BASIC CONCEPT OF MATHEMATICS

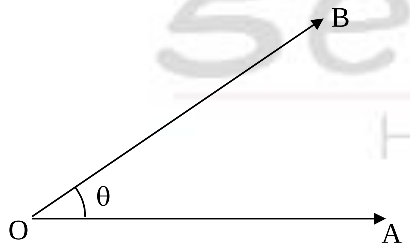
• TRIGONOMETRY

• INTRODUCTION

In this chapter we intend to study an important branch of mathematics called "Trigonometry". The word 'trigonometry' is derived from the Greek words : (i) trigonon and, (ii) metron.

• ANGLE

Consider a ray OA. If this ray rotates about its end point O and takes the position OB, then we say that the angle ∠AOB has been generated.



Thus an angle is considered as the figure obtained by rotating a given ray about its end–point.

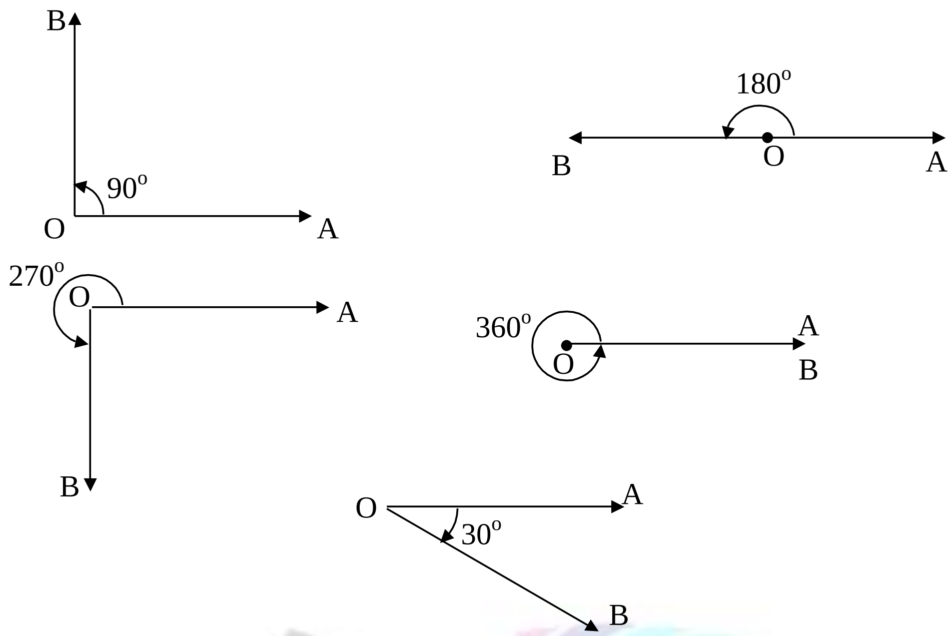
The revolving ray is called the generating line of the angle. The initial position OA is called the initial side and the final position OB is called the terminal side of the angle. The end point O about which the ray rotates is called the vertex of the angle.

Measure of an angle : The measure of an angle is the amount of rotation from the initial side to the terminal side.

• DEGREE MEASURE

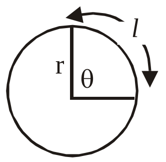
If a rotation from the initial side to terminal side is $(1/360)^{th}$ of a revolution, the angle is said to have a measure of one degree (1°). A degree is divided into 60 minutes and a minute divided into 60 seconds. One sixtieth of a degree is called minute (1′) and one sixieth of a minute is called second (1″).

Thus 1° = 60′; 1′ = 60″.



- ## RADIAN MEASURE

There is another unit for measurement of an angle, called the radian measure.



$\therefore\ \theta = \frac{l}{r}$ radian

***Illustration* 1:**

Convert 40° 20’ into radian measure

Solution:

$40°\ 20' = 40\frac{1}{3} = \frac{\pi}{180°} \times \frac{121}{3}$ radian $= \frac{121\pi}{540}$ radian

***Illustration* 2:**

Convert 6 radians into degree measure

Solution:

6 radians $= \frac{180}{\pi} \times 6$ degree

$= \frac{180 \times 6 \times 7}{22} = 343\frac{7}{11}$ degree

6 radians = 343° 38’ 11’’ approximately.

***Illustration* 3:**

Find the radius of the circle in which a central angle of 60° intercept an arc of length 37.4 cm $\left(\pi = \frac{22}{7}\right)$

Solution:

$$\theta = \frac{l}{r}$$

$$r = \frac{37.4 \times 3}{\pi} = 35.7 \text{ cm}$$

- ## RELATION BETWEEN DEGREE AND RADIAN

Since a circle subtends at the centre an angle whose radian measure is 2π and its degree measure is 360°. 2π radian = 360° or π radian = 180°.

$$1 \text{ radian} = \frac{180^\circ}{\pi} = 57.16^o \text{ approx.}$$

$$1^\circ = \frac{\pi}{180^\circ} \text{ radian} = 0.01746 \text{ radian}$$

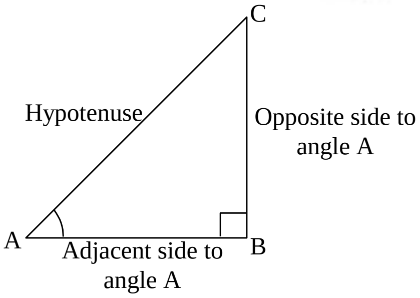
Radian measure = $(\pi/180) \times$ Degree measure

Degree measure = $(180/\pi) \times$ Radian measure

- ## TRIGONOMETRIC RATIOS

In this section we will define the trigonometric ratios for an acute angle of a right angled triangle.

