VECTORS AND SCALAR

Learning Objectives :

- ♦ Definition of scalar and vectors
- ♦ Representation of vector
- ♦ Types of vectors.
- ♦ Modulus of a vector
- ♦ Angle between vectors.
- ♦ Addition(or) substraction of two vectors.
- ♦ Laws of vector addition.
- ♦ Resultant of number of vectors.

Real life applications:

Φ Vector is used for finding displacement of a body in the easiest way.

Φ One of the most common uses of vectors is in the description of velocity.

Φ By utilizing vector addition on these different forces, mathematicians create an accurate estimate of the path of motion and distance traveled by the object.

Φ Vectors are mathematical constructs that include magnitude and direction. They can exist in any number of dimensions. Because of this, they are used to simply yet effectively convey information about objects or situations.

Φ Vectors are also used to plot trajectories. The movements of any thrown object, such as a football, can be mapped with vectors.

Φ Using multiple vectors allows for the creation of a model that encompasses external forces like wind.

♥ **Important Formulae:-**

1. unit vector $\hat{a} = \frac{Given\,Vector}{\text{mod}\,of\,Vector} = \frac{\vec{a}}{|\vec{a}|}$

2. For a vector $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$, magnitude or mod of vector $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

3. $R_x = R\cos\theta$, $R_y = R\sin\theta$, $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

4. when three forces ($\vec{a},\vec{b},\vec{c}$)are in equilibrium then resultant force is zero.

$$\vec{a} + \vec{b} + \vec{c} = 0$$

5. Resultant vector is sum of given vectors.

Eg : $\vec{a}$, $\vec{b}$, $\vec{c}$ resultant is $\vec{a} + \vec{b} + \vec{c}$

§§ CLASSIFICATION OF PHYSICAL QUANTITIES

All measurable quantities are called physical quantities. Most of the physical Quantities are classified into 'Scalars' and 'Vectors'.

Scalar:- Physical quantities having only magnitude are called Scalars.

Ex: Length, time, volume, density, temperature, mass, work, energy, electric charge, electric current, potential ,resistance, capacity, etc.....

Vector : - Physical quantities having both magnitude and direction and that obeys laws of vector addition are called vectors

(OR)

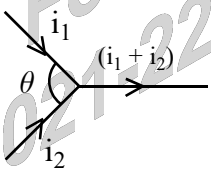
The vector, as a mathematical object, is defined as a directed line segment. It should also obey the laws of vector addition.

Eg : Displacement, velocity, acceleration, force, momentum, angular momentum, moment of force, Torque, magnetic moment, magnetic induction field, Intensity of electric field, etc ....

Note : 1) A physical quantity having magnitude and direction but not obeying laws of vector addition is treated as a scalar.

Ex : Electric current is a scalar quantity

Electric current is always associated with direction, but it is not a vector quantity. It does not obey law of vector addition for its addition.



The resultant of i_1 and i_2 is $(i_1 + i_2)$ by Kirchoff's current law. The resultant does not depend on angle between currents i_1 and i_2.

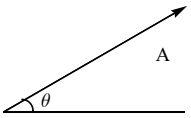
Note: Equations in vector form indicate both mathematical and geometrical relationships among the quantities. Physical laws in vector form are very compac and independent of choice of coordinate system.

Distinction between Scalars and Vectors

Scalars	Vectors
1.Scalars have magnitudes only.	1. Vectors have both magnitude and direction
2. Speed, mass, work etc. are examples of scalars.	2. Force, velocity, electric field etc. are examples of vectors
3. Scalars are represented by numerical values.	3. Vectors are represented by the segment of a straight line drawn in a specific direction.
4. Addition and substraction follow algebraic methods.	4. Geometric methods are used for addition and substraction
5. Two scalars are equal when they have the same magnitudes	5. Two vectors are equal when they have the same magnitude and the same direction.
6. Product of two scalars is a scalar.	Scalar product of two vectors is a scalar where as their vector product is a vector.
7. Simple letter symbols are used. Thus m can denote mass, t can denote time etc.	6. An arrowhead is placed over the letter symbol to denote vectors. Thus F can denote the force vector.

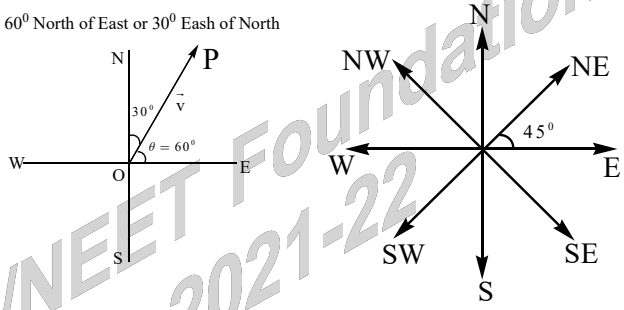
§§ GEOMETRICAL REPRESENTATION OF VECTORS

Any vector $\vec{A}$ can be represented geometrically as a directed line segment (an arrow), as shown in fig. The magnitude of $\vec{A}$ is denoted by $|\vec{A}|$, and the direction of $\vec{A}$ is specified by the sence of the arrow and the angle 'θ' that it makes with a fixed reference line.



