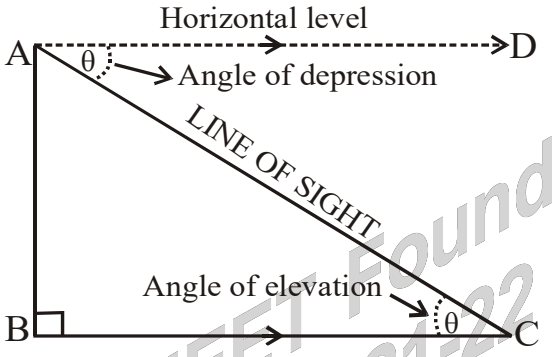
APPLICATIONS OF TRIGONOMETRY

INTRODUCTION :

Trigonometry is one of the most ancient subjects studied by scholars all over the world. The astronomers used trigonometry to calculate distance from the Earth to the planets and stars. Trigonometry is also used in geography to construct maps, determine the position of an island in relation to the longitudes and latitudes, etc.

HEIGHTS AND DISTANCES :

Let A be the top of a tower and C be the eye of a person from where he is observing the top of a tower, then AC is called the line of sight. The angle $\angle BCA$, so formed by the line of sight with the horizontal level is called the angle of elevation of the top of tower from the eye of a person.



Hence, the line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer, i.e., the **angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level.

Let C be an object and A be the eye of a person from where he is observing the object C, then AC is called the line of sight. The angle $\angle CAD$, so formed by the line of sight with the horizontal level is called the angle of depression of the object fro the eye of a person.

Hence, the line of sight is the line drawn from the eye of angle observer to the point in the object viewed by the observer, i.e., the angle so formed by the line of sight with the horizontal level is called the **angle of depression**. The angle of depression of a point on the object being viewed is the angle formed by the line of sight with the horizontal level when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed.

Now, in right Δ ABC, we have

BC || AD and AC is a transversal.

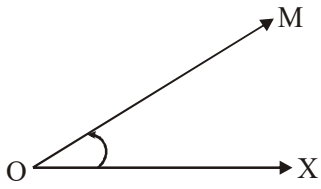
$\therefore \angle ACB = \angle CAD \quad [\text{Alternate angles}]$

$\Rightarrow \angle\theta = \angle\theta$

Hence, Angle of elevation = Angle of depression.

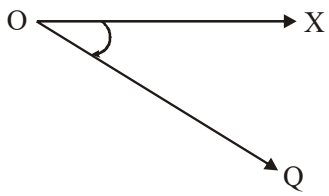
Angle of Elevation

$\angle XOM$ i.e., the angle in which the line joining the object and the eye makes with the horizontal through the eye is called the angle of elevation of M as seen from O.



Angle of Depression

$\angle XOP$ i.e., the angle in which the line joining the object and the eye makes with the horizontal through the eye is called the angle of depression of P as seen from O.



or

The angle between the horizontal line drawn through the observer's eye and the line joining the eye to any object is called, the angle of elevation of than object when it is at joining the eye to any object is called, the angle of elevation of than object when it is at higher level than the eye.

The angle of depression of the object when it is at a lower level than the eye.

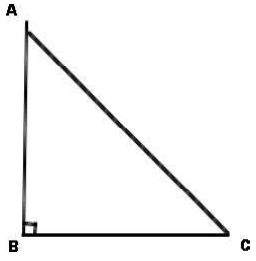
1. A boy observed the top of an electric pole at an angle of elevation of 60^0 when the observation point is 8 m away from the foot of the pole . Find the height of the pole.

sol: Let

AB = h m = height of the electric pole

BC= 8 m = Distance between pole to observation point

$\angle ACB = 60^0$ = Angle of elevation



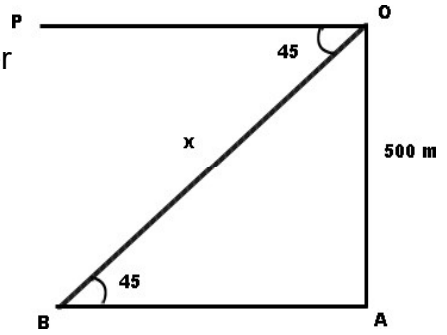
$In \ \ \Delta ABC$,

$$Tan\angle ACB = \frac{side\ \ opp.\ to\ \angle ACB}{Side\ \ adj.\ \ to\angle ACB}$$

$$Tan\ 60^0 = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{8}\ \ h = 8\sqrt{3}\ m$$

$h = 8\sqrt{3}\ m$

2. Raju Observes a person standing on the ground from a helicopter at an angle of depression 45^0 . If the helicopter flies at a height of 500 meters from the ground, what is the distance of the person from Raju?



Sol: Let AO = 500 m distance between ground to helicopter

OB = x m = Distance between Raju and person

$\angle POB = 45^0$ = Angle of depression from Raju to person

$AB \parallel OP$, BO is a transversal

$\angle POB = \angle ABO = 45^0$ (Alternate interior angles are equal)

$$Sin\angle ABO = \frac{Side\ opp.\ to\ \angle ABO}{Hypotenuse}$$

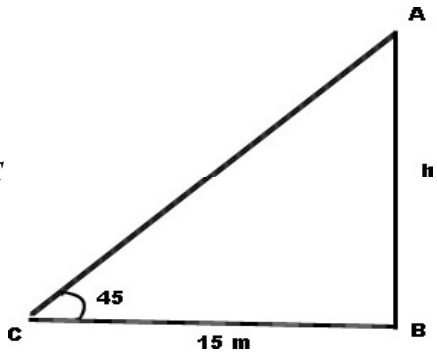
$$Sin\,45^0 = \frac{500}{x} \Rightarrow \frac{1}{\sqrt{2}} = \frac{500}{x}$$

$x = 500\sqrt{2}$ meters

3. A tower satands vertically on the ground. From a point which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is 45^0 . What is the height of the tower?

Sol: Let AB = h m = Height of the tower

BC = 15 m = Distance between foot of the tower to observation point



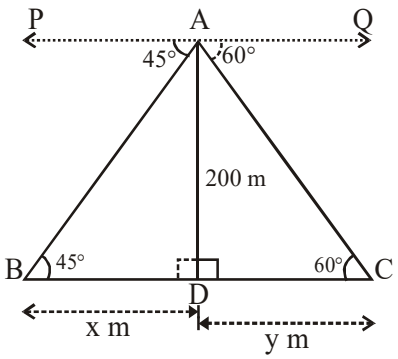
$\angle ACB = 45^0$ = Angle of elevation $In\ \Delta ABC$

$$Tan\angle ACB = \frac{Side\ \ opp.\ to \angle ACB}{Side\ adj.\ \ to\ \angle ACB}$$

$$Tan45^0 = \frac{AB}{BC} = \frac{AB}{15}$$

$$1 = \frac{AB}{15} \quad \Rightarrow AB = 15\,m$$

4 . An aeroplane at an altitude of 200 m observes the angles of depression of opposite points on the two banks of a river to be 45^o and 60^o. Find the width of the river.



Sol. Let A be an aeroplane and AD be an altitude of 200 m. The observer observes the angles of depression of opposite points B and C on the two banks of a river as 45^o and 60^o respectively. Let BD = x m and Cd = y m, then we have to find the width of the river, i.e., BC = (x + y) m in figure.

