VECTORS

LEARNING OBJECTIVES:

- Basic trigonometry
- Different types of vectors
- Components, rectangular components
- Addition rules for vectors
- Lami's theorm, Tangent law
- Dot product and Cross product

Real life applications:

Φ Vectors are mathematical constructs that include a length and a direction. They can exist in any number of dimensions. Because of this, they are used to simply yet effectively convey information about objects or situations.

Φ One of the most common uses of vectors is in the description of velocity.

Φ By using vectors, physicists describe the movement of a car in motion using a simple line on a geometric plane. This same principle is also applied by navigators to chart the movements of airplanes and ships.

Φ Vectors are also used to plot trajectories. The movements of any thrown object, such as a football, can be mapped with vectors.

Φ Using multiple vectors allows for the creation of a model that encompasses external forces like the wind.

Φ By utilizing vector addition on these different forces, mathematicians create an accurate estimate of the path of motion and distance traveled by the object

Important Formula:

1. A vector have both magnitude and direction
2. Maximum angle between two vectors is 180^0 and minimum is zero
3. If $\vec{A} = xi + yj + zk$, then $\left|\vec{A}\right| = \sqrt{x^2 + y^2 + z^2}$
4. Any vector $\vec{P}$ makes an angl θ with X- axis can be written as $\vec{P} = P\cos\theta\vec{i} + P\sin\theta\vec{j}$
5. If $\vec{a}$ & $\vec{b}$ acting at an angl θ then magnitude of their resultant is given by

   $\sqrt{a^2 + b^2 + 2ab\cos\theta}$ and dierectioin of resultant $\alpha = \tan^{-1}\left(\frac{b\sin\theta}{a + b\cos\theta}\right)$.
6. To find resultant parallalogram law, triangle law, polygon law, lami's theorm, tangent law are used.
7. Any two vecots $\vec{a}$ & $\vec{b}$ acting θ, then their dot product is given by $\vec{a}.\vec{b} = ab\cos\theta$
8. Any two vecots $\vec{a}$ & $\vec{b}$ acting θ, then their cross product is given by $\vec{a}\times\vec{b} = ab\sin\theta\vec{n}$

Worksheet-1

A physical quantity which requires two parameters namely magnitude and direction for its complete specification is known as vector. eg: force, displacement, velocity etc.

¶¶ **Representation**

A vector is graphically represented by a directed line segment. The length of the line segment is proportional to the magnitude of vector and arrow points along the direction.

¶¶ **Modulus of a vector**

The length or magnitude of a vector is known as the modulus of vector. If $\bar{a}$ is a vector, then length of magnitude is denoted by $|\bar{a}|$. This is read as Mod $\bar{a}$

§§ **TYPES OF VECTORS**

1. **EQUAL VECTORS:** Two vectors are said to be equal when their magnitude and direction are equal.
2. **NEGATIVE VECTOR:** Negative vectors are those which are equal in magnitude but opposite in direction.
3. **NULL VECTOR or ZERO VECTOR:** It is a vector whose magnitude zero and direction is unspecified.

   Ex: a) Displacement after one complete revolution.

   b) Velocity of vertically projected body at the highest point.
4. **UNIT VECTOR :** It is a vector whose magnitude is unity. A unit vector parallel to a given vector $\vec{r}$ is given by $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$
5. **REAL VECTOR or POLAR VECTOR :** If the direction of a vector is independent of the coordinate system, then it is called a polar vector.

   Ex: linear velocity, linear momentum, force etc.
6. **PSEUDO VECTOR or AXIAL VECTOR :** Vectors associated with rotation about an axis and whose direction is changed when the co-ordinate system is changed from left to right, are called pseudo vectors.

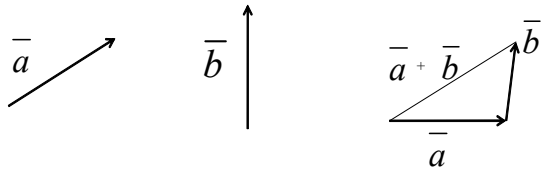
   Example : Torque, Angular momentum, Angular velocity etc.
7. **POSITION VECTOR:** It is a vector that represents the position of a particle with respect to the origin of a co-ordinate system. The Position Vector of a point (x, y, z) is $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

§§ **POSITION VECTOR IN 3D PLANE**

1. A point in space can be specified by a position vector $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$, x, y and z being the coordinates of the point in cartesian coordianate system.
2. Magnitude of position vector is given by $|\vec{R}| = \sqrt{x^2 + y^2 + z^2}$
3. The unit vector along $\vec{R}$ is given by $\hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$

¶¶ **Basic addition of vectors**

To add two vectors, one vector is drawn to scale and the tail of second vector is made to coincide with the head of first vector without changing its orientation.



Then the line joining the tail of first vector and the head of last vector represents the resultant both in mangnitude and direction.

¶¶ Laws of Vector Addition:

Vector addition follows commutative,associative and distributive laws.

a) Commutative law : $\bar{A} + \bar{B} = \bar{B} + \bar{A}$

b) Associative law : $\bar{A} + (\bar{B}+\bar{C}) = (\bar{A} + \bar{B}) + \bar{C}$

c) Distributive law : $m(\bar{A} + \bar{B}) = m\bar{A} + m\bar{B}$ where m is a scalar

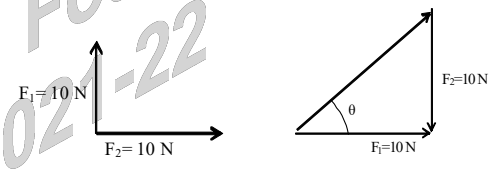
¶¶ Subtraction of a vector from another vector

The subtraction of one vector from the other is a special case of addition of two vectors in which one vector is compounded with the negative of another.

$\bar{a} - \bar{b} = \bar{a} + (-\bar{b})$

Example: Two forces F_1 and F_2 are acting on a body as shown in figure

find the resultant force vector



Solution: The magnitude of resultant force = $\sqrt{10^2 + 10^2}$ =10$\sqrt{2}$ N

This makes an angle 45^0 with $\overline{F_1}$

Note: 1) Subtraction of vectors is used to find

i) change in velocity of a particle.

ii) the relative velocity of one body with respect to another body.

2) Subtraction of vectors follows only distributive law.

¶¶ Angle between vectors

In order to find the angle between two vectors, two vectors must have common starting point (tail) or common end point (point)

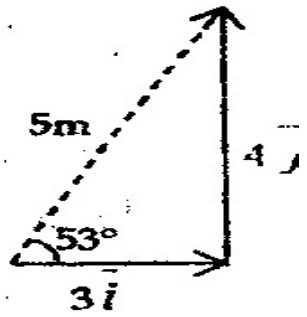
The lesser of the two angles between them is adopted as angle between the two vectors

$0^0 \le \theta \le 180^0$

EXAMPLE

Example:1 The displacement of a particle is $\bar{S} = 3\bar{i} + 4\bar{j}$ what is the magnitude of displacement

solution: displacement = $\sqrt{3^2+4^2}$ = 5m $\quad \tan\theta = \frac{4}{3}$



Example:2 Find the modulus of vector $\bar{i} + 2\bar{j}$

solution: $\left|\bar{i} + 2\bar{j}\right| = \sqrt{1^2 + 2^2} = \sqrt{5}$

example:3 Find the modulus of $2\bar{i}+3\bar{j}$

solution: $\left|2\bar{i} + 3\bar{j}\right| = \sqrt{2^2 + 3^2} = \sqrt{13}$

Example: 4 Find the unit vector along $\bar{i} + \bar{j}$

Solution: $\hat{\bar{a}} = \frac{\bar{a}}{|\bar{a}|} = \frac{\bar{i}+\bar{j}}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}\right)\bar{i} + \left(\frac{1}{\sqrt{2}}\right)\bar{j}$

Example: 5 Find the unit vector along the direction $\bar{i} + \sqrt{3}\bar{j}$

solution: $\hat{\bar{a}} = \frac{\bar{a}}{|\bar{a}|} = \frac{\bar{i}+\sqrt{3}\bar{j}}{\sqrt{1+3}} = \left(\frac{1}{2}\right)\bar{i} + \left(\frac{\sqrt{3}}{2}\right)\bar{j}$

Example:6 Find the value of x if the vector $0.8\hat{\bar{i}} + x\,\hat{\bar{j}}$ represents a unit vector

solution: $\hat{\bar{a}} = \frac{8}{10}\hat{\bar{i}} + x\,\hat{\bar{j}} \therefore \left|\hat{\bar{a}}\right| = 1$

$\sqrt{(0.8)^2 + x^2} = 1$ (or) $x^2 = 1 - 0.64 = 0.36 \Rightarrow x = \pm 0.6$

Example:7 Find the unit vector along the resultant of $\bar{i} + 2\bar{j}$ *and* $2\bar{i} + \bar{j}$

$$\text{Resultant}\left(\bar{R}\right)=\bar{i}+2\bar{j}\ +\ 2\bar{i}+\bar{j}=3\bar{i}+3\bar{j}$$

solution: $$=\frac{\bar{R}}{R}=\frac{3\bar{i}+3\bar{j}}{\sqrt{3^2+3^2}}=\frac{3\bar{i}+3\bar{j}}{3\sqrt{2}}=\left(\frac{1}{\sqrt{2}}\right)\bar{i}+\left(\frac{1}{\sqrt{2}}\right)\bar{j}$$

Example:8 Find the unit vector along the resultant of the vectors $\bar{a}=\bar{i}-\bar{j},\ \bar{b}=\bar{i}+2\bar{j},\ \bar{c}=4\bar{i}-\bar{j}$

solution: $\bar{R}=\bar{i}-\bar{j}+\bar{i}+2\bar{j}+4\bar{i}-\bar{j}=6\bar{i}$

$$\hat{\bar{R}}=\frac{\bar{R}}{R}=\frac{6\bar{i}}{\sqrt{6^2}}=\bar{i}$$

Example: 9 Find the vector $\bar{a}$ if the resultant of $\bar{a},\bar{b}=4\bar{i}+2\bar{j}$ and $\bar{c}=2\bar{i}-\bar{j}$ is $\bar{0}$

solution: $\bar{a}+4\bar{i}+2\bar{j}+2\bar{i}-\bar{j}=\bar{0}\Rightarrow\bar{a}=-6\bar{i}-\bar{j}$

Example: 10 $\bar{a}=\bar{i}-2\bar{j},\bar{b}=2\bar{i}+\bar{j},\bar{c}=x\bar{i}+y\bar{j}$ find $\hat{\bar{c}}$ if the resultant of $\bar{a},\bar{b}$ *and* $\bar{c}$ is $\bar{0}$

solution: $\bar{c}=-\left(\bar{a}+\bar{b}\right)\quad\bar{c}=-\left(2\bar{i}+\bar{j}+\bar{i}-2\bar{j}\right)$

$$\bar{c}=-3\bar{i}+\bar{j}$$

$$\hat{\bar{c}}=\frac{\bar{c}}{|\bar{c}|}\Rightarrow\hat{\bar{c}}=\frac{-3\bar{i}+\bar{j}}{\sqrt{3^2+1^2}}\Rightarrow\hat{\bar{c}}=\frac{-3\bar{i}+\bar{j}}{\sqrt{10}}$$

TEACHING TASK

Choose the correct option:

1. If $0.5\bar{i}+0.8\bar{j}+c\vec{k}$ is a unit vector, then C is

1) $\sqrt{89}$ 2) 0.2 3) 0.3 4) $\sqrt{0.11}$

2. If $\vec{P}=\bar{i}+3\bar{j}-7\vec{k}$ and $\vec{Q}=5\vec{i}-2\vec{j}+4\vec{k}$ then the length of the vector $\overline{PQ}$ is

1) $\sqrt{104}$ 2) $\sqrt{162}$ 3) -29 4) 20

3. If a body starts with a velocity $2\hat{i}-3\hat{j}+11\hat{k}\ ms^{-1}$ and moves with an accelaration of $10\hat{i}+10\hat{j}+10\hat{k}\ ms^{-2}$. then its velocity after 0.25s

1) $\frac{1}{2}\sqrt{811}ms^{-1}$ 2) $\sqrt{\frac{811}{2}}ms^{-1}$ 3) $\sqrt{811}ms^{-1}$ 4) $2\sqrt{811}ms^{-1}$

4. The value of 'm', if $\vec{i}+2\vec{j}-3\vec{k}$ is parallel to $3\vec{i}+m\vec{j}-9\vec{k}$ is

1) 12 2) 9 3) 6 4) 3

5. A fore $2\hat{i}+\hat{j}-\hat{k}$ newton acts on a body which is initially at rest. If the velocity of the body at the end of 20seconds is $4\hat{i}+2\hat{j}+2\hat{k}\ ms^{-1}$, the mass of the body

1) 20kg 2) 15kg 3) 10kg 4) 5kg

6. Two vectors $\overline{A}$ and $\overline{B}$ are related as $\overline{A}-2\overline{B}=-3\left(\overline{A}+\overline{B}\right)$. If $\overline{A}=6\,\bar{i}-2\,\bar{k}$ then $\overline{B}=$

1) $-24\,\bar{i}+8\,\bar{k}$ 2) $8\,\bar{i}-24\,\bar{k}$ 3) $2\,\bar{k}-6\,\bar{i}$ 4) $2\,\bar{k}+6\,\bar{i}$

Assertion A and Reason R:

◆ *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.*

7. **A :** A physical quantity is not called as a vector if its magnitude is zero

R : A vector has both magnitude and direction

A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

B) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'

C)'A' is true and 'R' is false

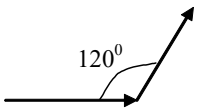
D) 'A' is false and 'R' is true

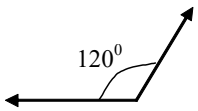
Match the following:

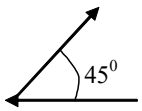
◆ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

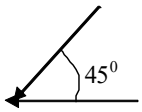
*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

8. Match by finding angle between the given vectors

a) 

b) 

c) 

d) 

1) 60^0 2) 120^0 3) 135^0 4) 45^0

A) a-1, b-2, c-3, d-4 B) a-4, b-3, c-2, d-1

C) a-2, b-3, c-1, d-4 D) a-1, b-2, c-4, d-1

Comprehention type:

◆ *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

9. If $\vec{A}$ and $\vec{B}$ are two vectors acting at an angle θ then find the angle between

a) $\vec{A}$ and $\vec{A}$ b) $\vec{B}$ and $\vec{B}$ c) $\vec{A}$ and $-\vec{B}$ d) $-\vec{A}$ and $\vec{B}$

e) $3\vec{A}$ and $\vec{A}$ f) $\vec{A}$ and $2\vec{B}$ g) $2\vec{A}$ and $3\vec{B}$ h) $-\vec{A}$ and $2\vec{B}$

i) $-\vec{A}$ and $-\vec{B}$ j) $-2\vec{A}$ abd $-\vec{B}$ k) $-3\vec{A}$ and $\vec{B}$ l) $-3\vec{A}$ and $-2\vec{B}$

Solve the following:

10. A motor cyclist drives his vehicle 8 km towards west then 5 km towards north and then 4 km towards west. What is his total displacement in entire journey.

11. Alex goes cruising on his dirt bike. He rides 700 m north, 300 m east, 400 m north, 600 m west, 1200 m south 300 m east and finally 100 m north. What was his displacement?

LEARNER'S TASK

BEGINNERS (Level - I)

Choose the correct option:

1. If $\bar{P} = \bar{i} - 2\,\bar{j} - 2\,\bar{k}$, the magnitude of $\vec{P}$ is

1)1 2) 2 3) 3 4) 9

2. The modulus of a vector $\bar{A} = 5\bar{i} + p\,\bar{j} + 4\sqrt{2}\,\bar{k}$ is 11. The value of p is

1) $+8$ 2) -8 3) ± 8 4) 6

3. A particle has a displacement of 12m towards east then 5m towards north and then 6m vertically upwards. The resultant displacement is

1) 10.04 m 2) 12.10 m 3) 14.32 m 4) 12.6 m

4. A force $\bar{F} = 6\,\bar{i} - 8\,\bar{j} + 10\,\bar{k}$ N Produces and acceleration of 1m/s² in a body. The mass of body would be

1) 200 kg 2) 20kg 3) $10\sqrt{2}kg$ 4) $6\sqrt{2}kg$

5. A vector has co-ordinates (1,1,2) for initial point and (2,1,3) as the terminal point. The length of the vector is

1) $\sqrt{2}$ 2) $\sqrt{34}$ 3) 2 4) $\sqrt{6}$

6. A vector is represented by $3\,\bar{i} + \bar{j} + 2\,\bar{k}$. Its length in xy plane is

1) 2 2) $\sqrt{14}$ 3) $\sqrt{10}$ 4) $\sqrt{5}$

7. If position vector is given by $\bar{R}$ = (-6,-4,-12) then the unit vector parallel to $\bar{R}$ is

1) $+\frac{1}{7}(3\mathrm{i}+2\bar{\mathrm{j}}+6\bar{\mathrm{k}})$ 2) $-\frac{1}{7}[3i + 2\,\bar{j} + 6\,\bar{k}]$ 3) $\pm\frac{1}{7}[3i + 2\,\bar{j} + 6\,\bar{k}]$ 4) $\frac{14}{[6i + 4\,\bar{j} + 12\,\bar{k}]}$

8. A man travels 1 mile due east, than 5 miles due south, then 2 miles due east and finally 9 miles due north. His displacement is

1) 3 miles 2) 5 miles 3) 4 miles 4) between 5 and 9 miles

ACHIEVERS (Level - II)

Solve the following:

1. A person walks 3m towards east and then 4m towards north. Find the displacement of the person.

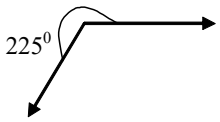
2. A boy walks 12m towards west and then 5m towards south. Find the displacement of the boy.

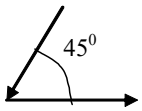
3. A car travels 10km towards south and then 24km towards east. Find the displacement of

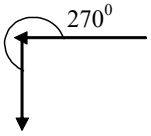
the car.

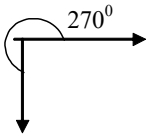
4. A train travels 60km towards north and then 80km towards west. Find the displacement of the car.

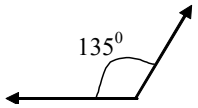
5. Find the angle between two vectors $\overline{A}\,\&\,\overline{B}$ as shown in the figures

a) 

b) 

c) 

d) 

e) 

6. What is the maximum possible angle between two vectors?

7. What is the minimum possible angle between two vectors?

8. The angle between two vectors is θ. If the direction of one of the vectors is reversed then the angle between the vectors becomes.

9. The angle between two vectors is θ. If the directions of both of the vectors are reversed then the angle between the vectors becomes

10. If $\vec{A}$ and $\vec{B}$ are two vectors acting at an angle 30^0 then find the abgle between

a) $\vec{A}$ and $\vec{A}$ b) $\vec{B}$ and $\vec{B}$ c) $\vec{A}$ and $-\vec{B}$ d) $-\vec{A}$ and $\vec{B}$

e) $3\vec{A}$ and $\vec{A}$ f) $\vec{A}$ and $2\vec{B}$ g) $2\vec{A}$ and $3\vec{B}$ h) $-\vec{A}$ and $2\vec{B}$

i) $-\vec{A}$ and $-\vec{B}$ j) $-2\vec{A}$ abd $-\vec{B}$ k) $-3\vec{A}$ and $\vec{B}$ l) $-3\vec{A}$ and $-2\vec{B}$

EXPLORERS (Level - III)

Assertion A and Reason R:

*This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.*

A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

B) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'

C)'A' is true and 'R' is false

D) 'A' is false and 'R' is true

1. **A**:-A null vector is a vector whose magnitudeis zero and direction is arbitray

R:-A null vector does not exist..

2. **A:-** $\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}$ is a unit vector

R:- The component vectors of a unit vector need not be unit vectors..

3. **A:** Electriccurrent isa vector

R: A physical quantity having magnitude and direction should be always a vector.

Multiple option type:

*This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D),out of which **ONE or MORE** is correct. Choose the correct options*

4. Which of the following is a null vector

a) velocity vector of a body moving in a circle with a uniform speed

b) velocity vector of a body moving in a straight line with a uniform speed

c) position vector of the origin of the rectangular co-ordinate system

d) displacement vector of a stationary object

A) both a & b B) both b & c C) a, b & c D) both c & d

5. Two vectors of same physical quantity are unequal if

a) They have the same magnitude and same direction

b) They have different magnitudes but same direction

c) They have same magnitude but different directions

d) They have different magnitudes and different directions

A) a & b are true B) b, c & d are true C) only c and d are true D) a, b, c & d are true

Match the following:

◆ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

6.

List-1	List-2
a.Force	1. Scalar
b.Mass	2. Axial Vector
c.Moment of Interia	3. Vector
d.Angular Momentum	4. Tensor

A) a-3,b-1,c-4,d-2 B)a-1,b-3,c-4,d-2 C) a-3,b-1,c-2,d-4 D)a-1,b-3,c-2,d-4

7.