When using graphical methods, the length of the arrow is proportional to the magnitude of the vector, and the arrow head represents the direction.

Ex:- The velocity vector $\vec{v}$ is represented by an arrow OP as shown in figure. The initial point of the vector is O, the final point of the vector is P. The length OP is the magnitude of the velocity and its direction is 60^0 north of east (or) 30^0 east of north .



§§ **POSITION VECTOR:** It is a vector that represents the position of a particle with respect to the origin of a co-ordinate system. The Position Vector of a point (x, y, z) is $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

§§ **Types of vectors:**

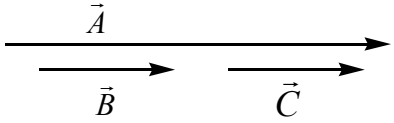
1) Polar Vectors : The vector whose direction does not change even though the co-ordinate system in which it is defined changes is called polar vector.

Eg : Force, momentum, Acceleration.

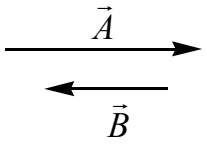
2) Axial Vectors : The vectors whose direction changes with the co-ordinate system in which it is defined changes is called axial vector.

Eg:Angular velocity, torque, angular momentum

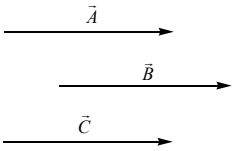
3) Like vectors (or) parallel vectors : Two or more vectors (representing same physical quantity) are called like vectors if they are parallel to each other, however their magnitudes may be different.



4) Unlike vectors (or) anti parallel vectors : Two vectors (representing same physical quantity) are called unlike vectors if they act in opposite direction however their magitudes can be different.

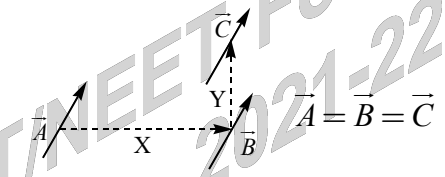


5) Equal vectors : Two or more vectors (representing same physical quantity) are called equal if their magnitudes and directions are same.

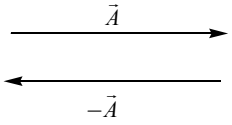


Eg : Suppose two trains are running on parallel tracks with same speed and direction. Then their velocity vectors are equal vectors.

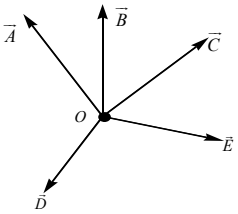
Note : *If a vector is displaced parallel to itself its magnitude and direction does not change.*



6) Negative Vector : A vector having the same magnitude and opposite in direction to that of a given vector is called negative vector of the given vector

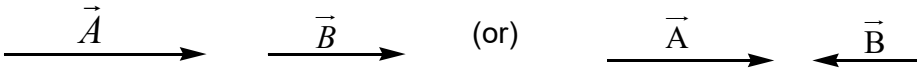


7) Co-initial vectors : The vectors having same initial point are called co-initial vectors.

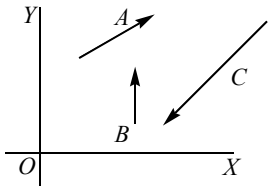


8) Collinear vectors : Two or more vectors are said to be collinear when they act along the same line however their magnitudes may be different.

Ex : Two vectors $\vec{A}$ and $\vec{B}$ as shown are collinear vectors.



9) Coplanar vectors : A number of vectors are said to be coplanar if they are in the same plane or parallel to the same plane. However their magnitudes may be different.



10) Unit vector : A vector whose magnitude equals one and used to specify a convenient direction is called a unit vector.

A unit vector has no units and dimensions. Its purpose is to specify the direction of given vector.

In cartesian coordinate system, unit vectors along positive x, y and z axis are symbolised as $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively. These three unit vectors are mutually perpendicular and their magnitudes $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.

If $\vec{A}$ is a non zero vector, then the unit vector in the direction of $\vec{A}$ is given by $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$.

§§ Angle between two vectors :

The angle between two vectors is represented by the smaller of the two angles between the vectors($0^0 \le \theta \le 180^0$) when they are placed tail to tail or head to head.

Eg: The angle between $\vec{A}$ and $\vec{B}$ is correctly represented in the following figures.