There are in all six different trigonometric ratios of an angle which are defined from the following figure as :



1. Sine A = $\frac{\text{Opposite side to angle A}}{\text{Hypotenuse}} = \frac{BC}{AC}$

2. Cosine A = $\frac{\text{Adjacent side to angle A}}{\text{Hypotenuse}} = \frac{AB}{AC}$

3. Tangent A = $\frac{\text{Opposite side to angle A}}{\text{Adjacent side to angle A}} = \frac{BC}{AB}$

4. Cosecant A = $\frac{\text{Hypotenuse}}{\text{Opposite side to angle A}} = \frac{AC}{BC}$

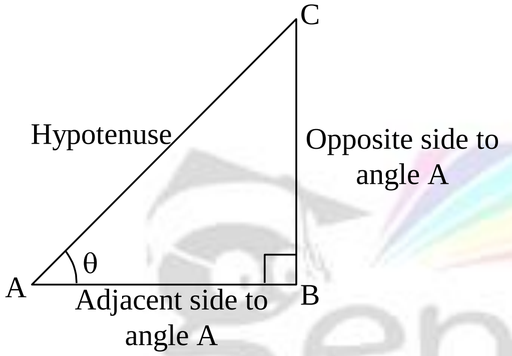
5. Secant A = $\frac{\text{Hypotenuse}}{\text{Adjacent side to angle A}} = \frac{AC}{AB}$

6. Cotangent A = $\dfrac{\text{Adjacent side to angle A}}{\text{Opposite side to angle A}} = \dfrac{AB}{BC}$

Note: The trigonometric ratio are generally written in short form as below
sin A is written for sine A
cos A is written for cosine A
tan A is written for tangent A
cosec A is written for cosecant A
sec A is written for secant A
and cot A is written for cotangent A

- **Relationships between Trigonometric Ratios of an Angle**

In the figure,



$\sin\theta = \dfrac{BC}{AC}$; $\cos\theta = \dfrac{AB}{AC}$; $\tan\theta = \dfrac{BC}{AB}$

$\text{cosec}\,\theta = \dfrac{AC}{BC}$; $\sec\theta = \dfrac{AC}{AB}$; $\cot\theta = \dfrac{AB}{BC}$

Now, let us derive some relations in between the above trigonometric ratios :

$$\text{cosec}\,\theta = \frac{AC}{BC} = \frac{1}{\left(\frac{BC}{AC}\right)} = \frac{1}{\sin\theta}$$

i.e., $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ or $\sin\theta = \dfrac{1}{\text{cosec}\,\theta}$

Similarly, we can show that

$\sec\theta = \dfrac{1}{\cos\theta}$ or $\cos\theta = \dfrac{1}{\sec\theta}$

and $\cot\theta = \dfrac{1}{\tan\theta}$ or $\tan\theta = \dfrac{1}{\cot\theta}$

Now, consider $\tan\theta = \dfrac{BC}{AB} = \dfrac{\left(\frac{BC}{AC}\right)}{\left(\frac{AB}{AC}\right)} = \dfrac{\sin\theta}{\cos\theta}$

Therefore, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Similarly, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

We can easily examine that

$\sin\theta \times \text{cosec}\,\theta = 1$

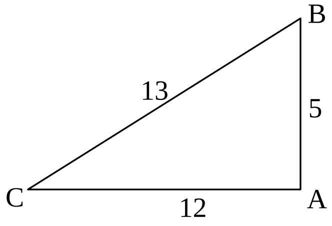
$\cos\theta \times \sec\theta = 1$

$\tan\theta \times \cot\theta = 1$

Illustration : 4

In a ΔABC, right angled at A, if AB = 5, AC = 12 and BC = 13, find sin B, cos C and tan B.

Solution



With reference to $\angle B$, we have

Base = AB = 5, perpendicular = AC = 12

and, hypotenuse = BC = 13.

$\therefore\ \sin B = \frac{AC}{BC} = \frac{12}{13}$

and $\tan B = \frac{AC}{AB} = \frac{12}{5}$

with reference to $\angle C$, we have

Base = AC = 12, perpendicular = AB = 5 and, hypotenuse BC = 13.

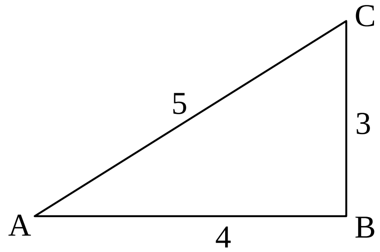
$\therefore\ \cos C = \frac{AC}{BC} = \frac{12}{13}$

Illustration : 5

If $\sin A = \frac{3}{5}$, find cos A and tan A.

Solution

We have, $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$



If we draw a triangle ABC, right angled at B such that

Perpendicular = BC = 3 units and

Hypotenuse = AC = 5 units

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow\ 5^2 = AB^2 + 3^2$

$\Rightarrow\ AB^2 = 16$

$\Rightarrow\ AB = 4$

When we consider the t–ratios of $\angle A$, we have

Base = AB = 4, perpendicular = BC = 3, hypotenuse = AC = 5.

$\therefore\ \cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$ and $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$

● TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES

1. Table of values of trigonometric ratios of 0º, 30º, 45º, 60º and 90º.

Trigonometric ratios of angle θ	θ = 0°	θ = 30°	θ = 45°	θ = 60°	θ = 90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
cosec θ	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
cot θ	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

2. Value of sin θ increases from 0 to 1 when θ increases from 0º to 90º.
3. Value of cos θ decreases from 1 to 0 when θ increases from 0º to 90º.
4. If A and B two unequal acute angles, then sin A ≠ sin B, cos A ≠ cos B and tan A ≠ tan B.

Illustration : 6

Evaluate in the simplest form

(i) $\sin 60^o \cos 30^o + \cos 60^o \sin 30^o$

(ii) $\tan 30^o \sec 45^o + \tan 60^o \sec 30^o$

Solution

(i) $\sin 60^o \cos 30^o + \cos 60^o \sin 30^o = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$

(ii) $\tan 30^o \sec 45^o + \tan 60^o \sec 30^o = \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} = \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} + \frac{\sqrt{3}+1}{2\sqrt{2}}$

● TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES

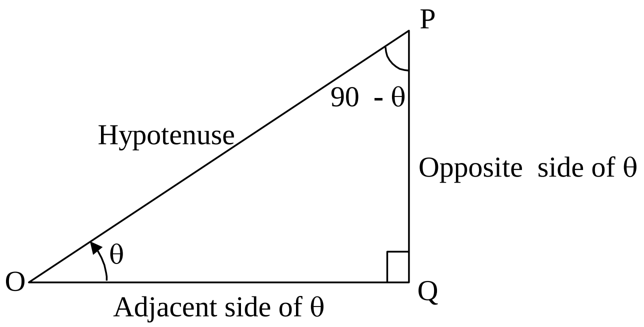
Let us construct a right angled ΔPOQ in which

∠POQ = θ, ∠OQP = 90º.