Column-I	Column-II
a) Equal vectors	1) same magnitude
b) Like vectors	2) same direction
c) Unlike vectors	3) different magnitude
d) Negative of a vector	4) opposite direction

A) a-1,2 b-2,3 c-3,4 d-1,4 B)a-1,4 b-3,4 c-1,4 d-2,3

C) a-3,4 b-1,3 c-2,4 d-1,4 D)a-1,2 b-3,4 c-2,3 d-1,4

Comprehention type:

◆ *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

8. If $\vec{a} = 4\vec{i} - 3\vec{j}$ and $\vec{b} = 8\vec{i} - 6\vec{j}$ then choose the correct magnitudes for the below

i) Magnitude of $\vec{a} + \vec{b}$ is

A) 5 B) 10 C) 15 D) $\sqrt{5}$

ii) Magnitude of $\vec{a} - \vec{b}$

A) 5 B) 10 C) 15 D) $\sqrt{5}$

iii) Magnitude of $\vec{b}-\vec{a}$

A) 5 B) 10 C) 15 D) $\sqrt{5}$

High order thinking skills (HOTS):

9. If $\bar{A}=3\bar{i}+5\bar{j}-2k$, and vector $\bar{B}=-3\bar{j}+6\bar{k}$. Find a vector $\bar{C}$ such that $2\bar{A}+7\bar{B}+4\bar{C}=0$

1) $-6\bar{i}+11\bar{j}-38\bar{k}$ 2) $11\bar{j}-4\bar{k}$ 3) $-1.5\bar{i}+2.75\bar{j}-9.5\bar{k}$ 4) $4\bar{j}-6\bar{k}$

10. If $\bar{A}=3\bar{i}+4\bar{j}$ and $\bar{B}=7\bar{i}+24\bar{j}$, then the vector having the same magnitude as that of $\bar{B}$ and parallel to $\bar{A}$ is

1) $15\bar{i}+20\bar{j}$ 2) $15\bar{i}-20\bar{j}$ 3) $20\bar{i}+15\bar{j}$ 4) $20\bar{i}-15\bar{j}$

11. The unit vector parallel to the resultant of vectors $\bar{A}=4\bar{i}+3\bar{j}+6\bar{k}$ and $\bar{B}=-\bar{i}+3\bar{j}-8\bar{k}$ is

1) $\frac{1}{7}(3\bar{i}+6\bar{j}-2\bar{k})$ 2) $-\frac{1}{7}(3\bar{i}+6\bar{j}-2\bar{k})$ 3) $\pm\frac{1}{7}(3\bar{i}+6\bar{j}-2\bar{k})$ 4) $\frac{1}{47}(3\bar{i}+6\bar{j}-2\bar{k})$

KEY

ΦΦ TEACHING TASK :

1) 4, 2) 2, 3) 1, 4) 3, 5) 3, 6) 1, 7) 4, 8) A, 9) 0^0, b) 0^0, c) 180-θ, d) 180-θ, e) 0^0, f) θ, g) θ, h) 180-θ, i) θ, j) θ, k) 180-θ, l) θ, 10) 13 km, 11) zero

ΦΦ LEARNER'STASK :

☐ **BEGINNERS :**

1) 3, 2) 3, 3) 3, 4) 3, 5) 1, 6) 3, 7) 3, 8) 2

☐ **ACHIEVERS :**

1) 5 m, 2) 13 m, 3) 26 km, 4) 100 km, 5) a) 135^0, b) 135^0, c) 90^0, d) 90^0, e) 135^0, 6) 180^0, 7) 0^0, 8) 180-θ, 9) θ, 10) 0^0, b) 0^0, c) 150^0, d) 150^0, e) 0^0, f) 30^0, g) 30^0, h) 150^0, i) 30^0, j) 30^0, k) 150^0, l) 30^0

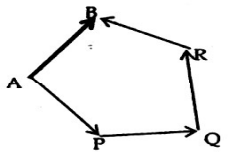
☐ **EXPLORERS :**

1) C, 2) B, 3) D, 4) D, 5) B, 6) C, 7) A, 8) i) C, ii) A, iii) A 9)3, 10)1, 11)3

Worksheet-2

¶¶ Components of a vector

Any vector can be expressed as the vector sum of some other vectors known as component vectors



eg: $\overline{AB}=\overline{AP}+\overline{PQ}+\overline{QR}+\overline{RB}$

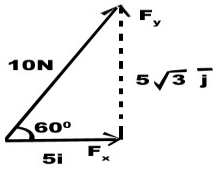
The maximum number of vector components that a given vector can be resolved into is infinite.

Illustration

A force of magnitude 10N is acting at an angle 60^0 with axis as shown in figure. If $\overline{F_x}$, $\overline{F_y}$ are the component vectors along X & Y directions. Then the given vector can be expressed as

$\overline{F_x} + \overline{F_y} = 10\cos 60^0 \bar{i} + 10\sin 60\, \bar{j}$

= 10 X $\frac{1}{2}\,\bar{i}$ + 10 X $\frac{\sqrt{3}}{2}\bar{j}$ = $5\bar{i}+5\sqrt{3}\,\bar{j}$



where $\bar{i}$ & $\bar{j}$ are the unit vectors along x and y - axes respectively. They just indicate the direction.

example: Find the angle made by the vector $3\bar{i} + 4\bar{j}$ with x-axis

solution: $\tan\theta = 4/3$

example: Find the angle made by the vector $\bar{i} + \sqrt{3}\bar{j}$ with x-axis

solution: $\tan\theta = \sqrt{3} \qquad \theta = 60^0$

§§ **RECTANGULAR COMPONENTS** :

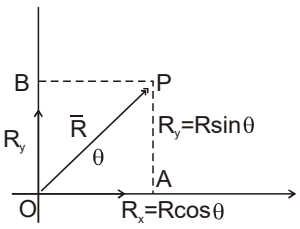
If a (plane) vector is resolved into two mutually perpendicular component vectors then these component vectors are called as the rectangular component vectors of that vector in 2 D coordinate system.

If a (spatial) vector is resolved into three mutually perpendicular component vectors then these component vectors are called as the rectangular component vectors of that vector in 3D coordinate system.

The advantage of splitting up a vector into rectangular components is we can find the magnitude of the vector by simply finding the square root of sum of the squares of the components.

§§ **RESOLUTION OF A VECTOR:**

1. A vector $(\overline{R})$ can be resolved into two mutually perpendicular components R_x and R_y.
2. The projection of $(\overline{R})$ along x-axis is called horizontal component (R_x) $R_x = R.\cos\theta$
3. The projection of $(\overline{R})$ along y-axis is called vertical component (R_y) $R_y = R.\sin\theta$



4. Magnitude of the resultant $|\bar{R}|=\sqrt{R_x^2+R_y^2}$

5. Direction of the resultant with x-axis $\theta=\tan^{-1}\left(\frac{R_Y}{R_X}\right)$

Illustration

1. The rectangular components of a vector are 3,4. Find the magnitude.

   solution: magnitude = $\sqrt{3^2+4^2}$ = 5m

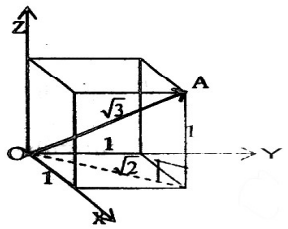
2. The rectangular components of a vector are 3, 4,12. Find the magnitude

   solution: magnitude = $\sqrt{3^2+4^2+12^2}$ =13

   $\overline{AB}=\overline{AC}+\overline{CB}$. Here $\overline{AC}$ & $\overline{CB}$ are at right angles to each other. Hence they are called as the rectangular component vectors of $\overline{AB}$

   The length of the vector $\overline{AC}$ = a

   The component of $\overline{AB}$ along X direction is 'a' .If the unit vector along X direction is taken as $\hat{i}$ then $\overline{AC}$ =a $\hat{i}$



   The length of the vector $\overline{CB}$ = b

   The component of $\overline{AB}$ along Y direction is 'b'

   If the unit vector along Y direction is taken as $\vec{j}$ then $\overline{CB}$ = b $\vec{j}$ hence $\overline{AB}$ = $a\vec{i}+b\vec{j}$

NOTE: If a,b & c are rectangular components of a given vector along X,Y & Z axes respectively then that vector can be represente d as $a\vec{i}+b\vec{j}+c\vec{k}$, where $\vec{k}$ is the unit vector along Z axis.

The vector $\overline{AB}=a\hat{i}+b\hat{j}$ can be drawn from any point in the XY plane provided it has components a & b along X and Y axes respecitvely. Similarly a spatial vector can also be drawn anywhere in the space of orthogonal triad without changing the orientation.

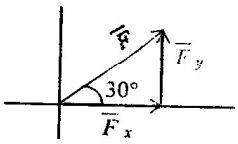
* A true vector can be slid paralld to itself. (This will be discussed later)

Example : Find the length of the body diagonal of a unit cube.

Solution : $\overline{OA}=\bar{i}+\bar{j}+\bar{k}$ $\quad OA=\sqrt{1^2+1^2+1^2}$ $\quad OA=\sqrt{3}$

Example : In the figure shown $\overline{F}_x$ & $\overline{F}_y$ are along two mutually $\perp$ directions. Express the $\overline{F}$ as sum of Rectangular Components vectors.

Solution : $\overline{F} = \overline{F_x} + \overline{F}_y$



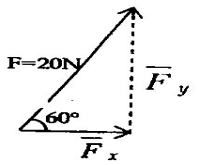
$\overline{F}$ = F cos 30^0 $\bar{i}$ +F sin 30^0 $\bar{j}$

Hence F_x & F_y are known as the rectangular components.

Example : A force of magnitude 20N is acting at an angle 60^0 with the +ve x-axis in anit - clock-wise direction as shown in figure. Express this force the vector sum of rectangular com-ponent vector.

Solution : $\overline{F} = F_x\bar{i} + F_y\bar{j}$

20N = 20 x cos60^0 $\bar{i}$ +20 x sin60^0 $\bar{j}$

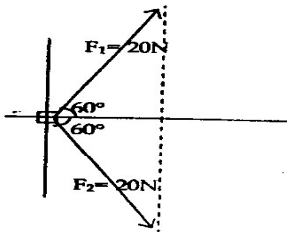


20N = 10 $\bar{i}$ + 10 $\sqrt{3}$ $\bar{j}$

Example : Two forces each of magnitude 20N are acting in horizontal plane on a particle of mass 5kg as shown in the figure. Find the acceleration of the particle.

Solution : $\overline{F_1} = 10\bar{i} + 10\sqrt{3}\bar{j}$

$\overline{F_2} = 10\bar{i} - 10\sqrt{3}\bar{j}$

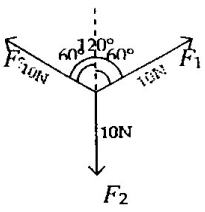


$\overline{R} = \overline{F_1} + \overline{F_2} = 20\bar{i}$

$\bar{a} = \frac{20}{5} = 4\bar{i}\, m/s^2$

Example : Three forces each of magnitude 10N are acting as shown in figure. Find the resultant force.

Solution : $\overline{F_1} = 10\cos 30\,\bar{i} + 10\sin 30\,\bar{j}$



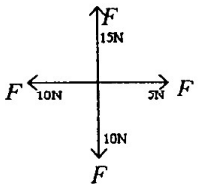
$\overline{R} = \overline{F_1} + \overline{F_2} + \overline{F_3} = 0$

Example : Find the resultant of the following

Solution : $\overline{R} = \overline{F_1} + \overline{F_2} + \overline{F_3} + \overline{F_3}$

$\overline{R} = 5\bar{i} - 10\bar{i} + 15\bar{j} - 10\bar{j}$

$\overline{R} = -5\bar{i} + 15\bar{j}$



$|\overline{R}| = \sqrt{(5)^2 + (5)^2}$

$|\overline{R}| = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}N$

Example : A boy moves 3m along positive x-axis, 4m along +ve y-axis, 5m along -ve x-axis and 2m along -ve, y-axis. Find the displacement.

Solution : $\overline{S} = 3\bar{i} + 4\bar{j} - 5\bar{i} - 2\bar{j}$

$\overline{S} = -2\bar{i} + 2\bar{j} \quad |\overline{S}| = -\sqrt{(2)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}m$

Example : A boy moves 3m eastward, $5\sqrt{2}$ m northeast and 5m west and 5m 60^0 north of west. Find the displacement.

Solution : $\overline{S} = 3\bar{i} + \left(5\bar{i} + 5\bar{j}\right) + \left(-5\bar{i}\right) + \left(-2.5\bar{i} + 2.5\sqrt{3}\,\bar{j}\right)$

$\overline{S} = 0.5\bar{i} + \left(5 + 2.5\sqrt{3}\right)\bar{j}$

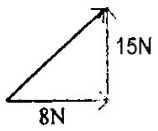
Example : A person moves 30m north, then 20m east and then $30\sqrt{2}$ m southwest. Find his displacement.

Solution : $\overline{S} = \left(30\bar{j}\right) + \left(20\bar{i}\right) + \left(-30\bar{i} + -30\bar{j}\right) \quad \overline{S} = -10\bar{j}$

Displacement = 10m west

Example : The horizontal and vertical components of a force are 8N and 15N respectively. Find the magnitude of force.

Solution : $\overline{F} = 8\bar{i} + 15\bar{j}$



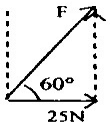
$|\overline{F}| = \sqrt{64+225} = \sqrt{289} = 17N$

Example : One of the two rectangular components of a force is 25N and it makes an angle of 60^0 with the force. Find the magnitude of other component.

Solution : F $\cos 60^0 = 25$

F = 25 x 2 = 50 N

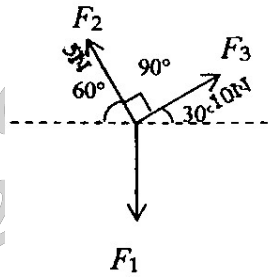
Other component = 50 sin 60^0 = $50\,50x\frac{\sqrt{3}}{2} = 25\sqrt{3}N$



Example : Find $\overline{F_1}$, if the resultant of $\overline{F_1}$, $\overline{F_2}$, $\overline{F_3}$ is zero.

Solution : $\overline{F_1}+\overline{F_2}+\overline{F_3}=0\bar{i}+o\bar{j}$

$$\overline{F_1}+5\bar{j}+5\sqrt{3}\,\bar{i}-\frac{5}{2}\bar{i}+\frac{5\sqrt{3}}{2}\bar{j}=0$$

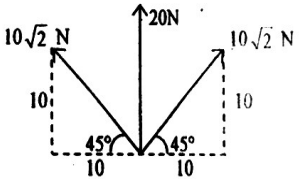


$$\overline{F_1}+\left(5+\frac{5\sqrt{3}}{2}\right)\bar{j}-\left(\frac{5}{2}-5\sqrt{3}\right)\bar{i}=0\bar{i}+0\bar{j}$$

$$\overline{F_1}+\left(-5-2.5\sqrt{3}\right)\bar{j}+\left(\frac{5}{2}-5\sqrt{3}\right)\bar{i}$$

$$|\overline{F_1}|+\sqrt{-\left(5+2.5\sqrt{3}\right)^2+\left(\frac{5}{2}-5\sqrt{3}\right)^2}$$

Example : Find the resultant of forces shown in figure



Solution: $\overline{F}=10\bar{i}+10\bar{j}-10\bar{i}+10\bar{j}+20\bar{j}$ = 40 $\bar{j}$

TEACHING TASK

Choose the correct option:

1. A room has dimensions 3mx4mx5m. A fly starting at one corner ends up at the diametrically opposite corner. The magnitude of the displacement of the fly is

1) 12m 2) 60 m 3) $2\sqrt{5}$ m 4) $5\sqrt{2}$ m

2. The horizontal and vertical components of a force are 8N and 15N respectively. The force is

1) 23 N 2) 20N 3) 17 N 4) 7 N

3. An aeroplane is heading north east at a speed of 141.4ms^{-1}. The northward component of its velocity is

1) 141.4 ms^{-1} 2) 100 ms^{-1} 3) zero 4) 50 ms^{-1}

4. The component of $\bar{A}$ along $\bar{B}$ is $\sqrt{3}$ times that of the component of $\bar{B}$ along $\bar{A}$. Then A:B is

1) $1:\sqrt{3}$ 2) $\sqrt{3}:1$ 3) $2:\sqrt{3}$ 4) $\sqrt{3}:2$

Assertion A and Reason R:

◆ *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which* ***ONLY ONE*** *is correct Choose the correct option.*

5. **A :** A vector is zero vector if any one of its compenents is zero

R : A vector in space is a sum of its three rectangular components

1) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

2) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'

3)'A' is true and 'R' is false

4) 'A' is false and 'R' is true

Multiple option type:

◆ *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D),out of which* ***ONE or MORE*** *is correct. Choose the correct options*

6. Consider the following statements A and B given below and identify the correct answer.

A) The projection of a vector gives the component.

B) The component of a vector is a scalar and can be converted to a vector components by multiplying with unit vector.

1) both A & B are correct 2) both A & B are wrong

3) A is correct but B is wrong 4) B is correct but A is wrong

Comprehension type:

◆ *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which* ***ONLY ONE i****s correct. Choose the correct option.*

7. A vector whose 2D rectangular components along X and Y the ratio 3:4

i) The vector is

A) 4i + 3j B) 3i + 4j C) i + j D) none

ii) Unit vector of the above vector is

A) (4i + 3j) / 5 B) (3i + 4j) / 5 C) (i + j) / $\sqrt{2}$ D) none

LEARNER'S TASK

BEGINNERS (Level - I)

1. If vertical component of a vector is equal to its horizontal component, the angle made by the vector with x-axis?

1) 0^0 2) 45^0 3) 90^0 4) 120^0

2. If one of the rectangular components of velocity 50ms^{-1} is 30ms^{-1}, the other component is

1) 20ms^{-1} 2) 40ms^{-1} 3) 80ms^{-1} 4) $\left(\frac{50}{\sqrt{2}} ms^{-1}\right)$

3. A force of 10N is resolved into the perpendicular components. If the first component makes 30^0 with the force the magnitudes of the components are

1) 5N, 5N 2) $5\sqrt{2}$N, 5 N 3) $5\sqrt{3}$N, 5N 4) 10 N, $10\sqrt{3}$N

4. A car weighing 100kg is on a slope that makes an angle 30° with the horizontal. The component of car's weight parallel to the slope is (g=10ms^{-2})

1) 500 N 2) 1000 N 3) 15,000 N 4) 20,000 N

ACHIEVERS (Level - II)

Solve the numericals:

1. A vector has components 3 units along X direction and 4 units along Y direction. Express the vector as sum of rectangular component vectors. What is the magnitude of the vector?

2. Find the magnitude of the vector . $\vec{A} = \vec{i} + 2\vec{j} - \vec{k}$

3. A vector is represented as $\left|3\hat{i} + 4\hat{j} + 5\hat{k}\right|$. Fine the length of the vector.

4. Find the unit vector along $3\bar{i} - 4\bar{j}$

5. Find the unit vector along the resultant of $8\bar{i} - 3\bar{j}$ *and* $4\bar{i} - 2\bar{j}$

6. Find the value of x if the 0.4i + xj represents a unit vector

7. Find the unit vector along the direction $3\bar{i} - 3\sqrt{3}\bar{j}$

8. A certain vector has rectangular compnents 5 and 3 in a reference system and in some other reference system the rectangular components are x and 2. Find X.

9. Find the vector $\bar{a}$ if the resultant vector of $\bar{a}$, $\bar{b} = 6\bar{i} - 9\bar{j}$ *and* $\bar{c} = 3\bar{i} + 7\bar{j}$ *is* $\bar{0}$

10. $\bar{a} = 4\bar{i} + 3\bar{j}$, $\bar{b} = -5\bar{i} + 2\bar{j}$, $\bar{c} = x\bar{i} + y\bar{j}$ find $\hat{\bar{c}}$ if the resultant of $\bar{a}, \bar{b}, and\ \bar{c}$ *is* $\bar{0}$

11. Find the unit vector along the resultant of the vectors $\bar{a}=6\bar{i}-7\bar{j}$, $\bar{b}=3\bar{i}-2\bar{j}$, $\bar{c}=5\bar{i}-8\bar{j}$

12. If $\hat{a}$ is the unit unit vector along $\bar{a}$ then find the angle between them

13. What is the sum of the squares of the rectangle components of the unit vector?

14. Find the unit vector along the direction of vector whose rectangular components are 3, 4 & 5 along X, Y and Z

EXPLORERS(Level - III)

Assertion A and Reason R:

This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ***ONLY ONE*** *is correct Choose the correct option.*

A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

B) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'

C)'A' is true and 'R' is false

D) 'A' is false and 'R' is true

1. **(A) :** Vector sum of three coplanar forces can be zero

(R) : Minimum number of unequal coplanar vectors required to keep a point in equilibrium is three

2. **(A) :** When a vector is rotated, its magnitude remains constant.

(R) : The magnitude of a vector is independent of the coordinate system.

3. **(A) :** The projection of $(3\hat{j}-4\hat{k})$ on the y–axis is 3 units

(R) : The projection of $\vec{\mathbf{A}}$ along y–axis is $\vec{\mathbf{A}}.\vec{\mathbf{j}}$.

4. **(A) :** A vector is described by $\bar{A}=A_x\bar{i}+A_y\bar{j}+A_z\bar{k}$ and 'A_y' is a scalar

(R) : The projection of a vector is a scalar

Multi option type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D),out of which ***ONE or MORE*** *is correct. Choose the correct options*

5. Consider the following statements A and B given below and identify the correct answer.

a) The projection of a vector gives the component.

b) The component of a vector is a scalar and can be converted to a vector components by multiplying with unit vector.

A) both a & b are correct B) both a & b are wrong

C) a is correct but b is wrong 4) b is correct but a is wrong

6. Which of the following statements is / are false ?

a) Mass, speed and energy are scalar quantities.

b) Momentum, force and torque are vector quantities.

c) Distance is a scalar quantity but displacement is a vector quantity.

d) A vector has only magnitude, where as a scalar has both magnitude and direction

A) only 'a' is false B) only 'b' is false C) only 'c' is false D) only 'd' is false

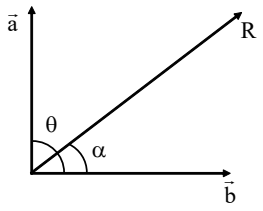
Match the following :

⬥ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

7.

