CBSE Maths Chapter: Arithmetic progressions

Exercise 5.3 Answers

1. Find the sum of the following APs.

(i) 2, 7, 12 ,…., to 10 terms.
(ii) – 37, – 33, – 29 ,…, to 12 terms
(iii) 0.6, 1.7, 2.8 ,…….., to 100 terms
(iv) 1/15, 1/12, 1/10, …… , to 11 terms

Answers:

(i) Given, 2, 7, 12, …, to 10 terms

For this Arithmetic progression, we can see that

first term, $a_1 = a = 2$.

And common difference, $d = a_2 - a_1 = 7-2 = 5$

n = 10 ….. (given)

We know that, the formula for sum of nth term in AP series is,

$S_n = n/2\ [2a + (n-1)d]$

$S_{10} = 10/2\ [2(2)+(10-1)\times 5]$

$= 5[4+(9)\times(5)]$

$= 5 \times 49 = 245$

(ii) Given, −37, −33, −29, …, to 12 terms

For this Arithmetic progression, we can see that the first term, a = −37

And common difference, $d = a_2 - a_1$

$d = (-33)-(-37)$

$= -33 + 37 = 4$

n = 12

We know that, the formula for sum of nth term in AP series is,

$S_n = n/2\ [2a+(n-1)d]$

$S_{12} = 12/2\ [2(-37)+(12-1)\times 4]$

$= 6[-74+11\times 4]$

$= 6[-74+44]$

= 6(-30) = -180

(iii) Given, 0.6, 1.7, 2.8 ,…, to 100 terms

For this Arithmetic progression, we can see that, the first term, a = 0.6

Common difference, d = $a_2 - a_1$ = 1.7 − 0.6 = 1.1

n = 100

We know that, the formula for sum of nth term in AP series is,

S_n = n/2[2a +(n-1)d]

S_{12} = 50/2 [1.2+(99)×1.1]

= 50[1.2+108.9]

= 50[110.1]

= 5505

(iv) Given, 1/15, 1/12, 1/10, …… , to 11 terms

For this Arithmetic progression, we can see that, the first term, a = 1/5

Common difference, d = $a_2 - a_1$ = (1/12)-(1/5) = 1/60

And number of terms n = 11

We know that, the formula for sum of nth term in AP series is,

$S_n = n/2\ [2a + (n - 1)\ d]$

S_{11} = (11/2) [2*1/5 + (11-1)(1/60)]

= (11/2)(2/15 + 10/60)

= (11/2)(9/30)

= 33/20

2. Find the sums given below:

(i) 7 + 10(1/2) + 14 +……………………… + 84
(ii) 34 + 32 + 30 + ……….. + 10
(iii) – 5 + (– 8) + (– 11) + ………… + (– 230)

Solutions:

(i)

For this given A.P.,$7+10\frac{1}{2}$ + 14 + ……………… +84,

First term, a = 7

n^{th} term, $a_n = 84$

Common difference, $d = a_2 - a_1 = \mathbf{10\frac{1}{2}} - 7 = \frac{21}{2} - \mathbf{7} = \frac{7}{2}$

Let 84 be the *n*th term of this A.P., then as per the n^{th} term formula,

$a_n = a(n-1)d$

84 = 7+(n – 1)×7/2

77 = (n-1)×7/2

22 = n−1

n = 23

We know that, sum of n term is;

$S_n = n/2\ (a + l)$, l = 84
$S_n = 23/2\ (7+84)$

$S_n = (23×91/2) = 2093/2$

(ii) Given, 34 + 32 + 30 + ……….. + 10

For this Arithmetic progression, we can see that, the first term, a = 34

common difference, $d = a_2−a_1 = 32−34 = −2$

n^{th} term, $a_n = 10$

Let 10 be the *n*th term of this A.P., therefore,

$a_n = a + (n−1)d$

10 = 34+(n−1)(−2)

−24 = (n −1)(−2)

12 = n −1

n = 13

We know that, sum of n terms is;

$S_n = n/2\ (a + l)$, l = 10

= 13/2 (34 + 10)

