K.E.

$\frac{K.E_1}{K.E_2} = \frac{m_2}{m_1}$

when K.E is same $P^2 \propto m \Rightarrow P \propto \sqrt{2m}$

Heavier body has more momentum.

(4)

Given $P = K.E$

$\Rightarrow P = \frac{P^{\cancel{2}}}{2m}$

$\Rightarrow P = 2m \Rightarrow \cancel{m}V = 2\cancel{m}$

$\Rightarrow V = 2\ m/s$

(5)

Given $\frac{d_1}{d_2} = \frac{1}{2}$; $\frac{r_1}{r_2} = \frac{1}{2}$; $\frac{V_1}{V_2} = \frac{2}{1}$

we know $K.E = \frac{1}{2}mv^2 = \frac{1}{2} d \times vol \times v^2$

$= \frac{1}{2} d \times \frac{4\pi}{3} r^3 \times v^2$

$K.E \propto d r^3 v^2$

$\Rightarrow \frac{K_1}{K_2} = \frac{d_1}{d_2} \left[\frac{r_1}{r_2}\right]^3 \left[\frac{V_1}{V_2}\right]^2$

$= \frac{1}{2}\left[\frac{1}{2}\right]^3\left[\frac{2}{1}\right]^2 = \frac{1}{4}$

(6)

we know $K.E = \frac{P^2}{2m}$

when $P =$ constant $K.E \propto \frac{1}{m}$

So $m_\alpha = 4m_p$ and m_e is very less as compared with α, deutron and proton.

For $K.E =$ constant $P^2 \propto m$

$P \propto \sqrt{m}$ so α-Particle has more momentum

(7)

We know Total energy $E = K.E + P.E$

$\Rightarrow E - P.E = K.E \Rightarrow E - U = \frac{1}{2}mv^2$ always positive

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(8)

let m_1, m_2 are masses of two bodies and their

K.E are k_1, k_2

$$k = \frac{p^2}{2m} \quad \text{when } p = \text{constant}$$

$$K.E \propto \frac{1}{m}$$ $\therefore$ lighter body has more K.E

(9)

We know momentum $p = mv$

$$K.E = \frac{1}{2} m v^2 = \frac{1}{2} \times (mv) v$$

$$\Rightarrow K.E = \frac{1}{2} \times pv$$

$$\Rightarrow 2 K.E = pv$$

(10)

Consider ground as zero Potential.

Initially $E = U + k = U + 0 = U$.

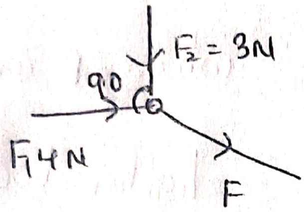
Finally $E = U + k = 0 + 0 = 0$.

so initially it had some potential energy, it was converted into kinetic energy by gaining speed it finally became zero so energy must have been lost as heat while applying breakes

Jee main's level

(1)

Given mass $m = 10\ kg$; initial speed $u = 0$; $t = 10 sec$



Resultant force $F = \sqrt{F_1^2 + F_2^2}$

$F = \sqrt{3^2+4^2} = \sqrt{9+16} = 5N$

$\therefore$ Acceleration of the body $a = \frac{F}{m} = \frac{5}{10} = \frac{1}{2}\ m/s^2$

From $v = u + at$ velocity at the end of 10 sec

is $v = 0 + (\frac{1}{2})(10) = 5\ m/s$

$\therefore\ \omega = \Delta k.E = \frac{1}{2}m(v^2 - u^2) = \frac{1}{2} \times 10 (5)^2 = 125J$

(2)

we know that $k.E = \frac{p^2}{2m}$

$k.E \propto p^2$

$\Rightarrow \frac{\Delta k.E}{kE} \times 100 = 2\frac{\Delta p}{p} = 2 \times 0.01 = 0.02\%$

(3)

Given momentum of the body increased by 100%

$p' = (\frac{100+100}{100})P = \frac{200}{100}P\% = 2P$

$k.E \propto p^2$

$\Rightarrow \frac{k'}{k} = [\frac{p'}{p}]^2 = [\frac{\frac{101}{100}p}{p}]^2$

$\frac{k'}{k} = [\frac{2P}{P}]^2$

$\Rightarrow k' = 4k$

$\Rightarrow k' = 400\%\ k$

$= [100 + 300]\%\ k$

$= 300\%$ is increase in k.E

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(4)

Given mass $m = 40$ kg, Energy $= 21$ kJ $= 21 \times 10^3$ J

efficiency $\eta = 28\%$

$$\Rightarrow \frac{E_{out}}{E_{in}} = 28\%$$

$$\Rightarrow E_{out} = 28\% \, E_{in} = \frac{28}{\cancel{100}} \times 21 \times 10^{\cancel{3}}$$

$$\Rightarrow mgh = 21 \times 28 \times 10$$

$$\Rightarrow \cancel{4}0 \times \cancel{10} \times h = 21 \times \overset{7}{\cancel{28}}\cancel{0}$$

$$h = \frac{21 \times 7}{10} = 14.7\,m \simeq 15\,m$$

(5)

Given force $= 3\hat{i} + 4\hat{j}$ N

$\vec{s} = 3\hat{i} + 4\hat{j}$ m

work done $= \vec{F} \cdot \vec{s} = (3\hat{i} + 4\hat{j}) \cdot (3\hat{i} + 4\hat{j})$

$= 3 \cdot 3 + 4 \cdot 4$

$= 9 + 16 = 25$ J

(6)

height from which water falling $\doteq 19.6$ m

loss of P.E = Gain in K.E of water.