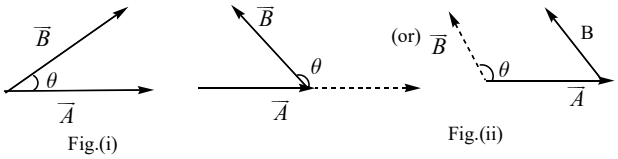
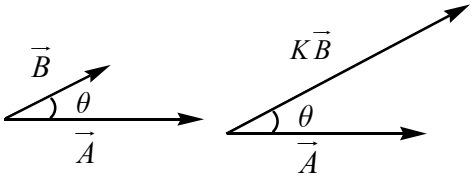
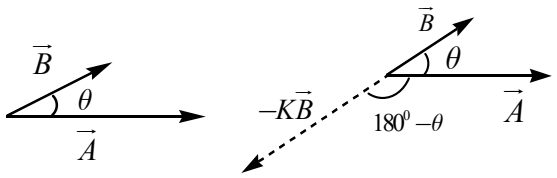


Fig.(i) Fig.(ii)

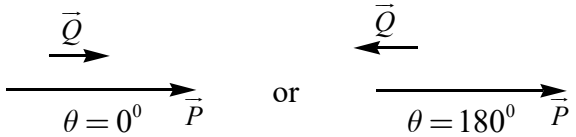
a) If the angle between $\vec{A}$ and $\vec{B}$ is θ then the angle between $\vec{A}$ and $K\vec{B}$ is also θ . Where 'K' is a positive constant.



b) If the angle between $\vec{A}$ and $\vec{B}$ is θ then the angle between $\vec{A}$ and $-K\vec{B}$ is $(180-\theta)$ Where K is a positive constant.



c) Angle between collinear vectors is always zero or 180^0C



EXAMPLE

√ **Example 1** :

A boy moves 3m along positive x-axis, 4m along +ve y-axis, 5m along -ve x-axis and 2m along -ve y-axis. Find the displacement.

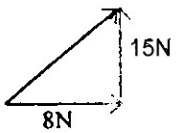
Solution : $\overline{S}=3\bar{i}+4\bar{j}-5\bar{i}-2\bar{j}$

$$\overline{S}=-2\bar{i}+2\bar{j} \qquad |\overline{S}|=-\sqrt{(2)^2+(2)^2}=\sqrt{8}=2\sqrt{2}m$$

√ **Example 2** :

The horizontal and vertical components of a force are 8N and 15N respectively. Find the magnitude of force.

Solution : $\overline{F}=8\bar{i}+15\bar{j}$



$$|\overline{F}|=\sqrt{64+225}=\sqrt{289}=17N$$

√ **Example 3:**

Find the unit vector along $\bar{i}+\bar{j}$

Solution: $\hat{\bar{a}}=\dfrac{\bar{a}}{|\bar{a}|}=\dfrac{\bar{i}+\bar{j}}{\sqrt{2}}=\left(\dfrac{1}{\sqrt{2}}\right)\bar{i}+\left(\dfrac{1}{\sqrt{2}}\right)\bar{j}$

√ **Example 4:**

Find the unit vector along the direction $\bar{i}+\sqrt{3}\bar{j}$

solution: $\hat{\bar{a}}=\dfrac{\bar{a}}{|\bar{a}|}=\dfrac{\bar{i}+\sqrt{3}\bar{j}}{\sqrt{1+3}}=\left(\dfrac{1}{2}\right)\bar{i}+\left(\dfrac{\sqrt{3}}{2}\right)\bar{j}$

√ **Example 5:**

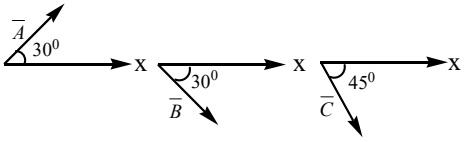
Find the value of x if the vector $0.8\hat{i}+x\hat{j}$ represents a unit vector

solution: $\vec{a}=\frac{8}{10}\vec{i}+x\vec{j}$ $\therefore |\vec{a}|$ =1

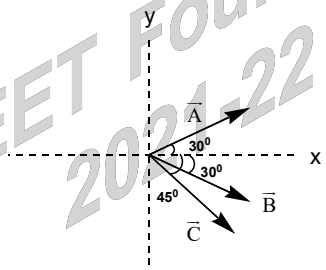
$$\sqrt{(0.8)^2+x^2}=1 \Rightarrow x^2=1-0.64=0.36 \Rightarrow x=\pm 0.6$$

√ **Example 6:**

Three vectors $\vec{A},\vec{B},\vec{C}$ are shown in the figure. Find angle between (i) $\vec{A}$ and $\vec{B}$ (ii) $\vec{B}$ and $\vec{C}$ (iii) $\vec{A}$ and $\vec{C}$.



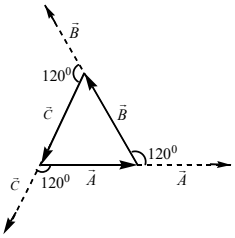
solution:To find the angle between two vectors we connect the tails of the two vectors. We can shift the vectors parallel to themselves such that tails of $\vec{A},\vec{B}$ and $\vec{C}$ are connected as shown in figure.



Now we observe that angle between $\vec{A}$ and $\vec{B}$ is 60°, $\vec{B}$ and $\vec{C}$ is 15° and between $\vec{A}$ and $\vec{C}$ is 75°

√ **Example-7:**

If $\vec{A},\vec{B},\vec{C}$ represents the three sides of an equilateral triangle taken in the same order then find the angle between i) $\vec{A}$ and $\vec{B}$ ii) $\vec{B}$ and $\vec{C}$ iii) $\vec{A}$ and $\vec{C}$.

