WS-2 Principle of homogeneity

Thrash

① Given that $F = at^{-1} + bt^{2}$

Acc to homogeneity principle

$[at^{-1}] = [F]$; $[bt^{2}] = [F]$

$\Rightarrow [a][t]^{-1} = [F]$; $[b][t]^{2} = MLT^{-2}$

$\Rightarrow [a]\,\cancel{T^{-1}} = MLT^{\cancel{-2}}$; $[b]\,T^{2} = MLT^{-2}$

$\Rightarrow [a] = MLT^{-1}$; $[b] = \dfrac{MLT^{-2}}{T^{2}}$

$[b] = MLT^{-4}$

③ The velocity of a body is given by $v = \frac{A}{B}(1 - e^{-Bt})$

exponential term are dimension less quantities.

$\therefore$ $[e^{-Bt}] = M^{0}L^{0}T^{0}$

$\Rightarrow [Bt] = 1 \Rightarrow [B][t] = 1$

$\Rightarrow [B]\,T = 1$

$\Rightarrow [B] = \frac{1}{T} = T^{-1}$

④ Given $s = a + bt + ct^{2}$

where 's' is displacement

According to homogeneity Principle.

$\Rightarrow [a] = [s]$; $[bt] = [s]$; $[ct^{2}] = [s]$

$\Rightarrow [a] = L$. $\Rightarrow [b][t] = [s]$: $[c][t^{2}] = [s]$

$\Rightarrow [b]\,T = L$; $[c]\,T^{2} = L$

$\Rightarrow [b] = LT^{-1}$; $[c] = LT^{-2}$.

⑥ Given $v = \frac{A}{B}[1 - e^{-Bt}]$ Acc to homogeneity rule

$[\frac{A}{B}] = [v]$ From Problem 3 we have $[B] = T^{-1}$

$\therefore$ $\frac{[A]}{[B]} = LT^{-1} \Rightarrow \frac{[A]}{T^{-1}} = LT^{-1} \Rightarrow [A] = LT^{-1}T^{-1}$

(17)

(7) Given $F = A\cos Bx + c\sin Dt$

All trigonometric functions are Dimensionless quantities

$\therefore [\cos Bx] = M^0L^0T^0$; $[\sin Dt] = M^0L^0T^0$

$\Rightarrow [Bx] = 1$; $[Dt] = 1$

$\Rightarrow$ ~~[B] [x]~~ ; $[D]$ ~~[t]~~ $T = 1$

$[B]\ L = 1$

$\Rightarrow [B] = L^{-1}$; $[D] = T^{-1}$

Acc to homogeneity Principle

$\therefore [A\cos Bx] = [F]$ $\qquad$ $[c\sin Dt] = [F]$

$\Rightarrow [A][\cos Bx] = MLT^{-2}$ $\qquad$ $\Rightarrow [c][\sin Dt] = MLT^{-2}$

$\Rightarrow [A] = MLT^{-2}$ $\qquad$ $\Rightarrow [c] = MLT^{-2}$

$\therefore \frac{A}{c} = \frac{\cancel{MLT^{-2}}}{\cancel{MLT^{-2}}} = 1 = M^0L^0T^0$

$\frac{D}{B} = \frac{T^{-1}}{L^{-1}} = LT^{-1}$ i.e M^0LT^{-1}

(9) Given $s = \frac{1}{2}ft^3$ where 's' is displacement 't' is time.

Any Numerical has no dimensions.

$\therefore [s] = [ft^3] \Rightarrow [s] = [f][t]^3$

$\Rightarrow L = [f]T^3$

$\Rightarrow [f] = LT^{-3}$ (or) M^0LT^{-3}

(10) Given $P = \frac{A}{B}\log(Bx^2 + c)$

logarithmic functions are dimensionless quantities

ie) $[Bx^2 + c] = M^0L^0T^0$

$\Rightarrow [Bx^2] = M^0L^0T^0$ and $[c] = M^0L^0T^0$

Because acc to homogeneity Principle two quantities can be added (or) subtracted if they have same dimensio

$\therefore [B][x^2] = 1$

$\Rightarrow [B] L^2 = 1 \Rightarrow [B] = L^{-2}$.

and $\left[\frac{A}{B}\right] = [P]$ where 'p' is Pressure

$[P] = ML^{-1}T^{-2}$

$\Rightarrow \frac{[A]}{[B]} = ML^{-1}T^{-2}$

$\Rightarrow [A] = [B] ML^{-1}T^{-2} \Rightarrow [A] = L^{-2} ML^{-1}T^{-2}$

$\Rightarrow [A] = ML^{-3}T^{-2}$

(12) Given $P = P_0 e^{-\alpha t^2}$

Here exponential terms are dimensionless quantities

$\therefore [\alpha t^2] = M^0 L^0 T^0$

$\Rightarrow [\alpha][t^2] = 1 \Rightarrow [\alpha] T^2 = 1$

$\Rightarrow [\alpha] = T^{-2}$

(14) Given equation $v = at^2 + b$

Acc to homogeneity Principle. $[at^2] = [v]$

$[a][t^2] = LT^{-1}$

$\Rightarrow [a] T^2 = LT^{-1}$

$\Rightarrow [a] = LT^{-3}$

unit of $a = m\ sec^{-3}$

and $[b] = [v]$

$[b] = LT^{-1}$

So unit for $b = m\ sec^{-1}$

$\therefore \frac{a}{b} = \frac{\cancel{m}\ sec^{-3}}{\cancel{m}\ \cancel{sec^{-1}}} = sec^{-2}$