Then we have, ∠OPQ = (90º – θ)

Angles θ and (90º – θ) are complementary angles.

In the figure, the sides are labeled corresponding to angle θ.



$\sin(90^o - \theta) = \frac{OQ}{OP}$ and $\cos\theta = \frac{OQ}{OP}$

$\therefore \quad \sin(90^o - \theta) = \frac{OQ}{OP} = \cos\theta$

Similarly, $\cos(90^o - \theta) = \frac{PQ}{OP} = \sin\theta$

$\tan(90^o - \theta) = \frac{OQ}{PQ} = \cot\theta$ $\quad$ $\operatorname{cosec}(90^o - \theta) = \frac{OP}{OQ} = \sec\theta$

$\sec(90^o - \theta) = \frac{OP}{PQ} = \operatorname{cosec}\theta$ $\quad$ $\cot(90^o - \theta) = \frac{PQ}{OQ} = \tan\theta$

Illustration : 7

Express sin 81^o + cos 71^o + tan 61^o in terms of trigonometric ratios of angles between 0^o and 45^o.

Solution

sin 81^o + cos 71^o + tan 61^o = $\sin(90^o - 9^o) + \cos(90^o - 19^o) + \tan(90^o - 29^o)$

$= \cos 9^o + \sin 19^o + \cot 29^o$

$\because \sin(90^o - \theta) = \cos\theta, \cos(90^o - \theta) = \sin\theta, \tan(90^o - \theta) = \cot\theta$

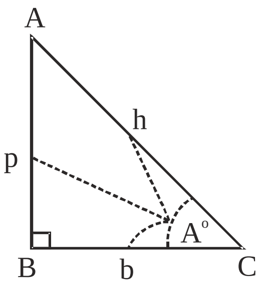
- **TRIGONOMETRIC IDENTITIES**

In ΔABC, right angled at B,

We have :

AC^2 = AB^2 + BC^2

[By Pythagoras Theorem] … (i)



[Dividing both sides by AC^2 in (i)]

$\Rightarrow \left(\frac{AC}{AC}\right)^2 = \left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2$

$\Rightarrow 1^2 = (\cos A)^2 + (\sin A)^2$

$\therefore \sin^2 A + \cos^2 A = 1$ ... (ii)

It is true for all $\angle A$ such that, $0^o \le A \le 90^o$

Again, we know that :

$AC^2 = AB^2 + BC^2$ [From (i)]

$$\Rightarrow \left(\frac{AC}{AB}\right)^2 = \left(\frac{AB}{AB}\right)^2 + \left(\frac{BC}{AB}\right)^2$$

[Dividing both sides by AB^2 in (i)]

$\Rightarrow 1 + \tan^2 A = \sec^2 A$ ... (iii)

Now, we again, know that :

$AC^2 = AB^2 + BC^2$ [From (i)]

$$\Rightarrow \left(\frac{AC}{BC}\right)^2 = \left(\frac{AB}{BC}\right)^2 + \left(\frac{BC}{BC}\right)^2$$

[Dividing both sides by BC^2 in (i)]

$\Rightarrow \text{cosec}^2 A = \cot^2 A + 1$

$\therefore 1 + \cot^2 A = \text{cosec}^2 A$ ... (iv)

Illustration : 8

Prove the following trigonometric identities :

(i) $(1 - \sin^2\theta) \sec^2\theta = 1$ (ii) $\cos^2\theta (1 + \tan^2\theta) = 1$

Solution

(i) $\text{LHS} = (1 - \sin^2\theta)\sec^2\theta$

$= \cos^2\theta \sec^2\theta = \cos^2\theta . \frac{1}{\cos^2\theta} = 1$ $(1 - \sin^2\theta = \cos^2\theta,\ \sec\theta = 1/\cos\theta)$

LHS = 1 = RHS

(ii) LHS = $\cos^2\theta (1 + \tan^2\theta)$

= $\cos^2\theta \sec^2\theta$ $(1 + \tan^2\theta = \sec^2\theta)$

= $\cos^2\theta . \frac{1}{\cos^2\theta} = 1$ $(\sec\theta = 1/\cos\theta)$

LHS = 1 = RHS

- ## DEFINITION AND LAWS OF LOGARITHMS

For a positive real number 'a' and a rational number m, let $a^m = b$ where b is a real number. In other words, the m^{th} power of base a is b. Another way of stating the same result is logarithm of b to base a is m. Symbolically

$\log_a b = m$; 'log' is the abbreviation of the word 'logarithm'

$\log_a b = m$ $b = a^m$. Hence the logarithm of a number is the power to which the base should be raised to get the number.

Thus, we have

$\log_2 16 = 4$ since $2^4 = 16$

$\log_5 625 = 4$ since $5^4 = 625$

$\log_3 27 = 3$ since $3^3 = 27$

$\log_6 1 = 0$ since $6^0 = 1$

Again $\sqrt{9} = 3$ i.e., $9^{1/2} = 3$ $\therefore$ $\log_9 3 = 1/2$

$\sqrt[3]{27} = 3 \Rightarrow (27)^{1/3} = 3$

i.e., $\log_{27} 3 = 1/3$

Laws of logarithms

In the following discussion, we shall take logarithms to any base 'a' ($a > 0$ and $a \neq 1$).