LIST - I	**LIST - II**
A) Component of $\vec{a}$ along $\vec{b}$	1) $a^2 + b^2$
B) Component of $\vec{b}$ along $\vec{a}$	2) 0
C) Component of R along $\vec{a}$	3) R sin α
D) Component of R along $\vec{b}$	4) R cos α



A) a – 1, b – 2, c – 3, d – 4 B) a – 2, b – 2, c – 3, d – 4

C) c – 2, d – 4, a – 2, d – 2 D) a – 2, b – 1, c – 4, d – 3

Comprehension type:

⬥ *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

8. A vector whose 3D rectangular components along X, Y and Z are in the ratio 1:1:1

i) The vector is

A) i + j + k B) 3i + 4j + 5k C) i + j + 2k D) none

ii) Unit vector of the above vector is

A) (i + j + k) / $\sqrt{3}$ B) (3i + 4j + 5k) / $5\sqrt{2}$ C) (i + j + 2k) / $\sqrt{6}$ D) none

High order thinking skills (HOTS):

9. If the components of a force are P and P along east and north directions, the force has magnitude of

1) P 2) P/2 3) $\sqrt{2}$ P 4) 2P

10. If a vector has an **x** component of -25.0units and a **y** component of 40.0 units, then the magnitude and direction of this vector is

1) $5\sqrt{89}\ units; \sin^{-1}\frac{-5}{\sqrt{89}}$ with x-axis 2) $5\sqrt{89}\ units; \cos^{-1}\frac{-5}{\sqrt{89}}$ with x-axis

3) $45\ units; \cos^{-1}\frac{-5}{9}$ with x-axis 4) $45\ units; \sin^{-1}\frac{-5}{9}$ with x-axis

11. Three forces F_1= a(i–j+k)1 F_2= 2i-3j+4k and F_3= 8i-7j+6k act simultaneously on a particle. If the particle is in equilibrium, the value of a is

1) 10 2) -10 3) 8 4) 2

KEY

ΦΦ TEACHING TASK :

1) 4, 2) 3, 3) 2, 4) 2, 5) 4, 6) 1, 7) i) B, ii) B

ΦΦ LEARNER'STASK :

☐ **BEGINNERS :**

1) 2, 2) 2, 3) 3, 4) 1

☐ **ACHIEVERS :**

1) 3i+4j, 5 units, 2) $\sqrt{6}$, 3) $5\sqrt{2}$, 4) (3i-4j)/5, 5) (12i-5j)/13,

6) $\sqrt{0.84}$, 7) (3i-3$\sqrt{3}$ j)/6, 8) 10/3, 9) -9i+2j, 10) (i-5j)/$\sqrt{26}$,

11) (14i -17j)/$\sqrt{485}$, 12) zero, 13) 1unit, 14) (3i+4j+5k)/$5\sqrt{2}$

☐ **EXPLORERS :**

1) A, 2) A, 3) A, 4) A, 5) A, 6) D, 7) B, 8) i) A, ii) A 9) 3, 10) 2, 11) 2

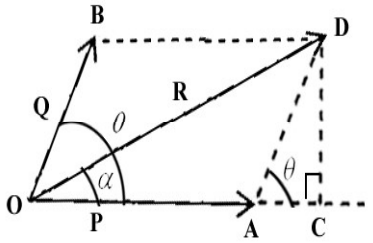
Worksheet-3

§§ Parallelogram law of vector addition

Statement: If two vectors are represented both in magnitude and direction as the adjacent sides of a parallelogram drawn from a point, the diagonal passing through that point represents the resultant both in magnitude and direction.

Let $\overline{OA}$ and $\overline{OB}$ represent two vectors $\overline{P}$ and $\overline{Q}$ both in magnitude and direction.

Let θ be the angle between the vectors. If we construct a parallelogram OABD with $\overline{P}$ and $\overline{Q}$ as adjacent sides then the diagonal $\overline{OD}$ represents the resultant both in magnitude and direction.



§§ MAGNITUDE OF THE RESULTANT

From the picture

$OD^2 = OC^2 + CD^2$

$= (OA+AC)^2+CD^2$

$= (OA+AD\cos\theta)^2+(AD\sin\theta)^2$

$R^2 = P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

§§ DIRECTION OF RESULTANT

Let the resultant $\overline{R}$ makes an angle α with the direction of $\overline{P}$. From the figure

In triangle OCD, $\tan\alpha = \frac{CD}{OC} \Rightarrow \tan\alpha = \frac{CD}{OA+AC}$

$\Rightarrow \tan\alpha = \dfrac{AD\ \sin\theta}{OA + AD\cos\theta} \Rightarrow \tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta}$

If β is the angle made by the resultant $\overline{R}$ with the direction of $\overline{Q}$ then

$\tan\beta = \dfrac{P\sin\theta}{Q+P\cos\theta}$

Note:

1) If $\overline{P}$ and $\overline{Q}$ are in the same direction i.e. $\theta=0^0$, $|\overline{R}|=|\overline{P}|+|\overline{Q}|$

2) If $\overline{P}$ and $\overline{Q}$ in opposite direction i.e. $\theta=180^0$ $|\overline{R}|=|\overline{P}|-|\overline{Q}|$

3) If $\overline{P}$ and $\overline{Q}$ are perpendicular to each other i.e. $\theta=90^0$ $|\overline{R}|=\sqrt{P^2+Q^2}$

4) If the magnitude of $\overline{P}$ is equal to the magnitude of $\overline{Q}$, then $|\overline{R}| = 2P\cos\dfrac{\theta}{2}$ and $\alpha = \dfrac{\theta}{2}$.

5) If $|\overline{P}| = |\overline{Q}|$ and $\theta=120^0$, then $|\overline{R}|=|\overline{P}|$.

6). Using parallelogram law of vectors $|(\overline{a}-\overline{b})| = \sqrt{a^2+b^2-2ab\cos\theta}$

7) The magnitude of resultant that can be obtained from the vectors of magnitude P & Q will in be between P+Q & P-Q

Example: 1

Two forces each of magnitude 10N act at a point with an angle 60^0 between them. Find the magnitude and direction of resultant.

solution: we know that R = $R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

R = $\sqrt{10^2+10^2+2\times10\times10\times\cos60^0} = 10\sqrt{3}N$

$\tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta} \Rightarrow \tan\alpha = \dfrac{10\sin60^0}{10+10\cos60^0} = \dfrac{1}{\sqrt{3}} \Rightarrow \alpha = 30^0$

Example:2

Find the maximum and minimum value of two forces that can be ontained from the forces 3N & 5N

solution: maximum value = 3+5 = 8N, minimum value = 5-3 = 2N

Example: 3 Maximum and minimum resultants of two forces are in the ratio 4:3. Find the ratio of the forces

Solution $\dfrac{F_1+F_2}{F_1-F_2} = \dfrac{4}{3} \Rightarrow 3F_1+3F_2 = 4F_1+4F_2 \Rightarrow 7F_2 = F_1 \Rightarrow \dfrac{F_1}{F_2} = \dfrac{7}{1} \Rightarrow F_1 : F_2 = 7:1$

Example: 4 The resultant of two forces 2p and $\sqrt{2}$ p is $\sqrt{10}$ p. Find the angle between the forces

solution: $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$$\left(\sqrt{10}p\right)^2 = (2p)^2 + \left(\sqrt{2}p\right)^2 + 2p \times \sqrt{2} \times 2 \times \cos\theta \Rightarrow 10p^2 = 4p^2 + 2p^2 + 4\sqrt{2}p^2\cos\theta$$

$$10p^2 - 6p^2 = 4\sqrt{2}p^2\cos\theta \Rightarrow \qquad \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \quad \therefore \theta = 45^0$$

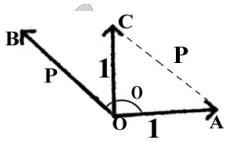
Example: 5 The greatest and least resultant of two forces acting at a point are 29N and 5N. If each force it increased by 3N and applied by 3N and applied at right angles on a particle then find the new resultant force

solution: $F_1 + F_2 = 29$, $\quad F_1 - F_2 = 05$ by sloving $2F_1 = 34$

$$F_1 = 17N \Rightarrow F_2 = 12N \qquad F_1^1 = 17 + 3 = 20N \qquad F_2^1 = 12 + 3 = 15N$$

Example 6: The resultant of P newton and 1N

is 1N and is perpendicular to 1N. Find the magnitude



and direction of P

solution: The resultant of $\overline{OA}$ & $\overline{OB}$ *is* $\overline{OC}$ which can be found using triangular law of vector addition by sliding $\overline{OB}$ as shown

The closing side $\overline{OC}$ of triangle OAC represents the resultant of P newton and 1N from the geometry $P^2 = 1^2 + 1^2 \quad \Rightarrow P = \sqrt{2} \quad \Rightarrow \theta = 135^0$

Example:7 The resultant of two forces at right angles is 17N. The maximum possible resultant with these forces is 23N. Find the greater force

solution: P+Q=23N

$$P^2 + Q^2 = 17^2 \Rightarrow P^2 + Q^2 = 289 \Rightarrow (P+Q)^2 = (23)^2 \Rightarrow P^2 + Q^2 + 2PQ = 529$$

$$2PQ = 529 - 289 \Rightarrow \quad PQ = \frac{240}{2} \Rightarrow \quad (P-Q) = P^2 + Q^2 - 2PQ = 289\text{-}240 = 49$$

P - Q = 7, P + Q = 23 by solving 2P = 30 $\Rightarrow$ P = 15N

Example: 8 Two forces F_1 and F_2 acting at a point have a resultant F. If F_2 is doubled F also doubled. If F_2 is reversed then also F is double. Find F_1:F_2:F

solution: $F^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \qquad ----------(1)$

$4F^2 = F_1^2 + 4F_2^2 + 4F_1F_2\cos\theta \qquad ---------(2)$

$4F^2 = F_1^2 + F_2^2 + 2F_1F_2\cos(180^0 - \theta)$

$4F^2 = F_1^2 + 4F_2^2 - 2F_1F_2\cos\theta \qquad ----------(3)$

$(1)+(3) \quad \rightarrow 5F^2 = 2F_1^2 + F_2^2 \quad \text{--------}(4)$

$(2)-2\times(1) \quad \rightarrow 2F^2 = -F_1^2 + 2F_2^2 \quad \text{--------}(5)$

$F_1 = F,\ F_2 = \frac{\sqrt{3}}{\sqrt{2}}F \qquad F_1:F_2:F = F:\frac{\sqrt{3}}{\sqrt{2}}F:F \Rightarrow F_1:F_2:F = \sqrt{2}:\sqrt{3}:\sqrt{2}$

TEACHING TASK

Choose the correct option

1. The maximum and minimum resultants of two forces are in the ratio 4:3. The forces are in the ratio

1) 7:1 2) 1:5 3) 4:7 4) 3:7

2. If three vectors $\bar{P}, \bar{Q}$ and $\bar{R}$ are related as $\bar{P} - \bar{Q} = \bar{R}$ and P-Q=R, the angle between the vectors $\bar{P}$ and $\bar{Q}$ is

1) 0 2) 30^0 3) 60^0 4) 180^0

3. The angle made by $\vec{j}+\vec{k}$ with y-axis is

1) 60^0 2) 30^0 3) 45^0 4) 90^0

4. A particle has a displacement of 12m towards east then 5m towards north and then 6m vertically upwards. The resultant displacement is

1) 10.04 m 2) 12.10 m 3) 14.32 m 4) 12.6 m

5. The resultant of two forces 2P and $\sqrt{2}$ P is $\sqrt{10}$ P. The angle between the forces is

1) 30^0 2) 60^0 3) 45^0 4) 90^0

6. If the angle between two vectors of equal magnitude P is θ, the magnitude of the difference of the vectors is

1) $2P\cos\frac{\theta}{2}$ 2) $2P\sin\frac{\theta}{2}$ 3) $P\cos\frac{\theta}{2}$ 4) $P\sin\frac{\theta}{2}$

7. The resultant of two forces is perpendicular to one of them. If the forces are 500gm wt and 250gm wt, the angle between the forces is

1) 60^0 2) 120^0 3) 150^0 4) 0^0

Assertion A and Reason R:

♦ *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which* ***ONLY ONE*** *is correct Choose the correct option.*

8. **A:** When $\vec{P}+\vec{Q}=\vec{R}$ and P + Q = R, the angle between $\vec{P}\ \&\ \vec{Q}$ must be 0^0.

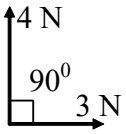
R : Here $\theta = 0^0,\ R = \sqrt{P^2+Q^2+2PQ\cos 0^0} = P+Q$

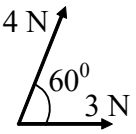
1) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'
2) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'
3)'A' is true and 'R' is false
4) 'A' is false and 'R' is true

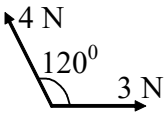
Multiple option type:

- *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D),out of which **ONE or MORE** is correct. Choose the correct options*

9. Set the resultant of forces in increasing order

a) 

b) 

c) 

1) b, a, c 2) a, b, c 3) b, c, a 4) c, a, b

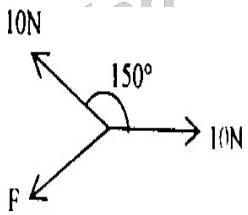
Comprehention type:

- *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

10. The forces 10 N, 10 N and F as shownare in equilibrium

i) The value of F should be

A) 10 N B) $\sqrt{3}$ N



C) $10\sqrt{3}$ N D) $10\sqrt{2-\sqrt{3}}$ N

ii) At what angle F is acting

A) Perpendicular to any 10 N B) at any angle with any 10 N

C) at 105^0 with any 10 N D) Data not sufficient

LEARNER'S TASK

BEGINNERS (Level - I)

Choose the correct option

1. If $|\bar{A}+\bar{B}|=|\bar{A}|+|\bar{B}|$ then the angle between the $\bar{A}$ and $\bar{B}$.

1) 0^0 2) 90^0 3) 180^0 4) 60^0

2. $|\bar{a}-\bar{b}|=|\bar{a}|+|\bar{b}|$ then the angle between $\bar{a}$ and $\bar{b}$ is

1) 90^0 2) 180^0 3) 45^0 4) 0^0

3. The resultant of $\bar{A}$ and $\bar{B}$ makes an angle α with $\bar{A}$ and angle β with $\bar{B}$, then $\alpha<\beta$

1) A < B 2) if A = B 3) if A > B 4) all the above

4. The minimum number of equal forces in a plane that can keep a particle in equilibrium is

1) 4 2) 2 3) 3 4) 5

5. The minimum number of unequal forces in a plane that can keep a particle in equilibrium is

1) 4 2) 2 3) 3 4) 6

6. The minimum number of non coplanar forces that can keep a particle in equilibrium is

1) 1 2) 2 3) 3 4) 4

7. If we add two vectors of equal magnitudes but in opposite directions, we get

1) Unit Vector 2) Unit Scalar 3) Zero Vector 4) Zero Scalar

8. The maximum resultant of two concurrent forces is 10N and their minimum resultant is 4N. The magnitude of large force is

1) 5N 2) 7N 3) 3N 4) 14N

9. If $\bar{P} + \bar{Q} = \bar{R}$ and $\bar{P} - \bar{Q} = \bar{S}$ then $R^2 + S^2$ is equal to

1) $P^2 + Q^2$ 2) $2(P^2 - Q^2)$ 3) $2(P^2 + Q^2)$ 4) $4PQ$

10. The resultant of forces $\vec{F}_1 = 3\vec{i}-4\vec{j}+2\vec{k}$, $\vec{F}_2 = 2\vec{i}+3\vec{j}-\vec{k}$, $\vec{F}_3 = 2\vec{i}+4\vec{j}-5\vec{k}$ is

1) $7\vec{i}-11\vec{j}-8\vec{k}$ 2) $7\vec{i}+3\vec{j}-6\vec{k}$ 3) $7\vec{i}+3\vec{j}-4\vec{k}$ 4) $7\vec{i}-3\vec{j}+4\vec{k}$

11. The maximum value of magnitude of $|(\bar{A}-\bar{B})|$ is

1) A - B 2) A + B 3) $A^2 + B^2$ 4) $A^2 - B^2$

12. If unit vectors $\bar{A}$ and $\bar{B}$ are inclined at an angle θ, then $|\bar{A}-\bar{B}|$

1) $2\cos\left(\frac{\theta}{2}\right)$ 2) $2\sin\left(\frac{\theta}{2}\right)$ 3) 1 4) 0

13. If $\bar{A} = \bar{B} + \bar{C}$ and the magnitudes of $\bar{A}, \bar{B}, \bar{C}$ are 5,4 and 3 units respectively the angle between $\bar{A}$ and $\bar{C}$ is

1) $Cos^{-1}\left(\frac{3}{5}\right)$ 2) $Cos^{-1}\left(\frac{4}{5}\right)$ 3) $\frac{\pi}{2}$ 4) $Sin^{-1}\left(\frac{3}{4}\right)$

14. To get a resultant displacement of 10 m two displacement vectors, one of magnitude 6m and another of 8m should be combined.

1) parallel 2) anti-parallel

3) at an angle 60^0 4) perpendicular to each other

15. Two forces each of 20N act on a body at 120^0. The magnitude and direction of the resultant is

1) $20N;\phi=60^0$ 2) $20\sqrt{2}N;\phi=60^0$ 3) $10\sqrt{2}N;\phi=0^0$ 4) $10\sqrt{2}N;\phi=120^0$

16. Two forces each of 10N act at an angle 60^0 with each other.The magnitude and direction of the resultant with respect to one of the vectors is

1) $\sqrt{10}N, 30^0$ 2) $10\sqrt{3}N, 30^0$ 3) $20N, 120^0$ 4) $10\sqrt{2}N, 120^0$

17. Two equal forces act at a point perpendicular to each other. If the resultant is 1414N, then magnitude of each force is

1) 500N 2)1000N 3) $1000\sqrt{2}$ N 4) $500\sqrt{2}$ N

18. Two vectors of magnitudes 8 and 6 are acting at a point so as to produce a resultant of magnitude 10. The angle between the two vectors is

1) $Cos^{-1}\left(\frac{3}{4}\right)$ 2) $Cos^{-1}\left(\frac{4}{3}\right)$ 3) $Cos^{-1}\left(\frac{7}{5}\right)$ 4) 90^0

19. Two forces each of magnitude 'f' and making an angle ϕ act on a body, then the resultant force is

1) $f\sqrt{2(1-\sin\phi)}$ 2) $f\sqrt{2(1+\sin\phi)}$ 3) $2f.\sin\left(\frac{\phi}{2}\right)$ 4) $2f.\cos\left(\frac{\phi}{2}\right)$

20. The vectors having same magnitude have a resultant equal to either, then the angle between the vectors will be

1) 30^0 2) 60^0 3) 90^0 4) 120^0

21. When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed then acceleration of the particle is

1) F_1/m 2) $F_1F_3/m\,F_1$ 3) $(F_2-F_3)/m$ 4) F_2/m

22. Two forces whose magnitudes arein the ratio 3:5 give a resultant of 35N. If the angle between them is 60^0, the magnitude of each force is

1) 3N, 5N 2) 9N, 25N 3) 15N, 25N 4) 21N, 35N

23. Two forces acting on a particle in opposite directions have a resultant of 10 kg wt. If they act at right angles to each other, the resultant is 60 kg wt. The forces are (in kg. wt)

1) 47.1, 37.1 2) 14.3, 24.3 3) 40, 30 4) 60, 70

24. A body suffers from two simultaneous displacements of 300 m and 400 m which are at right angles to each other. The magnitude of the resultant is

1) 500m 2) 700m 3) 900m 4) 1300m

25. Resultant of two vectors is $20\sqrt{3}$ and makes an angle 30^0 with a vector of magnitude 20. The magnitude of other vector is

1) 10 2) 20 3) $10\sqrt{3}$ 4) $20\sqrt{3}$

26. A particle is simultaneously acted upon by two forces of magnitude 3N and 4N. The net force on the particle is.

1) 7N 2) 5N 3) 1N 4) Between 1N and 7N

27. Two forces of magnitude 8N and 15N respectively act at a point. If the resultant force is 17N, the angle between the forces is.

1) 60^0 2) 45^0 3) 90^0 4) 30^0

ACHIEVERS (Level - II)

Solve the below numericals

1. Two forces each of magnitude 20N acting at a point at an angle 60^0 with each other find the magnitude and direction of resultant.

2. Two forces each of magnitude 15N are acting at a point at an angle 120^0 with each other find the resultant force.

3. The maximum and minimum values of possible results of two forces are 7N and 1N

respectively. Find the forces.

4. The maximum and minimum values of resultants of two vectors are 8N, 2N respectively. The magnitude of the vectors are .....and .....

5. The maximum and minimum values of resultant that can be produced by using the forces 10N and 5N is ..... and .....

6. The maximum value of magnitude of the vector $\bar{a}-\bar{b}$ is .....

7. $\sqrt{P^2+Q^2-PQ}$ the angle between $\bar{P}$ *and* $\bar{Q}$ is .....