(15) Given velocity $v = at + \frac{b}{t+c}$

Acc to homogeneity Principle two quantities can be added or subtracted if they have same dimensions.

i.e) Here t & c are adding so

$[c] = [t] \Rightarrow [c] = T$

$[at] = [v]$

$\Rightarrow [a][t] = LT^{-1}$

$\Rightarrow [a] T = LT^{-1}$

$\Rightarrow [a] = LT^{-2}$

$\left[\frac{b}{t+c}\right] = [v]$

$\frac{[b]}{[t+c]} = [v]$

$\frac{[b]}{T} = LT^{-1}$

$\Rightarrow [b] = LT^{-1}T$

$\Rightarrow [b] = L$

(16) Given $x = at + bt^2$

According to homogeneity Principle

$[at] = [x]$; $[bt^2] = [x]$

$\Rightarrow [b][t^2] = [x]$

$\Rightarrow [b] T^2 = L$

$\Rightarrow [b] = LT^{-2}$ so unit of b is ~~m/se~~

km/sec^2

(17) Given $[P + \frac{a}{v^2}][V - b] = \text{constant}$.

[Pressure] $= ML^{-1}T^{-2}$

[volume] $= L^3$

Acc to homogeneity Principle two quantities can be added or subtracted if they have same dimensions.

[velocity] $= LT^{-1}$

$\therefore [\frac{a}{v^2}] = [P]$

$\Rightarrow \frac{[a]}{[v]^2} = [P] \Rightarrow [a] = [P][v]^2$

$= ML^{-1}T^{-2}(LT^{-1})^2$

$= ML^{-1}T^{-2}L^2T^{-2}$

$[a] = MLT^{-4}$

$\therefore$ unit of a is kg m/sec^4

(18) Given equation $F = at + bt^2$

[Force] $= MLT^{-2}$

Acc to homogeneity Principle

$[at] = [F]$

$\Rightarrow [a][t] = [F]$

$\Rightarrow [a] T = MLT^{-2}$

$\Rightarrow [a] = MLT^{-3}$

LTask SAQs

① Given equation $\omega = as + \frac{b}{(c-s)^2}$

Acc to homogeneity Principle two quantities can be added (or) subtracted if they have same dimensions.

∴ Here c & s are subtracting means

$[c] = [s] \Rightarrow [c] = L.$

and also $[as] = [\omega]$; $[\frac{b}{(c-s)^2}] = [\omega]$

$\Rightarrow [a][s] = [\omega]$; $\frac{[b]}{[(c-s)]^2} = [\omega]$

$\Rightarrow [a] \cancel{L} = M L^{\cancel{2}} T^{-2}$; $\frac{[b]}{L^2} = M L^2 T^{-2}$

$\Rightarrow [a] = M L T^{-2}$

$\Rightarrow [b] = M L^2 T^{-2} L^2$

$\Rightarrow [b] = M L^4 T^{-2}$

③ Given, Pressure $P = \frac{(A-t^2)}{BS}$

∴ From homogeneity Principle two quantities can be added (or) subtracted when they have same dimensions. Here A & t^2 are subtracting means

$[A] = [t^2] \Rightarrow [A] = T^2$

④ Given equation $Y = A \sin(kt - \frac{x}{\lambda})$

Any trigonometric function is dimensionless quantity.

∴ $[\sin(kt - \frac{x}{\lambda})] = M^0 L^0 T^0$

∴ $[kt] = M^0 L^0 T^0 \Rightarrow [k][t] = M^0 L^0 T^0$

$\Rightarrow [k] T = M^0 L^0 T^0$

$\Rightarrow [k] = M^0 L^0 T^{-1}$

(5) The velocity of the body is $V = \frac{P}{Q}(1 - e^{Qt})$

i.e) $V = \frac{P}{Q} - \frac{P}{Q} e^{Qt}$

$\therefore$ Acc to homogeneity Principle

$$[\frac{P}{Q}] = [V]$$

$$\Rightarrow [\frac{P}{Q}] = M^0 LT^{-1}$$

(6) Here Pressure $P = \frac{A - x^2}{Bt}$

Here A & x^2 are substracting. This is possible only if they have same dimensions.

$\therefore$ $[A] = [x^2] \Rightarrow [A] = L^2$ (or) $M^0 L^2 T^0$

(7) Given force $= \frac{x}{density} + y$

From homogeneity Principle $\frac{[x]}{[density]} = [Force]$

$$\Rightarrow \frac{[x]}{ML^{-3}} = MLT^{-2}$$

$$\Rightarrow [x] = ML^{-3} MLT^{-2}$$

$$\Rightarrow [x] = M^2 L^{-2} T^{-2}$$

and $[Y] = [force] \Rightarrow [Y] = MLT^{-2}$

(9) Given $S = A(1 - e^{-Bxt})$

Any exponential term is dimensionless quantity.

i.e) $[e^{-Bxt}] = M^0 L^0 T^0$

$\Rightarrow [Bxt] = 1 \Rightarrow [B][x][t] = 1$

$\Rightarrow [B] LT = 1$

$\Rightarrow [B] = L^{-1} T^{-1}$

So the unit of B is $m^{-1} s^{-1}$