I. First law

Prove that $\log_a mn = \log_a m + \log_a n$

Proof: Let $\log_a m = x$ and $\log_a n = y$

$m = a^x$ and $n = a^y$

$\therefore$ $mn = a^x \cdot a^y = a^{x+y}$

$\therefore$ $a^{x+y} = mn$

$\therefore$ $\log_a (mn) = x + y = \log_a m + \log_a n$

i.e., Logarithm of the product of two numbers = Sum of their logarithms

II. Second law

Prove that $\log_a \frac{m}{n} = \log_a m - \log_a n$

Proof: Let $\log_a m = x$ $m = a^x$

Let $\log_n a = y$ $\therefore$ $n = a^y$

$\therefore$ $\frac{m}{n} = \frac{a^x}{a^y} = a^{x-y}$

$\therefore$ $\log_a \frac{m}{n} = x - y = \log_a m - \log_a n$

$\therefore$ logarithm of a quotient = logarithm of numerator – logarithm of denominator:

III Third law

Prove that $\log_a m^n = n \log_a m$

Let $\log_a m = x$ $\therefore$ $m = a^x$

$\therefore$ $m^n = \left(a^x\right)^n = a^{nx}$

$\therefore \quad \log_a m^n = nx = n\log_a m$

IV. Prove that logarithm of 1 with respect to any base = 0

Proof: Let a be any base

$a^0 = 1 \quad \therefore \quad \log_a 1 = 0$

V. Prove that logarithm of base of itself = 1

Proof: Let a be any base

$a^1 = a$

$\therefore \quad \log_a a = 1$

VI. Prove that $\log_b a = \dfrac{1}{\log_a b}$

Let $\log_b a = x$

$\therefore \quad a = b^x$

Taking logarithms of both sides with respect to base 'a'

$\log_a a = \log_a b^x = x\log_a b$

$1 = x\log_a b \quad \therefore \quad x = \dfrac{1}{\log_a b}$

$\log_b a = \dfrac{1}{\log_a b}$

VII. Prove that $\log_a b = \dfrac{\log_c b}{\log_c a}$

Proof: Let $\log_a b = x \qquad \therefore \quad b = a^x$

Taking logarithms of both sides with respect to base c.

$\log_c b = \log_c a^x = x\log_c a$

i.e., $x\log_c a = \log_c b$

$\therefore \quad x = \dfrac{\log_c b}{\log_c a}$

i.e., $\log_a b = \dfrac{\log_c b}{\log_c a}$

- **COMMON LOGARITHM**

  Logarithms to base 10 are called common logarithms. When the logarithm of a number is written as the sum of a positive decimal fraction and a certain positive or negative integral, the integral part is called the **characteristic** and the decimal part is called the **mantissa**. If the characteristic is negative, we usually write '–' above the integral part

  $\therefore \ \bar{3}.6452$ stands for $-3 + 0.6452$

- **RULES FOR FINDING THE CHARACTERISTIC OF A NUMBER**

(i) **The characteristic of a number greater than 1**

$10^o = 1$ $\quad\therefore \log_{10}1 = 0$

$10^1 = 10$ $\quad\therefore \log_{10} 10 = 1$

$10^2 = 100$ $\quad\therefore \log_{10} 100 = 2$

$10^3 = 1000$ $\quad\therefore \log^{10} 1000 = 3$

85.36 is a number between 10 and 100 (i.e.,) $10 < 85.36 < 100$

$\therefore \log_{10} 10 < \log_{10} 85.36 < \log_{10} 100$

$1 < \log_{10} 85.36 < 2$

$\therefore$ the $\log_{10} 85.36$ lies between 1 and 2

$\text{Log}_{10}\ 85.36 = 1 + f$ where f is fractional decimal part (Here the characteristic is one less than the number of digits in the integral part i.e., 85)

$\log_{10} 100 < \log_{10} 952.25 < \log_{10} 1000$

i.e., $2 < \log_{10} 952.25 < 3.$

$\therefore$ the $\log_{10} 952.25$ lies between 2 and 3

$\therefore$ $\log 952.25 = 2 + f_1$ where f_1 is the fractional decimal part

(ii) **The characteristic of the logarithm of a number less than 1**

$10^o = 1$

$\log_{10} 1 = 0$

$10^{-1} = \frac{1}{10} = 0.1$

$\therefore \log_{10} 0.1 = -1$

$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

$\therefore \log_{10} 0.01 = -2$

$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$

$\log_{10} 0.001 = -3$

$0.01 < 0.08 < 0.1$

$\log_{10} 0.01 < \log_{10} 0.08 < \log_{10} 0.1$

$-2 < \log_{10} 0.08 < -1$

$\log_{10} 0.08$ lies between -2 and -1

$\log_{10}0.08 = -2 + f$ where f is fractional decimal part

Characteristic $-2 = -(1 + 1) = -$ (number of zeros after decimal part + 1)

$0.001 < 0.008 < 0.01$

$\log_{10} 0.001 < \log_{10} 0.008 < \log_{10} 0.01$

$-3 < \log_{10} 0.008 < -2$

∴ $\log_{10} 0.008 = -3 + f$, where f is the fractional decimal part

characteristic of $\log_{10} 0.008 = -3 = -(2 + 1)$ = –(number of zeros after decimal place + 1)

If a number is less than 1, count the number of zeros after the decimal place.

If the number of zeros after the decimal point is n, then the characteristic of the logarithm of the number = $-(n + 1)$

- **NATURAL LOGARITHMS**

In theoretical investigation, the base that is usually employed is e, which is nearly equal to 2.71828. Logarithms to the base 'e' are called natural logarithms. $\log_e x$ is usually denoted by ln x.

It may be noted that $\log_{10} e = 0.4343$ and $\log_e 10 = \frac{1}{\log_{10} e} = 2.303$

Illustration : 9

Find the value of $\log_2\left(\frac{1}{4}\right)$

Solution

Let $\log_2\left(\frac{1}{4}\right) = x$

$\Rightarrow 2^x = \frac{1}{4} = \frac{1}{2^2} = 2^{-2}$

$\Rightarrow 2^x = 2^{-2}$

$\Rightarrow x = -2$

Illustration : 10

Find the value of $\log_2 32$

Solution

Let $\log_2 32 = x$

$2^x = 32 = 2^5$

$\Rightarrow x = 5$

Illustration : 11

Find the value of $\log_5(0.04)$

Solution

Let $\log_5(0.04) = x$

$\Rightarrow 5^x = 0.04 = \frac{4}{100} = \frac{1}{25}$

$\Rightarrow 5^x = \frac{1}{5^2} = 5^{-2}$

$\Rightarrow x = -2$

Illustration : 12

If $\log_x 6\frac{1}{4} = 2,$ find the value of x.

Solution

$\log_x 6\frac{1}{4} = 2$

$\therefore \quad x^2 = \left(\frac{5}{2}\right)^2 \quad \therefore \quad x = \frac{5}{2} = 2\frac{1}{2}$

- **THE USE OF TABLES OF COMMON LOGARITHMS**

Illustration : 13

Find the logarithm of 768.9 from Four–Figure tables.