8. The magnitude of $\bar{A}+\bar{B}$ is ..... (pick the right options)

1) equal to A+B 2) not greater than A+B

3) not greater than or equal to A+B

4) may be equal to A+B 5) not less than |A-B|

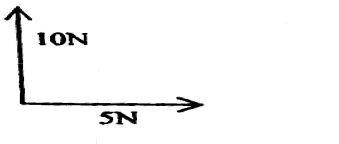
9. A man travels 1 mile due east then 5mile due south then 2mile due east and finally 9 mile due north ; how far is he from the starting point?

10. The resultant of two forces, one double the other in magnitude, is perpendicular to the smaller of the two forces.What is the angle between the two forces?

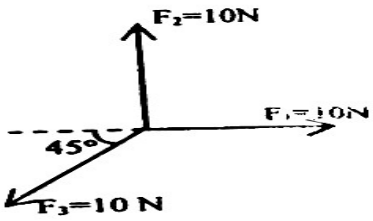
11. Two forces F_1 = 5N and F_2 = 10N are acting on a body at right angles to each other as shown in figure

1) Find the resultant force

2) If the resultant force makes an angle θ with 5N force then tan θ =..



12. Find the magnitude of resultant of the following forces acting at a point as shown below

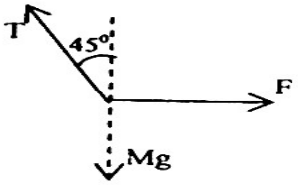


13. The sum of the magnitude of 2 forces acting at a point is 18 and the magnitude of their resultant is 12 if the resultant is at 90^0 with the force of smaller magnitude, what are the magnitudes of force?

14. The resultant of two forces acting at an angle of 150^0 is 10 kgwt, perpendicular to one of the forces. What is the smaller force?

15. If $\bar{A}=\bar{B}+\bar{C}$ and the magnitudes of $\bar{A},\bar{B}\,\&\,\bar{C}$ are 5,4 and 3 units respectively, what is the angle between $\bar{A}\,\&\,\bar{C}$?

16. If the forces shown in figure are in equilibrium then F/Mg =



EXPLORERS (Level - III)

Assertion A and Reason R:

♦ *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.*

A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

B) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'

C)'A' is true and 'R' is false

D) 'A' is false and 'R' is true

1. **A:** Two forces 7N and 5N are acting at a point and their resultant can be 3N.

R:If two vectors P and Q are acting at a point their resutant can have a value between P–Q to P + Q.

2. **A:** The resultant of $\overline{P}$ and $\overline{Q}$ makes the angles α and β with $\overline{P}$ and $\overline{Q}$ respectively. If $|\overline{P}| > |\overline{Q}|$ then $\beta > \alpha$

R: Resultant is always closer to the vector of larger magnitude

Multi choice type (Choose the correct)

♦ *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D),out of which **ONE or MORE** is correct. Choose the correct options*

3. If $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$ then

a) The angle between $\vec{A}$ and $\vec{B}$ is 90^0 b) $\vec{B} = 0$

c) $\vec{A} = 0$ d) The angle between $\vec{A}$ and $\vec{B}$ is 120^0

A) a, b & c are correct B) b, c & d are correct

C) c, d & a are correct D) d, a & b are true

4. Consider the following statements A and B given below and identify the correct answer.

A) The sum and difference of two vectors will be equal in magnitude when two vectors are perpendicular to each other.

B) The sum and difference of two vectors will have the same direction, when vectors of unequal magnitudes act in the same direction.

A) both A and B are true B) A is true but B is false

C) B is true but A false D) both A and B are false

Match the following

♦ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

5. The relations between the three vectors A, B, C are given column I. The angles between the

vectors are given in column II in a random order.

a) $\vec{A}-\vec{B}=\vec{C}$ and A – B = C

b) $\vec{A}+\vec{B}=\vec{C}$ and A – B = C

c) $\vec{A}-\vec{B}=\vec{C}$ and $A^2 + B^2 = C^2$

d) $\vec{A}+\vec{B}=\vec{C}$ and A= B =C

e) π

f) $2\pi/3$

g) 0

h) $\pi/2$

A) $a \to g; b \to h; c \to f; d \to e$

B) $a \to h; b \to e; c \to g; d \to f$

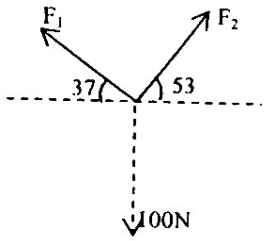
C) $a \to f; b \to e; c \to h; d \to g$

D) $a \to g; b \to e; c \to h; d \to f$

Comprehention type questions:

- *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

6. Give F_1, F_2 and 100N are in equilibrium



i) Find the velue of F_1

A) 60 N B) 80 N C) 100 N D) 0

ii) Find the value of F_2

A) 60 N B) 80 N C) 100 N D) 0

iii) What is the angle between F_1 and F_2

A) 0^0 B) 90^0 C) 180^0 D) 360^0

High order thinking skills (HOTS):

7. A vector $\vec{Q}$ has a magnitude of 8 is added to the vector $\vec{P}$ which lies along the X-axis. The resultant of these two vectors is a third vector $\vec{R}$ which lies along the Y-axis and has a magnitude twice that of $\vec{P}$. The magnitude of $\vec{P}$ is

1) $6/\sqrt{5}$ 2) $8/\sqrt{5}$ 3) $12/\sqrt{5}$ 4) $16/\sqrt{5}$

8. Two forces of 1 N and P N act at a point so that the magnitude of resultant is 1 N and it is perpendicular to 1 N. The value of P in newtons and the angle between the 1 N and P N are

1) $\sqrt{2},135^0$ 2) $\sqrt{2},120^0$ 3) $1,45^0$ 4) $2,150^0$

9. The angle between two Forces (X+Y) and (X-Y), if their resultant is $\sqrt{2\left(x^2+y^2\right)}$

1) 90^0 2)60^0 3)45^0 4)30^0

10. Two vectors $\overline{A}$ and $\overline{B}$ have precisely equal magnitudes. If magnitude of $\overline{A}+\overline{B}$ to be larger than the magnitude of $\overline{A}-\overline{B}$ by a factor n, the angle between them is

1) $2\tan^{-1}\left(1/n\right)$ 2) $\tan^{-1}\left(1/n\right)$ 3) $\tan^{-1}\left(1/2n\right)$ 4) $2\tan^{-1}\left(1/2n\right)$

11. The greatest and least resultant of two forces acting at a point are 29 Kg wt. and 5 Kg wt. respectively. If each force is increased by 3 Kg wt. the magnitude of the resultant of new forces acting at right angles to each other is

1) 45 kg wt. 2) 35 kg wt. 3) 25 kg wt. 4) 15 kg wt.

KEY

ΦΦ TEACHING TASK :

1)1, 2)1, 3)3, 4) 3, 5) 3, 6) 2, 7) 2, 8) 1, 9) 4, 10) i) D, ii) C

ΦΦ LEARNER'STASK :

☐ **BEGINNERS :**

1)1, 2) 2, 3) 3, 4) 2, 5) 3, 6) 4, 7) 3, 8) 2, 9) 3, 10) 3, 11) 2, 12) 2, 13) 1, 14) 4, 15) 1, 16) 2, 17) 2, 18) 4, 19) 4, 20) 4, 21) 1, 22) 3, 23)1, 24)1, 25) 2, 26)4, 27) 3

☐ **ACHIEVERS :**

1.$20\sqrt{3}$, $\tan^{-1}(\sqrt{20}/3)$, 2.15 N, 3.8 N, 6 N, 4.5 N, 3 N 5.15 N, 5 N, 6.a + b, 7.120^0, 8.2,4,5, 9.5 miles, 10.120^0 ,11.5$\sqrt{5}$ N, 2, 12.zero, 16.1

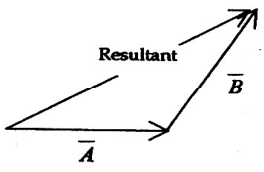
☐ **EXPLORERS :**

1) A, 2) A, 3) A, 4) A, 5) D, 6) i) A, ii) B, iii) B 7)2, 8)1, 9)1, 10)1, 11)3

Worksheet-4

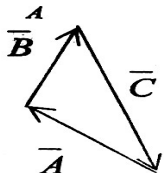
§§ **Triangular law of vector addition**

statement - 1: If two vectors are represented both in magnitude and direction as the adjacent sides of a triangle taken in order, then the closing side taken in reverse order



represents the resultant both in magnitude and direction.

Statement - 2: If three vectors are such that their resultant is zero vector or null vector then they can be represented both in magnitude and direction as the adjacent sides of a triangle taken in order, $\overline{A},\overline{B},\overline{C}$ are acting at a point such that

$\overline{A}+\overline{B}+\overline{C}=\overline{0}$ as shown in figure 

If the resultant of three vectors is a null vector, then they can be

represented both in direction and magnitude as the adjacent sides

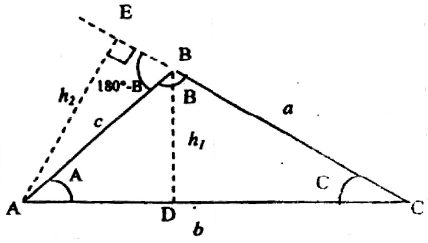
of a triangle taken in order $\overline{a}+\overline{b}+\overline{c}=\overline{0}$

NOTE: A zero vector or null vector has no magnitude and possesses inderterminate (undefined) direction

¶¶ **Sine rule:** In any triangle the length of the side is proportional to the sine of the angle opposite to it

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

In $\Delta ABD, \sin A=\frac{h_1}{c}\Rightarrow h_1=c\,SinA$



$\Delta BDC, \sin C=\frac{h_1}{a}\Rightarrow h_1=a\,SinC$ c sin A = a sin C

$\frac{c}{\sin C}=\frac{a}{\sin A}$ $\quad -----(1)$ $\quad \Delta ACE, h_2=b\sin C$

$\Delta ABE, h_1=c\sin(180^0-B)=c\sin B$ that means b sin C = c sin B

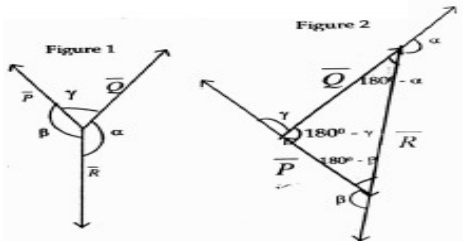
$\frac{b}{\sin B}=\frac{c}{\sin C}$ $\quad --------(2)$ from (1) and (2) $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

§§ **LAMIS THEOREM**

If the resultant of three coplanar vectors is a null vector then the magnitude of each vector is proportional to sine of the angle between other two vectors

If the resultant of three coplanar vectors $\overline{P},\overline{Q}\,\&\,\overline{R}$ is a null vector, the angle between the vectors $\overline{R}\,\&\,\overline{Q}$ is α the angle between $\overline{P}\,\&\,\overline{R}$ is β and the angle between $\overline{P}\,\&\,\overline{Q}\,is\,\gamma$ then according to Lami's theorem

$$\frac{P}{\sin\alpha}=\frac{Q}{\sin\beta}=\frac{R}{\sin\gamma}$$



Consider three coplanar vectors $\overline{P},\overline{Q}\,\&\,\overline{R}$ acting at sa point as shown in the figure

($\overline{P},\overline{Q}\,\&\,\overline{R}$ are in equilibrium)

The vectors $\overline{P},\overline{Q}\,\&\,\overline{R}$ can be represented both in direction and magnitude as the adjacent sides of a triangle taken in order as shown in figure 2 (from triangle Law) from sine-rule

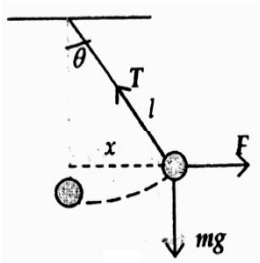
$$\frac{P}{\sin\left(180^0-\alpha\right)}=\frac{Q}{\sin\left(180^0-\beta\right)}=\frac{R}{\sin\left(180^0-\gamma\right)}\Rightarrow\frac{P}{\sin\alpha}=\frac{Q}{\sin\beta}=\frac{R}{\sin\gamma}$$

§§ **TANGENT LAW (APPLICATION OF LAMI'S THEOREM)**

Let a bob of mass m suspended to the ceiling by means of a light string of length l is pulled by a horizontal force F till the weight of the bob, the force F and the tension T are in equilibrium.

Let the string now makes an angle θ with vertical and horizontal displacement of the bob is x.

The equilibrium position is shown in figure

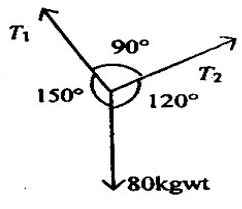


$$\frac{F}{\sin\left(180^0-\theta\right)}=\frac{mg}{\sin\left(90^0+\theta\right)}=\frac{T}{\sin 90^0}\Rightarrow\frac{F}{\sin\theta}=\frac{mg}{\cos\theta}=T$$

$T=\frac{mg}{\cos\theta}$ (or) $F=mg\tan\theta$ (tangent law) ------- (1)

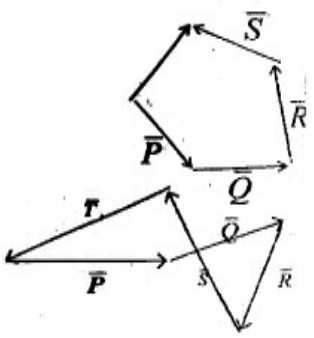
can also be written in terms of l & x as $\frac{F}{\left(\frac{x}{l}\right)}=\frac{mg}{\left(\frac{\sqrt{l^2-x^2}}{l}\right)}=T$

Example: 1 A man of mass 80kg is supported by two cables as shown in figure. Find the ratio of $T_1:T_2$



solution: $\frac{T_1}{\sin 120^0}=\frac{T_2}{\sin 150^0}\Rightarrow\frac{T_1}{T_2}=\frac{\sin 120^0}{\sin 150^0}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\Rightarrow T_1:T_2=\sqrt{3}:1$

§§ POLYGON LAW OF VECTOR ADDITION



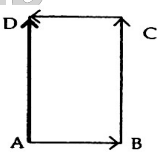
If several vectors are represented both in magnitude and direction as the adjacent sides of a polygon taken in order then the closing side taken in reverse order represents the resultant both in magnitude and direction.

Polygon law can also be stated like this. If the vector sum of several vectors is a null vector, then they can be represented both in magnitude and direction as the adjacent sides of a closed polygon taken in order

Note

1. If n forces each of same magnitude are acting at a point with the angle 360/n between them then the resultant is zero.
2. If n-1 forces of equal magnitude are acting at a point such that each vector makes an angle 360/n with the preceding one then the magnitude of resultant force is equal to the magnitude of force acting.

Example: A person follows the path $A \to B \to C \to D$.



The coordinates of A, B, C & D are respectively.

$(0,0)(2,0)(2,6)\&(0,6)$. Find the magnitude of displacement.

Solution: The vectors $\overline{AB}, \overline{BC} \& \overline{CD}$ are represented both in magnitude and direction as the adjacent sides of a polygon taken in order. Therefore $\overline{AD}$ the vector taken in reverse order represents resultant displacement both in magnitude and direction.

TEACHING TASK

Choose the correct option:

1. The vectors $2\bar{i}+3\bar{j}+4\bar{k},\ 3\bar{i}+2\bar{j}-3\bar{k},\ \ 5\bar{i}+5\bar{j}+\bar{k}$ form

1) An equilateral triangle 2) An isosceles tringle

3) Right angled triangle 4) Circle

2. An iron sphere of mass 100 kg is suspended freely from a rigid support by means of a rope of length 2m. The horizontal force required to displace it horizontally through 50cm is nearly

1) 980 N 2) 490 N 3) 245 N 4) 112.5 N

3. Two light strings of length 4 cm and 3cm are tied to a bob of weight 500gm. The free ends of the strings are tied to pegs in the same horizontal line and separated by 5cm. The ratio of tension in the longer string to that in the shorter string is

1) 4 : 3 2) 3 : 4 3) 4 : 5 4) 5 : 4

4. In case of three vector quantities of same type where resultant can be zero?

1) 10,10,10 2) 10,10,20 3) 10,20,30 4) 10,20,40

5. Three forces $\vec{A}=\{\hat{i}+\hat{j}+\hat{k}\}$, $\vec{B}=\{2\hat{i}-\hat{j}+3\hat{k}\}$ and $\vec{C}$ acting on a body. To keep it in equilibrium value of $\vec{c}$ is

1) $-\left\{3\hat{i}+4\hat{k}\right\}$ 2) $-\left\{4\hat{i}+3\hat{k}\right\}$ 3) $3\hat{i}+4\hat{j}$ 4) $2\hat{i}-3\hat{k}$

6. 4 forces each of magnitude 10N are acting at a point. The angle between successive vectors is 90^0. Find the resultant force

1) 0 2) 10 N 3) $10\sqrt{2}$ 4) none

Assertion A and Reason R:

*This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.*

7. **A :** If n forcs each of same magnitude are acting at a point with the angle 360/n between them then the resultant is zero.

R: If n-1 forces of equal magnitude are acting at a point such that each vector makes an angle 360/n with the preceding one then the magnitude of resultant force is equal zero.

A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

B) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'

C)'A' is true and 'R' is false

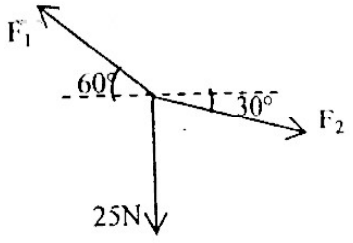
D) 'A' is false and 'R' is true

Match the following:

*This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

8. Forces in the figure shown keep the particle in equilibrium. Then match the following



a) angle between F_1 and F_2 is 1) 60^0

b) angle between 25 N and F_2 is 2) 150^0

c) Force F_1 = 3) $25\sqrt{3}$ N

d) Force F_2 = 4) 25 N

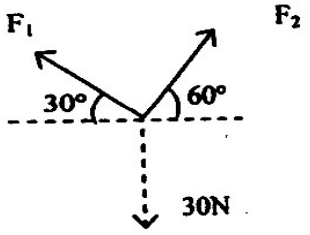
A) a-1, b-2, c-3, d-4 B) a-2, b-1, c-3, d-4

C) a-2, b-1, c-4, d-3 D) a-4, b-3, c-2, d-1

Comprehention type:

*This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

9. Forces in the figure shown are in equilibrium



i) Force F_1 is

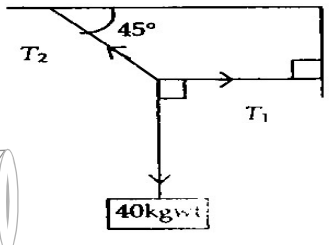
A) 15 N B) $15\sqrt{3}$ N C) $\sqrt{3}$ N D) zero

ii) Force F_2 is

A) 15 N B) $15\sqrt{3}$ N C) $\sqrt{3}$ N D) zero

Solve the following:

10. Find the forces T_1 & T_2 in the figure shown if the forces keep the particle in equilibrium



LEARNER'S TASK

BEGINNERS (Level - I)

Choose the correct option:

1. The vectors $\bar{a} = 2\hat{i} + \hat{j} + \hat{k}$ & $\bar{b} = -4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\bar{c}$ are represented as the adjacent sides of a triangle taken in order. Then $\vec{c}$ =

1) $-(-2i - j + 4k)$ 2) $-(2i + j - 4k)$ 3) $-2i - j + 4k$ 4) $6i + 3j + 4k$

2. Two vectors $\bar{a} = 2\hat{i} + \hat{j} - \hat{k}$ & $\bar{b} = 3\hat{i} + 2\hat{j} + \hat{k}$ are represented as the adjacent sides of a triangle taken in order. Write the closing side taken in reverse order

1) $5i + 3j$ 2) $-(5i + 3j)$ 3) $3i + 5j$ 4) $-(3i + 5j)$

3. If a particle possesses three velocities simultaneously, represented by the three sides of a triangle taken in order, then what is velocity of the particle

1) zero 2) non zero

3) can't be decided 4) data not sufficient

4. When 3 coplanar forces are in equilibrium then magnitude of each force is proportional to the ..... of the angles between the other two forces

1) sine 2) cosine 3) tangent 4) cosecant

5. A bob of mass m suspended to the ceiling by means of a light string is pulled by a horizontal force F and the tension T are in equilibrium. If the string now makes an angle θ with vertical then F = ...

1) mg cos θ 2) mg sin θ 3) mg cot θ 4) mg tan θ

6. An iron sphere of mass 100kg is supended freely from a rigid support by means of a rope of length 2m. The horizontal force required to displace it through an angle 60^0 from the mean position is

1) $490\sqrt{3}N$ 2) $980\sqrt{3}N$ 3) $\frac{490}{\sqrt{3}}N$ 4) $\frac{980}{\sqrt{3}}N$

7. Three forces $F_1 = 2\bar{i} + \bar{j} - \bar{k}$ N $F_2 = 2\bar{i} + 3\bar{j} - 3\bar{k}$ N $F_3 = a\left(\bar{i} + \bar{j} - \bar{k}\right)$ N act simultaneously on a particle. The value of 'a' so that particle may be in equilibrium is

1) 4 2) - 4 3)2 4)6

8. If the magnitudes of $\bar{A}, \bar{B}, \bar{C}$ are 12,5,13 units respectively and $\bar{A} + \bar{B} = \bar{C}$, then the angle between $\bar{A}, \bar{B}$ is

1) 0 2) π 3) $\pi/2$ 4) $\pi/4$

9. Five equal forces each of 20N are acting at a point in the same plane. If the angles between them are same, the resultant of these forces is

1) 0 2) 40N 3) 20N 4) $20\sqrt{2}$

10. 11 forces each of magnitude 5N are acting on a particle simultaneously. If each force makes an angle 30^0 with the previous one, find the resultant of all the forces.