$$\Rightarrow \cancel{m}gh = \frac{1}{2}\cancel{m}v^2$$

$$\Rightarrow \frac{v^2}{2} = gh \Rightarrow v^2 = 2gh$$

$$\Rightarrow v^2 = 2 \times 9.8 \times 19.6 = 19.6 \times 19.6$$

$$\Rightarrow v^{\cancel{2}} = (19.6)^{\cancel{2}} \Rightarrow v = 19.6\,m/s$$

(7)

% loss of K.E $= \frac{h_1 - h_2}{h_1}$ Given $h_1 = 12.4$

$\Rightarrow$ From $v^2 = u^2 - 2gh$

$\Rightarrow v^2 = u^2 + 2(9.8)(12.4) = u^2 + 243.04.$

K.E of the ball when it just hits the wall $= \frac{1}{2}mv^2$

$= \frac{1}{2}m(u^2 + 243.04]$

K.E of the ball after the impact $= \frac{(100-15)}{100} \times \frac{1}{2}m(u^2 + 243.04$

$\Rightarrow \frac{85}{100} \times \frac{1}{2}m(u^2 + 243.04]$

let v_2 be the velocity just after collision with the floor

So $\frac{1}{2}mv_2^2 = \frac{85}{100} \times \frac{1}{2}m(u^2 + 243.04)$

For upward motion $v = 0$; $u = v_2$; $v^2 = u^2 - 2gh$

$\Rightarrow \frac{85}{100}(u^2 + 243.04) - 2(9.8 \times 12.04)$

$\Rightarrow \frac{85}{100}u^2 = 36.46 \rightarrow u^2 = 42.89$

$\Rightarrow u = 6.55$ m/s

(8)

Given $h = 10m$

K.E = P.E

$\Rightarrow \frac{1}{2}\cancel{m}v^2 = \cancel{m}gh$

$\Rightarrow v^2 = 2gh$

$\Rightarrow v = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = 10\sqrt{2}$

$\Rightarrow v = 14.14$ m/s $\simeq 14$ m/s

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9

For a vertically projected body $T.E = P.E + K.E = \frac{1}{2}mu^2$

Maximum height reached $h = \frac{u^2}{2g} \rightarrow u^2 = 2gh$

$\therefore T.E = \frac{1}{2}m(2gh) = mgh$

at a height $\frac{3h}{4}$ $P.E = mg(\frac{3h}{4}) = \frac{3}{4}mgh$

$K.E = T.E - P.E = mgh - \frac{3mgh}{4}$

$\Rightarrow K.E = \frac{1}{4}mgh$

$\therefore \frac{K.E}{P.E} = \frac{\frac{1}{4}mgh}{\frac{3}{4}mgh} = \frac{1}{3}$

10

Given $\frac{P.E}{K.E} = \frac{5}{2}$ height $h = 400m$.

$\Rightarrow \frac{mgh}{\frac{1}{2}mv^2} = \frac{5}{2}$

$\Rightarrow \frac{2gh}{v^2} = \frac{5}{2} \Rightarrow v^2 = \frac{2}{5} \times 2gh = \frac{4}{5} \times 10 \times 400$

$\Rightarrow v^2 = \sqrt{3200} = 40\sqrt{2} \simeq 56\ m/s$

Advanced level

9

Given $m = 5kg$; $P = 10\ kg\,m/s$; Force $F = 0.2N$; $t = 10sec$

we know $F = \frac{dp}{dt} \Rightarrow$ change in momentum $dp = F\,dt$

$dp = 0.2 \times 10 = 2\ kg$

$P_F - P_I = 2 \Rightarrow P_F = 2 + P_I = 2 + 10$
$= 12\ kg\,m/s$

From $v = u + at$

$v = 0 + \frac{F}{m} \times 10 \Rightarrow v = \frac{0.2}{5} \times 10 = 0.4\ m/s$

9th continuation

$k.E_F = \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times (0.4)^2$

$= \frac{1}{2} \times 5 \times 0.16 = 0.4$

$P = 10$

$mu = 10 \to u = \frac{10}{m} = \frac{10}{5} = 2$ m/s

$k.E_I = \frac{1}{2} m u^2 = \frac{1}{2} \times 5 \times 2^2 = 10$

$\Delta k.E = \frac{1}{2} m(v^2 - u^2) = k.E_I - k.E_F = 10 - 0.4 = 9.6$ J

(10)

K.E at the ground = P.E at the top

$\Rightarrow \quad = mgh$

Now the ball will go say x m up with initial velocity 10 m/s.

At highest point $v = 0$.

From $2gx = v^2 - u^2 = 0 - 100 \Rightarrow x = 5$

So the ball will be 25m up the ground when it comes to rest

$P.E = mgh = 10 \times 10^1 \times 25 = 25$ J

(11)

Given $m = 250$ kg. height $h = 2$ m

work done $= mgh = 250 \times 9.8 \times 2$

$= 4900$ J

$= 4.9$ kJ

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(8)

Sol we know $T.E = K.E + P.E = mgh$ = constant at any point

(a) at a height $\frac{h}{2}$ $P.E = mg\left(\frac{h}{2}\right) = \frac{mgh}{2}$

$K.E = T.E - P.E = mgh - \frac{mgh}{2} = \frac{mgh}{2}$

$K.E = P.E$

(c) at a height $\frac{h}{3}$ $P.E = mg\left(\frac{h}{3}\right) = \frac{mgh}{3}$

$K.E = T.E - P.E = mgh - \frac{mgh}{3} = \frac{2mgh}{3}$

$K.E = 2\, P.E$

(b) at a height $\frac{2h}{3}$ $\Rightarrow$ $P.E = mg\left(\frac{2h}{3}\right) = \frac{2mgh}{3}$

$K.E = T.E - P.E \Rightarrow mgh - \frac{2mgh}{3} = \frac{mgh}{3}$

$\Rightarrow P.E = 2\, K.E$