Solution

The characteristic of the logarithm of 768.9 is 2 (Number of integers in integeral part – 1) i.e, 3 – 1 = 2.

To get the mantissa look down the first column of the table till 76 is reached and find the number corresponding to the row containing 76 and the column containing 8. It is found to be 8854, which is actually 0.8854. In the same row, find the number corresponding to the mean difference column containing 9. It is 0.0005. Adding these two numbers together i.e., 0.8854 + 0.0005 = 0.8859. The maintissa is 0.8859. Hence the required logarithm is 2.8859.

The above working is illustration below:

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
76								8854											5

Use of table of anti–logarithms to find the number whose logarithm is given

Illustration : 14

Find the number whose logarithm is (i) 1.4376 and (ii) $\bar{2}.4376$

Solution : Antilogarithm

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
0.43								8854								4			

Find the number corresponding to the row containing 0.43 and the column containing 7. It is found to be 2735. In the same row, find the number corresponding to the mean difference column containing 6. It is found to be 4. Adding these two numbers 2735 + 4 = 2739 logarithm of the number is 1.4376

The characteristic is 1. In the number there should be 2 digits in the integral place. The required number is 27.39 (antilogarithm of 0.4376 is 2739).

❖❖❖

KEY POINTS

- $\sin^2 A + \cos^2 A = 1$
- $\sin^2 A = 1 - \cos^2 A$
- $\cos^2 A = 1 - \sin^2 A$
- $1 + \tan^2 A = \sec^2 A$
- $1 = \sec^2 A - \tan^2 A$
- $\tan^2 A = \sec^2 A - 1$
- $1 + \cot^2 A = \operatorname{cosec}^2 A$
- $1 = \operatorname{cosec}^2 A - \cot^2 A$
- $\cot^2 A = \operatorname{cosec}^2 A - 1$
- $\sin\left(90^o - A\right) = \cos A$
- $\cos\left(90^o - A\right) = \sin A$
- $\tan\left(90^o - A\right) = \cot A$
- $\cot\left(90^o - A\right) = \tan A$
- $\sec\left(90^o - A\right) = \operatorname{cosec} A$
- $\operatorname{cosec}\left(90^o - A\right) = \sec A$
- Any number (integers) is divided by zero is equal to not defined.
- If $\log_a b = m \Rightarrow b = a^m$. Hence the logarithm of a number is the power to which the base should be raised to get the number.
- $\log_a mn = \log_a m + \log_a n$
- $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$
- $\log_a m^n = n \log_a m$
- logarithm of 1 with respect to any base = 0 i.e. $\log_a 1 = 0$
- logarithm of base of itself = 1 i.e. $\log_a a = 1$
- $\log_b a = \frac{1}{\log_a b}$
- $\log_a b = \frac{\log_c b}{\log_c a}$
- $\log_{a^k} N = \frac{1}{k} \log_a N$
- Logarithms to base 10 are called common logarithms. When the logarithm of a number is written as the sum of a positive decimal fraction and a certain positive or negative integral, the integral part is called the **characteristic** and the decimal part is called the **mantissa**.

ASSIGNMENT – I

1. Find the radian measure corresponding to the following degree measures
   (i) 240° (ii) –22° 30’ (iii) 175° 45’

2. Find in the degrees the angle subtended at the centre of a circle of diameter 50 cm by an arc of length 11 cm.

3. If cosec A = $\sqrt{10}$, find other five trigonometric ratios.

4. If tan A = $\sqrt{2}-1$, show that sin A cos A = $\frac{\sqrt{2}}{4}$

5. If $\tan\theta=\frac{12}{13}$, evaluate $\frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$

6. Evaluate the following expressions
   (i) $2\sin^2 30^o \tan 60^o - 3\cos^2 60^o \sec^2 30^o$ (ii) $\operatorname{cosec}^2 30^o \sin^2 45^o - \sec^2 60^o$

7. Prove that : $\frac{\cos 30^o + \sin 60^o}{1+\cos 60^o + \sin 30^o} = \frac{\sqrt{3}}{2}$

8. Evaluate (i) $\frac{\cos 37^o}{\sin 53^o}$ (ii) $\frac{\sin 41^o}{\cos 49^o}$

9. Prove that $\sin 35^o \sin 55^o - \cos 35^o \cos 55^o = 0$

10. Given sin θ = $\frac{p}{q}$, find cos θ + sin θ in terms of p and q.

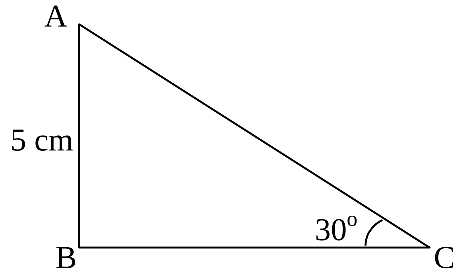
11. Evaluate the following without using the tables.
    (i) $\log_{\sqrt{2}} 64$ (ii) $\log_{0.01} 100$
    (iii) $\frac{\log 1000}{\log 100}$ (iv) $\frac{\log 125}{\log 25}$
    (v) $\log_{\frac{1}{3}} 243$ (vi) $\log_{64} 128$

12. Prove that $\log\left(a^{-1}+b^{-1}\right)=\log(a+b)-\log a-\log b$

ASSIGNMENT – II

1. In ΔABC right angled at B, BC = 5 cm and AC – AB = 1 cm. Evaluate $\frac{1+\sin C}{\cos C}$.

2. In ΔABC right angled at B, AB = 5 cm and $\angle ACB = 30^o$. Determine the lengths of the sides BC and AC.



3. Prove the following identities

   (i) $\cos^2\theta + \frac{1}{1+\cot^2\theta} = 1$ (ii) $\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta} = \frac{\sec\theta+1}{\sec\theta-1}$

