WS-14 for class 8

Ttask

1. Given P, Q are two forces

Maximum value $= P+Q$

minimum value $= P-Q$

$\therefore \frac{P+Q}{P-Q} = \frac{3}{1}$

$P+Q = 3(P-Q)$

$\Rightarrow P+Q = 3P-3Q$

$\Rightarrow 4Q = 2P$

$\Rightarrow \frac{P}{Q} = \frac{2}{1}$

$\Rightarrow P = 2Q$

2. Given $F_1 = F_2$

$F = 141.4\ N$

$\theta = 120^\circ$

when F_1 & F_2 are perpendicular

$\theta = 90^\circ$

$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 90^\circ}$

$\Rightarrow 141.4 = \sqrt{F_1^2 + F_1^2}$

$\Rightarrow 141.4 = \sqrt{2}\, F_1$

$\Rightarrow F_1 = 100$

when $\theta = 120^\circ$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120^\circ}$

$R = F_1 = 100\ N.$

3. let $F_1 = 2P$

$F_2 = \sqrt{2}\, P$

$R = \sqrt{10}\, P$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$\Rightarrow \sqrt{10}\, P = \sqrt{4P^2 + 2P^2 + 4\sqrt{2}\, P^2 \cos\theta}$

$\Rightarrow 10P^2 = 6P^2 + 4\sqrt{2}\, P^2 \cos\theta$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$; $\theta = 45^\circ$

4. $F_{net} = 0$ ie

$N = \frac{360^\circ}{\theta} = \frac{360}{30} = 12.$

11 equal forces given

mean 10 forces balance

1 force Not balance ie,

$F_{net} = 5\ N.$

(5) Longest side length

$<$ sum of two other sides

(8) satisfy the above condit

$6 < 2+5$

so these three forces keep the Particle in equilibrium Because they are satisfying Triangle law

(6) Given $F_1 = P$; $F_2 = P$

$\theta = 90^\circ$; $R = 1$

$R = \sqrt{F_1^2 + F_2^2 + 2P\cos\theta}$

$1 = 1 + P^2 - \frac{1}{P} \times P(2)$

$\Rightarrow 1 = P^2 - 1$

$P = \sqrt{2}$ N

$\tan\beta = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$

$\tan 90 = \frac{P\sin\theta}{1 + P\cos\theta}$

$\Rightarrow \infty = \frac{P\sin\theta}{1+P\cos\theta} \Rightarrow 1 + P\cos\theta = \frac{P\sin\theta}{\infty}$

$\Rightarrow 1 + P\cos\theta = 0$

$\Rightarrow \cos\theta = -\frac{1}{P} = -\frac{1}{\sqrt{2}} \Rightarrow \theta = 135^\circ$

(7) Given $R = Q$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\Rightarrow Q^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\Rightarrow -P^2 = 2PQ\cos\theta$

$\Rightarrow -P = 2Q\cos\theta$

when Q is doubled ie $2Q$

$\tan\alpha = \frac{2Q\sin\theta}{P + 2Q\cos\theta}$

$\Rightarrow \tan\alpha = \frac{2Q\sin\theta}{-2Q\cos\theta + 2Q\cos\theta}$

$\Rightarrow \tan\alpha = \infty$

$\alpha = 90^\circ$

(8) let $F_1 = 4N$; $F_2 = 3N$

R is their resultant

$R^2 = F_1^2 + F_2^2 = 4^2 + 3^2$

$R = 5N$.

Now it is increased by 12

$R' = 5 + 12 = 17N$

Given $R'^2 = 12 + R^2$

$\Rightarrow 16 + 9 + 2 \times 4 \times 3\cos\theta = 12 + 25$

$\Rightarrow 24\cos\theta = 12$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 60^\circ$

(9) Given $P = (n+1)$; $Q = 1$

$R = \sqrt{P^2 + Q^2} \Rightarrow R = \sqrt{(n+1)^2 + 1^2} \rightarrow (1)$

now After rotating through an angle 60° $P' = n$; $Q' = 3$

$R = \sqrt{n^2 + 3^2} \rightarrow (2)$

$\therefore$ From (1) & (2)

$\sqrt{n^2 + 3^2} = \sqrt{(n+1)^2 + 1^2}$

$\Rightarrow n^2 + 9 = n^2 + 2n + 1 + 1$

$\Rightarrow 9 = 2n + 2$

$\Rightarrow n = \frac{7}{2} = 3.5$

(15) $\theta = 0^\circ$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{5^2 + 3^2 + 2(5)(3)\cos 0}$

$= \sqrt{25 + 9 + 30} \Rightarrow \sqrt{64}$

$= 8$ N

(16) Given $P = 20$, $Q = 25$ N

$R = 39$ N; $\theta = ?$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\Rightarrow (39)^2 = (20)^2 + (25)^2 + 2(20)(25)\cos\theta$

$\Rightarrow 1521 = 400 + 625 + 1000\cos\theta$

$\Rightarrow \cos\theta = \frac{500}{1000}$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\theta = 60^\circ$