Now, In right ΔADB, we have $\tan 45^o = \frac{AD}{BD}$

$\Rightarrow 1 = \frac{200}{x}$

x = 200 m… (i)

Again, in right ΔADC, we have $\tan 60^o = \frac{AD}{DC}$

$\Rightarrow \sqrt{3} = \frac{200}{y}$

$\Rightarrow y = \frac{200}{\sqrt{3}}$ … (ii)

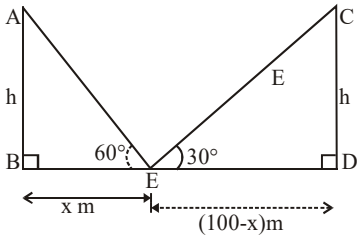
Adding (i) and (ii), we get $x + y = 200 + \frac{200}{\sqrt{3}}$

$\Rightarrow x + y = \frac{200\left(\sqrt{3}+1\right)}{\sqrt{3}}$

$\Rightarrow x + y = 200\left(1 + \frac{1}{\sqrt{3}}\right)m$ $\quad x + y = 315.4$ m

Hence, the width of the river BC = (x + y) m = 315.4 m

5. Two pillars of equal height are on either side of a road, which is 100 m wide. The angles of elevation of the top of the pillar are 60^o and 30^o at a point on the road between the pillars. Find the position of the point between the pillars and the height of each pillar.



Sol.

Let AB and CD be two pillars of equal height h m. Let E be a point on the road such that BE = x m. BD (width of the road) = 100 m. Then ED = (100 – x) m. $\angle AEB = 60^\circ$ and $\angle CED = 30^\circ$ are given in figure.Now, In right ΔABE, we have

$$\tan 60^\circ = \frac{AB}{BE}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \ldots \text{(i)}$$

Again, in right DCDE, we have

$$\tan 30^\circ = \frac{CD}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100 - x}$$

$$\Rightarrow 100 - x = \sqrt{3}\,h \ldots \text{(ii)}$$

Substituting the value of x from equation (i) in equation (ii), we get

$$100 - x = \sqrt{3}\,h$$

$$\Rightarrow 100 - \frac{h}{\sqrt{3}} = \sqrt{3}h \Rightarrow \frac{100\sqrt{3} - h}{\sqrt{3}} = \sqrt{3}\,h$$

$$\Rightarrow 100\sqrt{3} - h = 3h \Rightarrow 100\sqrt{3} = 4h$$

$$\Rightarrow h = 25\sqrt{3}m \qquad \ldots \text{(iii)}$$

$$\Rightarrow h = 25 \times 1.732m$$

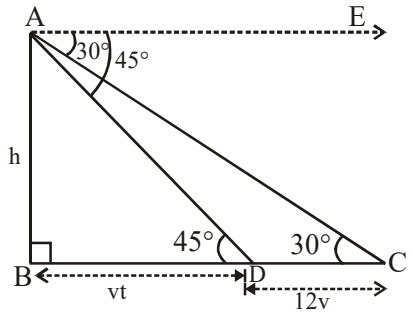
h = 43.3 m

Again substituting the value of h in equation (i) from (iii), we get

$$x = \frac{h}{\sqrt{3}} \Rightarrow x = \frac{25\sqrt{3}}{\sqrt{3}}m = 25m$$

Hence, the position of the point E from the first pillar is 25 m and 75 m from the second pillar. The height of the pillars = hm = 43.3 m

6. A man on the top of a vertical tower observes a car having at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to charge from 30^o to 45^o, how soon after this, will the car reach the tower? Give your answer to the nearest second.



Solution:

Let AB be the tower of height h m. Let C be the initial position of the car and after 12 minutes the car be at D. The angles of depression at C and D are 30^o and 45^o respectively. Let the speed of the car be v m/s in figure.

$\therefore$ Distance traveled by the car in 12 minutes = 12 v m

[$\because$ distance = speed × time]

Let the car take t minutes to reach the tower AB from D. Then DB = vt m.Now, in right ΔABD, we have

$$\tan 45^o = \frac{AB}{BD} \Rightarrow 1 = \frac{h}{vt} \Rightarrow h = vt \quad \ldots (i)$$

Again, in right Δ ABC, we have

$$\tan 30^o = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{vt + 12v}$$

$$\Rightarrow \sqrt{3}\,h = vt + 12v \quad \ldots (ii)$$

Substituting the value of h from equation (i) in equation (ii), we get

$$\sqrt{3}\,h = vt + 12v$$

$$\Rightarrow \sqrt{3} \times vt = vt + 12v \qquad \Rightarrow \sqrt{3}\,t = t + 12$$

$$\Rightarrow \sqrt{3}\,t - t = 12 \qquad \Rightarrow t\left(\sqrt{3} - 1\right) = 12$$

$$\Rightarrow t = \frac{12}{\sqrt{3} - 1} = \frac{12}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{12\left(\sqrt{3} + 1\right)}{3 - 1} = 6\left(\sqrt{3} + 1\right)$$

$$\Rightarrow t = 691.732 + 1 \qquad \Rightarrow t = 6\left(2.732\right)$$

$\Rightarrow$ t = 16.39 minutes

$\therefore$ t = 16 minutes 23 seconds. $[\because 0.39 \times 60 = 23]$

Hence, the car will reach the tower from D in 16 minutes and 23 seconds

TEACHING TASK

I. MCQ's with single correct answers

1. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground by making 30^0 angle with the ground.The distance between the foot of the tree and the top of the tree on the ground is 6m. Then the height of the tree before falling down is .....

   A) $6\sqrt{3}\ m$ B) $6\sqrt{3}\ c.m.$ C) $3\sqrt{3}\ m.$ D) $3\sqrt{6}\ m.$

2. You want to erect a pole of height 10 m with the support of three ropes. Each rope has to make an angle 30^0 with the pole. What should be the length of the rope

   A) 34.64 m B) 34.46 m C) 31.34 m D) None

3. An electrician wants to repair an electric connection on a pole of height 9 m.He needs to reach 1.8 m below the top of the pole to do repair work. What should be the length of the ladder which he should use, when he climbs it at an angle of 60^0 with the ground?

   A) 5.06 ft B)4.1568 ft C) 4.0068 ft D) 51.96 ft

4. A boat has to cross a river , it crosses the river by making an angle of 60^0 with the bank of the river due to the stream of the river and travels a distance of 600 m to reach the another side of the river . What is the width of the river

   A) $100\sqrt{3}\ m.$ B) $200\sqrt{3}\ m.$ C) $600\sqrt{3}\ m.$ D) $300\sqrt{3}\ m.$

5. A T.V tower stands vertically on the side of a road . From a point on the other side directly opposite to the tower, the angle of elevation of thetop of tower is 60^0 .From another point 10 m away from this point , on the line joining this point to the foot of the tower , the angle of elevation of the top of the tower is 30^0 , then the height of the tower and the width of the road

   A) $5\sqrt{3}\ m.\ \ 5m$ B) $\sqrt{3}\ m.\ \ 5m$ C) $5\sqrt{3}\ m.\ \ 5\sqrt{3}m$ D) $\sqrt{3}\ m.\ \ \sqrt{3}m$

6. A 1.5 m tall boy is looking at the top of a temple which is 30 meter in height from a point at certain distance. The angle of elevation from his eye to the top of the crown of the temple increases from 30^0 to 60^0 as he walks towards the temple. Then the distance he walked towards the temple.....

   A) 39.208 m B) 31.908 m C)32.908 m D) None

7. From the top a building , the angle of elevation of the top of a cell tower is 60^0 and the angle of depression to its foot is 45^0 . If distance of the building from the tower is 7 m. Then the height of the tower is .....

   A) 19.124 m B) 19.114 m C) 18.124 m D) 18.114 m

8. The angle of elevation of the top of a building from the foot of the tower is 30^0 and the angle of elevation of the top of the tower from the foot of the building is 60^0.If the tower is 30 m high, then the height of the building is.....

   A) 10 m B) 5 m C) 15 m D) 20 m

9. Two poles of equal heights are standing opposite to each other on either side of the road , which is 120 feet wide. From a point between them on the road, the angles of elevation of the top of the poles are 60^0 and 30^0 respectively . Then the height of the poles and the distances of the point from the poles is .....

   A) 50ft , 30ft B)51.96 ft, 30 ft C) 52 ft , 30 ft D) 30 ft , 30 ft

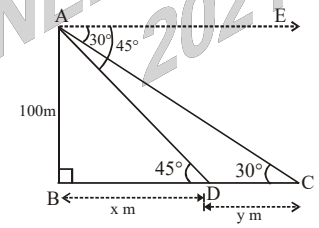
10. The angles of elevation of the top of a tower from two points at a distances of 4 m and 9 m , then the height of the tower from the base of the tower and in the same straght line with it are complementary.....