4. Simplify the expression $(\sec\theta+\tan\theta)(1-\sin\theta)$

5. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

6. In a right triangle ABC, right angled at B, the ratio of AB to AC is $1:\sqrt{2}$. Find the values of

   (i) $\frac{2\tan A}{1+\tan^2 A}$ and (ii) $\frac{2\tan A}{1-\tan^2 A}$

7. If $\sin\theta = \frac{3}{4}$, prove that $\sqrt{\frac{\operatorname{cosec}^2\theta-\cot^2\theta}{\sec^2\theta-1}} = \frac{\sqrt{7}}{3}$

8. Find the acute angle θ, when $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

9. Evaluate : $\frac{2\cos 67^o}{\sin 23^o} - \frac{\tan 40^o}{\cot 50^o} - \cos 0^o$

10. Prove that $\tan 10^o \tan 15^o \tan 75^o \tan 80^o = 1$

11. If $\log_{10}(x + 3) = -\log_{10}2$, then prove that $x = -2\frac{1}{2}$

12. Prove that $\log_2 5 . \log_{25} 8$ is smaller than 2.

❖❖❖

KEY & HINTS

Basic Concepts of Mathematics

Answer KEY

ASSIGNMENT – I

1. (i) $(4\pi/3)^c$ (ii) $(-\pi/8)^c$ (iii) $\left(\frac{703}{720}\pi\right)^c$
2. 25°12'

10. $\frac{p+\sqrt{q^2+p^2}}{q}$

11. (i) 12. (ii) –1 (iii) 3/2 (iv) 3/2 (v) –5 (vi) 7/6

ASSIGNMENT – II

1. 5
2. BC = $5\sqrt{3}$ cm, AC = 10 cm

Hints and Solutions

ASSIGNMENT – I

1. (i) 180° = π^c

$240° = \left(\frac{\pi}{180}\times 240\right)^c = \left(\frac{4\pi}{3}\right)^c$

(ii) 180° = π^c

$(-22°\ 30') = -\left(\frac{\pi}{180}\times\frac{45}{2}\right)^c = -\left(\frac{\pi}{8}\right)^c$

(iii) $175°\ 45' = 175° + \left(\frac{45}{60}\right)^\circ = (703/4)°$

$= \left(\frac{703}{4}\right)\times\frac{\pi}{180}$ radian

2. Here r = 25 cm and l = 11 cm

$\therefore\ \theta = \left(\frac{l}{r}\right)^c = \left(\frac{11}{25}\right)^c = \left(\frac{11}{25}\times\frac{180}{\pi}\right)^\circ$

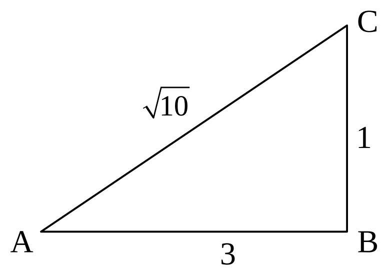
$= \left(\frac{11}{25}\times\frac{7}{22}\times 180\right)^\circ = \left(\frac{126}{5}\right)^\circ = 25°\ 12'$

3. We have cosec A = $\frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\sqrt{10}}{1}$

So, we draw a right triangle ABC, right–angled at B such that

Perpendicular = BC = 1 unit and

Hypotenuse = AC = $\sqrt{10}$ units.



By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \left(\sqrt{10}\right)^2 = AB^2 + 1^2$

$\Rightarrow AB^2 = 10 - 1 = 9$

$\Rightarrow AB = \sqrt{9} = 3$

When we consider the trigonometric ratios of ∠A, we have

Base = AB = 3, Perpendicular = BC = 1, and Hypotenuse = AC = $\sqrt{10}$

$\therefore$ sin A = $\frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{\sqrt{10}}$

cos A = $\frac{\text{Base}}{\text{Hypotenuse}} = \frac{3}{\sqrt{10}}$

tan A = $\frac{\text{Perpendicular}}{\text{Base}} = \frac{1}{3}$

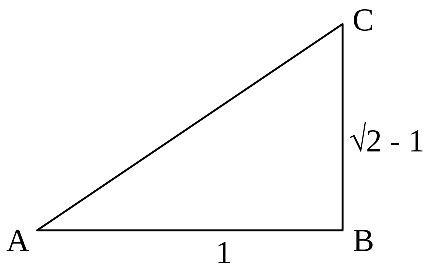
sec A = $\frac{\text{Hypotenuse}}{\text{Base}} = \frac{\sqrt{10}}{3}$

and cot A = $\frac{\text{Base}}{\text{Perpendicular}} = \frac{3}{1} = 3$

4. We have, tan A = $\frac{\text{Perpendicular}}{\text{Base}} = \frac{\sqrt{2}-1}{1}$

So, we draw a right triangle ABC, right angled at B such that

Base = AB = 1 and perpendicular = BC = $\sqrt{2}-1$



By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 1^2 + \left(\sqrt{2}-1\right)^2$

$\Rightarrow AC^2 = 1 + 2 + 1 - 2\sqrt{2}$

$\Rightarrow AC^2 = 4 - 2\sqrt{2}$

$\Rightarrow AC = \sqrt{4-2\sqrt{2}}$

Now, $\sin A = \frac{BC}{AC} = \frac{\sqrt{2}-1}{\sqrt{4-2\sqrt{2}}}$ and $\cos A = \frac{AB}{AC} = \frac{1}{\sqrt{4-2\sqrt{2}}}$

$\therefore$ sin A cos A = $\frac{\sqrt{2}-1}{\sqrt{4-2\sqrt{2}}} \times \frac{1}{\sqrt{4-4\sqrt{2}}} = \frac{\sqrt{2}-1}{4-2\sqrt{2}}$

$\Rightarrow \sin A \cos A = \frac{\sqrt{2}-1}{2\sqrt{2}\left(\sqrt{2}-1\right)} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

5. We have $\frac{2\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta} = \frac{\frac{2\sin\theta\cos\theta}{\cos^2\theta}}{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}$ (dividing Nr and Dr by $\cos^2\theta$)

$= \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\times\left(\frac{12}{13}\right)}{1-\left(\frac{12}{13}\right)^2} = \frac{312}{25}$

6. (i) $2\sin^2 30^o \tan 60^o - 3\cos^2 60^o \sec^2 30^o$

$= 2\left(\sin 30^o\right)^2 \tan 60^o - 3\left(\cos 60^o\right)^2\left(\sec 30^o\right)^2$