= (13×44/2) = 13 × 22

= 286

(iii) Given, (−5) + (−8) + (−11) + ………… + (−230)

For this Arithmetic progression, we can see that, the first term, a = −5

nth term, a_n= −230

Common difference, d = $a_2 - a_1$ = (−8)−(−5)

⇒d = − 8+5 = −3

Let −230 be the n^{th} term of this A.P., and by the n^{th} term formula we know,

$a_n = a+(n-1)d$

$-230 = -5+(n-1)(-3)$

$-225 = (n-1)(-3)$

$(n-1) = 75$

$n = 76$

And, Sum of n terms of this A.P. is ,

$S_n = n/2\ (a + l)$ where '**a**' is the first term and **'l'** is the last term or nth term or a_n

The formula can be written as $S_n = n/2\ (a + a_n)$

= 76/2 [(-5) + (-230)]

= 38(-235)

= -8930

3. In an AP
(i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.
(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13}.
(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.
(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.
(v) Given $d = 5$, $S_9 = 75$, find a and a_9.
(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.
(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.
(viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.
(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.
(x) Given $l = 28$, $S = 144$ and there are total 9 terms. Find a.

Solutions:

(i) Given that, $a = 5$, $d = 3$, $a_n = 50$

As we know, from the formula for the nth term of an AP,

$a_n = a + (n - 1)d,$

Therefore, substituting the given values, we get,

⇒ $50 = 5+(n -1)\times 3$

⇒ $3(n -1) = 45$

⇒ $n -1 = 15$

⇒ $n = 16$

Now, sum of n terms,

$S_n = n/2\ (a + a_n)$

$S_n = 16/2\ (5 + 50) = 440$

(ii) Given that, $a = 7$, $a_{13} = 35$

As we know, from the formula of the nth term in an AP,

$a_n = a+(n-1)d$,

Therefore, substituting the given values, we get,

⇒ $35 = 7+(13-1)d$

⇒ $12d = 28$

⇒ $d = 28/12 = 2.33$

Now, $S_n = n/2\ (a+a_n)$

$S_{13} = 13/2\ (7+35) = 273$

(iii) Given that, $a_{12} = 37$, $d = 3$

As we know, from the formula of the n^{th} term in an AP,

$a_n = a+(n-1)d$,

Therefore, putting the given values, we get,

⇒ $a_{12} = a+(12-1)3$

⇒ $37 = a+33$

⇒ $a = 4$

Now, sum of nth term,

$S_n = n/2\ (a+a_n)$

$S_n = 12/2\ (4+37)$

$= 246$

(iv) Given that, $a_3 = 15$, $S_{10} = 125$

As we know, from the formula of the nth term in an AP,

$a_n = a + (n-1)d$,

Therefore, substituting the given values, we get,

$a_3 = a + (3-1)d$

$15 = a + 2d$ ………………………….. **(i)**

Sum of the nth term,

$S_n = n/2\ [2a + (n-1)d]$

$S_{10} = 10/2\ [2a + (10-1)d]$

$125 = 5(2a + 9d)$

$25 = 2a + 9d$ ……………………….. **(ii)**

On multiplying equation **(i)** by **(ii)**, we will get;

$30 = 2a + 4d$ ……………………………… **(iii)**

By subtracting equation **(iii)** from **(ii)**, we get,

$-5 = 5d$

$d = -1$

From equation **(i)**,

$15 = a + 2(-1)$

$15 = a - 2$

$a = 17$ = First term

$a_{10} = a + (10-1)d$

$a_{10} = 17 + (9)(-1)$

$a_{10} = 17 - 9 = 8$

(v) Given that, $d = 5$, $S_9 = 75$

As, sum of n terms in AP is,

$S_n = n/2\ [2a + (n-1)d]$

Therefore, the sum of first nine terms are:

$S_9 = 9/2\ [2a + (9-1)5]$

$25 = 3(a+20)$

$25 = 3a+60$

$3a = 25-60$

$a = -35/3$

As we know, the n^{th} term of an A.P. can be written as;