    A) 2m B) 5 m C) 6 m D) None

11. The angle of elevation of a jet plane from a pont A o0n the ground is 60^0.After a flight of 15 seconds , the angle of elevation changes to 30^0. If the jet plane is flying at a constant height of $1500\sqrt{3}$ meter, Then the speed of the jet plane .....

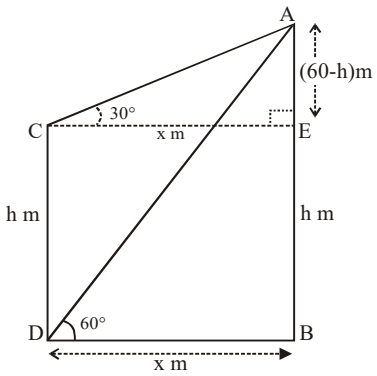
    A) $100\ m/s$ B) $200\ m/s$ C) $300\ m/s$ D) None

12. As observed from the top of a light house, 100 m above area level, the angle of depression of a ship, sailing directly towards it, changes from 30° to 45°. Determine the distance traveled by the ship during the period of observation.



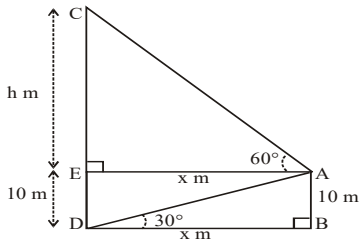
   A) 10 m (B) 7.32 m (C) 40 m (D) 21 m

13.. From the top of a building 60 m high the angles of depression of the top and the bottom of a tower are observed to be 30° and 60°. Find the height of the tower.



(A) 20 M (B) 25 M C) 60 M (D) 40 M

14. A man is standing on the deck of a ship, which is 10 m above water level. He observes the angle of elevation of the top of a hill as 60^o and the angle of depression of the base of the hill as 30^o. Calculate the distance of the hill from the ship and the height of the hill.



(A) 10 M (B) 40 M C) 80 (D) 35 M

15. The angle of elevation of the top of a tower from the top and bottom of a building of height 'a' are 30^0 and 45^0 respectively. If the tower and the building stand at the same level, the height of the tower is

(A) $a\sqrt{3}$ (B) $a(\sqrt{3}-1)$ (C) $a\frac{(3+\sqrt{3})}{2}$ (D) $a(\sqrt{3}+1)$

16. The angles of elevation of the top of a tower at two points which are at distances a and b from the foot in the same horizontal line and on the same sides of the tower, are complementary. The height of the tower is-

(A) ab (B) $\sqrt{ab}$ (C) $\sqrt{a/b}$ D) $\sqrt{b/a}$

17. From the top of h metres high cliff, the angles of depression of the top and bottom of a tower are observed to be 30° and 60° respectively. The heigth of the tower is-

(A) $\frac{2h}{3}$ (B) $\frac{h}{3}$ (C) $\frac{2h}{\sqrt{3}}$ (D) $h\sqrt{3}$

18. The angle of elevation of the top of a tower at a distance of 500 metres from its foot is 30^0. Then height of the tower is

(A) $\frac{500\sqrt{3}}{3}$m (B) $\frac{500(\sqrt{3}-1)}{3}$m (C) $\frac{500(\sqrt{3}+1)}{3}$m (D) 500 m

19. A tower subtends an angle of 30° at a point on the same level as the foot of the tower. At a second point, h metre above first, the depression of the foot of the tower is 60°, the horizontal distance of the tower from the point is-

(A) h cos 60° (B) (h/3) cot 30° (C) (h/3) cot 60° (D) h cot 30°

20. The tops of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle 30° with the horizontal, then the length of the wire is

(A) 12 m (B) 10 m (C) 8 m (D) 14 m

II. MCQ'S WITH MULTIPLE CORRECT ANSWERS

1. The angles of elevation of the top of a rock from the top and foot of a 100m. High tower are respectively 30° and 45°. What is the height of the rock?

(A) $\frac{\sqrt{3}-1}{100\sqrt{3}}$m (B) $\frac{100\sqrt{3}}{\sqrt{3}-1}$m (C) 246.5 m (D) 236.5 m

2. A hedgehog wishes to cross a road without being runover. He observer the angle of elevation of a lamp post on the other side of the road to be 45° from the edge of the road and 30° from a point 10m back from the road. how wide is the road?

(A) 13.90 m (B) $\frac{10}{\sqrt{3}}$m (C) $\frac{10}{\sqrt{3}-1}$m (D) 13.66 m

III. MATRIX MATCH TYPE

1. From a window, h metres high above the ground, of a house in a street, the angles of elevation and depression of the top and bottom of another house on the opposite side of the street are ? and ?, respectively, then match them correctly

Column I		Column II	
(A) DB	[ ]	(p)	$h(1+\tan\alpha\cot\beta)$
(B) CE	[ ]	(q)	$h\sin\beta$
(C) CE	[ ]	(r)	$h\tan\cot\beta$
(D) AD	[ ]	(s)	$h\cot\beta$

2. A ladder (length = *l*) rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance d, so that it slides a distance h down the wall, making an angle β with the horizontal, then match the column

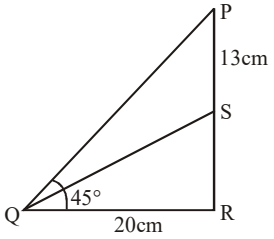
Column I		Column II	
(A) d	[ ]	(p)	$l(\sin\alpha-\sin\beta)$
(B) h	[ ]	(q)	$l(\cos\alpha-\cos\beta)$
(C) d/h	[ ]	(r)	$\frac{\cos\alpha-\cos\beta}{\sin\beta-\sin\alpha}$
(D) dh/l^2	[ ]	(s)	$\sin(\alpha+\beta)-\cos(\alpha-\beta)$

IV. INTEGER TYPE

This section contains questions where the answer to each of the question is a single digit integer, ranging from 0 to 9.

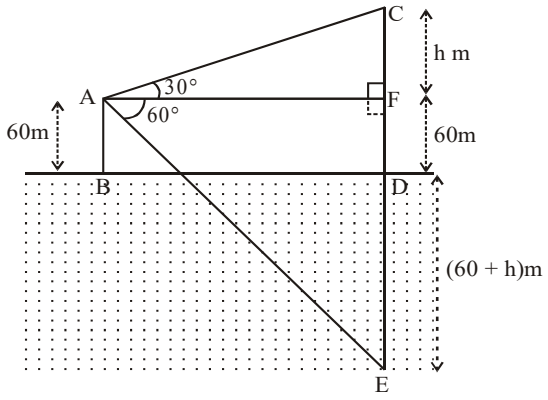
1. A bridge across the river makes an angle of 30° with the river bank. The length of the bridge across the river is 60m. If width of the river is k meter, then what is the value of k/l 15m.

2. A statue 1.46 m tall, stands on top of a pedestal. From a point on the ground the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. What is the height of the pedestal?
3. The angle of depression of point C, when observed from point A is 45°. If BC = 1m determine $\frac{AB}{1m}$.
4. The tops of two poles of height 16m and 10m are connected by a wire. If the wire makes an angle of 30° with the horizontal and x is the length of wire then find $\frac{x}{4m}$
5. In the given figure QR = 20cm, PS = 13 cm, $\angle PQR = 45^\circ$, then value of $\frac{SR}{7cm}$



SOLVE THE FOLLOWING

1. A tower subtends an angle of 30^0 at a point on the same level as the foot of the tower. At a second point h metres above the first, the depression of the foot of the tower is 60^0. Find The horizontal distance of the tower from the point.
2. A man is standing on the deck of a ship, which is 8m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the distance of the hill from the ship and the height of the hill.
3. An aeroplane flying horizontally at a height of 1.5 km above the ground is observed at a certain point on earth to subtend an angle of 60º. After 15 seconds, its angle of elevation is observed to be 30º. Calculate the speed of the aeroplane in km/h $\left[\sqrt{3} = 1.732\right]$.
4. The angle of elevation of a cloud from a point 60 m above a lake is 30^o and the angle of depression of the reflection of cloud in the lake is 60^o. Find the height of the cloud.