1) zero 2) 5 N 3) 25 N 4) 55 N

11. 6 forces each of magnitude 10N are acting at a point. The angle between successive vectors is 60^0 Find the resultant force

1) zero 2) 6 N 3) 10 N 4) 60 N

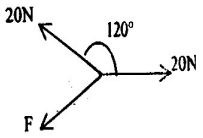
12. 3 forces each of magnitude 5N are acting on a particle simultaneously. If each force makes an angle 90^0 with the previous one, find the resultant of all the forces

1) zero 2) 5 N 3) 3 N 4) 15 N

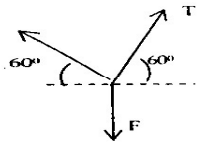
ACHIEVERS (Level - II)

Solve the following

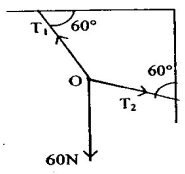
1. If the forces 20N, 20N and F are in equilibrium as shown then find F



2. Find T and F if these forces along with 150N keeps the particle in equilibrium



3. Find the forces T_1 & T_2 in the figure shown if the forces keep the particle in equilibrium



EXPLORERS(Level - III)

Assertion A and Reason R:

♦ *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which* ***ONLY ONE*** *is correct Choose the correct option.*

Opions: A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

B) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'

C)'A' is true and 'R' is false

D) 'A' is false and 'R' is tr

1. **A :** Vector sum of three coplanar forces can be zero

R : Minimum number of unequal coplanar vectors required to keep a point in equilibrium is three

2. **A :** When a vector is rotated, its magnitude remains constant.

R : The magnitude of a vector is independent of the coordinate system.

Multi choice type (Choose the correct)

♦ *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D),out of which* ***ONE or MORE*** *is correct. Choose the correct options*

3. Which of the following set of forces may produce a zero resultant?

a) 3N, 7N, 11N b) 4N, 6N, 5N c) 6N, 7N, 12N d) 10N, 6N, 7N

A) a, b only correct B) b, c only correct

C) a, b, c only correct D) b, c, d only correct

4. Lami's theorem is applicable for the vectors which are

a) 3 in number b) coplanar c) concurrent d) in equilibrium

A) a, b only correct B) a, b, c only correct

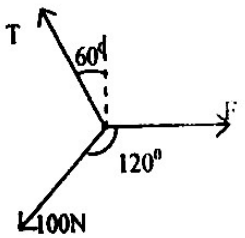
C) a, b, c only correct D) all are correct

Match the following

♦ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in* ***Column–I*** *have to be matched with statements (p, q, r, s) in* ***Column–II****. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

5. In the figure the particle in equilibrium. Then match the follwoing



a) net force on the particle 1) zero

b) angle between T and 100 N is 2) 90^0

c) Value of F is 3) 200 N

d) value of T is 4) $50\sqrt{3}$ N

A) a-1,b-2,c-3,d-4 B) a-4,b-3,c-2,d-1

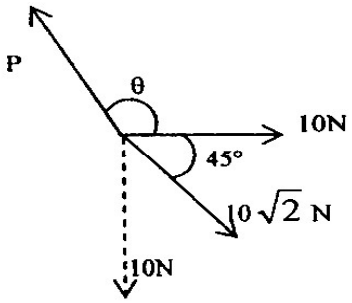
C) a-1,b-2,c-4,d-3 D) a-3,b-1,c-3,d-2

Comprehention type questions:

♦ *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which* ***ONLY ONE i****s correct. Choose*

the correct option.

6. All the forces are in equilibrium in the figure shown



i) The angle θ should be

A) 45^0 B) 90^0 C) 135^0 D) 180^0

ii) The velue of P should be

A) zero B) $10\sqrt{2}$ N C) $20\sqrt{2}$ N D) $\sqrt{2}$ N

High order thinking skills (HOTS):

7. A person follows the path $A \to B \to C \to D \to E$ The coordinates A, B, C, D and E are respectively (0,0) (0,3) (3,4) (2,5) & (1,0) Find the magnitude of displacement.

1) (1, 0) 2) (0, 0) 3) (4, 0) 4) (2, 3)

8. A body of mass m is suspended by a string of length *'l'* and pulled to a side through horizontal a distance 'r' by means of horizontal force. Then the tension in the string is

1) $\frac{mg\sqrt{l^2-r^2}}{l}$ 2) $\frac{mgr}{l}$ 3) $\frac{mgl}{r}$ 4) $\frac{mgl}{\sqrt{l^2-r^2}}$

9. If a particle possesses several velocities simultaneously, represented by the adjacent sides ofa polygon taken in order, then what is velocity of the particle.

1) zero 2) sevaral velocities

3) one none zero velocity 4) none

10. The plane which can be formed with the vectors $\bar{a} = 3\bar{i} - 4\bar{j} + 2\bar{k}$, $\bar{b} = 2\bar{i} - \bar{j} - 6\bar{k}$, $\bar{c} = 5\bar{i} - 5\bar{j} - 4\bar{k}$ is.

1) Quadralateral 2) Triangle 3) Circle 4) Hyperbola

KEY

ΦΦ TEACHING TASK :

1)3, 2)3, 3)2, 4)1, 5)1, 6)1, 7)C, 8)B, 9)i) A,ii) B, 10) 80 N, $40\sqrt{2}$ N

ΦΦ LEARNER'STASK :

☐ **BEGINNERS :**

1) 1, 2) 1, 3) 1, 4) 1, 5) 4, 6) 2, 7) 2, 8) 3, 9) 1, 10) 2, 11) 1, 12) 2

☐ **ACHIEVERS :**

1.20 N , 2.150 N, $150\sqrt{3}$ N, 3.$60\sqrt{3}$ N, 60 N

☐ **EXPLORERS :**

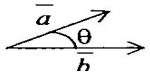
1) A, 2) A, 3) D, 4) D, 5) A, 6) i) C, ii) C, 7) 1, 8) 4, 9) 1, 10) 2

Worksheet-4

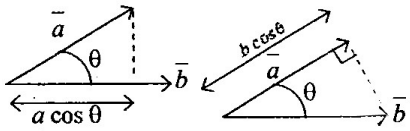
§§ **Dot product (scalar product)**

The dot or scalar product two vectors is defined as the product of the magnitudes of the vectors and cosine of the angle between them.

$\bar{a}.\bar{b} = ab\cos\theta$ (or)



It is also defined as the product of magnitude of one of the vectors and the component of the other vector along the direction of first vector.



$\bar{a}.\bar{b} = ab\ \cos\ \theta$ (or) $\bar{a}\ .\bar{b} = (a\ \cos\ \theta\ )b$

§§ **Properties of dot product**

1. The dot product yields a scalar (even though the two quantities being multiplied are vectors)
2. Dot product obeys commutative law i.e. , $\bar{a}\ .\bar{b} = \bar{b}\ .\bar{a}$
3. Dot product between two vectors can be positive, zero or negative according as the angle between them is acute, ninety or obtuse.

   thus if is θ acute then θ cos=+ve $\quad \therefore \bar{a}\ .\ \bar{b} = |\bar{a}||\bar{b}|\cos\theta = +ve$

   If θ=90^0 then θ cos=0 $\quad \therefore \bar{a}\ .\ \bar{b} = |\bar{a}||\bar{b}|\cos 90^0 = 0$

   This forms the condition of perpendicularity (or orthogonality) of two vectors. Thus if the dot product of two non zero, then it necessarily implies that the two are perpendicular to each other

   As $\therefore \bar{a}\ .\ \bar{b} \Rightarrow |\bar{a}||\bar{b}|\cos 0 = 0 \quad \therefore |\bar{a}| \neq 0; \cos\ \theta = 0 \Rightarrow \theta = 90^0$

   If θ is obtuse (greater than 90^0 & less than 180^0) then θ cos=-ve

   $\therefore |\bar{a}|.|b| = |\bar{a}||\bar{b}|\cos\ \theta = -ve$
4. Dot product obeys distributive law irrespective of the order

   $\bar{a}\ .(\bar{b}+\bar{c}+\bar{d}) = \bar{a}.\bar{b}+\bar{a}.\bar{c}+\bar{a}.\bar{d}$ (or) $\bar{a}\ .(\bar{b}+\bar{c}+\bar{d}) = \bar{b}.\bar{a}+\bar{c}.\ \bar{a}+\bar{d}.\bar{a}$

   (or) $\bar{a}\ .(\bar{b}+\bar{c}+\bar{d}) = \bar{c}.\bar{a}+\bar{b}.\ \bar{a}+\bar{d}.\bar{a}...$
5. The dot product of a vector with itself is the square of the modulus of the vector. (angle between the two same vector is 0^0)

   $\therefore \bar{a}\ .\bar{a} = |a||a|\cos\theta^0 \quad \therefore \bar{a}\ .\bar{a} = |\bar{a}|^2\ or\ a^2$
6. Dot θ product between two vectors of fixed magnitude can be maximum or minimum according cos is maximum minimum $\quad Cos\,\theta_{\max} = 1, so\ \therefore (\bar{a}.\ \bar{b})_{\max} = |\bar{a}||\bar{b}|$

7. $\hat{i}\cdot\hat{i}=\left|\hat{i}\right|\left|\hat{i}\right|\cos 0=1\times1\times1=1 \qquad \hat{i}\cdot\hat{j}=\left|\hat{i}\right|\left|\hat{j}\right|\cos 90^0=1\times1\times0=0$

$\hat{j}\cdot\hat{j}=\left|\hat{j}\right|\left|\hat{j}\right|\cos 0=1\times1\times1=1 \qquad \hat{k}\cdot\hat{j}=\left|\hat{k}\right|\left|\hat{j}\right|\cos 90^0=1\times1\times0=0$

$\hat{k}\cdot\hat{k}=\left|\hat{k}\right|\left|\hat{k}\right|\cos 0=1\times1\times1=1 \qquad \hat{i}\cdot\hat{k}=\left|\hat{i}\right|\left|\hat{k}\right|\cos 90^0=1\times1\times0=0$

8. Dot product of two vectors in terms of unit vectors ($\hat{i},\hat{j}\ \&\ \hat{k}$)

If $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ & $\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ then $\bar{a}.\bar{b}=a_1b_1+a_2b_2+a_3b_3$

9. Angle between two vectors in terms of their dot product

$\therefore \bar{a}\cdot\bar{b}=\left|\bar{a}\right|\left|\bar{b}\right|\cos\theta \qquad \therefore \cos\theta=\dfrac{\bar{a}\cdot\bar{b}}{\left|\bar{a}\right|\left|\bar{b}\right|}$

in terms of unit vectors ($\hat{i},\hat{j}\ \&\ \hat{k}$)

If $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ & $\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ then $\cos\theta=\dfrac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$

Solved Examples

1. Find the dot product of the vectors ($2\bar{i}+\bar{j}+\bar{k}$) and ($3\bar{i}+2\bar{j}+2\bar{k}$)

($2\bar{i}+\bar{j}+2\bar{k}$).($3\bar{i}+2\bar{j}+2\bar{k}$)=2X3+1X2+2X2=12

2. Find the dot product of the vectors $4\bar{i}-3\bar{j}+3\bar{k}$ and $3\bar{i}+8\bar{j}+4\bar{k}$

($4\bar{i}-3\bar{j}+3\bar{k}$).($3\bar{i}+8\bar{j}+4\bar{k}$)=4X3+(-3X8)+3X4=0

3. Find the dot product of the vectors $a_1\bar{i}+a_2\bar{j}+a_3\bar{k}$ and $b_1\bar{i}+b_2\bar{j}+b_3\bar{k}$

$a_1\bar{i}+a_2\bar{j}+a_3\bar{k}\ .\ b_1\bar{i}+b_2\bar{j}+b_3\bar{k} = a_1\times b_1+a_2\times b_2+a_3\times b_3 = a_1b_1+a_2b_2+a_3b_3$

4. If the angle between the vectors ($\bar{i}-2\bar{j}-3\bar{k}$) and ($-2\bar{i}-c\bar{j}+\bar{k}$) is 90^0 then find the value of 'c' $\quad -2+2c-3=0 \Rightarrow 2c=5 \Rightarrow c=2.5$

5. If $\left|\bar{A}+\bar{B}\right|=\left|\bar{A}-\bar{B}\right|$ then find the angle between $\bar{A}$ and $\bar{B}$

Given $\left|\bar{A}+\bar{B}\right|=\left|\bar{A}-\bar{B}\right| \Rightarrow \quad \left|\bar{A}+\bar{B}\right|^2=\left|\bar{A}-\bar{B}\right|^2 \Rightarrow \left|\bar{A}+\bar{B}\right|\cdot\left|\bar{A}+\bar{B}\right|=\left|\bar{A}-\bar{B}\right|\cdot\left|\bar{A}-\bar{B}\right|$

$\Rightarrow \bar{A}\cdot\bar{A}+2\bar{A}\cdot\bar{B}+\bar{B}\cdot\bar{B}=\bar{A}\cdot\bar{A}-2\bar{A}\cdot\bar{B}+\bar{B}\cdot\bar{B} \Rightarrow \bar{A}^2+2\bar{A}\cdot\bar{B}+\bar{B}^2=\bar{A}^2-2\bar{A}\cdot\bar{B}+\bar{B}^2$

$\Rightarrow 2AB\cos\theta = -2AB\cos\theta \Rightarrow \quad 4AB\cos\theta = 0 \Rightarrow \quad \cos\theta = 0$

the angle between $\bar{A}$ and $\bar{B}$ is 90^0

§§ **Angle between vectors**

The angle between two vectors $\bar{a}$ and $\bar{b}$ is given by $\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}$

Example:1 Find the angle between the vectors $(\bar{i}+2\bar{j}+\bar{k})$ and $2\bar{i}+\bar{j}+\bar{k}$

$$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} = \frac{(\bar{i}+2\bar{j}+\bar{k})(2\bar{i}+\bar{j}+\bar{k})}{\sqrt{6}\ \sqrt{6}} = \frac{2+2+1}{6} = \frac{5}{6} \Rightarrow \quad \theta = \cos^{-1}\left(\frac{5}{6}\right)$$

Example:2 Find the angle between the vectors $(\bar{i}-2\bar{j}+\bar{k})$ and $2\bar{i}-2\bar{j}+\bar{k}$

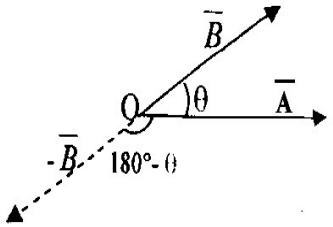
$$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} = \frac{(\bar{i}-2\bar{j}+\bar{k})(2\bar{i}-2\bar{j}+\bar{k})}{\sqrt{6}\ \sqrt{9}} = \frac{2+4+1}{3\sqrt{6}} = \frac{7}{3\sqrt{6}} \Rightarrow \quad \theta = \cos^{-1}\left(\frac{7}{3\sqrt{6}}\right)$$

Example:3 $\bar{A}$ and $\bar{B}$ be two vectors with an angle θ between them. Find the magnitude of the resultant.

$|\bar{A}+\bar{B}|^2 = (\bar{A}+\bar{B})\cdot(\bar{A}+\bar{B}) \quad \bar{A}\cdot\bar{A} + 2\,\bar{A}\cdot\bar{B} + \bar{B}\cdot\bar{B} \Rightarrow A^2 + 2AB\cos\theta + B^2$

$\Rightarrow |\bar{A}+\bar{B}| = \sqrt{A^2+2AB\cos\theta+B^2}$

Example:4 If the angle between $\bar{A}$ and $\bar{B}$ is θ then find $|\bar{A} - \bar{B}|$



solution $|\bar{A} + (-\bar{B})|^2 = (\bar{A} + (-\bar{B}))\cdot(\bar{A} + (-\bar{B}))$

$\bar{A}\cdot\bar{A} + 2\bar{A}\cdot(-\bar{B}) + (-\bar{B})\cdot(-\bar{B})$

$A^2 + 2AB\cos(180^0-\theta) + B^2$

$A^2 - 2AB\cos\theta + B^2$

$|\bar{A}-\bar{B}| = \sqrt{A^2-2AB\cos\theta+B^2}$

Example:5 If $|\bar{A} - \bar{B}| = \sqrt{A^2 + AB + B^2}$ then find the angle between $\bar{A}$ and $\bar{B}$

solution: let angle between $\bar{A}$ and $\bar{B}$ be θ

$|\bar{A}-\bar{B}| = \sqrt{A^2-2AB\cos\theta+B^2} \Rightarrow |\bar{A} - \bar{B}| = \sqrt{A^2 + AB + B^2}$

$\Rightarrow -2AB\cos\theta = AB \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = -120^0$

the angle between $\overline{A}$ and $\overline{B}$ is 120^0

Example:6 if $\left|\overline{A} + \overline{B}\right| = \sqrt{A^2 - 2AB + B^2}$ then find the angle between $\overline{A}$ and $\overline{B}$

solution: $\sqrt{A^2 - 2AB + B^2} = \sqrt{A^2 + 2AB\cos\theta + B^2}$

$2AB\cos\theta = -2AB \Rightarrow \cos\theta = -1 \Rightarrow \theta = 180^0$

the angle between $\overline{A}$ and $\overline{B}$ is 180^0

Example: 7 If $\left|\overline{A} + \overline{B}\right| = \sqrt{A^2 - 2AB\cos\theta + B^2}$ then find the angle between $\overline{A}$ and $\overline{B}$

solution: given $\left|\overline{A} + \overline{B}\right| = \sqrt{A^2 - 2AB\cos\theta + B^2}$

we know that $\left|\overline{A} + \overline{B}\right| = \sqrt{A^2 + 2AB\cos\alpha + B^2}$

then $\cos\alpha = -\cos\theta$ *then* $\theta = -\alpha \Rightarrow \alpha = 180^0 - \theta$

Example:**8** If $\overline{A} + \overline{B} = \overline{C}$ and $\left|\overline{A}\right| = \left|\overline{B}\right| = \left|\overline{C}\right| = \sqrt{3}\left|\overline{A}\right|$ then find the angle between $\overline{A}$ and $\overline{B}$

solution: $\Rightarrow A^2 + 2AB\cos\theta + B^2 = C^2 \Rightarrow A^2 + 2A^2\cos\theta = 0 \Rightarrow A^2(1 + 2\cos\theta) = 0$

$\Rightarrow 2\cos\theta = -1 \Rightarrow \cos\theta = \frac{-1}{2} \Rightarrow \theta = 120^0$ the angle between $\overline{A}$ and $\overline{B}$ is 120^0

Example:9 If $\overline{A} + \overline{B} = \overline{C}$ and $\left|\overline{A}\right| = \left|\overline{B}\right| \& \left|\overline{C}\right| = \sqrt{3}\left|\overline{A}\right|$ then find the angle between $\overline{A}$ and $\overline{B}$

solution: $\left(\overline{A} + \overline{B}\right)^2 = \overline{C}^2 \Rightarrow A^2 + 2AB\cos\theta + B^2 = \left(\sqrt{3}A\right)^2 \Rightarrow 2A^2 + 2A^2\cos\theta = 3A^2$

$\Rightarrow 2A^2(1 + 2\cos\theta) = 3A^2 \Rightarrow 1 + \cos\theta = \frac{3}{2} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^0$

the angle between $\overline{A}$ and $\overline{B}$ is 60^0

TEACHING TASK

Choose the correct option:

1. If two vectors $\overline{P} = 2\vec{i} + x\vec{j} + \vec{k}$ and $\overline{Q} = 4\vec{i} - 2\vec{j} - 2\vec{k}$ are perpendicular to each other, the value of x is

1) 4 2) 3 3) 1 4) -3

2. If $\overline{A}$ = 7i+2j-3k and $\overline{B}$= i+j+2ck are perpendicular vectors, the value of c is

1) -1.5 2) 1.5 3) 3 4) zero

3. If $\vec{A} = i + j$ and $\vec{B} = j - k$ then angle between $\vec{A}$ and $\vec{B}$ is

1) 30^0 2) 45^0 3) 90^0 4) 60^0

4. If $\vec{A} = 2\hat{i} + 3\hat{j}$ its component along $\vec{B} = 2\hat{j} + 3\hat{k}$ is

1) 6 2) 1/6 3) 2 4) $6/\sqrt{13}$

5. If $\vec{A} = 5\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} + 2\hat{k}$, component of $\vec{B}$ along $\vec{A}$ is

1) $\frac{\sqrt{14}}{38}$ 2) $\frac{28}{\sqrt{38}}$ 3) $\frac{\sqrt{28}}{38}$ 4) $\frac{14}{\sqrt{38}}$

6. The vlaue of dot product of two vectors $\vec{a}$ and $\vec{b}$ is 'x', then

1) 0 < x < ab 2) – ab < x < 0 3) – ab $\leq$ x $\leq$ ab 4) – ab < x < ab

7. A vector perpendicular to the vector (3i+5j) is

1) 5i-3k 2) 5i+3j 3) 3i-5j 4) 5k

8. The magnitude of two vectors are 3 and 4 units respectively. Their scalar product is 6 untis, the angle between them is

1) $\pi/3$ 2) $\pi/6$ 3) $\pi/2$ 4) $\pi/4$

9. The angle between the vectors (i + j + k) and (i - j - k) is

1) $\sin^{-1}\frac{\sqrt{8}}{3}$ 2) $\cos^{-1}\left(-\frac{1}{3}\right)$ 3) $\cos^{-1}\frac{\sqrt{8}}{3}$ 4) $\cos^{-1}\sqrt{\frac{8}{3}}$

10. Because of the application of force $\bar{F}$ = 4i-5j newton, the velocity of a body at any instant is given by $\bar{v}$ =2i-7j ms^{-1}. The instantaneous power is

1) 27 W 2) 43 W 3) 21.5 W 4) zero

Assertion A and Reason R:

◆ *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which* ***ONLY ONE*** *is correct Choose the correct option.*

A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

B) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'

C)'A' is true and 'R' is false

D) 'A' is false and 'R' is true

11. **A :** $\bar{A}.(\bar{B} + \bar{C})$ is a scalar

R : The dot product of two vectors is a scalar

12. **A :** The peripendicular vector of $(i + j + k)$ is $(i + 2j + k)$

R : Two vectors are peripendicular if their dot product is zero.