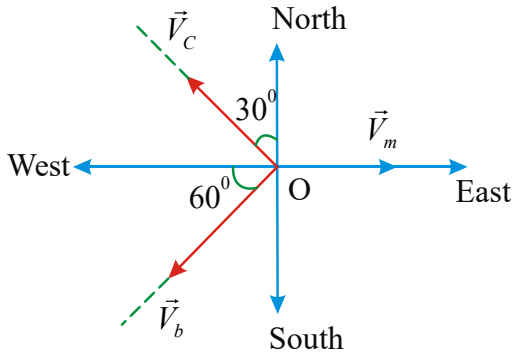


solution: From the diagram the angle between the vectors $\vec{A}$ and $\vec{B}$ is 120^0, the angle between $\vec{B}$ and $\vec{C}$ is 120^0, the angle between $\vec{A}$ and $\vec{C}$ is 120^0

√ **Example-8:**

A man walks towards east with certain velocity. A car is travelling along a road which is 30^0 West of north.While a bus is travelling in another road which is 60^0 South of west.Find the angle between velocity vector of

a) man and car b) car and bus c) bus and man.



solution:

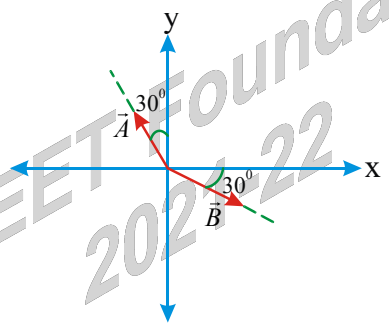
From the diagram the angle between velocity vector of man and car is $90^0+30^0=120^0$

The angle between velocity vector of car and bus is $60^0+60^0=120^0$

The angle between velocity vector of bus and man is $30^0+90^0=120^0$

√ **Example-9:**

A vector $\vec{A}$ makes an angle 30^0 with the y-axis in anticlockwise direction.Another vector $\vec{B}$ makes an angle 30^0 with the x-axis in clockwise direction. Find angle between vectors $\vec{A}$ and $\vec{B}$.



solution:

From the diagram the angle between $\vec{A}$ and $\vec{B}$ is $60^0+90^0+60^0=210^0$ (or) -150^0

TEACHING TASK

I) Choose the correct option:

1. If $0.5\bar{i}+0.8\bar{j}+C\bar{k}$ is a unit vector, then C is

1) $\sqrt{89}$ 2) 0.2 3) 0.3 4) $\sqrt{0.11}$

2. A particle has a displacement of 12m towards east then 5m towards north and then 6m vertically upwards. The resultant displacement is

1) 10.04 m 2) 12.10 m 3) 14.32 m 4) 12.6 m

3. The modulus of a vector $\vec{A}=5\vec{i}+p\vec{j}+4\sqrt{2}\vec{k}$ is 11. The value of p is

1) $+8$ 2) -8 3) ± 8 4) 6

4. If position vector is given by $\bar{R}$ = (-6,-4,-12) then the unit vector parallel to $\bar{R}$ is

1) $+\frac{1}{7}(3\text{i}+2\bar{\text{j}}+6\bar{\text{k}})$ 2) $-\frac{1}{7}[3i+2\bar{j}+6\bar{k}]$

3) $\pm\frac{1}{7}[3i+2\bar{j}-6\bar{k}]$ 4) $\frac{14}{[6i+4\bar{j}+12\bar{k}]}$

5. A force $\bar{F}=6\bar{i}-8\bar{j}+10\bar{k}$ N Produces and acceleration of 1m/s² in a body. The mass of body would be

1) 200 kg 2) 20kg 3) $10\sqrt{2}kg$ 4) $6\sqrt{2}kg$

6. A man travels 1 mile due east, than 5 miles due south, then 2 miles due east and finally 9 miles due north. His displacement is

1) 3 miles 2) 5 miles 3) 4 miles 4) between 5 and 9 miles

7. Find the unit vector whose 3D rectangular components along X and Y the ratio 1:1:1

1)[(i+j+k)/$\sqrt{3}$] 2)[$(2i+j+k)/\sqrt{3}$]

3)[(i+2j+k)/$\sqrt{3}$] 4)[(i+j+3k)/$\sqrt{3}$]

8. If $\left|x\vec{i}-3\vec{j}+5\vec{k}\right|=\sqrt{98}$, the value of x is

1) ± 2 2) ± 4 3) ± 6 4) ± 8

II) Multiple option type:

- *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options*

9. Any vector should have

a) perticular magnitude b) perticular direction

c) perticular colour d) perticular shape

A) only a, b B) only b, c C) only c, d D) only a, d

10. Let $\vec{A}=3i+4j$ and $\vec{B}=12i+5j$

a) magnitude of $\vec{A}$ is 5 b) magnitude of $\vec{B}$ is 13

c) $\vec{B}$ is a unit vector d) unit vector of $\vec{A}$ is $(3i+4j)/5$

A) only a, b, c correct B) only a, b, d correct

C) only a, c, d correct D) all are correct

III) Fill in the blanks:

11. Any vector should have both magnitude and .....

12. Magnitude of null vector is equal to .....

13. Magnitude of unit vector is .....

14. Time is ..... quantity

IV) Match the following

- *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have*

*to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

15. a) negative vector 1) act in same plane
b) null vector 2) Have the same magnitude but act in the opposite direction
c) Colinear vector 3) Have zero magnitude and arbitary direction
d) Coplanar vector 4) act along straight parallel lines
A) a - 1, b -2 ,c - 3, d - 4 B) a -4 , b -3, c -1, d - 2
C) a- 4, b- 3 ,c- 2, d- 1 D) a - 2, b - 3, c - 4, d- 1

V) Comprehention type:

- *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

16. If the two vector $\vec{A} = 2\vec{i} + 2\vec{j}$ and $\vec{B} = 2\vec{i} - 4\vec{j}$ so that

i) Find the magnitude of $\vec{B}$

A) $2\sqrt{5}$ B) $5\sqrt{2}$ C) $\sqrt{20}$ D) Both A&C

ii) Find the magnitude of $\vec{A}$

a) $\sqrt{8}$ B) $2\sqrt{2}$ C) $\sqrt{5}$ D) Both A&B

iii) Unit vector of $\vec{A}$ is

A) $(2i+2j)/\sqrt{8}$ B) $(2i+2j)/\sqrt{2}$ C) $(i+j)/\sqrt{8}$ D) $(2i+2j)$

KEY

ΦΦ TEACHING TASK :

1) 4, 2) 3, 3) 3, 4) 2, 5) 3, 6) 2, 7) 1, 8) 4, 9) A, 10) B,
11) direction , 12) zero, 13) one unit, 14) scalar, 15) D,
16) i) C, ii) B, iii) A

LEARNER'S TASK

BEGINNERS (Level - I)

I) Choose the correct option:

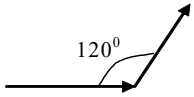
1. Choose the scalar quantity
1) force 2) mass 3) velocity 4) impulse

2 Magnitude of vector con not be
1) unity 2) positive 3) negative 4) zero

3. Which one of the following is meanigful?
1)vector/vector 2) scalar/vector 3) scalar+vector 4) vector/scalar

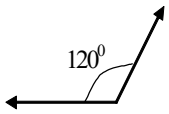
4. A vector is not changed if
1) it is divided by a scslar 2) it is multiplied by a scalar
3) it is slided parallel to itself 4) none

5. Find the angle between two vectors $\overline{A}\ \&\ \overline{B}$ as shown in the figures



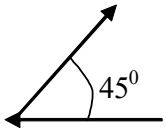
1) 60^0 2)80^0 3) 70^0 4) 20^0

6. Find the angle between two vectors $\overline{A}\ \&\ \overline{B}$ as shown in the figures



1) 120^0 2)60^0 3) 50^0 4) 110^0

7. Find the angle between two vectors $\overline{A}\ \&\ \overline{B}$ as shown in the figures



1) 130^0 2)350^0 3) 135^0 4) 120^0

8. The magnitude of vector $\vec{a} = \vec{i} + 3\vec{j}$ is

1) $\sqrt{4}$ 2) $\sqrt{6}$ 3) $\sqrt{10}$ 4) $\sqrt{15}$

9. The magnitude of vector $\vec{a} = 3\vec{i} + 4\vec{j} - 5\vec{k}$ is

1) 12 2) - 4 3) 50 4) $5\sqrt{2}$

10. The magnitude of $\vec{i} + \vec{j} + \vec{k}$ is

1) 6 2) 1 3) 3 4) $\sqrt{3}$

11. Set the following vectors in the increasing order of their magnitudes.

a) 3i+4j b) 2i+4j+6k c) 2i+2j+2k

1) b, a, c 2) c, a, b 3) a, c, b 4) a, b, c

12. Find the unit vector along the direction of vector whose rectangular components are 3, 4 & 5 along X, Y and Z

1) [(3i+4j+5k)/ $5\sqrt{2}$] 2) (3i+4j+5k) 3) [(3i+4j+5k)/5] 4) none

13. Which of the following statement is true?

1) a scalar quantity is added to the vector

2) it is possible for the magnitude of a vector to be equal zero even through one componet is non zero.

3) scalar quantities are path dependent, while vtector are not.

4) scalar quantities and vector can both be added algebraically.

14. If $\hat{n}$ is unit vector along the direction of $\vec{A}$ then

1) $\hat{n} = \frac{\vec{A}}{|\vec{A}|}$ 2) $\hat{n} = A + \vec{A}$ 3) $\hat{n} = A - \vec{A}$ 4) $\hat{n} = A / \vec{A}$

15. What will be the unit vector in the direction of $\vec{A} = 5\vec{i} + \vec{j} - 2\vec{k}$

1) $\frac{5\vec{i}+\vec{j}-2\vec{k}}{\sqrt{15}}$ 2) $\frac{5\vec{i}+\vec{j}-2\vec{k}}{\sqrt{30}}$ 3) $\frac{\vec{i}-5\vec{j}+2\vec{k}}{\sqrt{30}}$ 4) $\frac{\vec{i}-5\vec{j}+2\vec{k}}{\sqrt{15}}$