LEARNER'S TASK

BEGINNERS (Level - I)

1. MCQ'S with single correct answer

1. A contractor wants to set up a slide for the children to play in the park. He wants to set it up at the height of 2 m and by making an angle of 30^0 with the ground . What should be the length of the slide.....

   A) 1 m B) 2 m C) 3 m D) 4 m

2. Length of the shadow of a 15 meter high pole is $5\sqrt{3}$ meters at 7 O' clock in the morning . Then what is the angle of elevation of the sun rays with the ground at the time .....

   A) 30^0 B) 45^0 C) 60^0 D) 90^0

3. Suppose you are shooting an arrow from the top of a building at an height of 6 m to a target on the ground at an angle of depressions of 60^0 , Then the distance between you and the object.....

   A) $2\sqrt{3}\,m$ B) $3\sqrt{3}\,m$ C) $4\sqrt{3}\,m$ D) $5\sqrt{3}\,m$

4. An Observer of height 1.8 m is 13.2 m away from a palm tree. The angle of elevation of the top of the tree from his eyes is 45^0 Then the height of the palm tree is ....

   A) 12 m B) 14 m C) 15 m D) 16 m

5. Two men on either side of a temple of 30 meter height observe its top at the angles of elevation 30^0 and 60^0 respectively . Then the distance between the two men is .....

   A) $10\sqrt{3}\,m$ B) $40\sqrt{3}\,m$ C) $60\sqrt{3}\,m$ D) $80\sqrt{3}\,m$

6. A Straight highway leads to the foot of a tower. Ramaiah standding at the top of the tower observes a car at an angle of deoression 30^0 .The car is approaching the foot of the tower with a uniform speed. Six seconds later , the angle of depression of the car is found to be 60^0, then the time taken by the car to reach the foot of the tower from this point.....

   A) 6 Seconds B) 3 seconds C) 4 seconds D)2 seconds

7. A statue stands on the top of a 2 m tall pedestal . From a point on the ground, the angle of elevation of the top of the statue is 60^0 and from the same point , the angle of elevation of the top of the pedestal is 45^0, then the height of the statue is .....

   A) 1.464 M B) 1.646 M C) 14.64 M D)16.46 M

8. A wire of length 18 m had been tied with electric pole at an angle of elevation 30^0 with the ground .Because it was covering a long distance, it was ut and tied at an angle of elevation 60^0 with the ground . then the length of the wire was cut .....

   A) 6. 708 m B) 8.706 m C) 7.608 m D) None

9. The angle of elevation of the top of a tower from the foot of the building is 30^0 and the angle of elevation of the top of the building from the foot of the tower is 60^0, then the ratio of heights of tower and building is ....

   A) 1 : 2 B) 2 : 1 C) 1 :3 D) 3 :1

ACHIEVERS (Level - II)

II. MCQ'S WITH MULTIPLE CORRECT ANSWERS

1. A pole 15m long rests against a vertical wall at an angle of 60° with the ground. How high up the wall will the pole reach?

(A) $\frac{15\sqrt{3}}{2}$m (B) $8\sqrt{3}$m (C) 12.99 m (D) 14.38 m

2. A man in a boat rowing away from light house 100 m. He takes 2 minute to change the angle of elevation of the top of light house from 60° to 45°. What is the speed of the boat?

(A) $\frac{5}{18}\left(3-\sqrt{3}\right)$m/s (B) $\frac{18}{5}\left(\sqrt{3}-1\right)$m/s (C) 2.364 m/hr. (D) 1.268 km/h

3. From a point on the ground the angle of elevation of the bottom and top of a transmission elevation of the bottom and top of a transmission tower fixed at the top of a 20m. High building are 45° and 60° respectively. What is the height of the tower?

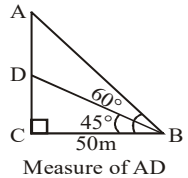
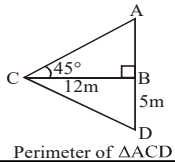
(A) $20\sqrt{3}$m (B) $20\left(\sqrt{3}-1\right)$m (C) 14.64 m (D) 14.94 m

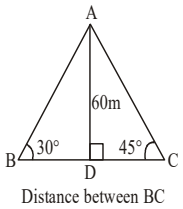
III. MATCH THE FOLLOWING

1. From the top of a cliff 50 m high, the angles of depression of the top and the bottom of a tower are observed to be 30º and 45º respectively, match them correctly

Column I		Column II
(A) Horizontal distance of point of observation from top of tower....	[ ]	(p) $\left(50-\frac{50}{\sqrt{3}}\right)$m
(B) Height of tower.....	[ ]	(q) 50 m
(C) Horizontal distance of bottom of cliff from bottom of tower....	[ ]	(r) $50\sqrt{2}$m
(D) Distance of top of cliff from bottom of tower	[ ]	(s) $50/\sqrt{2}$m

2.

Column I		Column II
(A)  Measure of AD	[ ]	(p) $6\left(5+2\sqrt{2}\right)$m
(B)  Perimeter of ΔACD	[ ]	q) 10 m

(C) 

Distance between BC [ ] (r) $50\left(\sqrt{3}-1\right)$ m

(D) If the shadow of a tower is 30m. When the sun's elevation is 30°, Find the length of the shadow when sun's elevation is 60° [ ] (s) $60\left(\sqrt{3}+1\right)$ m

COMPREHENSION TYPE

The following question is based on the paragraph given below

1. A guard observes an enemy boat, from an observation tower at a height of 200m above sea level to be an angle of depression of 30°.
   (i) What is the distance of the boat from the foot of the observation tower.
   (ii) If the boat is 200m from the observation tower, then what is the angle of depression.

EXPLORERS (Level - III)

SOLVE THE FOLLOWING

1. A vertical to stands on a horizontal plane and is surrounded by a vertical flag staff of height 6 meters. At point on the plane, the angle of elevation of the bottom and the top of the flag staff are respectively 30° and 60°. Find the height of tower.
2. The angles of depressions of the top and bottom of 8 m tall building from the top of a multistoried building are 30° and 45° respectively. Find the height of multistoried building and the distance between the two buildings.
3. The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight of 15 sec, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 metres, find the speed of the aeroplane.
4. From the top of a light house 60 m high with its base at the sea level the angle of depression of a boat is 15°. Then find the distance of the boat from the foot of light house.
5. At a distance 2h meter from the foot of a tower of height h metre the top of the tower and a pole at the top of the tower subtend equal angles. Find the Height of the pole
6. A house subtends a right angle at the window of an opposite house and the angle of elevation of the window from the bottom of the first house is 60°. If the distance between the two houses be 6 m, then find the height of the first house.
7. The base of a cliff is circular. From the extremities of a diameter of the base of angles of elevation of the top of the cliff are 30° and 60°. If the height of the cliff be 500 m, then find the diameter of the base of the cliff .
8. A boat is being rowed away from a cliff 150 m high. At the top of the cliff the angle of depression of boat changes from 60° to 45° in 2 min. Then find the speed of the boad (in m/h)

9. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60° and after 10s the elevation is observed to be 30°. Then find uniform speed of the aeroplane(in km/h)

KEY

ΦΦ TEACHING TASK :

MCQ'S with single correct answer

1. A 2. A3. B 4. D 5. A 6. C 7. A 8. A 9. B 10. C 11.B
12. B 13. D 14. B 15. C 16. B 17. A 18. A 19. B 20. D 21. B
22. B 23. A 24. D