Multiple option type:

◆ *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D),out of which* ***ONE or MORE*** *is correct. Choose the correct options*

13. Consider the following statements A and B given below and identify the correct answer

A) If $\vec{A}$ is a vector then the magnitude of the vector is given by $\sqrt{\vec{A}.\vec{A}}$

B) If $\vec{a} = m\vec{b}$ where 'm' is a scalar the value of 'm' is equal to $\frac{\vec{a}.\vec{b}}{b^2}$

1) both A & B are correct 2) A is correct but B is wrong
3) A is wrong but B is correct 4) both A and B are wrong

14. If $\vec{A}$ = 4i + 2j + 6k, set the following values in increasng order.

a) $\vec{A}.\hat{i}$ b) $\vec{A}.\hat{j}$ c) $\vec{A}.\hat{k}$

1) a, b, c 2) b, c, a 3) a, c, b 4) b, a, c

Match the following:

◆ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in* ***Column–I*** *have to be matched with statements (p, q, r, s) in* ***Column–II****. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

15. A certain vector is given by $\vec{P} = 3\hat{i} + 4\hat{j} + 7\hat{k}$. Then choose the correct

a) Magnitude of the vector is 1) $\sqrt{74}$

b) angle made by it with x-axis is 2) $Cos^{-1}\left(\frac{3}{\sqrt{74}}\right)$

c) angle made by it with y-axis is 3) $Cos^{-1}\left(\frac{4}{\sqrt{74}}\right)$

d) angle made by it with z-axis is 4) $Cos^{-1}\left(\frac{7}{\sqrt{74}}\right)$

A) a-1, b-2, c-3, d-4 B) a-4, b-3, c-2, d-1 C) a-1, b-3, c-2, d-1 D) a-4, b-2, c-3, d-1

Comprehention type:

◆ *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which* ***ONLY ONE i****s correct. Choose the correct option.*

16. If $\vec{a}\cdot(\vec{a}-\vec{b})=0$ then

i) $\vec{a}\cdot\vec{b}$ =

A) a B) b C) a^2 D) b^2

ii) The component of $\vec{b}$ along the direction of $\vec{a}$ is

A) a B) b C) a^2 D) b^2

KEY

ΦΦ TEACHING TASK :

1) 2, 2) 2, 3) 4, 4) 4, 5) 4, 6) 3, 7) 4,
8) 1, 9) 1, 10) 2, 11)A, 12) D, 13) 1,
14) 4, 15) 1, 16) i) C, ii) A

LEARNER'S TASK

BEGINNERS (Level - I)

Choose the correct option:

1. The angle between the two vectors $\bar{A} = i + 2\bar{j} - \bar{k}$ and $\bar{B} = -i + \bar{j} - 2\bar{k}$ is

1) 90^0 2) 30^0 3) 45^0 4) 60^0

2. The angle between two vectors $\bar{A} = (4, -2, 5)$ and $\bar{B} = (3, 1, -2)$ is

1) 60^0 2) 30^0 3) 90^0 4) 45^0

3. The scalar product of two vectors $\bar{A} = (4, -2, 0)$ and $\bar{B} = (1, 2, 3)$ is

1) 0 2)-2 3)6 4)-6

4. Velocity and acceleration vectors of charged particle moving perpendicular to the direction of a magnetic field at a given instant of time are $\vec{v} = 2\hat{i} + c\hat{j}$ and $\vec{a} = 3\hat{i} + 4\hat{j}$ respectively. Then the value of 'c' is

1) 3 2) 1.5 3) -1.5 4) -3

5. The vector $\bar{A} = \bar{i} + \bar{j}$ where $\bar{i}, \bar{j}$ are unit vectors along X and Y axes respectively makes an angle of ..... with X-axis

1) 0^0 2) 45^0 3)60^0 4)90^0

6. The angle made by the vector $\bar{A} = 2\bar{i} + 3\bar{j}$ with Y-axis is

1) $\tan -1\left(\frac{3}{2}\right)$ 2) $\tan -1\left(\frac{2}{3}\right)$ 3) $\sin^{-1}\left(\frac{2}{3}\right)$ 4) $\cos^{-1}\left(\frac{3}{2}\right)$

7. The angle between the two vectors $-2i + 3\bar{j} - \bar{k}$ and $i + 2\bar{j} + 4\bar{k}$ is

1) 0^0 2) 90^0 3)180^0 4)45^0

8. The angle between the vectors $\left(\vec{i} + \vec{j} + \vec{k}\right)$ and $\left(\vec{i} - \vec{j} - \vec{k}\right)$ is

1) $\sin^{-1}\frac{\sqrt{8}}{3}$ 2) $\sin^{-1}\left(\frac{1}{3}\right)$ 3) $\cos^{-1}\frac{\sqrt{8}}{3}$ 4) $\cos^{-1}\sqrt{\frac{8}{3}}$

9. The angle between two vectors $\bar{P}=3i+2\bar{j}+3\bar{k}$, $\bar{Q}=2i-\bar{j}+3\bar{k}$ is

1) $Cos^{-1}\left[\frac{6.5}{\sqrt{77}}\right]$ 2) $Cos^{-1}\left[\frac{\sqrt{77}}{6.5}\right]$ 3) $Sin^{-1}\left[\frac{6.5}{\sqrt{77}}\right]$ 4) $Sin^{-1}\left[\frac{\sqrt{77}}{6.5}\right]$

10. If $\bar{A}=ai+2\bar{j}-5\bar{k}, \bar{B}=2i-j-4\bar{k}$ are perpendicular of each other, the value of 'a' is

1) 9 2) -9 3) 4 4) -4

11. Given two vectors $\overline{A}=\hat{i}-2\hat{j}-3\hat{k}$ and $\overline{B}=4\hat{i}-2\hat{j}+6\hat{k}$. The angle made by $\left(\overline{A}+\overline{B}\right)$ with X-axis is

1. 30^0 2) 45^0 3) 60^0 4) 90^0

12. Two vectors $\bar{A}$ and $\bar{B}$ are at right angles to each other when

1) $\bar{A}+\bar{B}=0$ 2) $\bar{A}-\bar{B}=0$ 3) $\bar{A}\,x\,\bar{B}=0$ 4) $\bar{A}.\bar{B}=0$

13. If $\bar{A}.\bar{B}=0$ the angle between the vectors A and B will be

1) 0^0 2) 60^0 3) 90^0 4)180^0

14. The angle between $V_1=\bar{i}+2\bar{j}+2\bar{k}$, $V_2=2\bar{i}+3\bar{j}-4\bar{k}$ is

1) 0^0 2)45^0 3)90^0 4)120^0

15. A force $\bar{F}=2\bar{i}+4\bar{j}+\bar{k}$ acts on a body and produces a displacement of $\bar{S}=3\bar{i}+2\bar{j}+5\bar{k}$ work done is

1) 10 units 2) 19 units 3) 29 units 4) zero

16. The angle between vectors $\bar{A}=i-5\bar{j}$ and $\bar{B}=2i-10\bar{j}$ is

1) 90^0 2) 180^0 3) 60^0 4) 0^0

17. The angle made by the vector $\bar{A}=\bar{i}+\bar{j}$ with x-axis is

1) 90^0 2)45^0 3)22.5^0 4)30^0

18. Angle (in rad) made by the vector $\sqrt{3}\hat{i}+\hat{j}$ with the X-axis

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

19. If A and B are two perpendicular vectors given by $\bar{A}=5\bar{i}+10\bar{j}-3\bar{k}$, and $\bar{B}=2\bar{i}+4\bar{j}-c\bar{k}$ then the value of c is

1) 1/6 2) 3/2 3) 5/6 4) -50/3

20. The angle between two vectors $6\bar{i}+6\bar{j}-3\bar{k}$ and $7\bar{i}+4\bar{j}-4\bar{k}$ is

1) $Cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 2) $Cos^{-1}\left(\frac{5}{\sqrt{3}}\right)$ 3) $Sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ 4) $Cos^{-1}\left(\frac{26}{27}\right)$

21. The vector $\bar{P} = a\bar{i} + a\bar{j} + 3\bar{k}$ and $\bar{Q} = a\bar{i} - 2\bar{j} - \bar{k}$ are perpendicular to each other. The positive value of 'a' is

1) 3 2)2 3)1 4) 0

22. A force $\bar{F} = 3\bar{i} + c\bar{j} + 2\bar{k}$ N acting on a particle causes a displacement $\bar{S} = -4\bar{i} + 2\bar{j} - 3\bar{k}$ m. If the work done is 6 joule, the value of c is

1) 0 2) 1 3) 12 4)6

23. The angle between the two vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is

1) 30^0 2)45^0 3)60^0 4)90^0

24. $\vec{A} + \vec{B} = \vec{C}, |\vec{A}| = |\vec{B}| = |\vec{C}|$, then the angle between $\bar{A}$ and $\bar{B}$ is

1) 45^0 2) 60^0 3) 90^0 4)120^0

25. A force of 1200 N acting on a stone by means of a rope slides the stone through a distance of 10m in a direction inclined at 60^0 to the force. The work done by the force is

1) $6000\sqrt{3}J$ 2) $6000\,J$ 3) $12000\,J$ 4) $8000\,J$

ACHIEVERS (Level - II)

Solve the following:

1. Find the angle made by the vector $\bar{i} + \sqrt{3}\,\bar{j}$ with Y-axis.

2. Find angle made by $\bar{i} + \bar{j} + 2\bar{k}$ with Z-axis.

3. Find the angle made by the vector $\bar{i} + \bar{j} + 2\bar{k}$ with X-axis.

4. Find the angle made by the vector $\bar{i} - \bar{j}$ with Z-axis

5. What is the angle made by the $\bar{A} = 2\bar{i} + 3\bar{j}$ with Y-axis.

6. A particle displaces from the position vector $\bar{i} + \bar{j} - 2\bar{k}$ to $2\bar{i} - \bar{j} + 3\bar{k}$ what is the angle made by the displacement vector with X-axis ?

7. If $|\bar{A} - \bar{B}| = |A - B|$ then find angle between $\bar{A}\ and \bar{B}$.

8. What is the angle between the vectors $(\bar{i} + \bar{j})\&(\bar{j} + \bar{k})$?

9. Find the angle between the vectors $\sqrt{3}\bar{i} + \bar{j}$ and $\bar{i} + \sqrt{3}\,\bar{j}$.

10. If the vectors $2\bar{i} + \bar{j} + c\,\bar{k}$ and $\bar{i} - 2\bar{j} + \bar{k}$ are perpendicular to each other then find the value c.

11. Find the angle between the vectors $2\hat{\bar{i}} + \hat{\bar{j}} + 3\bar{k}\,;\,\hat{\bar{i}} + \hat{\bar{j}} + \bar{k}$

EXPLORERS (Level - III)

Assertion A and Reason R:

*This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.*

A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

B) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'

C)'A' is true and 'R' is false

D) 'A' is false and 'R' is true

1. **A :** Scalar product of two vectors can be zero

R : If two vectors are peripendicular to each other, their scalar product is zero.

2. **A :** If $\vec{P} . \vec{Q} = PQ$, then angle between $\vec{P}$ & $\vec{Q}$ is 0^0.

R : Angle between $\vec{P}$ & $\vec{Q}$ can be from 0^0 to 180^0.

Multiple option type:

*This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D),out of which **ONE or MORE** is correct. Choose the correct options*

3. Choose the correct statements.

a) The product of a scalar and a vector is a vector quantity.

b) Two vectors of different magnitude can be combined to give a zero resultant.

c) Three vectors of different magnitudes may be combined to give a zero resultant.

d)All the above are wrong statements.

1) a & b are correct 2) a & d are correct

3) a & c are correct 4) b & c are correct

4. Dot product is used in the determination of

a) Work done by a force

b) Power developed by an automobile moving with uniform velocity.

c) The normal flux linked with a coil kept in magnetic field.

d) The force acting on a conductor carrying current kept in a magnetic field.

1) a, d are true 2) b, d are true 3) a, b, c are true 4) c, d are true

5. Set the angles made by following vectors with X-axis in the increasing order.

a) 3i + 4j b) 4i + 3j c) i + j

1) a, b, c 2) c, b, a 3) b, c, a 4) a, c, b

Match the following:

*This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

6) a) $\bar{i}.\bar{i} = \bar{j}.\bar{j} = \bar{k}.\bar{k} =$ 1) 0

b) $\bar{i}.\bar{j} = \bar{j}.\bar{k} = \bar{k}.\bar{i} =$ 2) 1

c) $\bar{i}.\bar{i} + \bar{j}.\bar{j} + \bar{k}.\bar{k} =$ 3) 2

d) $\bar{i}.(\bar{i} + \bar{j} + \bar{k}) + \bar{j}.(\bar{i} + \bar{j} + \bar{k}) =$ 4) 3

A) a-1, b-2, c-3, d-4 B) a-2, b-1, c-4, d-3

C) a-4, b-3, c-2, d-1 D) a-1, b-2, c-4, d-3

Comprehention type:

- *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which* ***ONLY ONE i****s correct. Choose the correct option.*

7. A force $F = 5\hat{i} - 3\hat{j} + 2\hat{k}$ moves a particle from $\bar{r_1} = 2\hat{i} + 7\hat{j} + 4\hat{k}$ to $\bar{r_2} = 5\hat{i} + 2\hat{j} + 8\hat{k}$

i) Displacement of the particle is

1) $5\hat{i} + 2\hat{j} + 8\hat{k}$ 2) $2\hat{i} + 7\hat{j} + 4\hat{k}$ 3) $3\hat{i} - 5\hat{j} + 4\hat{k}$ 4) $3\hat{i} - 5\hat{j} - 4\hat{k}$

ii) then workdone is

1) 48 units 2) 32 units 3) 38 units 4) 24 units

8. The angle between the vectors $\bar{a}$ & $\bar{b}$ is 60^0.

i) $\bar{a}.\bar{b} =$

1)) 0 2) ab 3) ab/2 4) 1

ii) The component vector of $\bar{b}$ along $\bar{a}$ is

1) $b\hat{\bar{a}}$ 2) $a\hat{\bar{b}}$ 3) $2b\hat{\bar{a}}$ 4) $\frac{b}{2}\hat{\bar{a}}$

High order thinking skills (HOTS):

9. $\bar{a}$ and $\bar{b}$ are unit vectors then $\frac{1-\bar{a}.\bar{b}}{1+\bar{a}.\bar{b}}$ =

1) $Sin^2\frac{\theta}{2}$ 2) $Cos^2\frac{\theta}{2}$ 3) $\tan^2\frac{\theta}{2}$ 4) $Cot^2\frac{\theta}{2}$

10. The angle between the diagonals of a cube with edges of length is

1) $\sin^{-1}(1/3)$ 2) $\cos^{-1}(1/3)$ 3) $\tan^{-1}(1/3)$ 4) $\cot^{-1}(1/3)$

11. If $\bar{a} = 9i - 7\bar{j} + 5\bar{k}$ and $\bar{b} = 3i - 2\bar{j} - 6\bar{k}$ then the value of $\left(\bar{a}+\bar{b}\right).\left(\bar{a}-\bar{b}\right)$ is

1) 206 2) 128 3) 106 4) -17

KEY

ΦΦ LEARNER'STASK :

☐ **BEGINNERS :**

1) 4, 2) 3, 3) 1, 4) 3, 5) 2, 6) 2, 7) 2, 8) 1, 9) 1, 10) 2, 11) 2, 12) 4,
13) 3, 14) 3, 15) 2, 16) 4, 17) 2, 18) 1, 19) 4, 20) 4, 21) 3, 22) 3, 23) 3, 24) 4,

25) 2.

☐ **ACHIEVERS :**

1) 30^0, 2) $\cos^{-1}\left(2/\sqrt{6}\right)$, 3) $\cos^{-1}\left(1/\sqrt{6}\right)$, 4) 90^0, 5) $\cos^{-1}\left(3/\sqrt{13}\right)$, 6) $\cos^{-1}\left(1/\sqrt{30}\right)$, 7) 0^0, 8) 60^0, 9) 30^0, 10) zero, 11) $\cos^{-1}\left(\sqrt{6/7}\right)$.

☐ **EXPLORERS :**

1) A, 2) B, 3) 3, 4) 3, 5) 2, 6) B, 7)i)3,ii)3, 8)i)3, ii)4 9)3, 10)2, 11)3

Worksheet-5

§§ CROSS PRODUCT (or) VECTOR PRODUCT

The cross product of two vectors $\vec{a}\ and\ \vec{b}$ with an angle θ between them is a vector of magnitude $\left|\vec{a}\right|\left|\vec{b}\right|\sin\ \theta$, normal to the plane containing both $\vec{a}$ & $\vec{b}$

$$\vec{a} \times \vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\sin\ \theta\ \hat{n}$$

whose direction is given by right hand screw rule or right hand thumb hole

¶¶ Right hand screw rules

It states that, if a right handed screw be placed normal to the plane containing the two vectors $\left(say\ \bar{a}\ and\ \bar{b}\right)$ and rotated such that the first vector $\left(\bar{a}\right)$ is turned towards the second vector $\left(\bar{b}\right)$ through the smaller angle, then the screw would advance along the direction of cross product of the two vectors $\left(i.e.\ \bar{a}\times\bar{b}\right)$

¶¶ Right hand thumb rule:

To find the direction of $\bar{a}\times\bar{b}$,the first vector $\left(\bar{a}\right)$ is rotated on to the direction of the second vector $\left(\bar{b}\right)$ through the smaller angle. Make the curling of right hand fingers coincide with the direction of rotation. The direction of thumb then represents the direction of $\left(\bar{n}\right)$

¶¶ Right hand system:

In the right hand system the three unit vectors along the $\hat{i},\hat{j}\ \&\ \hat{k}$ three mutually perpendicular directions are so chosen that $\hat{i}\times\hat{j}=\hat{k}$ or $\hat{j}\times\hat{k}=\hat{i}$ or $\hat{k}\times\hat{i}=\hat{j}$

¶¶ Properties:

1. The quantity obtained as a cross product of two vectors is a vector quantity again.
2. $\left|\bar{a}\times\bar{b}\right|$ is maximum when $\sin\theta$ is maximum = 1 $\therefore \left|\bar{a}\times\bar{b}\right|_{max} = \left|\bar{a}\right|\left|\bar{b}\right| when\ \theta = 90^0$

$|\vec{a}\times\vec{b}|$ is minimum when $\sin\theta$ is min $(for\ 0\le\theta\le\pi)$ $\sin\theta$ is minimum for $for\ \theta=0\ or\ \pi$

$(\sin\theta)_{min}=0 \quad \therefore |\vec{a}\times\vec{b}|_{min}=0$

This forms the condition of parallelism of two vectors. Thus of two non-zero vectors are collinear (or parallel) with either $for\ \theta=0\ or\ \pi$ then their cross product is necessarily zero

3. Cross product does not obey commutative law

$\vec{a}\times\vec{b}=|\vec{a}|\,|\vec{b}|\sin\theta\,\hat{n}$ is a unit vector along $(\vec{a}\times\vec{b})=-(\vec{b}\times\vec{a})$ and

$\vec{b}\times\vec{a}=|\vec{b}|\,|\vec{a}|\sin\theta(-\hat{n})$ $\qquad \therefore \vec{a}\times\vec{b}\neq\vec{b}\times\vec{a}$

thus the vector $(\vec{a}\times\vec{b})$ and $(\vec{b}\times\vec{a})$ are two vectors of equal magnitudes but opposite directions.

4. Vector products obeys distributive law provided, the order is maintained

eg: $\vec{a}\times(\vec{b}+\vec{c}+\vec{d}.....)=(\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})+.....\neq(\vec{b}\times\vec{a})+(\vec{c}\times\vec{a})+....$

5. Vector product of two vectors is zero vector. Thus $\vec{a}\times\vec{a}=|\vec{a}|\,|\vec{a}|\sin 0\,\hat{n}=0$

6. Cross product of unit vectors $(\hat{i},\hat{j}\ \&\ \hat{k})$

$\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0\ \ \hat{i}\times\hat{j}=\hat{k}\ ;\ \hat{j}\times\hat{k}=\hat{i}\ ;\ \hat{k}\times\hat{i}=\hat{j}\qquad \hat{j}\times\hat{i}=-\hat{k}\ ;\hat{k}\times\hat{j}=-\hat{i}\ ;\ \hat{i}\times\hat{k}=-\hat{j}$

7. cross product of two vectors in terms of $\hat{i},\hat{j}\ \&\ \hat{k}$

let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ then

$\vec{a}\times\vec{b}=\left(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}\right)\times\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)=(a_2b_3-a_3b_2)\vec{i}-(a_1b_3-a_3b_1)\vec{j}+(a_1b_2-a_2b_1)\vec{k}$

using $\hat{i}\times\hat{i}=0\ ;\ \hat{j}\times\hat{k}=\hat{i}$ etc.

also $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\a_1&a_2&a_3\\b_1&b_2&b_3\end{vmatrix}$

8. If θ be the angle between $\vec{a}$ and $\vec{b}$ then $\left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta \Rightarrow \sin\theta = \dfrac{\left|\vec{a}\times\vec{b}\right|}{\left|\vec{a}\right|\left|\vec{b}\right|}$

in terms of unit vectors $\sin\theta = \dfrac{\sqrt{\sum\left(a_2b_3 - a_3b_2\right)^2}}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$

Note: from the above formula for $\sin\theta$ the value θ cannot be evaluated

$\therefore$ *for* θ lying in [0, 180^0] for each value of $\sin\theta$ there are two vectors of θ.

the values of θ can be determined from $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$

* The torque or turning effect of a force $\bar{F}$ is given by

$\bar{\tau} = \bar{r}\times\bar{F}$ where $\bar{r}$ is the position vector of point of application of force.