16. Vector $\overline{A}$ has an x and y components of -8.70 cm and 15.0 cm respectively. $\overline{B}$ vector has an x and y components of 13.2cm and -6.60cm respectively. If $\overline{A}-\overline{B}+3\overline{C}=0$ then x and y components of $\overline{C}$

1) -7.20cm and 7.30cm 2) 7.30cm and -7.20cm
3) -7.30cm and 7.20cm 4) -7.30cm and -7.20cm

17. If a body starts with a velocity $2\hat{i}-3\hat{j}+11\hat{k}\ ms^{-1}$ and moves with an accelaration of $10\hat{i}+10\hat{j}+10\hat{k}\ ms^{-2}$. then its velocity after 0.25s

1) $\frac{1}{2}\sqrt{811}ms^{-1}$ 2) $\sqrt{\frac{811}{2}}ms^{-1}$ 3) $\sqrt{811}ms^{-1}$ 4) $2\sqrt{811}ms^{-1}$

18. The value of 'm', if $\vec{i}+2\vec{j}-3\vec{k}$ is parallel to $3\vec{i}+m\vec{j}-9\vec{k}$ is

1) 12 2) 9 3) 6 4) 3

ACHIEVERS (Level - II)

Solve the following

1. If $\vec{A}$ and $\vec{B}$ are two vectors acting at an angle θ then find the angle between

a) $\vec{A}$ and $\vec{A}$ b) $\vec{B}$ and $\vec{B}$ c) $\vec{A}$ and $-\vec{B}$ d) $-\vec{A}$ and $\vec{B}$

e) $3\vec{A}$ and $\vec{A}$ f) $\vec{A}$ and $2\vec{B}$ g) $2\vec{A}$ and $3\vec{B}$ h) $-\vec{A}$ and $2\vec{B}$

i) $-\vec{A}$ and $-\vec{B}$ j) $-2\vec{A}$ abd $-\vec{B}$ k) $-3\vec{A}$ and $\vec{B}$ k) $-3\vec{A}$ and $-2\vec{B}$

2. Find the magnitude of the vector $\overline{A}=\overline{i}+2\overline{j}-\overline{k}$.

3. A vector is represented as $3\hat{i}+4\hat{j}+5\hat{k}$. Fine the length of the vector.

4. Find the unit vector along $3\overline{i}-4\overline{j}$

5. Find the unit vector whose 2D rectangular components along X and Y the ratio 3:4.

6. A vector is represented by $3\overline{i}+\overline{j}+2\overline{k}$. Its length in xy plane is ?

7. If $\hat{a}$ is the unit unit vector along $\overline{a}$ then find the angle between them

8. A person walks 3m towards east and then 4m towards north. Find the displacement of the person.

9. A boy walks 12m towards west and then 5m towards south. Find the displacement of the boy.

10. A car travels 10km towards south and then 24km towards east. Find the displacement of the car.

EXPLORERS (Level - III)

I) Multiple option type:

- *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options*

1. The angle between two vectors can be

a) minimum zero b) maximum 180^0

c) any between 0^0 to 180^0 d) any above 180^0

A) only a, b, c are correct B) only a, c, d are correct

C) only b, c, d are correct D) all a, b, c, d are correct

2. Choose the correct in the following

a) null vector have zero magnitude b) unit vector have unit magnitude

c) equal vectors have equal magnitude and same direction

d) opposite vectors have equal magnitude and equal direction

A) only a, b, c are correct B) only a, c, d are correct

C) only b, c, d are correct D) all a, b, c, d are correct

3. Choose the scalars

a) velocity b) denisty c) area d) volume

A) only a, b, c B) only b, c, d C) only a, c, d D) all a, b, c, d

4. Choose the vectors

a) distance b) displacement c) speed d) velocity

A) only a, b B) only b, c C) only a, c D) only b, d

II) Fill in the blanks:

5. Vector multiplid by scalar gives ..... value

6. Mass is ..... quantity

7. Angle between $\vec{A}$ & $\vec{A}$ is .....

8. If angle between $\vec{A}$ & $\vec{B}$ is θ, then angle between $\vec{A}$ & $-\vec{B}$ is .....

9. A particle moves 3 units aloing positive x-axis direction, and 2 units in negative y-axis direction. It can be represented in vector form as .....

10. 0.8 i + x j is a unit vector, the value of x is .....