Multi correct answers : 1) B, D 2) B , D

Matching : 1) s, r, p, q 2) q, p , r , s

Integer type : 1) 22) 2 3) 1 4) 3 5) 1

ΦΦ LEARNER'STASK :

□ BEGINNERS :

MCQ'S with single correct answer

1. D 2. A3. C 4. C 5. B 6. B 7. A 8. C 9. C

□ ACHIEVERS :

Multi correct answers : 1) A , C 2) A , D 3) B , C

Matching : 1) q, p , q , r 2) r , p , s, q

Comprehension : 1 **(i)** 346.4 (ii) 45^0

HINTS & SOLUTIONS

Teaching Task

Single correct answer

12. Let C and D be the two positions of the ship Let BD = x m be the distance traveled by the ship during the period of observation, i.e BD = x m. Let observer be at A, the top of the light house AB = 100m. The angles of depression from A of C and D are 30^0 and 45^0 respectively i.e,

$\angle C = 30^0 \, and \, \angle D = 45^0$ in figure

Now, in right ΔABD, we have

$$\tan 45^0 = \frac{AB}{BD} \Rightarrow 1 = \frac{100}{BD}$$

$\therefore BD(x) = 100m$

Again, in right ΔABC, we have

$$\tan 30^0 = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BD + DC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x + y}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x + y} \Rightarrow 100 + y = 100\sqrt{3} \Rightarrow y = 100\sqrt{3} - 100$$

$$\Rightarrow y = 100\left(\sqrt{3} - 1\right) \Rightarrow y = 100(1.732 - 1)$$

$$\Rightarrow y = 100 \times 732 \Rightarrow y = 7.32m$$

13. Let AB be the building and its height is 60m, and CD is a tower Let CD = h m and CE be horizontal from C. The angles of depression of the top C and the bottom D of the tower CD are 30^0 and 60^0 respectively

Let DB = CE = xm

Now In right ΔAEC, we have $\tan 30^0 = \frac{AE}{CE}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60 - h}{x}$$

$\Rightarrow x = (60 - h)\sqrt{3}m$ .....(i)

Again in right ΔABD, we have $\tan 60^0 = \frac{AB}{BD}$

$$\Rightarrow \sqrt{3} = \frac{60}{x}$$

$\therefore x = \frac{60}{\sqrt{3}}m$ .....(ii)

From (i) and (ii) we get $(60 - h)\sqrt{3} = \frac{60}{\sqrt{3}}m$

$\Rightarrow (60 - h) \times 3 = 60$

$\Rightarrow 60 - h = 20$

$\Rightarrow$ 60-20 = h

$\therefore$ h = 40m

Hence, the height of the tower (CD) = h = 40m

14. Let the mass who is stranding on the deck of a ship at point A and CD be the hill. It is also given that the angle of depression of the base D of the hill DC observed from A is 30^0 and the angle of elevation of the top C of the hill CD observed from A is 60^0.

i.e $\angle EAC = 60^0, \angle BDA = 30^0$ and AB=ED =10m in figure

Now, in right ΔAEC, we have $\tan 60^0 = \frac{CE}{AE}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow h - \sqrt{3}x$ .....(i)

Again, in right ΔABD, we have $\tan 30^0 = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}$

$\Rightarrow x = 10\sqrt{3}$ .....(ii)

Now, substitiruting the value of x from equation (ii) equation(i), we get

$h = \sqrt{3}x$

$\Rightarrow h = \sqrt{3} \times 10\sqrt{3} \Rightarrow h = 30m$

$\therefore$ h (CE)=30m

and CE=CE+ED = 30m +10m

$\therefore$ CD=40m

Hence, the distance of the hill from the ship BD = xm = $10\sqrt{3}$

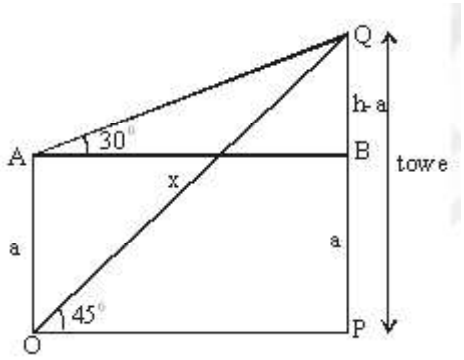
and the height of the hill (CD) = 40m

15. $\frac{h}{x} = \tan 45^0 = 1 \quad \therefore h = x$

$\frac{h-a}{x} = \tan 30^0 = \frac{1}{\sqrt{3}}$

$h - a = \frac{x}{\sqrt{3}} = \frac{h}{\sqrt{3}}$

$\therefore h\left(1 - \frac{1}{\sqrt{3}}\right) = a \quad \therefore h = \frac{\sqrt{3}a}{\sqrt{3}-1}$



16. Let PQ be given tower of height h. If A, B be given points then suppose $\angle PAQ = a$ and $\angle PBQ = b$

Then $a + b = \frac{x}{2}$

Now in $\Delta PAQ, \tan a = \frac{h}{a}$ .....(i)

In $\Delta PBQ, \tan b = \frac{h}{b}$ .....(ii)

(i), (ii) $\Rightarrow \tan a \tan b = \frac{h}{a}\frac{h}{b} \Rightarrow \left(\frac{\pi}{2} - \alpha\right) = \frac{a^2}{ab} \Rightarrow h = \sqrt{ab}$

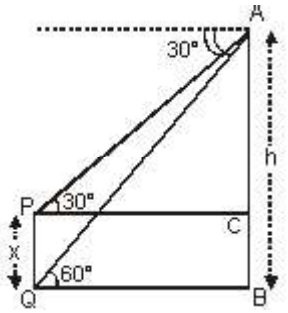
17. Let AB and PQ be given cliff and tower respectivley. If PQ =x, then as given from adjoinning diagram

$\angle APC = 30^0 \, and \, \angle AQB = 60^0$

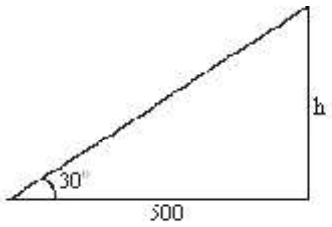
Now in triangles ABQ and ACP

$\frac{h}{QB} = \tan 60^0 = \sqrt{3} \quad and \, \frac{h-x}{PC} = \tan 30^0 \Rightarrow \frac{h-x}{QB} = \frac{1}{\sqrt{3}}$

$(2)+(1) \Rightarrow \frac{h-x}{h} = \frac{1}{3} \Rightarrow x = \frac{2h}{3}$



18. $\frac{h}{500} = \tan 30^0 = \frac{1}{\sqrt{3}} \quad \therefore h = \frac{500}{\sqrt{3}} = \frac{500\sqrt{3}}{3} m$



19. Let the tower OP subtend an angle of 30^0 at A a point on the same level as the foot of the tower. From a point B at the height h, vertically above A, from where the angle of depression of foot O of the tower is 60^0

i.e $\angle AOP = 60^0$

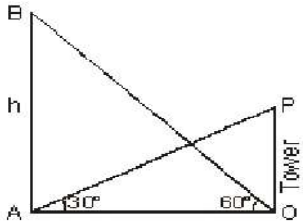
From DOAP, OA = h cot 60^0

Also from DOAP, OA = OP cot 30^0

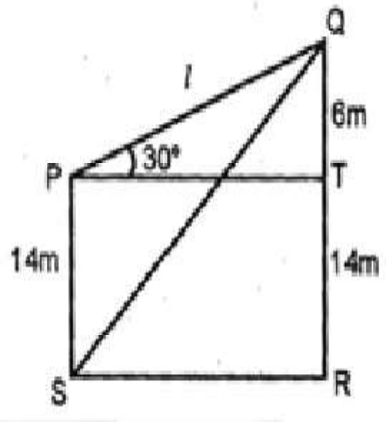
$\therefore$ $\text{OA} = \text{h cot } 60^0 = \text{OP cot} 30^0$