$= 2\times\left(\frac{1}{2}\right)^2 \times\sqrt{3} - 3\times\left(\frac{1}{2}\right)^2\times\left(\frac{2}{\sqrt{3}}\right)^2$

$2\times\frac{1}{4}\times\sqrt{3} - 3\times\frac{1}{4}\times\frac{4}{3} = \frac{\sqrt{3}}{2} - 1 = \frac{\sqrt{3}-2}{2}$

(ii) We have, $\operatorname{cosec}^2 30^o \sin^2 45^o - \sec^2 60^o$

$= \left(\operatorname{cosec} 30^o\right)^2\left(\sin 45^o\right)^2 - \left(\sec 60^o\right)^2 = (2)^2\times\left(\frac{1}{\sqrt{2}}\right)^2 - (2)^2 = 2 - 4 = -2$

7. We have, $\frac{\cos 30^o + \sin 60^o}{1+\cos 60^o + \sin 30^o} = \frac{\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}{1+\frac{1}{2}+\frac{1}{2}} = \frac{\sqrt{3}}{2}$

8. (i) $\frac{\cos 37^o}{\sin 53^o} = \frac{\cos\left(90^o - 53^o\right)}{\sin 53^o} = \frac{\sin 53^o}{\sin 53^o} = 1$ $\left(\because \cos\left(90^o-\theta\right) = \sin\theta\right)$

(ii) $\frac{\sin 41^o}{\cos 49^o} = \frac{\sin\left(90^o - 49^o\right)}{\cos 49^o} = \frac{\cos 49^o}{\cos 49^o} = 1$

9. LHS = $\sin 35^o \sin 55^o - \cos 35^o \cos 55^o$

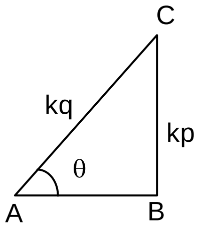
$= \sin\left(90^o - 55^o\right)\sin\left(90^o - 55^o\right) - \cos 35^o \cos 55^o$

$= \cos 55^o \cos 35^o - \cos 35^o \cos 55^o = 0$ = RHS

10. Draw a right triangle ABC, right angled at B

Let ∠CAB = θ. Then, sin θ = BC/AC

sin θ = p/q ⇒ BC/AC = p/q (given)



If BC = kp, then AC = kq where k is a positive number.

In right triangle ABC, we have

$AC^2 = AB^2 + BC^2$ [By Pythagorals theorem]

$\Rightarrow (kq)^2 = AB^2 + (kp)^2$

$\Rightarrow AB^2 = k^2q^2 - k^2p^2 = k^2(q^2 - p^2)$

$\Rightarrow AB = k\sqrt{q^2 - p^2}$

Now, $\cos\theta = \frac{AB}{AC} = \frac{k\sqrt{q^2-p^2}}{kq} = \frac{\sqrt{q^2-p^2}}{q}$

$\therefore \sin\theta + \cos\theta = \frac{p}{q} + \frac{\sqrt{q^2-p^2}}{q} = \frac{p+\sqrt{q^2-p^2}}{q}$

11. (i) Let $\log_{\sqrt{2}} 64 = x$

$\therefore \left(\sqrt{2}\right)^x = 64$

i.e. $\left(2^{1/2}\right)^x = 2^{x/2} = 2^6$

$\therefore \frac{x}{2} = 6$ or $x = 12$.

(ii) Let $\log_{0.01}100 = x$

$\therefore (0.01)^x = 100$ i.e. $\left(\frac{1}{100}\right)^x = 100$

i.e., $(100^{-1})^x = 100^{-x} = 100^1$

$\Rightarrow -x = 1$ or $x = -1$

(iii) $\frac{\log_{10} 1000}{\log_{10} 100}$

$\log a^m = m \log a$

$\therefore \frac{\log_{10} 1000}{\log_{10} 100} = \frac{\log_{10} 10^3}{\log_{10} 10^2} = \frac{3\log_{10} 10}{2\log_{10} 10} = = \frac{3}{2}[\because \log_{10} 10 = 1]$

(iv) $\frac{\log 125}{\log 25}$

We know that $\log a^m = m \log a$

$\frac{\log 125}{\log 25} = \frac{\log 5^3}{\log 5^2} = \frac{3\log 5}{2\log 5} = \frac{3}{2}$

(v) $\log_{1/3} 243$

We know that $\log_b a = \frac{\log_c a}{\log_c b}$

$\therefore \log_{1/3} 243 = \frac{\log_3 243}{\log_3 1/3} = \frac{\log_3 3^5}{\log_3 3^{-1}}$

$= \frac{5\log_3 3}{-1\log_a 3} = -5$

(vi) $\log_{63}128$

We known that $\log_b a = \frac{\log_c a}{\log_c b}$ and $\log a^m = m\log a$

$$\therefore \quad \log_{64} 128 = \frac{\log_2 2^7}{\log_2 2^6} = \frac{7\log_2 2}{6\log_2 2} = \frac{7}{6}$$

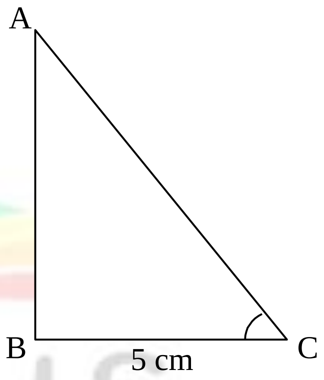
12. $\log\left(a^{-1} + b^{-1}\right) = \log\left(\frac{1}{a} + \frac{1}{b}\right)$

$$= \log\left(\frac{a+b}{ab}\right) = \log(a+b) - \log(ab)$$

= log(a + b) – (log a + b) = log (a + b) – log a – log b

ASSIGNMENT – II

1. In the figure, ΔABC is right angled at B.
BC = 5 and AC – AB = 1 cm.
i.e. AC = AB + 1 cm
Using Pythagoras theorem, we have



$AC^2 = AB^2 + BC^2 \Rightarrow (AB + 1)^2 = AB^2 + BC^2$

$\Rightarrow AB^2 + 2AB + 1 = AB^2 + (5)^2$

$\Rightarrow 2AB + 1 = 25$ or $2\,AB = 24$

$\Rightarrow AB = 12$ cm

$\Rightarrow AC = 12$ cm $+ 1 = 13$ cm

From ΔABC, we have

$\sin C = \frac{AB}{AC} = \frac{12}{13} \Rightarrow 1 + \sin C = 1 + \frac{12}{13} = \frac{25}{13}$

Also $\cos C = \frac{BC}{AC} = \frac{5}{13}$

Then $\frac{1 + \sin C}{\cos C} = \frac{\left(\frac{25}{13}\right)}{\left(\frac{5}{13}\right)} = 5$

2. We are given ΔABC right angled at B, $\angle ACB = 30^\circ$ and AB = 5 cm. First of all, we will find the length of the hypotenuse AC.