$a_n = a+(n-1)d$

$a_9 = a+(9-1)(5)$

$= -35/3+8(5)$

$= -35/3+40$

$= (35+120/3) = 85/3$

(vi) Given that, $a = 2$, $d = 8$, $S_n = 90$

As, sum of n terms in an AP is,

$S_n = n/2\ [2a +(n -1)d]$

$90 = n/2\ [2a +(n -1)d]$

$\Rightarrow 180 = n(4+8n -8) = n(8n-4) = 8n^2-4n$

$\Rightarrow 8n^2-4n -180 = 0$

$\Rightarrow 2n^2-n-45 = 0$

$\Rightarrow 2n^2-10n+9n-45 = 0$

$\Rightarrow 2n(n -5)+9(n -5) = 0$

$\Rightarrow (n-5)(2n+9) = 0$

So, $n = 5$ (as n only be a positive integer)

$\therefore a_5 = 8+5\times4 = 34$

(vii) Given that, $a = 8$, $a_n = 62$, $S_n = 210$

As, sum of n terms in an AP is,

$S_n = n/2\ (a + a_n)$

$210 = n/2\ (8 +62)$

$\Rightarrow 35n = 210$

$\Rightarrow n = 210/35 = 6$

Now, $62 = 8+5d$

$\Rightarrow 5d = 62-8 = 54$

$\Rightarrow d = 54/5 = 10.8$

(viii) Given that, n[th] term, $a_n = 4$, common difference, $d = 2$, sum of n terms, $S_n = -14$.

As we know, from the formula of the n[th] term in an AP,

$a_n = a+(n-1)d$,

Therefore, substituting the given values, we get,

$4 = a+(n-1)2$

$4 = a+2n-2$

$a+2n = 6$

$a = 6 - 2n$ …………………………………………. **(i)**

As we know, the sum of n terms is;

$S_n = n/2\ (a+a_n)$

$-14 = (n/2)\ (a+4)$

$-28 = n\ (a+4)$

$-28 = n\ (6-2n+4)$ -------------------->(From equation **(i)**)

$-28 = n\ (-2n+10)$

$-28 = -2n^2+10n$

$2n^2 - 10n - 28 = 0$

$n^2 - 5n - 14 = 0$

$n^2 - 7n + 2n - 14 = 0$

$n(n-7) + 2(n-7) = 0$

$(n-7)(n+2) = 0$

Either $n - 7 = 0$ or $n + 2 = 0$

$n = 7$ or $n = -2$

However, n can neither be negative nor fractional.

Therefore, $n = 7$

From equation **(i)**, we get

$a = 6-2n$

$a = 6-2(7)$

$= 6-14$

$= -8$

(ix) Given that, first term, $a = 3$,

Number of terms, $n = 8$

And sum of n terms, $S = 192$

As we know,

$S_n = n/2\ [2a+(n-1)d]$

$192 = 8/2\ [2\times3+(8-1)d]$

$192 = 4[6+7d]$

$48 = 6+7d$

$42 = 7d$

$d = 6$

(x) Given that, $l = 28, S = 144$ and there are total of 9 terms.

Sum of n terms of an A.P. formula,

$S_n = n/2\ (a + l)$

$144 = (9/2)\ (a+28)$

$(144)\times(2/9) = a+28$

$32 = a+28$

$a = 4$

4. How many terms of the AP. 9, 17, 25 … must be taken to give a sum of 636?

Solutions:

Assume that n terms of the AP. 9, 17, 25 …has to be taken to get Sum as 636.

For this A.P.,

First term, $a = 9$

Common difference, $d = a_2 - a_1 = 17 - 9 = 8$

As, the sum of n terms, is;

$S_n = n/2\ [2a+(n-1)d]$

$636 = n/2\ [2\times a+(8-1)\times 8]$

$636 = n/2\ [18+(n-1)\times 8]$

$636 = n\ [9 + 4n - 4]$

$636 = n\ (4n + 5)$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

$n\ (4n + 53) - 12\ (4n + 53) = 0$

$(4n + 53)(n - 12) = 0$

Either $4n+53 = 0$ or $n-12 = 0$

$n = (-53/4)$ or $n = 12$

n cannot be negative or a fraction or a decimal number, therefore, $n = 12$ only.