$\Rightarrow$ OP = h/3

$\therefore$ $\text{OA} = (\text{h}/3)\,\text{cot} 30^0$



20. $\frac{6}{l} = \sin 30^0 = \frac{1}{2} \Rightarrow l = 12m$



Multi correct answer type

1. Let AB= (h meters) be the rock

Let CD =(100 m) be the tower

Now, In right triagles ABD, $\tan 45^0$ = AB

$\Rightarrow 1 = \frac{h}{x}$

$\Rightarrow x = 1$ ..... (i)

In tright triangle AEC, wer have

$\tan 30^0 = \frac{AE}{EC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB - EB}{EC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h - 100}{x}$

$\Rightarrow x = \sqrt{3}h - 100\sqrt{3}$ .....(ii)

On equating the value of x from equation (i) and equation (ii) we get

h= $\sqrt{3}$ h-1--$\sqrt{3}$

$\Rightarrow$ h= 236.5 m

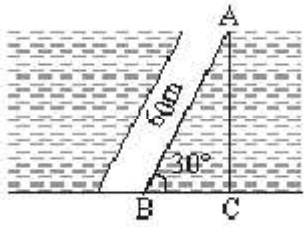
Integer Type

1. $\sin 30^0 = \frac{AC}{AB}$

$\Rightarrow \frac{1}{2} = \frac{AC}{60}$

$\Rightarrow$ AC=30

$\Rightarrow$ K= 30



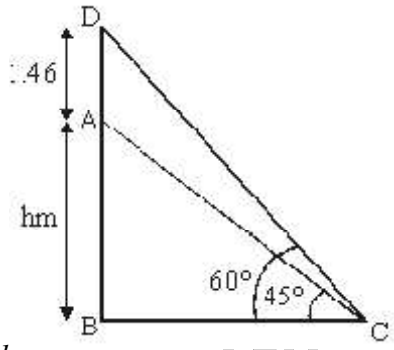
So, $\frac{K}{15} = \frac{30}{15} = 2m$

2. AB= hm

AD = 1.46m

In ΔDBC

$\tan 60^0 = \frac{DB}{BC}$



$\tan 60^0 = \frac{DA + AAB}{BC}$ $\sqrt{3} = \frac{1.464 + h}{BC}$ .....(i)

$\Rightarrow BC = \frac{1.464 + h}{\sqrt{3}} m$ In right triangle ABC,

$\tan 45^0 = \frac{AB}{BC} \Rightarrow 1 = \frac{h}{BC}$

$\Rightarrow$ BC = hm .....(ii)

from equation (i) and equation (ii), we get

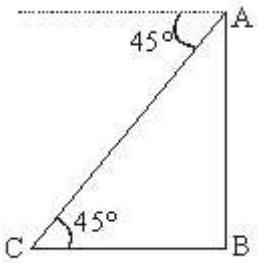
$\frac{1.464 + h}{\sqrt{3}} = h$ $\frac{1.464}{\left(\sqrt{3} - 1\right)} = 2$

3. BC=1 m

tan45^0

$1 = \frac{AB}{1}$

$\therefore \frac{AB}{1} = \frac{1}{1} 1m \Rightarrow AB = m$



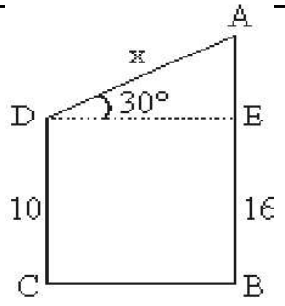
4. AB = 16m

DC = 10m

AE = 4m

New tan right triangle

AED, $\sin 30^0 = \frac{AE}{DA}$



$\Rightarrow \frac{1}{2}\frac{4}{x}$ $\Rightarrow x = 8m$

then $\frac{x}{4m} = \frac{8}{4} = 2m$

5. In ΔPQR

$\angle PQR = 45^0$

$\therefore$ QR - PR = 20cm

$\because$ PS = 13 cm

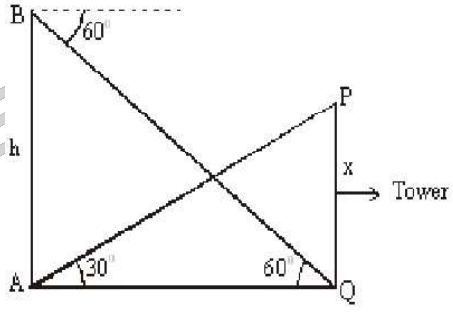
$\therefore$ SR = PR - PS 20 - 13 = 7cm

$\therefore \frac{SR}{7cm} = \frac{7cm}{7cm} - 1$

Solve the following

1. Let PQ = x meters denote the tower so that $\angle PAQ = 30^0$

$\frac{h}{AQ} = \tan 60^0 = \sqrt{3}$



$\therefore PQ - \frac{h}{\sqrt{3}} = h\cot 60^0$

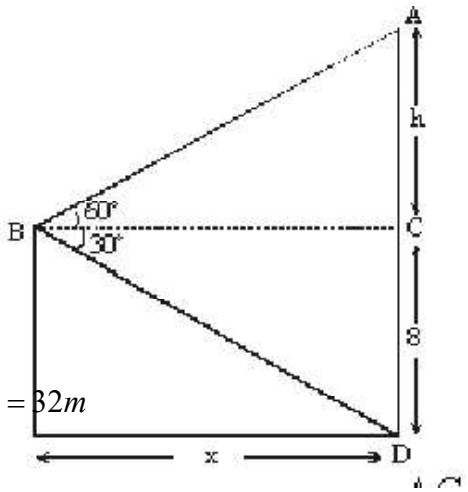
3. Let x be the distance of hill from man
and h +8 be height of hill which is required

In rt. DACD $\tan 60^0 = \frac{AC}{BC} = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x}$

In rt. DBCD,

$\tan 30^0 = \frac{CD}{BC} = \frac{8}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{8}{x} \Rightarrow x = 8\sqrt{3}$

$\therefore$ Height of hill $= h + 8\sqrt{3}x + 8 = \left(\sqrt{3}\right)\left(8\sqrt{3}\right) + 8 = 32m$



4. Let PX be the horizontal ground

Let A be the initial position of the aeroplane Let P be the point of observation on the ground

Draw AC $\perp$ PX

Then, $\angle ACP = 90^0$

And AC = 1.5 km

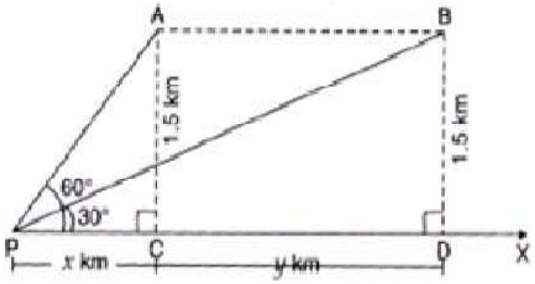
The angle of elevation of the point A at point P is 60^0 i.e $\angle ACP = 60^0$

After 15 second let B be the positic

Draw BD $\perp$ PX then $\angle BDP = 90^0$

As the plane is flying horizontally,

BD = 1.5 km



The angle of elecation of the point B at point P is 30^0 i.e, $\angle BPD = 30^0$

Let PC =x km and CD = y km

Then, PD = PC+CD = (x+y)km .....(i)

in right $\angle trina\lg e$ ACP, we have

$$\tan 60^0 = \frac{AC}{PC}$$

$$\Rightarrow \sqrt{3} - \frac{1.5}{x}$$

$$\Rightarrow x = \frac{1.5}{\sqrt{3}} \qquad \text{.....(ii)}$$

in right triangle BDP, we have

$$\tan 30^0 = \frac{BD}{PD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1.5}{x+y} \qquad \text{(using (i))}$$

$$\Rightarrow x+y=1.5\sqrt{3}$$

$$\Rightarrow y = 1.5\sqrt{3} - \frac{1.5}{\sqrt{3}}$$

$$\Rightarrow h = 4\sqrt{3}\left(\sqrt{3}+1\right) \Rightarrow h = 4\left(3+\sqrt{3}\right)$$