§§ **Geometrical interpretation:**

Let $\vec{a}$ and $\vec{b}$ be any two vectors shown by $\overline{OA}$ and $\overline{OB}$ respectively, with θ as the angle between them then

$\left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta = \left(\left|\vec{a}\right|\right)\left(\left|\vec{b}\right|\sin\theta\right)$=(OA)(OB $\sin\theta$)=(OB)(BM)

= area of the parallelogram OACB

Thus the modulus of the cross product between two vectors, is the area of the parallelogram formed with the two vectors drawn in magnitude and direction as its two adjacent sides.

Note: 1. The area of the triangle formed by two vectors $\vec{a}$ and $\vec{b}$ taken as the two sides will be $\dfrac{1}{2}\left|\vec{a}\times\vec{b}\right|$

2. $\dfrac{1}{2}\left(\vec{a}\times\vec{b}\right)$ or $\left(\vec{a}\times\vec{b}\right)$ will denote the vector areas of the triangle or parallelogram respectively formed by taking the two vectors as the adjacent sides drawn

Example:1 Two vectors having magnitudes 5 units and 12 units respectively. Find their cross product if the angle between them is 30^0

$\vec{a}\times\vec{b} = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta\,\hat{n} \Rightarrow \vec{a}\times\vec{b} = (5)(12)\sin 30^0\,\hat{n} = 30\,\hat{n}$

thus the cross product is a vector of magnitude 30 units, along the direction normal to the plane containing the two vectors in accordance with the right hand screw rule.

Example:2 Two vectors are given $6\hat{i}+3\hat{j}-2\hat{k}$ and $3\hat{i}+12\hat{j}-4\hat{k}$ what is the angle between them?

$\vec{a} = 6\hat{i}+3\hat{j}-2\hat{k}\,;\vec{b} = 3\hat{i}+12\hat{j}-4\hat{k}$, $\sin\theta = \dfrac{\left|\vec{a}\times\vec{b}\right|}{\left|\vec{a}\right|\left|\vec{b}\right|}$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 6 & 3 & -2\\ 3 & 12 & -4\end{vmatrix}=\hat{i}(-12+24)-\hat{j}(-24+6)+\hat{k}(72-9)=12\hat{i}-18\hat{j}+63\hat{k}$$

$$\therefore\left|\vec{a}\times\vec{b}\right|=\sqrt{12^2+18^2+63^2}=\sqrt{4437}=3\sqrt{493}$$

and $\left|\vec{a}\right|=\sqrt{6^2+3^2+2^2}=7,\quad \left|\vec{b}\right|=\sqrt{3^2+12^2+(-14)^2}=13$

$$\sin\theta=\frac{4\sqrt{493}}{7\times13}=\frac{3}{91}\sqrt{493}$$

alternatively $\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}=\dfrac{18+36+8}{7\times13}=\dfrac{62}{91}$ AB = 62 AC=91

$$BC=\sqrt{91^2-62^2}=\sqrt{4437}=3\sqrt{493}\qquad \sin\theta=\frac{BC}{AC}=\frac{3\sqrt{493}}{91}$$

Example:3 If $2\hat{i}-3\hat{j}+4\hat{k}$ and $3\hat{i}+\lambda\hat{j}+\mu\hat{k}$ be collinear vectors, then find the values of μ *and* λ

solution $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{b}=3\hat{i}+\lambda\hat{j}+\mu\hat{k}$ given $\vec{a}$ *and* $\vec{b}$ *are collinear*

$\therefore\vec{a}\times\vec{b}=0$ $(see\ property\ 2)$ i.e. $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 4\\ 3 & \lambda & \mu\end{vmatrix}=\vec{0}$

$$\Rightarrow(-3\mu-4\lambda)\hat{i}-(2\mu-12)\hat{j}+(2\lambda+9)\hat{k}=\vec{0}$$

equating the coefficients of $\hat{i},\hat{j}$ *and* $\hat{k}$ on both sides of the equation

we have $-3\mu-4\lambda-12=0$ *and* $2\lambda+9=0\ \therefore\mu=6,\lambda=-9/2$

Example:4 $\vec{a}$ *and* $\vec{b}$ are two vectors with moduli 1 and 5 respectively. If the magnitude of their cross product be 3 units; then find the tangent of the angle between $\vec{a}$ *and* $\vec{b}$

solution $\vec{a}\times\vec{b}=\left|\vec{a}\right|\left|\vec{b}\right|\sin\theta$ using $\sin\theta=\dfrac{\left|\vec{a}\times\vec{b}\right|}{\left|\vec{a}\right|\left|\vec{b}\right|}$ $\therefore\tan\theta=\dfrac{3}{1.5}=\dfrac{3}{5}$

$$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{(3/5)}{\sqrt{1-(3/5)^2}} = \frac{(3/5)}{\pm(4/5)} = \pm\frac{3}{4}$$

Example:5 If $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = 5\hat{i} - \hat{j} - \hat{k}$ then what is the area and vector are of the triangle formed by taking $\vec{a}$ and $\vec{b}$ as its two sides?

solution: vector area is given by $\frac{1}{2}(\vec{a}\times\vec{b}) = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 2 \\ 5 & -1 & -1 \end{vmatrix}$

$$= \frac{1}{2}\left[(2+2)\hat{i} - (-3-10)\hat{j} + (-3+10)\hat{k}\right] = \frac{1}{2}\left[4\hat{i} + 13\hat{j} + 7\hat{k}\right] sq.units$$

TEACHING TASK

Choose the correct option

1. The cross product of the vectors (2i -3j+ 4k) and (i + 4j - 5k) is

1) - i - 14j + 11k 2) i + 14j + 11k 3) - i + 14j + 11k 4) - i + 14j + 5k

2. Cross product of two unit vectors cannot be more than

1) 0.5 2) 1 3) 2 4) 3

3. The radius vector r =2i+5k and the angular velocity of a particle is $\overline{w} = 3i - 4k$. Then the linear velocity of the particle is

1) 15i - 8j + 6k 2) 2i + 3j + k 3) 15i + 8j - 6k 4) - 23j

4. (i+j) x (i-j) =

1) -2k 2) 2k 3) zero 4) 2i

5. The area of the parallelogram whose adjacent sides are (3i-j+2k) and (i+4j-6k) is

1) 614 units 2) $\sqrt{614}$ units 3) $\sqrt{573}$ units 4) 32 units

6. If the dot product of two vectors is equal to the magnitude of the cross product of the same vectors, then the angle between the vectors is

1) $\frac{\pi}{2}$ rad 2) $\frac{\pi}{4}$ rad 3) $\frac{\pi}{3}$ rad 4) π rad

7. If a,b are the magnitudes of vectors $\overline{a}$ & $\overline{b}$ and $\overline{a}.\overline{b} = 0$ the value of $|\overline{a}\times\overline{b}|$ is

1) 0 2) ab 3) $\sqrt{ab}$ 4) a/b

8. If $\overline{A} = 4N, \overline{B} = 3N$ the value of $|\overline{A}\times\overline{B}|^2 + |\overline{A}.\overline{B}|^2$ then

1) 5 N 2) 25 N 3) 144 N 4) 169 N

9. The resultant of vectors of magnitude 3 units and 4 units is 1 unit then the magnitude of their cross product is

1) -12 units 2) 7 units 3) -1 unit 4) zero

Assertion A and Reason R:

◆ *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which* ***ONLY ONE*** *is correct Choose the correct option.*

Opions: A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

B) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'

C)'A' is true and 'R' is false D) 'A' is false and 'R' is tr

10. **A :** If $\vec{A}$ is parallel to $\vec{B}$, then $\vec{A}\times\vec{B}$ is a null vector

R : The cross product of two vectors is given by $\vec{A}\times\vec{B} = AB\sin\theta\hat{n}$

11. **A :** If $\vec{A}\times\vec{B} = \vec{C}\times\vec{B}$, then $\vec{C}$ must be equal to $\vec{A}$.

R : The magnitude of the cross product of two vectors depends upon the angle between them.

Multiple option type:

◆ *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D),out of which* ***ONE or MORE*** *is correct. Choose the correct options*

12. If $\vec{P}\times\vec{Q} = \vec{R}; \vec{Q}\times\vec{R} = \vec{P}$ and $\vec{R}\times\vec{P} = \vec{Q}$ then

a) $\vec{P}.\vec{Q}$ and $\vec{R}$ are coplanar

b) angle between $\vec{P}$ and $\vec{Q}$ may be less than 90^0

c) $\vec{P}+\vec{Q}+\vec{R}$ cannot be equal to zero.

d) $\vec{P},\vec{Q}$ and $\vec{R}$ are mutually perpendicular

1) both c & d 2) a, b & c 3) both a & b 4) both d & b

Match the following:

◆ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in* ***Column–I*** *have to be matched with statements (p, q, r, s) in* ***Column–II****. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

13. Study the following table.

a) $\vec{i}.(\vec{j}\times\vec{k}) + \vec{j}.(\vec{k}\times\vec{i}) + \vec{k}.(\vec{i}\times\vec{j}) =$ e) 0

b) $(\vec{i}.\vec{i}) + (\vec{i}\times\vec{i}) =$ f) 1

c) $(\vec{A}.\vec{B})(\vec{A}\times\vec{B}) + (\vec{B}.\vec{A})(\vec{B}\times\vec{A}) =$ g) 3

d) $(\vec{i}+\vec{j}).(2\vec{i}+2\vec{k}) =$ h) 2

1) $a \to e; b \to f; c \to g; d \to h$ 2) $a \to g; b \to h; c \to f; d \to e$

3) $a \to g; b \to f; c \to e; d \to h$ 4) $a \to f; b \to g; c \to h; d \to e$

Comprehention type:

- *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

14. Two vectors $\bar{A}, \bar{B}$ are such that $\bar{A} = 4\bar{i} - \bar{j} - \bar{k}$ and $\bar{B} = 4\bar{i} + \bar{j} - 4\bar{k}$

i) $\bar{A} \times \bar{B} =$

1) $5\bar{i} - 5\bar{j} + 5\bar{k}$ 2) $\pm(5\bar{i} + 12\bar{j} + 8\bar{k})$ 3) $\bar{i} + 3\bar{j} + \bar{k}$ 4) $5\bar{i} - 5\bar{j} + 5\bar{k}$

ii) The unit vector perpendicular to the plane containing $\bar{A}, \bar{B}$ is

1) $\frac{5\bar{i} - 5\bar{j} + 5\bar{k}}{\sqrt{3}}$ 2) $\pm\frac{5\bar{i} + 12\bar{j} + 8\bar{k}}{\sqrt{233}}$ 3) $\frac{\bar{i} + 3\bar{j} + \bar{k}}{11}$ 4) $5\bar{i} - 5\bar{j} + 5\bar{k}$

LEARNER'S TASK

BEGINNERS (Level - I)

Choose the correct option

1. $|A \times B|^2 + (A \cdot B)^2$ is equal to

1) $A^2 + B^2$ 2) A^2B^2 3) $2A^2B^2$ 4) Zero

2. Two sides of a triangle are given by $\hat{i} + \hat{j} + \hat{k}$ and $-\hat{i} + 2\hat{j} + 3\hat{k}$, then area of triangle is

1) $\sqrt{26}$ 2) $\sqrt{26}/2$ 3) $\sqrt{46}$ 4) 26

3. Area of the triangle whose base is given by $\bar{A} = i + j + k$ and one of the two sides by $\bar{B} = 4j + 3k$ is

1) $\sqrt{26}$ 2) $\sqrt{26}$ /2 3) 5/2 4) 5

4. If the radius vector is $\bar{r} = 6i + j + 3k$ and the force $\bar{F} = i - 4j + k,$ the torque is

1) 11i + 9j + 25k 2) -11i + 9j + 25k 3) -11i - 9j - 25k 4) 13i - 3j - 25k

5. Area of a parallelogram formed by vectors $(3\hat{i} - 2\hat{j} + \hat{k})$m and $(\hat{i} + 2\hat{j} + 3\hat{k})$ m is

1) $3\sqrt{8}\ m^2$ 2) 24 m^2 3) $8\sqrt{3}m^2$ 4) $4\sqrt{3}m^2$

6. The unit vector perpendicular to $\bar{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\bar{B} = \hat{i} - \hat{j} + \hat{k}$ is

1) $\frac{4\hat{i}-\hat{j}-5\hat{k}}{\sqrt{42}}$ 2) $\frac{4\hat{i}-\hat{j}+5\hat{k}}{\sqrt{42}}$ 3) $\frac{4\hat{i}+\hat{j}+5\hat{k}}{\sqrt{42}}$ 4) $\frac{4\hat{i}+\hat{j}-5\hat{k}}{\sqrt{42}}$

7. Of the vectors given below, the parallel vectors are,

$\vec{A}=6\hat{i}+8\hat{j}$ $\vec{B}=210\hat{i}+280\hat{k}$ $\vec{C}=5.1\hat{i}+6.8\hat{j}$ $\vec{D}=3.6\hat{i}+8\hat{J}+48\hat{k}$

1) $\vec{A}$ and $\vec{B}$ 2) $\vec{A}$ and $\vec{C}$ 3) $\vec{A}$ and $\vec{D}$ 4) $\vec{C}$ and $\vec{D}$

8. On rotating a wheel of radius 4m, a force of 20N is applied at an angle 30^0 to the radius at point of application. The resulting torque on the wheel is

1) 80 Nm 2) 60 Nm 3) 40 Nm 4) 20 Nm

9. The area of the triangle whose adjacent sides are represented by the vector $\left(4\bar{i}+3\bar{j}+4\bar{k}\right)$ and $5\bar{i}$ in sq. units is

1) 25 2) 12.5 3) 50 4) 45

10. The angular velocity of a rotating body is $\bar{\omega}=4\bar{i}+\bar{j}-2\bar{k}$. The linear velocity of the body whose position vector $2\bar{i}+3\bar{j}-3\bar{k}$ is

1) $5\bar{i}+8\bar{j}+14\bar{k}$ 2) $3\bar{i}+8\bar{j}+10\bar{k}$ 3) $8\bar{i}-3\bar{j}+2k$ 4) $-8\bar{i}+3\bar{j}+2\bar{k}$

11. If $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}$, then the angle between $\vec{A}$ and $\vec{B}$ is .....

1) π 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$

12. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin then

1) $\vec{r}.\vec{F}=0$ and $\vec{F}.\vec{\tau}\neq 0$ 2) $\vec{r}.\vec{\tau}\neq 0$ and $\vec{F}.\vec{\tau}=0$

3) $\vec{r}.\vec{\tau}\neq 0$ and $\vec{F}.\vec{\tau}\neq 0$ 4) $\vec{r}.\vec{\tau}=0$ and $\vec{F}.\vec{\tau}=0$

13. The area of the parallelogram whose adjacent sides are $\bar{P}=3\bar{i}+4\bar{j}$, $\bar{Q}=-5\bar{i}+7\bar{j}$ is (in sq. units)

1) 20.5 2) 82 3)41 4) 46

ACHIEVERS (Level - II)

Solve the following:

1. If $5\bar{i}-4\bar{j}-m\bar{k}$ & $2\bar{i}+2\bar{j}+4\bar{k}$ are normal to each other then find the value of m

2. If the scalar product between $3\bar{i}+4\bar{j}-7\bar{k}$ & $2\bar{i}+4\bar{j}+p\bar{k}$ is 8 then find p

3. If $\bar{P}\cdot\bar{Q}=0$ & find the angle between $\bar{P}$ & $\bar{Q}$

4. Find the angle made by the vector $\bar{i} - \sqrt{3}\,\bar{j}$

5. Find the component vector of $2\bar{i} + \bar{j} - \bar{k}$ along $\bar{i} + \bar{j} + \bar{k}$

EXPLORERS (Level - III)

Assertion A and Reason R:

♦ *This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which* ***ONLY ONE*** *is correct Choose the correct option.*

Opions: A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

B) Both 'A' and 'R' are true and 'R' is not correct explanation of 'A'

C)'A' is true and 'R' is false D) 'A' is false and 'R' is tr

1. **A :** If $\left|\vec{P}\times\vec{Q}\right| = PQ$, then $\vec{P}$ must be perpendicular to $\vec{Q}$.

R : It will hold even if $\vec{P} \| \vec{Q}$.

2. **A :** $\bar{A}x\bar{B} = \bar{C}$ where $\bar{C}$ is a vector normal to both $\bar{A}$ and $\bar{B}$

R : Direction of cross product obeys right hand thumb rule

Multi correct option type

This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D),out of which ***ONE or MORE*** *is correct. Choose the correct options*

3. If vectors $\vec{A}$ and $\vec{B}$ are given by $\vec{A} = 5\hat{i} + 6\hat{j} + 3\hat{k}$ and $\vec{B} = 6\hat{i} - 2\hat{j} - 6\hat{k}$ which is / are of the followings correct ?

a) $\vec{A}$ and $\vec{B}$ are mutually perpendicular b) product of $\vec{A}\times\vec{B}$ is same as $\vec{B}\times\vec{A}$

c) The magnitude of $\vec{A}$ and $\vec{B}$ are equal d) The magnitude of $\vec{A}.\vec{B}$ is zero.

1) a, d are correct 2) b, c are correct 3) c, d are correct 4) b, a are correct

4. Which of the following statements is/are correct?

a) The magnitude of the vector $3\hat{i} + 4\hat{j}$ is 5

b) A force $(3\hat{i} + 4\hat{j})$ N acting on a particle causes a displacement $6\hat{j}$ m. The work done by the force is 30Joule

c) If $\vec{A}$ and $\vec{B}$ represent two adjacent sides of a parallelogram, then $|\vec{A}\times\vec{B}|$ gives the ares of that parallelogram.

d) A force has magnitude 20N. It component in a direction making an angle 60^0 with the force is $10\sqrt{3}$ N.