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Solution:

Given that,

first term, a = 5

last term, l or $a_n = 45$

Sum of the AP, $S_n = 400$

As we know, the sum of AP formula is;

$S_n = n/2\ (a + l)$

$400 = (n/2)\ (5 + 45)$

$400 = (n/2)\ (50) \Rightarrow 8 = n/2 \Rightarrow 16 = n$.

Therefore, number of terms, n =16

As we know, the last term of AP series can be written as;

$a_n = a+(n-1)d$

$45 = 5 + (16 - 1)d$

$40 = 15d$

Common difference, $d = 40/15 = 8/3$

6. The first and the last term of an AP are 17 and 350, respectively. If the common difference is 9, how many terms are there and what is their sum?

Solution:

Given that,

First term, a = 17

Last term, l = 350

Common difference, d = 9

Let there be *n* terms in the A.P., thus the formula for last term or n^{th} term can be written as:

$l = a + (n - 1)d$

$350 = 17 + (n - 1)9$

$333 = (n - 1)9$

$(n - 1) = 37$

$n = 38$

$S_n = n/2\ (a + l)$

$S_{38} = 13/2\ (17+350)$

$= 19 \times 367$

$= 6973$

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22^{nd} term is 149.
Solution:

Given,

Common difference, d = 7

22^{nd} term, $a_{22} = 149$

Sum of first 22 term, $S_{22} = ?$

By the formula of nth term,

$a_n = a+(n-1)d$

$a_{22} = a+(22-1)d$

$149 = a+21\times 7$

$149 = a+147$

$a = 2$ = First term

Sum of n terms,

$S_n = (n/2)(a+a_n)$

$S_{22} = (22/2)(2+149)$

$= 11\times 151$

$= 1661$

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18, respectively.
Solution:

Given that,

Second term, $a_2 = 14$

Third term, $a_3 = 18$

Common difference, $d = a_3-a_2 = 18-14 = 4$

$a_2 = a+(2\text{-}1)d = a + d$

$14 = a+4$

$a = 10$ = First term

Sum of n terms;

$S_n = n/2\ [2a + (n-1)d]$

$S_{51} = 51/2\ [2\times 10\ (51\text{-}1)\ 4]$

$= 51/2\ [2+(20)\times 4]$

$= 51 \times 220/2$

$= 51 \times 110$

$= 5610$

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first *n* terms.

Solution:

Given that,

$S_7 = 49$

$S_{17} = 289$

We know, Sum of n terms;

$S_n = n/2\ [2a + (n - 1)d]$

Therefore,

$S_7 = 7/2\ [2a + (n - 1)d]$

$S_7 = 7/2\ [2a + (7 - 1)d]$

$49 = 7/2\ [2a + 6d]$

$7 = (a + 3d)$

$a + 3d = 7$ …………………………………… **(i)**

Similarly,

$S_{17} = 17/2\ [2a + (17 - 1)d]$

$289 = 17/2\ (2a + 16d)$

$17 = (a + 8d)$

$a + 8d = 17$ ………………………………… **(ii)**

Subtracting equation **(i)** from equation **(ii)**,

$5d = 10$

$d = 2$

From equation **(i)**, we can write it as:

$a + 3(2) = 7$

$a + 6 = 7$

$a = 1$

Hence,

$S_n = n/2[2a + (n - 1)d]$

$= n/2[2(1) + (n - 1) \times 2]$

$= n/2(2 + 2n - 2)$

$= n/2(2n)$

$= n^2$

10. Show that a_1, a_2 … , a_n , … form an AP where a_n is defined as below

(i) $a_n = 3+4n$
(ii) $a_n = 9-5n$
Also find the sum of the first 15 terms in each case.