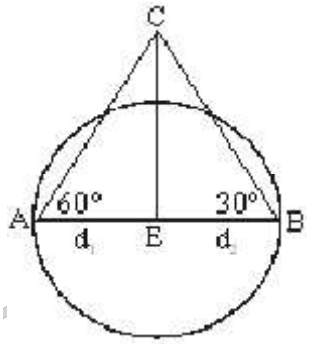
$$\Rightarrow y = 1.5\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow y = 1.5\left(\frac{3-1}{\sqrt{3}}\right) = \frac{1.5\times 2}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} \qquad \text{(iii)}$$

$$speed = \frac{Dis\tan ce}{time} = \frac{y\,km}{15\sec onds} = \frac{\sqrt{3}\,km}{15\sec onds} \quad \text{[Using (iii)]}$$

$= \frac{\sqrt{3}}{15}$ x 3600 km / hour = 415.68 km / hour

Thus, the speed of the aeroplane is 415.68 km/hour.



and in $\Delta BEC, \tan 30^0 = \frac{500}{d_2}$

$$\Rightarrow d_2 = 500\sqrt{3}m$$

Required diameter,

$$AB = d_1 + d_2 = \frac{500}{\sqrt{3}} + 500\sqrt{3} = \frac{500}{\sqrt{3}}(1+3) = \frac{2000}{\sqrt{3}}m$$

$$200 = h\left(\frac{\sin\alpha}{\sin 2\alpha \sin\alpha}\right) \qquad \text{....(v)}$$

From equ (iv) and (v)

$$\frac{100}{200} = \frac{\sin\alpha}{\sin 3\alpha} \Rightarrow \sin 3\alpha = 2\sin\alpha \Rightarrow 3\sin a - 4\sin^3\alpha = 2\sin\alpha$$

$$\Rightarrow 4\sin^3\alpha - \sin\alpha = 0 \Rightarrow \sin\alpha = 0$$

or

$$\sin^2\alpha = \frac{1}{4} = \sin^2\left(\frac{\pi}{6}\right) \Rightarrow \alpha - \frac{\pi}{6}$$

hence h = 200 $\sin\frac{\pi}{3} = 200\frac{\sqrt{3}}{2} = 100\sqrt{3}$

4. Le BD is the surface of the lake and A be the point of observation such that AB = 60m

Let C be the position of the cloud and E be its reflection in the lake.

Then CD = ED.

Let AF be perpendicular from A on CE then

$\angle CAF = 30^0 \; and \angle EAF = 60^0$

Let CF = hm

Now, CD = (60+h) m.

similarly ED = (60+h)m

Now In right ΔCFA, we have

$$\tan 30^0 = \frac{CF}{AF} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AF}$$

$$\Rightarrow AF = \sqrt{3}h$$

Again, in right ΔEFA we have $\tan 60^0 = \frac{EF}{AF}$

$$\Rightarrow \sqrt{3} = \frac{ED + DF}{AF} \quad \Rightarrow \sqrt{3}AF = 60 + h + 60$$

$$\Rightarrow AF = \frac{120 + h}{\sqrt{3}} \qquad \text{....(ii)}$$

From (i) and (ii), we get $\sqrt{3}h = \frac{120 + h}{\sqrt{3}}$

$\Rightarrow$ 3h = 120 +h 3h - h =120

h = 60m

Now, CD = CF +FD=h+60=60+60

$\therefore$ CD = 120 m

Hence, the hight of the cloud from the surface of the lake = CD = 120 M

Learners Task

Comprehension

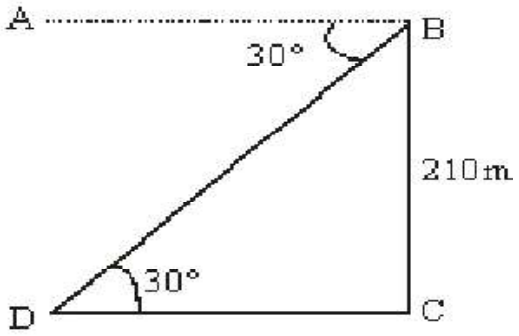
1. i) In triangle BDC

$$\tan 30^0 = \frac{BC}{DC}$$

$$\frac{1}{\sqrt{3}} = \frac{200}{DC}$$

DC = 200 $\sqrt{3}$ m = 346.4 m

ii) Let give DC = 200m,



$\tan\theta = \frac{200}{200} = 1$

$\therefore \theta = 45^0$ so, angle of depression = 45^0

Solve the following

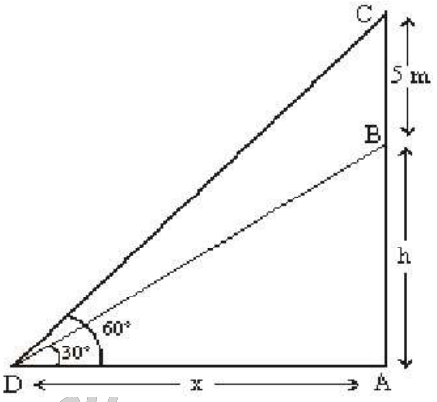
1. Let AB be the tower of height h meter and BC be the height of flag surmounted on te tower

In DABD

$\tan 30^0 = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}h$ .....(i)

In DADC,

$\tan 60^0 = \frac{AC}{AD} \Rightarrow \sqrt{3}\frac{5+h}{x} \Rightarrow x = \frac{5+h}{\sqrt{3}}$ .....(ii)



From (i) and (ii)

$\sqrt{3}h = \frac{5+h}{\sqrt{3}} \Rightarrow 3h = 5+h \Rightarrow 2h = 5$

$\Rightarrow h = \frac{5}{2} = 2.5m$

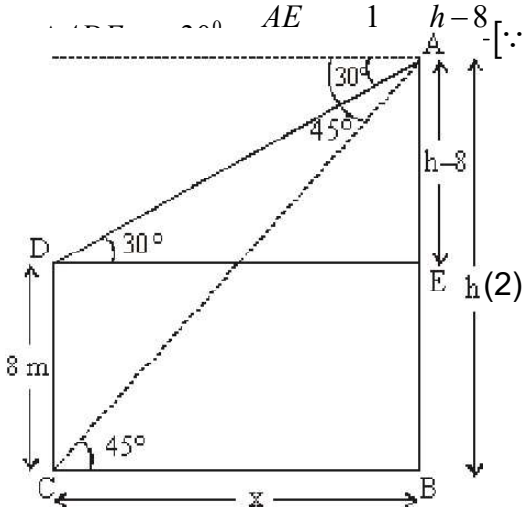
So, that height of tower = 2.5 m

2. Let AB be the multistoried building of height h and let the distance between two building be meters

$\angle XAC = \angle ACB = 45^0$ (Alernative angles)

$\angle XAD = \angle ADE = 30^0$ (Alternative angles)

$\frac{AE}{} = \frac{1}{} = \frac{h-8}{}$ $\left[\because CD = DE = x\right]$



$\Rightarrow \sqrt{3}$ h-8$\sqrt{3}$ =h

$\Rightarrow \sqrt{3}h - h = 8\sqrt{3} \quad \Rightarrow h\left(\sqrt{3}-1\right) = 8\sqrt{3}$

$\Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1}\frac{\left(\sqrt{3}+1\right)}{\sqrt{3}+1} \Rightarrow h = \frac{8\sqrt{3}(\sqrt{3}+1)}{2}$

$\Rightarrow h = 4\sqrt{3}\left(\sqrt{3}+1\right) \qquad \Rightarrow h = 4\left(3+\sqrt{3}\right)$ meters

From (2), x=h

So, $x = 4(3+\sqrt{3})$ meters

Hence, height of multostired building = $4(3+\sqrt{3})$ meters distance between two building $4(3+\sqrt{3})$ meters