11. Angle between two vectors can not be more than .....

III) Match the following:

- *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

  *If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

13.

a) scalar	1) having equal magnitude and same direction
b) vector	2) having unit magnitude
c) unit vector	3) having magnitude as well as direction
d) equal vector	4) having only magnitude and no direction

A) a-1, b-2, c-3, d-4 B) a-4 , b-3, c-1, d-2
C) a-4, b-3 ,c-2, d-1 D) a-2, b-1, c-4, d-3

14. a) polar vector 1) torque

b) axial vector 2) $\sqrt{x^2+y^2+z^2}$

c) unit vector 3) magnitude unity

d) magnitude of xi+yj+zk 4) force

A) a-1, b-2, c-3, d-4 B) a-4 , b-1, c-3, d-2
C) a-4, b-3 ,c-2, d-1 D) a-2, b-1, c-4, d-3

15. If $\vec{A}$ and $\vec{B}$ are two vectors acting at an angle 30^0 then find the agle between

a) $\vec{A}$ and $\vec{A}$ 1) 150^0

b) $\vec{A}$ and - $\vec{B}$ 2) 30^0

c) $\vec{A}$ and $2\vec{B}$ 3) 180^0

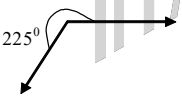
d) $\vec{A}$ and $-2\vec{A}$ 4) 0^0

A) a-1, b-2, c-3, d-4 B) a-4 , b-1, c-2, d-3 C) a-1, b-4, c-3, d-2 D) a-4, b-2, c-1, d-3

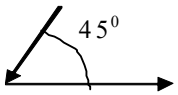
IV) Comprehention type:

- *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

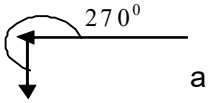
16. In order to find the angle between two vectors, they must have common starting point or common end point. The lesser of the two angles between them is adopted as angle between the two vectors $0^0 \le \theta \le 180^0$

i) 

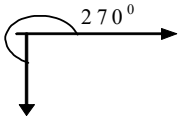
a) 10^0 b)135^0 c) 70^0 d) 120^0

ii) 

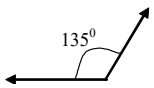
a) 20^0 b)80^0 c) 135^0 d) 30^0

iii) 

a) 40^0 b)80^0 c) 70^0 d) 90^0

iv) 

a) 80^0 b)90^0 c) 70^0 d) 20^0

v) 

a) 135^0 b)80^0 c) 70^0 d) 30^0

17. A vector has components 3 units along positive X direction and 4 units along positive Y direction.

i) Express the vector as sum of rectangular component vectors.

A) 3i + 4j B) 3i-4j C) i + j D) 4i - 3j

ii) What is the magnitude of the vector?

A) 5 unis B) 3 units C) 2 units D) zero

18. A vector is represented as $3\hat{i}+4\hat{j}+5\hat{k}$.

i) Find length of vector in xy plane

A) 5 units B) $\sqrt{41}$ units C) $\sqrt{34}$ units D) $5\sqrt{2}$ units

ii) Find length of the vector in xz plane

A) 5 units B) $\sqrt{41}$ units C) $\sqrt{34}$ units D) $5\sqrt{2}$ units

iii) Find length of the vector in yz plane

A) 5 units B) $\sqrt{41}$ units C) $\sqrt{34}$ units D) $5\sqrt{2}$ units

iv) Find length of the vector

A) 5 units B) $\sqrt{41}$ units C) $\sqrt{34}$ units D) $5\sqrt{2}$ units

KEY

ΦΦ LEARNER'S TASK :

☐ **BEGINNERS :**

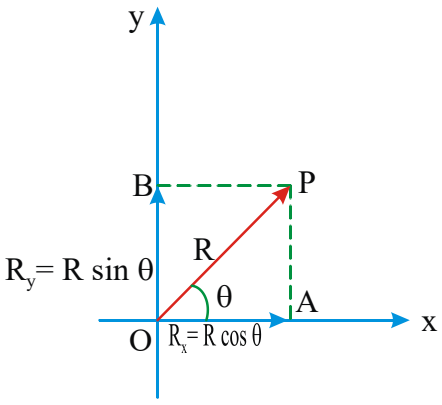
1) 2, 2) 3, 3) 4, 4) 3, 5) 1, 6) 1, 7) 3, 8) 3, 9) 4, 10) 4, 11) 2, 12) 2, 13) 3, 14) 1, 15) 2 16) 2, 17) 1, 18) 3

☐ **ACHIEVERS :** *1)* a) 0^0, b) 0^0, c) 180-θ, d) 180-θ, e) 0^0, f) θ, g) θ, h) 180-θ i) θ, j) θ, k) θ, *2)* $\sqrt{6}$ *3)* $5\sqrt{2}$ *4)*(3i-4j)/5 *5)*(3i+4j)/5 *6)* $\sqrt{14}$ *7)*zero *8)*5 m *9)*13 m *10)*26 km

☐ **EXPLORERS :** 1) A, 2) A, 3) B, 4) D, 5) vector, 6) scalar, 7) 0^0, 8) 180^0, 9) 3i-12j, 10) 0.6, 11) 180^0, 13) C, 14) B, 15) B, 16) i) b, ii) c, iii) d, iv) b, v) a, 17) i) A, ii) A, 18) i) A, ii) C, iii) B, iv) D.

§§ Resolution of a Vector into Components in two Dimensions

A vector $(\vec{R})$ can be resolved into two mutually perpendicular components R_x and R_y in a plane say x - y



The projection of $(\vec{R})$ along x-axis is called horizontal component (R_x) $R_x = R\cos\theta$

The projection of $(\vec{R})$ along y-axis is called vertical component (R_y) $R_y = R\sin\theta$

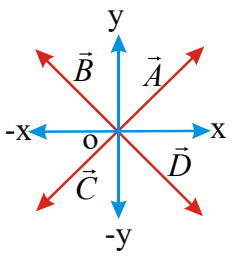
Component of a vector is a scalar quantity.