Solutions:

(i) $a_n = 3+4n$

$a_1 = 3+4(1) = 7$

$a_2 = 3+4(2) = 3+8 = 11$

$a_3 = 3+4(3) = 3+12 = 15$

$a_4 = 3+4(4) = 3+16 = 19$

We can see here, the common difference between the terms is:

$a_2 - a_1 = 11-7 = 4$

$a_3 - a_2 = 15-11 = 4$

$a_4 - a_3 = 19-15 = 4$

Hence, $a_{k+1} - a_k$ is the same every time. Therefore, this is an AP with common difference as 4 and first term as 7.

Now, as we know, the sum of nth term of an A.P. is

$S_n = n/2[2a+(n-1)d]$

$S_{15} = 15/2[2(7)+(15-1)\times4]$

$= 15/2[(14)+56]$

$= 15/2(70)$

$= 15\times35$

$= 525$

(ii) $a_n = 9-5n$

$a_1 = 9-5\times1 = 9-5 = 4$

$a_2 = 9-5\times2 = 9-10 = -1$

$a_3 = 9-5\times3 = 9-15 = -6$

$a_4 = 9-5\times4 = 9-20 = -11$

We can see here, the common difference between the terms are;

$a_2 - a_1 = -1-4 = -5$

$a_3 - a_2 = -6-(-1) = -5$

$a_4 - a_3 = -11-(-6) = -5$

Hence, $a_{k+1} - a_k$ is same every time. Therefore, this is an A.P. with common difference as −5 and first term as 4.

Now, we know, the sum of nth term is;

$S_n = n/2\ [2a + (n-1)d]$

$S_{15} = 15/2[2(4) + (15-1)(-5)]$

$= 15/2[8 + 14(-5)]$

$= 15/2(8-70)$

$= 15/2(-62)$

$= 15(-31)$

$= -465$

11. If the sum of the first *n* terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the10th and the *n*th terms.

Solution:

Given that,

$S_n = 4n-n^2$

First term, $a = S_1 = 4(1) - (1)^2 = 4-1 = 3$

Sum of first two terms $= S_2 = 4(2)-(2)^2 = 8-4 = 4$

Second term, $a_2 = S_2 - S_1 = 4-3 = 1$

Common difference, $d = a_2-a = 1-3 = -2$

Nth term, $a_n = a+(n-1)d$

$= 3+(n-1)(-2)$

$= 3-2n+2$

$= 5 - 2n$

Therefore, $a_3 = 5-2(3) = 5-6 = -1$

$a_{10} = 5-2(10) = 5-20 = -15$

Hence, the sum of first two terms is 4. The second term is 1.

The 3rd, the 10th, and the n^{th} terms are −1, −15, and $5 - 2n$ respectively.

12. Find the sum of first 40 positive integers divisible by 6.

Solution:

The positive integers that are divisible by 6 are 6, 12, 18, 24 ….so on

We can find here, that this series forms an A.P. whose first term is 6 and common difference is 6.

$a = 6$

$d = 6$

$S_{40} = ?$

By the formula of sum of n terms, we know,

$S_n = n/2\ [2a + (n-1)d]$

Therefore, putting n = 40, we get,

$S_{40} = 40/2\ [2(6)+(40-1)6]$

$= 20[12+(39)(6)]$

$= 20(12+234)$

$= 20 \times 246$

$= 4920$

13. Find the sum of first 15 multiples of 8.

Solution:

The multiples of 8 are 8, 16, 24, 32…

The series is in the form of AP, having first term as 8 and common difference as 8.

Therefore, $a = 8$

$d = 8$

$S_{15} = ?$

By the formula of sum of n^{th} term, we know,

$S_n = n/2\ [2a+(n-1)d]$

$S_{15} = 15/2\ [2(8) + (15-1)8]$

= 15/2[6 +(14)(8)]

= 15/2[16 +112]

= 15(128)/2

= 15 × 64

= 960

14. Find the sum of the odd numbers between 0 and 50.

Solution:

The odd numbers between 0 and 50 are 1, 3, 5, 7, 9 … 49.

Therefore, we can see that these odd numbers are in the form of A.P.