(17) $F_1 = F_2 = F$; $R = 1414$ N

$\theta = 90^\circ$

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$1414 = \sqrt{F^2 + F^2 + \underbrace{2FF\cos 90}_{0}}$

$\Rightarrow 1414 = \sqrt{F^2 + F^2} = \sqrt{2}F$

$\Rightarrow F = \frac{1414}{\sqrt{2}} = \frac{1414}{1.414}$

$\Rightarrow F = 1000 = 10^3$ N

(18) $F_1 = 4x$; $F_2 = 3x$

$F_1 + F_2 = 7$

$\Rightarrow 4x + 3x = 7$

$\Rightarrow 7x = 7$

$\Rightarrow x = 1$

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(19) $C = \sqrt{A^2+B^2+2AB\cos\theta}$

$C^2 = A^2+B^2+2AB\cos\theta \rightarrow (1)$

(a) $C = A-B \Rightarrow C^2 = A^2+B^2-2AB \rightarrow (2)$

$\therefore$ From (1) and (2)

$\cancel{A^2+B^2}+2AB\cos\theta = \cancel{A^2+B^2}-2AB$

$\Rightarrow 2AB\cos\theta = -2AB$

$\cos\theta = -1$

$\Rightarrow \theta = \pi$

(b) $|\vec{C}|^2 = |\vec{A}+\vec{B}|$

$C^2 = A^2+B^2+2AB\cos\theta \rightarrow (3)$

(1) & (2)

$A^2+B^2+2AB\cos\theta = A^2+B^2-2AB$

$\Rightarrow \cos\theta = -1$

$\Rightarrow \theta = \pi$

(c) $C^2 = A^2+B^2 \rightarrow (4)$

From (3) & (4)

$\cancel{A^2+B^2} = \cancel{A^2+B^2}+2AB\cos\theta$

$\Rightarrow 2AB\cos\theta = 0$

$\Rightarrow \theta = 90^\circ$

(d) From (3)

$C^2 = A^2+B^2+2AB\cos\theta$

$\Rightarrow C^2 = C^2+C^2+2C^2\cos\theta$

$\Rightarrow -C^2 = 2C^2\cos\theta$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\theta = 120^\circ = \frac{2\pi}{3}$

L Task

(1) $|\vec{P}| = |\vec{Q}| = 1$

$|\vec{R}| = 1$

$\Rightarrow R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$\Rightarrow R^2 = P^2+Q^2+2PQ\cos\theta$

$\Rightarrow 1^2 = 1^2+1^2+2(1)(1)\cos\theta$

$\Rightarrow 1 = 1+1+2\cos\theta$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\theta = 120^\circ$

Magnitude of difference

$R' = \sqrt{P^2+Q^2-2PQ\cos\theta}$

$R' = \sqrt{1^2+1^2-2(1)(1)\left(-\frac{1}{2}\right)}$

$R' = \sqrt{3}$

(2) $\vec{R} = |\vec{P}+\vec{Q}|$

$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$R^2 = P^2+Q^2+2PQ\cos\theta \rightarrow (1)$

$\vec{S} = \vec{P}-\vec{Q}$

$S = \sqrt{P^2+Q^2-2PQ\cos\theta}$

$S^2 = P^2+Q^2-2PQ\cos\theta$

$R^2+S^2 = P^2+Q^2+\cancel{2PQ\cos\theta}$
$\quad + P^2+Q^2-\cancel{2PQ\cos\theta}$

$R^2+S^2 = 2(P^2+Q^2)$

(3) Given

$F_1+F_2 = 7$

$F_1-F_2 = 3$

$2F_1 = 10$

$F_1 = 5N$

$\Rightarrow F_2 = 2N$

They gave the two forces are increased by 3

$F_1' = 5+3 = 8N$

$F_2' = 2+3 = 5N$

$\theta = 60^\circ$

$R = \sqrt{F_1'^2+F_2'^2+2F_1'F_2'\cos\theta}$

$= \sqrt{8^2+5^2+2(8)(5)\cos 60}$

$= \sqrt{64+25+2(40)\frac{1}{2}}$

$= \sqrt{129}\ N$

(4) Given $\theta = 0^\circ$

$C = \sqrt{a^2+b^2+2ab\cos\theta}$

$C = \sqrt{a^2+b^2+2ab\cos 0^\circ}$

$C = \sqrt{a^2+b^2+2ab}$

$C = \sqrt{(a+b)^2}$

$C = a+b$

(6) Given $F_1+F_2 = 29$

$F_1-F_2 = 5$

$2F_1 = 34$

$F_1 = 17N$

$F_2 = 12N$

Both are increased by 3

$F_1' = 17+3 = 20$

$F_2' = 12+3 = 15$

$\theta = 90^\circ$

$R = \sqrt{F_1'^2+F_2'^2} = \sqrt{20^2+15^2}$

$R = 25$ kg wt

(7) The net force F_{net} should be in b/w maximum Resultant and minimum resultant

$F_1 = 4N$; $F_2 = 3N$.