1) a & b are correct 2) a & c are correct 3) a & d are correct 4) all are correct

5. Consider the following statements A and B given below and identify the correct answer.

A)If $|\vec{A}+\vec{B}|=\vec{A}.\vec{B}$, then angle between $\vec{A}$ and $\vec{B}$ is 90^0

B) If $\vec{A}\times\vec{B}$ = 0, then $\vec{A}.\vec{B}$ = AB

1) both A and B are true
2) A is true but B is false
3) B is true but A is false
4) both A and B are false

Match the following:

♦ *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

6.

a) $\vec{P}.\vec{Q}$	I) pseudo vector
b) $\vec{P}\times\vec{Q}$	II) undefined
c) $\vec{P}.(\vec{Q}.\vec{R})$	III) A scalar
d) $\vec{P}\times(\vec{Q}\times\vec{R})$	IV) A vector

1) a-IV, b-III, c-I, d-II
2) a-II, b-I, c-III, d-IV
3) a-III, b-IV, c-II, d-I
4) a-I, b-II, c-IV, d-III

7.

a) $\|\vec{A}+\vec{B}\|=\|\vec{A}-\vec{B}\|$	e) 45^0	1) $a\to e; b\to g; c\to f; d\to h$
b) $\|\vec{A}\times\vec{B}\|=\vec{A}.\vec{B}$	f) 30^0	2) $a\to g; b\to e; c\to h; d\to f$
c) $\vec{A}.\vec{B}=\frac{AB}{2}$	g) 90^0	3) $a\to g; b\to f; c\to e; d\to h$
d) $\|\vec{A}\times\vec{B}\|=\frac{AB}{2}$	h) 60^0	4) $a\to e; b\to g; c\to h; d\to f$

8. Match the follwoing

a) Addition of vectors	e) relative velocity
b) dot product	f) resultant velocity
c) cross product	g) work done
d) subtraction of vectors	h) torque

1) $a\to e; b\to f; c\to g; d\to h$
2) $a\to h; b\to g; c\to f; d\to e$
3) $a\to f; b\to g; c\to h; d\to e$
4) $a\to g; b\to h; c\to e; d\to f$

Comprehention type questions:

♦ *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

9. Two vectors $\vec{A} = 3i - 2j + k \;\&\; \vec{B} = i + 2j + 3k$ are acting as parallel sides then

i) Area of triangle formed by them is

A) $3\sqrt{8}$ sq units B) 24 Sq units C) $4\sqrt{3}$ sq units D) none

ii) The area of parallelogram formed is

A) $6\sqrt{8}$ sq units B) 42 Sq units C) $8\sqrt{3}$ sq units D) none

High order thinking skills (HOTS):

10. If $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{B} = 3\hat{i} - 6\hat{j} + 2\hat{k}$ then vector perpendicular to both $\vec{A}$ and $\vec{B}$ has magnitude K times that of $6\hat{i} + 2\hat{j} - 3\hat{k}$. Then K =

1) 1 2) 3 3) 7 4) 9

11. Three vectors $ai - 4j + 5k, i - 2j + 3k$ and $2i - j + k$ are coplanar. The value of 'a' is

1) 3 2) 4 3) –5 4) 5

12. A vector $\bar{A}$ has magnitude of 3 units and points towards east while another vector $\bar{B}$ has magnitude of 4 units and points towards north. The ratio between $\bar{A}.\bar{B}$ and $|\bar{A} \times \bar{B}|$ is

1)1:1 2) 1:0 3) 0:12 4)12:0

RESEARCHERS (Level - IV)

I. Choose the correct option:

1. The component of vector $\bar{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ along the direction of $\hat{i} - \hat{j}$ is **(2008E)**

1) $a_x - a_y + a_z$ 2) $a_x - a_y$ 3) $\frac{a_x - a_y}{\sqrt{2}}$ 4) $(a_x + a_y + a_z)$

2. Velocity and acceleration vectors of charged particle moving perpendicular to the direction of a magnetic field at a given instant of time are $\vec{v} = 2\hat{i} + c\hat{j}$ and $\vec{a} = 3\hat{i} + 4\hat{j}$ respectively. Then the value of 'c' is

1) 3 2) 1.5 3) -1.5 4) -3

3. At given instant of time two particles are having the position vectors $4\vec{i} - 4\vec{j} + 7\vec{k}$ and $2\vec{i} + 2\vec{j} + 5\vec{k}$ m respectively. If the velocity of the first particle is $0.4\vec{i}\ ms^{-1}$, the velocity of second particle in ms^{-1} if they collide after 10 sec is **[2004E]**

1) $6(\vec{i} - \vec{j} + \frac{1}{\sqrt{5}}\vec{k})$ 2) $0.6(\vec{i} - \vec{j} + \frac{\vec{k}}{3})$ 3) $0.6(\vec{i} + \vec{j} + \frac{\vec{k}}{3})$ 4) $0.6(\vec{i} + \vec{j} - \frac{\vec{k}}{3})$

4. Two particles having position vectors $\vec{r}_1 = (3\vec{i} + 5\vec{j})m$ and $\vec{r}_2 = (-5\vec{i} + 3\vec{j})m$ are moving with velocities $\vec{V}_1 = (4\vec{i} + 3\vec{j})ms^{-1}$ and $\vec{V}_2 = (-a\vec{i} + 4\vec{j})ms^{-1}$. If they collide after 2 seconds,

the value of 'a' is

1) -2 2) -4 3) - 6 4) - 8

5. The angle between two vectors $6\bar{i}+6\bar{j}-3\bar{k}$ and $7\bar{i}+4\bar{j}-4\bar{k}$ is **(1999E)**

1) $Cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ 2) $Cos^{-1}\left(\frac{5}{\sqrt{3}}\right)$ 3) $Sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ 4) $Cos^{-1}\left(\frac{26}{27}\right)$

6. A force $\bar{F}=3\bar{i}+c\bar{j}+2\bar{k}$ N acting on a particle causes a displacement $\bar{S}=-4\bar{i}+2\bar{j}-3\bar{k}$ m. If the work done is 6 joule, the value of c is **(1997 E&M)**

1) 0 2) 1 3) 12 4)6

7. A particle is moving east wards with a velocity of 15ms^{-1}. Sudenly, it movs towards north and moves with same speed in a time 10 s. The average acceleration during this time in ms^{-2} is **(1997E)**

1) $\frac{3}{\sqrt{2}}$ NE 2) $3.\sqrt{2}$ NE 3) $\frac{3}{\sqrt{2}}$ NW 4) $3.\sqrt{2}$ NW

8. If $0.5\bar{i}+0.8\bar{j}+C\bar{k}$ is a unit vector, then C is **(1994E)**

1) $\sqrt{89}$ 2) 0.2 3) 0.3 4) $\sqrt{0.11}$

9. $\left|\bar{A}+\bar{B}\right|=\left|\bar{A}-\bar{B}\right|$ then the angle betwen $\bar{A},\bar{B}$ is **(1993E)**

1) 120^0 2)0^0 3)90^0 4)180^0

10. If the magnitudes of $\bar{A}$ and $\bar{B}$ are a and b respectively, the magnitude of the resultant vector is always **(1993E)**

1) equal to $(a+b)$ 2) less than $(a+b)$

3) greater than $(a+b)$ 4)not greater than $(a+b)$

11. If A and B are two perpendicular vectors given by $\bar{A}=5\bar{i}+7\bar{j}+3\bar{k},$ and $\bar{B}=2\bar{i}+2\bar{j}+c\bar{k},$ then the value of c is **(1991E)**

1) - 2 2)8 3) -7 4) -8

12. If A and B are two perpendicular vectors given by $\bar{A}=5\bar{i}+10\bar{j}-3\bar{k},$ and $\bar{B}=2\bar{i}+4\bar{j}-c\bar{k}$ then the value of c is **(1991E)**

1) 1/6 2) 3/2 3) 5/6 4) -50/3

13. If $\bar{A}=3\bar{i}+4\bar{j}$ and $\bar{B}=7\bar{i}+24\bar{j}$, then the vector having the same magnitude as that of $\bar{B}$ and parallel to $\bar{A}$ is **(1989E)**

1) $15\bar{i}+20\bar{j}$ 2) $15\bar{i}-20\bar{j}$ 3) $20\bar{i}+15\bar{j}$ 4) $20\bar{i}-15\bar{j}$

14. The vector $\bar{A}=\bar{i}+\bar{j}$ where $\bar{i},\bar{j}$ are unit vectors along X and Y axes respectively makes an angle of ..... with X-axis **(1988E)**

1) 0^0 2) 45^0 3)60^0 4)90^0

15. Three forces $\vec{A}=\{\hat{i}+\hat{j}+\hat{k}\}$, $\vec{B}=\{2\hat{i}-\hat{j}-3\hat{k}\}$ and $\vec{C}$ acting on a body to keep it in equilibrium is **(2008M)**

1) $-\{3\hat{i}+4\hat{k}\}$ 2) $-\{4\hat{i}+3\hat{k}\}$ 3) $3\hat{i}+4\hat{j}$ 4) $2\hat{i}-3\hat{k}$

16. Given two vectors $\bar{A}=\hat{i}-2\hat{j}-3\hat{k}$ and $\bar{B}=4\hat{i}-2\hat{j}+6\hat{k}$. The angle made by $(\bar{A}+\bar{B})$ with X-axis is **(2007M)**

1) 30^0 2) 45^0 3) 60^0 4) 90^0 [7]

17. Of the vectors given below, the parallel vectors are, **(2006M)**

$\vec{A}=6\hat{i}+8\hat{j}$ $\vec{B}=210\hat{i}+280\hat{k}$ $\vec{C}=5.1\hat{i}+6.8\hat{j}$ $\vec{D}=3.6\hat{i}+8\hat{J}+48\hat{k}$

1) $\vec{A}$ and $\vec{B}$ 2) $\vec{A}$ and $\vec{C}$ 3) $\vec{A}$ and $\vec{D}$ 4) $\vec{C}$ and $\vec{D}$

18. Angle (in rad) made by the vector $\sqrt{3}\hat{i}+\hat{j}$ with the X-axis **(2005M)**

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

19. A vector $\vec{Q}$ has a magnitude of 8 is added to the vector $\vec{P}$ which lies along the X-axis. The resultant of these two vectors is a third vector $\vec{R}$ which lies along the Y-axis and has a magnitude twice that of $\vec{P}$. The magnitude of $\vec{P}$ is **[2004M]**

1) $6/\sqrt{5}$ 2) $8/\sqrt{5}$ 3) $12/\sqrt{5}$ 4) $16/\sqrt{5}$

20. A stationary body of mass 3 kg explodes into three equal pieces. Two of the pieces fly of at right angle to each other, one with a velocity $2\vec{i}\ ms^{-1}$ and the other with a velocity $3\vec{j}\ ms^{-1}$. If the explosion takes place in 10^{-5}s, the average force acting on the third piece in newtons is **[2003M]**

1) $(2\vec{i}+3\vec{j})\times10^{-5}$ 2) $-(2\vec{i}+3\vec{j})\times10^{5}$ 3) $(3\vec{i}-2\vec{j})\times10^{5}$ 4) $(2\vec{i}-3\vec{j})\times10^{-5}$

21. The unit vector parallel to the resultant of vectors $\bar{A}=4\bar{i}+3\bar{j}+6\bar{k}$ and $\bar{B}=-\bar{i}+3\bar{j}-8\bar{k}$ is **(2000M)**

1) $\frac{1}{7}(3\bar{i}+6\bar{j}-2\bar{k})$ 2) $-\frac{1}{7}(3\bar{i}+6\bar{j}-2\bar{k})$ 3) $\frac{1}{49}(3\bar{i}+6\bar{j}-2\bar{k})$ 4) $\frac{1}{47}(3\bar{i}+6\bar{j}-2\bar{k})$

22. A body of mass 5kg starts from origin with an initial velocity $30\bar{i}+40\bar{j}\ ms^{-1}$. If a constant force $\bar{F}=-6\bar{i}-5\bar{j}$ N acts on the body, the time in which the y-component of velocity become zero is **(2000M)**

1) 5s 2) 20s 3)40s 4)80s

23. $\vec{A}+\vec{B}=\vec{C}, |\vec{A}|=|\vec{B}|=|\vec{C}|$, then the angle between $\bar{A}$ and $\bar{B}$ is **(1999M)**

1) 45^0 2) 60^0 3) 90^0 4)120^0

24. The vector $\bar{P}=a\bar{i}+a\bar{j}+3\bar{k}$ and $\bar{Q}=a\bar{i}-2\bar{j}-\bar{k}$ are perpendicular to each other. The positive

value of 'a' is **(1998)**

1) 3 2)2 3)1 4) 0

25. The vectors having same magnitude have a resultant equal to either, then the angle between the vectors will be **(1996M)**

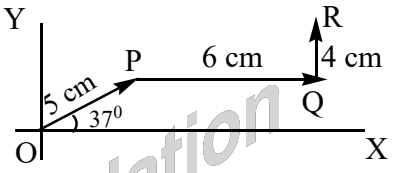
1) 30^0 2)60^0 3)90^0 4)120^0

26. Two forces each of magnitude 'f' and making an angle ϕ act on a body, then the resultant force is **(1996M)**

1) $f\sqrt{2(1-\sin\phi)}$ 2) $f\sqrt{2(1+\sin\phi)}$ 3) $2f.\sin\left(\frac{\phi}{2}\right)$ 4) $2f.\cos\left(\frac{\phi}{2}\right)$

II. Additional worksheet for practice:

1. The resultant of the three displacement vectors $\overline{0P}, \overline{PQ}$ and $\overline{QR}$ is



1) 15 cm 2) 17 cm

3) $\sqrt{149}$ cm 4) $\sqrt{107}$ cm

2. The square of the resultant of two forces 4N and 3N exceeds the square of the resultant of the two forces by 12 when they are mutually perpendicular. The angle between the vectors is

1) 30° 2) 60° 3) 90° 4) 120°

3. The greatest and least resultant of two forces acting at a point are 29N and 5N. If each force is increased by 3N and applied at right angles on a particle, the new resultant force is

1) 35N 2) 25N 3) $\sqrt{433}$ N 4) zero

4. The resultant of two equal forces is 141.4N when they are mutually perpendicular. When they are inclined at an angle 120º, then the resultant force will be

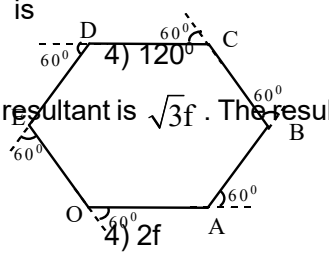
1) 100 N 2) 141.4 N 3) 196 N 4) Zero

5. The resultant of two forces at right angle is 17N. If the maximum resultant is 23N, the greater force is

1) 15N 2) 8N 3) 7N 4) 23N

6. Resultant of two vectors of magnitudes P and Q is of magnitude 'Q'. If the magnitude of $\vec{Q}$ is doubled now, then the angle made by new resultant with $\vec{P}$ is

1) 30^0 2) 60^0 3) 90^0 4) 120^0



7. When $\overline{f_1}$ and $\overline{f_2}$ each of magnitude f, inclined to each other resultant is $\sqrt{3}f$. The resultant of $\overline{f_1}$ & $-\overline{f_2}$ is

1) f 2) $\sqrt{2}f$ 3) $\sqrt{3}f$ 4) 2f

8. Which of the following are the rectangular components of $3\vec{i}+4\vec{j}$

1) –3, 4 2) $\sqrt{5}, 2\sqrt{5}$ 3) 1, $2\sqrt{6}$ 4) all of the above

9. Four forces of magnitude p, 2p, 3p, 4p act along the four sides of a square ABCD in cyclic order. The magnitude of resultant is

1) $\sqrt{2}P$ 2) $\sqrt{8}P$ 3) $\sqrt{4}P$ 4) $\sqrt{3P}$

10. Resultant of two forces of magnitudes a and b when act at angle θ is $5(\sqrt{a^2+b^2})$. When the angle is $90-\theta$, then resultant becomes $3(\sqrt{a^2+b^2})$. Then, $\tan\theta$

1) 1/2 2) 1/3 3) $1/\sqrt{2}$ 4) $1/\sqrt{3}$

11. The rectangular components of two vectors are (4.8, 6.4) and (–17.32, 10), then the angle between the two vectors is

1) 90^0 2) 97^0 3) 83^0 4) 45^0

12. A person moving on a motor cycle in a ground takes a turn through 60^0 on his left after every 50 m. Then find the magnitude of

displacement suffered by him

after 9th turn

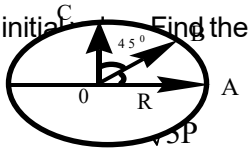
1) 100 m 2) 50 m

3) $50\sqrt{3}$ m 4) 20 m

13. ABCDEF is a regular hexagon, then find $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}$. (O is the centre of regular hexagon)

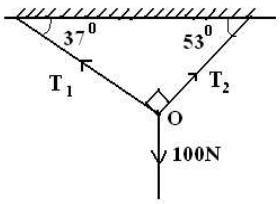
1) $6\overrightarrow{OA}$ 2) $6\overrightarrow{OB}$ 3) $6\overrightarrow{OC}$ 4) $6\overrightarrow{OD}$

14. Two vectors of equal magnitude are inclined at some angle such that the difference in magnitude of resultant and magnitude of either of the vectors is 0.732 times either of the magnitude of vectors. If the angle between them is increased by half of its initial value. Find the magnitude of difference of the vectors



1) 2 P 2) $\sqrt{2}P$ 3) 3 P $\sqrt{3}P$

15. Two balls are moving on a table. One has velocity components $V_x = \sqrt{3}$ ms^{-1} and V_x = 3ms^{-1} and the other has compnents U_x = 2ms^{-1} and U_y = 2ms^{-1}, then find the angle between the directions of motion



1) 30^0 2) 45^0 3) 15^0

16. If $\overline{A}+\overline{B}+\overline{C}=\overline{0}$ out of $\overline{A},\overline{B},\overline{C}$, if two of them are having equal ma g $\sqrt{2}$ times the magnitude of either of the two, angles between $\overline{A}\&B$, $B\&C$, $C\&A$ is

1) 30^0 60^0 90^0 2) 120, 120, 120 3) 45, 45, 90^0 4) 90^0, 135^0, 135^0

17. If a,b and g are the angles made by a vector with X,Y and Z-axes then $\sin^2a+\sin^2b+\sin^2 g$ is

1) 1 2) 2 3) 3 4) 4

18. A certain vector in the xy plane has an x component of 4m and a y component of 10m. It is then rotated in the xy plane so its x component is doubled. Its new y component is about:

1) 20 m 2) 7.2 m 3) 5.0 m 4) 4.5 m

19. The resultant of the three vectors $\overline{0A}, \overline{0B}$ and $\overline{0C}$ has magnitude (R=Radius of the circle)

1) $\sqrt{2}$ R 2) (1+$\sqrt{2}$) R

3) ($\sqrt{2}$ -1) R 4) 2R

20. Two light strings of length 4cm and 3cm are tied to a bob of weight 500gm. The free ends of the strings are tied to pegs in the same horizontal line and separated by 5cm. The ratio of tension in the longer string to that in the shorter string is

1) 4:3 2) 3:4 3) 4:5 4) 5:4

21. If 'O' is in equilibrium then tensions T_1 and T_2 are $\left(\text{Tan}\,37^0 = \frac{3}{4}\right)$

1) 30 N, 50 N 2) 60 N, 80 N

3) 80 N, 60 N 4) 20 N, 40 N

22. For any two vectors A and B. If $\left|\overline{A}.\overline{B}\right| = \left|\overline{A}\times\overline{B}\right|$, the magnitude of $\vec{C} = \vec{A} + \vec{B}$ is

1) $\sqrt{A^2 + B^2}$ 2) $\left|\overline{A} + \overline{B}\right|$ 3) $\sqrt{A^2 + B^2 + \frac{AB}{\sqrt{2}}}$ 4) $\sqrt{A^2 + B^2 + \sqrt{2}AB}$

23. If $\overline{A} = \overline{B}$ x $\overline{C}$ the component of $\overline{A}$ along $\overline{B}$ is x, if θ is angle between $\overline{B}$ & $\overline{C}$. Then x is

1) $A\cos\theta$ 2) $B\cos\theta$ 3) $C\cos\theta$ 4) 0

24. (i + j + k) x (i + j - k) is equal to

1) 2 (i + j) 2) -2 (i - j) 3) -2 (i + k) 4) 2(j + k)

25. The unit vector perpendicular to vectors $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{j} - 5\hat{k}$ is

1) $\pm\frac{\left(\hat{i} - 3\hat{j} + \hat{k}\right)}{\sqrt{11}}$ 2) $\pm\frac{3\hat{i} + \hat{j}}{\sqrt{11}}$ 3) $\frac{2\hat{i} - \hat{j} - 5\hat{k}}{\sqrt{30}}$ 4) $\frac{i + j}{\sqrt{2}}$

26. If $\overline{a}$ is a unit vector along the axis of rotation of a purely rotating body and $\overline{b}$ is a unit vector along the direction of linear velocity of a particle of the body away from axis, then the value $\overline{a}.\overline{b}$ is

1) 1 2) -1 3) 0 4) 1/2

27. If $\vec{A}, \vec{B}, \vec{C}$ are coplanar. then. $\vec{A}.\left(\vec{B}\times\vec{C}\right)$ is

1) $-\vec{A}$ 2) $\vec{B}$ 3) 1 4) 0

28. If $\vec{A} = \left(2\hat{i} + 3\hat{j}\right)$ and $\vec{B} = \left(\hat{i} + \hat{j}\right)$ then component of $\vec{A}$ along $\vec{B}$ is

1) $\frac{5}{2}\left(\hat{i} + \hat{j}\right)$ 2) $\frac{5}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$ 3) $\frac{\sqrt{5}}{2}\left(\hat{i} + \hat{j}\right)$ 4) $\frac{5}{\sqrt{2}}\left(\hat{i} + \hat{k}\right)$

29. $\vec{A}$ and $\vec{B}$ are two vectors such that they are inclined at an angle of 60^0 and have equal magnitude of 10 unit. Then find $\left(\vec{A} + \vec{B}\right).\vec{C}$ where $\vec{C}$ is another vector of same magnitude as $\vec{A}$ and $\vec{B}$ making an angle 60^0 with the plane containing $\vec{A}$ and $\vec{B}$

1) 50 units 2) $50\sqrt{2}$ units 3) $50\sqrt{3}$ units 4) $\frac{50}{\sqrt{3}}$ units

30. Diagonals of a parallelogram are given by $\vec{d}_1 = 2i + 3k$ and $\vec{d}_2 = i - j + 2k$, the area of the triangle formed by the sides is

1) $\sqrt{7.0}$ 2) $\sqrt{3.5}$ 3) $\sqrt{14.0}$ 4) zero

KEY

ΦΦ TEACHING TASK :

1) 3, 2) 2, 3) 4, 4) 1, 5) 3, 6) 2, 7) 2, 8) 3, 9) 4, 10) A, 11) B, 12)1, 13) 3, 14) i) 2, ii) 2

ΦΦ LEARNER'STASK :

☐ **BEGINNERS :**

1) 2, 2) 2, 3) 2, 4) 2, 5) 4, 6) 1, 7) 2, 8) 3, 9) 3, 10) 2, 11) 1, 12) 4, 13) 3,

☐ **ACHIEVERS :** 1.1/2, 2.2 3.90^0 4.120^0 5.2/$\sqrt{3}$

☐ **EXPLORERS :** 1) C, 2) A, 3) 1, 4) 2, 5) 3, 6) 3, 7) 2, 8) 3, 9) i) C, ii) C 10) 3, 11) 4, 12) 3

☐ **RESEARCHERS :**

I. 1) 2, 2) 3, 3) 2, 4) 4, 5) 4, 6) 3, 7) 3, 8) 4, 9) 3, 10) 4, 11) 4, 12) 1, 13) 2, 14) 2, 15) 1, 16) 2, 17) 2, 18) 1, 19) 2, 20) 2, 21) 1, 22) 3, 23) 4, 24) 3, 25) 4, 26) 4

II. 1) 3, 2) 2, 3) 2, 4) 1, 5) 1, 6) 3, 7) 1, 8) 1, 9) 2, 10) 4, 11) 2, 12) 1, 13) 4, 14) 2, 15) 3, 16) 4, 17) 2, 18) 1, 19) 2, 20) 2, 21) 2, 22) 4, 23) 4, 24) 2, 25) 1, 26) 3, 27) 4, 28) 1, 29) 3, 30) 2