Hence,

First term, a = 1

Common difference, d = 2

Last term or n^{th} term, l or a_n= 49

By the formula of last term, we know,

l or $a_n = a+(n-1)\ d$

$49 = 1+(n-1)2$

$48 = 2(n - 1)$

$n - 1 = 24$

$n = 25$ = Number of terms

By the formula of sum of nth term, we know,

$S_n = n/2(a + l)$

$S_{25} = 25/2\ (1+49)$

= 25(50)/2

=(25)(25)

= 625

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Solution:

We can see that the given penalties are in the form of A.P. having first term as 200 and common difference as 50.

Therefore, $a = 200$ and $d = 50$

Penalty that should be paid if contractor has delayed the work by 30 days = S_{30}

By the formula of sum of n terms, we know that

$S_n = (n/2)[2a+(n-1)d]$

Therefore,

$S_{30} = (30/2)[2(200)+(30-1)50]$

$= 15[400+1450]$

$= 15(1850)$

$= 27750$

Therefore, the contractor will have to pay Rs 27750 as penalty for the delay of work by 30 days.

16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Solution:

Let the cost of 1st prize be Rs. x.

Cost of 2nd prize = Rs. $x - 20$

And cost of 3rd prize = Rs. $x - 40$

We can see that the cost of these prizes is in the form of A.P., having common difference as −20 and first term as x.

Therefore, a = x and d = −20

Given that, $S_7 = 700$

By the formula of sum of n terms of an A.P. , we know,

$S_n = (n/2)\ [2a + (n-1)d]$

$7/2\ [2a + (7-1)d] = 700$

$$\frac{[2a + (6)(-20)]}{2} = 100$$

$a + 3(-20) = 100$

$a - 60 = 100$

$a = 160$……so, x = 160.

Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Solution:

It can be observed that the number of trees planted by the students is in an AP.

1, 2, 3, 4, 5………………..12

First term, $a = 1$

Common difference, $d = 2-1 = 1$

$S_n = (n/2)\ [2a + (n-1)d]$

$S_{12} = (12/2)\ [2(1)+(12-1)(1)]$

= 6(2+11)

= 6(13)

= 78

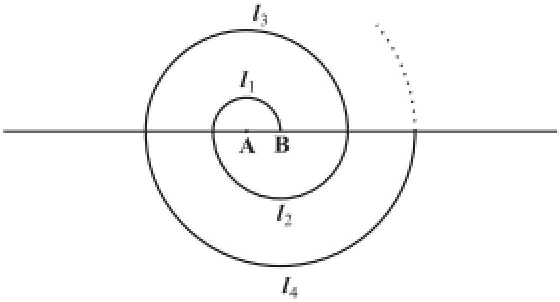
Therefore, number of trees planted by 1 section of the classes = 78

Number of trees planted by 3 sections of the classes = 3×78 = 234

Therefore, 234 trees will be planted by the students.

18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ……… as

shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take π = 22/7)



Solution:

We know,

Perimeter of a semi-circle = πr

Therefore,

Perimeter of first semi-circle, $P_1 = \pi(0.5) = \pi/2$ cm

Perimeter of second semi-circle, $P_2 = \pi(1) = \pi$ cm

Perimeter of third semi-circle, $P_3 = \pi(1.5) = 3\pi/2$ cm

Where, P_1, P_2, P_3 are the lengths of the semi-circles…..and all when added, would give us the length of the total wire together.

Hence, we will get a series of perimeters here, as,

$\pi/2, \pi, 3\pi/2, 2\pi, \ldots$

$P_1 = \pi/2$ cm

$P_2 = \pi$ cm

Common difference, $d = P_2 - P_1 = \pi - \pi/2 = \pi/2$

First term = $P_1 = a = \pi/2$ cm

By the sum of n term formula, we know,

$S_n = n/2\ [2a + (n - 1)d]$

Therefor, Sum of the length of 13 consecutive circles is;