$F_{max} = F_1+F_2 = 7N$

$F_{min} = F_1 \sim F_2 = 1N$

$\therefore$ So F_{net} in between 7 and 1.

(8) Given $P=8$; $Q=6$; $R=10$.

$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$\Rightarrow R^2 = P^2+Q^2+2PQ\cos\theta$

$\Rightarrow 10^2 = 8^2+6^2+2\times8\times6\cos\theta$

$\Rightarrow 100 = 64+36+96\cos\theta$

$\Rightarrow \cos\theta = 0$

$\theta = 90^\circ$

(9) Given $P=Q$; $R=Q$.

$R=\sqrt{P^2+Q^2+2PQ\cos\theta}$

$\Rightarrow Q=\sqrt{Q^2+Q^2+2QQ\cos\theta}$

$\Rightarrow Q^2=2Q^2+2Q^2\cos\theta$

$\Rightarrow \cos\theta=-\frac{1}{2}$

$\theta=120^\circ$

(18) $R=\sqrt{P^2+Q^2+2PQ\cos\theta}$

$\theta=180^\circ$. For minimum value

$\Rightarrow R=\sqrt{P^2+Q^2+2PQ\cos180^\circ}$

$\Rightarrow R=\sqrt{P^2+Q^2+2PQ(-1)}$

$\Rightarrow R=\sqrt{P^2+Q^2-2PQ}$

$\Rightarrow R=\sqrt{(P-Q)^2}$

$\Rightarrow R=P-Q=10-5$

$R=5N$ $[P=10N, Q=5N]$

(10) Given $\vec{A}=\vec{B}$

$(\vec{A}+\vec{B})=n(\vec{A}-\vec{B})$

$\Rightarrow \sqrt{A^2+B^2+2AB\cos\theta}=n\sqrt{A^2+B^2-2AB\cos\theta}$

$\Rightarrow A^2+B^2+2AB\cos\theta=n^2(A^2+B^2-2AB\cos\theta)$

$\Rightarrow A^2+A^2+2AA\cos\theta=n^2(A^2+A^2-2AA\cos\theta)$

$\Rightarrow 2A^2+2A^2\cos\theta=n^2(2A^2-2A^2\cos\theta)$

$\Rightarrow 2A^2(1+\cos\theta)=n^2 2A^2(1-\cos\theta)$

$\Rightarrow \cancel{2}\cos^2\frac{\theta}{2}=n^2\cancel{2}\sin^2\frac{\theta}{2}$

$\Rightarrow \cos\frac{\theta}{2}=n\sin\frac{\theta}{2}$

$\Rightarrow \frac{1}{n}=\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

$\Rightarrow \tan\frac{\theta}{2}=\frac{1}{n}$

$\Rightarrow \frac{\theta}{2}=\tan^{-1}(\frac{1}{n})$

$\Rightarrow \theta=2\tan^{-1}(\frac{1}{n})$

(17) $P=Q=15N$ $\theta=120^\circ$

$R=5x$

$R=\sqrt{P^2+Q^2+2PQ\cos\theta}$

$\Rightarrow 5x=\sqrt{15^2+15^2+2(15)(15)\cos120}$

$\Rightarrow 5x=\sqrt{15^2+15^2+2(15)^2(-\frac{1}{2})}$

$\Rightarrow 5x=\sqrt{15^2+15^2-15^2}$

$\Rightarrow 5x=15$

$\Rightarrow \underline{x=3}$

(19) Given $P=2Q$

R is ⊥er to Q

$\beta=90^\circ$

$\tan\beta=\frac{P\sin\theta}{Q+P\cos\theta}$

$\Rightarrow Q+P\cos\theta=\frac{P\sin\theta}{\tan90}$

$\Rightarrow Q+P\cos\theta=0$

$\Rightarrow P\cos\theta=-Q$

$\Rightarrow 2Q\cos\theta=-Q$

$\Rightarrow \cos\theta=-\frac{1}{2}$

$\theta=120^\circ$

Given $\theta=\frac{2\pi}{n}$

$\Rightarrow \frac{2\pi}{n}=\frac{2\pi}{3}$

$\Rightarrow n=3$.

(14) (15) (16)

Given $F_1=8N$; $F_2=6N$

$\theta=90^\circ$

$F=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$

$F=\sqrt{8^2+6^2+2(8)(6)\cos90}$

$F=\sqrt{8^2+6^2}=10N$

if $\theta=120^\circ$

$F'=\sqrt{8^2+6^2+2(8)(6)\cos120^\circ}$

$=\sqrt{64+36+2(48)(-\frac{1}{2})}$

$=\sqrt{100-48}$

$=\sqrt{52}$

$=7.2N$

$\tan\alpha=\frac{Q\sin\theta}{P+Q\cos\theta}$

$=\frac{6\sin90}{8+6\cos90}$

$\tan\alpha=\frac{6}{8}$

$\tan\alpha=\frac{3}{4}$

$\underline{\alpha=37^\circ}$