We have $\frac{AB}{AC} = \sin 30^\circ \Rightarrow \frac{5}{AC} = \frac{1}{2}$

$\Rightarrow AC = 10$ cm

Now, $\frac{BC}{AC} = \cos 30^\circ \Rightarrow \frac{BC}{10} = \frac{\sqrt{3}}{2}$

$\Rightarrow BC = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$ cm

3. (i) LHS = $\cos^2\theta + \frac{1}{1+\cot^2\theta} = \cos^2\theta + \frac{1}{\operatorname{cosec}^2\theta} = \cos^2\theta + \sin^2\theta = 1$ = RHS

(ii) LHS = $\frac{\tan\theta + \sin\theta}{\tan\theta - \sin\theta} = \frac{\frac{\sin\theta}{\cos\theta} + \sin\theta}{\frac{\sin\theta}{\cos\theta} - \sin\theta} = \frac{\sin\theta\left(\frac{1}{\cos\theta} + 1\right)}{\sin\theta\left(\frac{1}{\cos\theta} - 1\right)}$

$$= \frac{\left(\frac{1}{\cos\theta}+1\right)}{\left(\frac{1}{\cos\theta}-1\right)} = \frac{\sec\theta+1}{\sec\theta-1} = \text{RHS}$$

4. $(\sec\theta+\tan\theta)(1-\sin\theta) = \left(\frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}\right)(1-\sin\theta)$

$= \frac{(1+\sin\theta)}{\cos\theta}\times(1-\sin\theta) = \frac{1^2-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos\theta} = \cos\theta$

5. LHS = $m^2-n^2=(m+n)(m-n)$

$= \{(\tan\theta+\sin\theta)+(\tan\theta-\sin\theta)\}\{(\tan\theta+\sin\theta)-(\tan\theta-\sin\theta)\}$

$= \{2\tan\theta\}\times\{2\sin\theta\} = 4\tan\theta\sin\theta$

$= 4\sqrt{\tan^2\theta\sin^2\theta} = 4\sqrt{(\sec^2\theta-1)\sin^2\theta}$

$= 4\sqrt{\sec^2\theta\sin^2\theta-\sin^2\theta} = 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}-\sin^2\theta}$

$= 2\sqrt{\tan^2\theta-\sin^2\theta} = 4\sqrt{(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)}$

$= 4\sqrt{mn}$ = RHS

6. We have, AB : AC = $1:\sqrt{2}$ $\Rightarrow$ $\frac{AB}{AC} = \frac{1}{\sqrt{2}}$

$\therefore$ AB = x and AC = $\sqrt{2}\,x$, for some x.

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow$ $\left(\sqrt{2}\,x\right)^2 = x^2 + BC^2$

$\Rightarrow$ $BC^2 = 2x^2 - x^2 = x^2$

$\Rightarrow$ BC = x

$\therefore$ tan A = $\frac{BC}{AB} = \frac{x}{x} = 1$

(i) We have $\frac{2\tan A}{1+\tan^2 A} = \frac{2\times1}{1+1^2} = \frac{2}{2} = 1$

(ii) We have $\frac{2\tan A}{1-\tan^2 A} = \frac{2\times1}{1-1} = \frac{2}{0}$, which is undefined.

8. We have, $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

$\Rightarrow$ $\frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\cos\theta+\sin\theta}{\cos\theta}} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$ (dividing numerator and denominator by cos θ)

$\Rightarrow$ $\frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

$\Rightarrow$ $\tan\theta = \sqrt{3}$ $\Rightarrow$ $\tan\theta = \tan 60^o$

$\Rightarrow$ $\theta = 60^o$

9. $$\frac{2\cos 67^\circ}{\sin 23^\circ}-\frac{\tan 40^\circ}{\cot 50^\circ}-\cos 0^\circ=\frac{2\cos\left(90^\circ-23^\circ\right)}{\sin 23^\circ}-\frac{\tan\left(90^\circ-50^\circ\right)}{\cot 50^\circ}-\cos 0^\circ$$
$$=\frac{2\sin 23^\circ}{\sin 23^\circ}-\frac{\cot 50^\circ}{\cot 50^\circ}-\cos 0^\circ=2\times1-1-1=0$$

10. LHS = $\tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ$
$= \tan\left(90^\circ-80^\circ\right)\tan\left(90^\circ-75^\circ\right)\tan 75^\circ \tan 80^\circ$
$= \cot 80^\circ \cot 75^\circ \tan 75^\circ \tan 80^\circ$ $\left(\because \tan\left(90^\circ-\theta\right)=\cot\theta\right)$
$=\left(\cot 80^\circ \tan 80^\circ\right)\left(\cot 75^\circ \tan 75^\circ\right)=1\times1=1=\text{RHS}$ $\left(\because \cot\theta.\tan\theta=1\right)$

11. $\log_{10}(x + 3) = -\log_{10}2 = \log_{10}2^{-1} = \log_{10}(1/2)$
$\therefore$ $x + 3 = ½$
$\therefore$ $x = ½ - 3 = -2\frac{1}{2}$

12. $$\log_2 5\cdot\log_{25}8=\frac{\log_k 5}{\log_k 2}\cdot\frac{\log_k 8}{\log_k 25}$$
$$=\frac{\log_k 5}{\log_k 2}\cdot\frac{\log_k 2^3}{\log_k 5^2}=\frac{\log_k 5}{\log_k 2}\cdot\frac{3\log_k 2}{2\log_k 5}$$
$$=\frac{3}{2}\text{ which is }<2$$