$S_{13} = 13/2\ [2(\pi/2) + (13 - 1)\pi/2]$

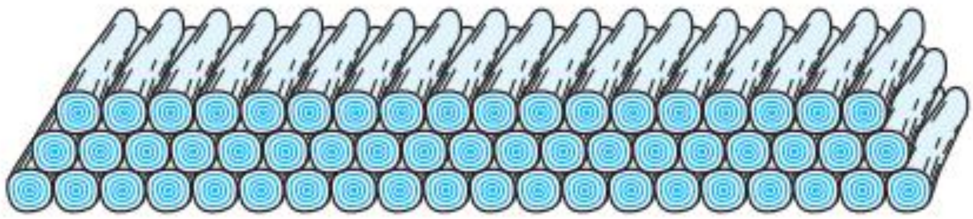
$= 13/2\ [\pi + 6\pi]$

$= 13/2\ (7\pi)$ Here, put $\pi = 22/7$

$= 13/2 \times 7 \times 22/7$

= 143 cm

19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Solution:

We can see that the numbers of logs in rows are in the form of an A.P.

20 in first row, 19 in second row, 18 in third row……….and so on

For the given A.P.,

First term, $a = 20$ and common difference, $d = a_2 - a_1 = 19 - 20 = -1$

Assume a total of 200 logs are placed in n rows.

Thus, Sum of the logs placed in all the rows together, $S_n = 200$.

By the sum of nth term formula,

$S_n = n/2\ [2a + (n - 1)d]$

$S_{12} = 12/2\ [2(20) + (n - 1)(-1)]$

$400 = n\ (40 - n + 1)$

$400 = n\ (41 - n)$

$400 = 41n - n^2$

$n^2 - 41n + 400 = 0$

$n^2 - 16n - 25n + 400 = 0$

$n(n - 16) - 25(n - 16) = 0$

$(n - 16)(n - 25) = 0$

Here, $(n - 16) = 0$ or $n - 25 = 0$

$n = 16$ or $n = 25$

By the nth term formula,

$a_n = a + (n - 1)d$

$a_{16} = 20+(16-1)(-1)$

$a_{16} = 20-15$

$a_{16} = 5$

Similarly, the 25th term could be written as;

$a_{25} = 20+(25-1)(-1)$

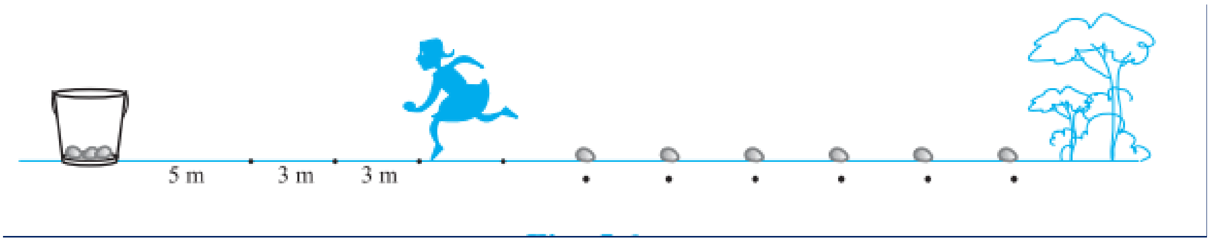
$a_{25} = 20-24$

$= -4$ This is not valid….because number of logs in a row can't be negative. So, n is not 25.

Therefore, n is 16. It can be taken as the number of logs in 16th row is 5.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?
[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2×5+2×(5+3)]

Solution:

The distances of potatoes from the bucket are respectively 5, 8, 11, 14…, which is in the form of AP.

Given, the distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept….because competitor has to run to and fro….to take the potato and to come back and put it in the bucket. Again moving to the next potato….picking it up, then coming to the bucket.

Therefore, distances to be run could be written as:

5*2, 8*2, 11*2, 14*2, 17*2, ……….

⇨ 10, 16, 22, 28, 34…………

Hence, the first term, a = 10 and common difference, d = 16−10 = 6

S_{10} =?

By the formula of sum of n terms, we know,

S_{10} = (10/2) [2(20)+(n -1)(-1)]

= 5[20+54]

= 5(74)

= 370

Therefore, the competitor will run a total distance of 370 m.