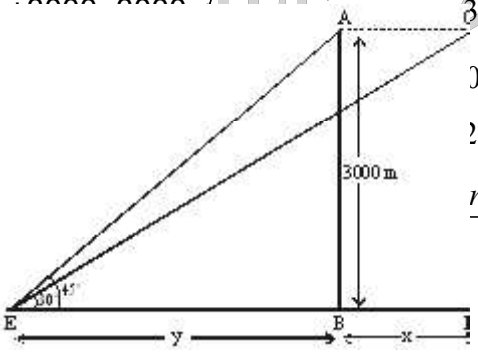
3. Let the point ion the ground is E which is y meters from point B and left after15 sec. fight it covers x meters distance

In DAEB, $\tan 45h0 = \frac{AB}{EB} \quad \Rightarrow 1 = \frac{3000}{y} \Rightarrow y = 3000.....(1)$

In DCED, $\tan 30^0 = \frac{CD}{ED} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3000}{x+y} \quad (\because AB = CD)$

$\Rightarrow$ x+y=3000 $\sqrt{3}$ .....(2)

From eq.(1) and (2)



3000 $\sqrt{3}$ - 3000

)00 x (1.732-1)

?19 m

red

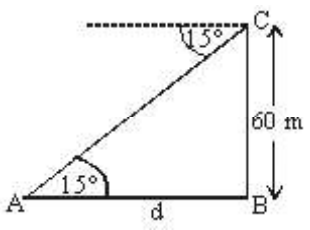
$= \frac{2196}{15} m/\sec = 146.4 m/\sec = \frac{2196}{15} \times \frac{18}{5} km/hr = 527.04 km/hr$

Hence, the speed of aeroplane is 527.04 km/hr

4. Let BC be the light house

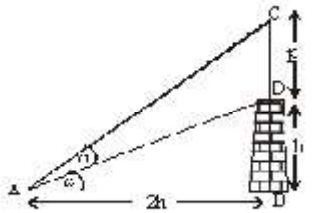
In $\Delta ABC, \tan 15^0 = \frac{60}{d}$

$\Rightarrow d = 60\cot 15^0 = 60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)m$



5. In $\Delta ABD,\ \tan\alpha = \frac{h}{2h} = \frac{1}{2}$

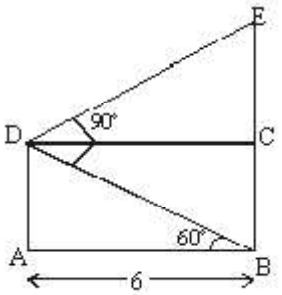
In $\Delta ABC,\ \tan 2\alpha = \frac{h+p}{2h}$



$$\Rightarrow \frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} = \frac{h+p}{2h} \Rightarrow \frac{4}{3} = \frac{h+p}{2h}$$

$$\Rightarrow 8h = 3h + 3p \quad \Rightarrow 5h = 3p \Rightarrow == \frac{5h}{3} m$$

6. In $\Delta ABD, \cos 60^0 = \frac{6}{BD} \Rightarrow BD = \frac{6}{1/2} = 12$

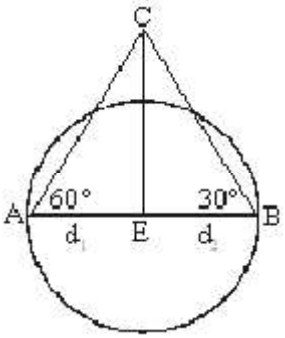


In $\Delta DBE, \sin 60^0 = \frac{BD}{BE}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{12}{h} \Rightarrow h = 8\sqrt{3} m$$

In $\Delta AEC, \tan 60^0 = \frac{500}{d_1} \Rightarrow d_1 = \frac{500}{\sqrt{3}} m \quad ....(i)$

7. and in $\Delta BEC, \tan 30^0 = \frac{500}{d_2}$



$$\Rightarrow d_2 = 500\sqrt{3} m$$

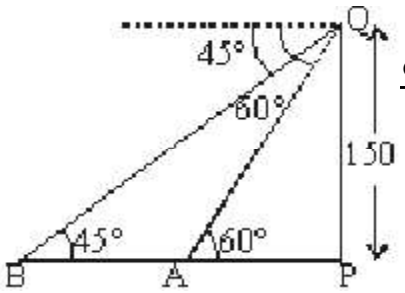
Requited diameter,

AB= $d_2 + d_2 = \frac{500}{\sqrt{3}} + 500\sqrt{3} = \frac{500}{\sqrt{3}}(1+3) = \frac{2000}{\sqrt{3}} m$

8. $\because$ PQ=150 m

In $\Delta APQ, \tan 600 = \frac{PQ}{AP} \Rightarrow AP = \frac{150}{\sqrt{3}} ....(i)$

and in $\Delta APQ, \tan 45^0 = \frac{PQ}{AB + AP}$



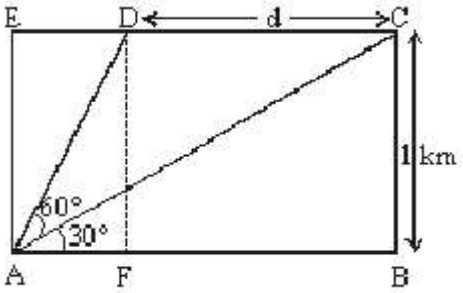
$\Rightarrow AB+\frac{150}{\sqrt{3}}=150\Rightarrow AB=\frac{150}{\sqrt{3}}\left(\sqrt{3}-1\right)$

The speed of boad

$=\frac{AB}{2}=\frac{1}{2}\times\frac{150}{\sqrt{3}}\left(\sqrt{3}-1\right)\times 60m/h=\frac{4500}{\sqrt{3}}\left(\sqrt{3}-1\right)m/h$

9. In $\Delta ADF, \tan 60^0=\frac{1}{AF}$

$AF=\cot 60^0=\frac{1}{\sqrt{3}}$



In $\Delta ABC, \tan 30=\frac{1}{AB}\Rightarrow AB=\cot 30^0$

$\Rightarrow AF+FB=\sqrt{3}$

$\Rightarrow d=\sqrt{3}-\frac{1}{\sqrt{3}}=\frac{2}{\sqrt{3}}$

$\therefore speed=\frac{dis\tan ce\ from\ D\ to\ C}{time\ taken}=\frac{\frac{2}{\sqrt{3}}}{10}\times 60\times 60=240\sqrt{3}$ **km/h**

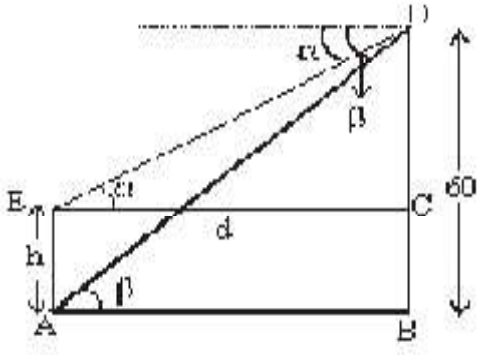
10. In $\Delta ABD, \tan\beta=\frac{60}{d}$, $d=\cos\beta$ **..(i)**

In $\Delta DEC, \tan\alpha=\frac{DC}{EC}\Rightarrow DC=d\tan\alpha$

$\Rightarrow 60-h=d\tan\alpha \quad [\because BC=EA=h]$

$\Rightarrow 60-h=60\cot\beta\tan\alpha \quad [from(i)]$

$\Rightarrow h=60\left(1-\frac{\cos\beta}{\sin\beta}\bullet\frac{\sin\alpha}{\cos\alpha}\right)$



$\Rightarrow h=\frac{60\sin(\beta-\alpha)}{\cos\alpha\sin\beta}$

$\Rightarrow \frac{60\sin(\beta-\alpha)}{x}=\frac{60\sin(\beta-\alpha)}{\cos\alpha\sin\beta}(given)$

$\Rightarrow x=\cos\alpha\sin\beta$