Magnitude of the resultant $|\vec{R}| = \sqrt{R_x^2 + R_y^2}$

Direction of the resultant with x-axis is

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Note :



a) If the vector $\vec{A}$ is in first quadrant then it can be written as $\vec{A} = A_x\hat{i} + A_y\hat{j}$

b) If the vector $\vec{B}$ is in second quadrant then

$$\vec{B} = -B_x\hat{i} + B_y\hat{j}$$

c) If the vector $\vec{C}$ is in third quadrant then

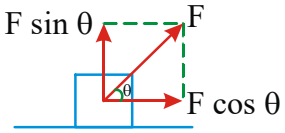
$$\vec{C} = -C_x\hat{i} - C_y\hat{j}$$

d) If the vector $\vec{D}$ is in fourth quadrant then

$$\vec{D} = D_x\hat{i} - D_y\hat{j}$$

§§ Applications on resolution of vector :

1)

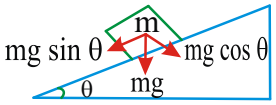


A book is placed on smooth horizontal surface and pulled by a force $\vec{F}$ making an angle `θ' with horizontal.

Component of force along horizontal =F$\cos\theta$.

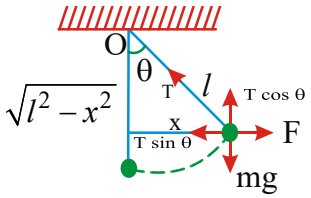
Component of force along vertical =F$\sin\theta$.

2)



A block of mass `m' is placed on an inclined plane of angle `θ'then the component of weight parallel to the inclined plane is `mg $\sin\theta$' , the component of weight perpendicular to the inclined plane is mg$\cos\theta$.

3)



A simple pendulum having a bob of mass `m' is suspended from a rigid support and it is pulled by a horizontal force `F' . The string makes an angle θ with the vertical as shown in figure.

The horizontal component of tension = T sin θ

The vertical component of tension = T cos θ

when the bob is in equilibrium

T sin θ =F .....(1)

T cos θ = mg .....(2)

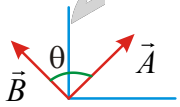
$$T=\frac{mg}{\cos\theta}=\frac{mgl}{\sqrt{l^2-x^2}}$$

From equation (1) and (2)

$$Tan\theta=\frac{F}{mg}\Rightarrow F=mgTan\theta=mg\frac{x}{\sqrt{l^2-x^2}}$$

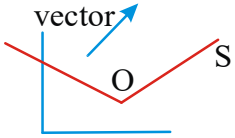
$$T=\sqrt{F^2+(mg)^2}$$

Note: If a vector is rotated through an angle other then integral multiple of 2π (or 360^0) its direction changes, but magnitude does not change. so vector changes.



$$\vec{A}\neq\vec{B}$$

Note : If the frame of reference is rotated the vector does not change (though its components may change).



EXAMPLE

√ **Example-1:**

The components of a vector along the x and y directions are (n+1) and 1 respectively. If the coordinate system is rotated by an angle 60^0 then the components changes to n and 3. Find the value of n.

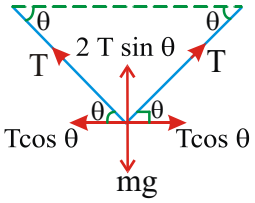
Sol. Length of the vector does not change on rotation.

$$\sqrt{(n+1)^2+1^2}=\sqrt{n^2+3^2} \Rightarrow n=\frac{7}{2}=3.5$$

√ **Example-2:**

A weight mg is suspended from the middle of a rope whose ends are rigidly clamped at the same level.The rope is no longer horizontal.What is the minimum tension required to completely straighten the rope

Sol. From the diagram



$$2T\sin\theta=mg \Rightarrow T=\frac{mg}{2\sin\theta}$$

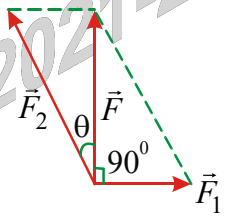
The rope will be straight when $\theta=0^0$

$$T=\frac{mg}{2\sin 0^0}=\infty$$

The tension required to completely straighten the rope is infinity.

√ **Example-3:**

The sum of magnitudes of two forces acting at a point is 16 N.If their resultant is normal to the smaller force and has a magnitude of 8N.Then the forces are



Sol. let $\vec{F}$ be the resultant of two forces $\vec{F_1}$ *and* $\vec{F_2}$ as shown in figure with $F_2 > F_1$

$F_2\sin\theta=F_1$ ...(i) $F_2\cos\theta=F=8$ ...(ii)

Squaring and adding Eqs (i) and (ii), we get

$F_2^2=F_1^2+64$ .....(iii)

Given $F_1+F_2=16$ .....(iv)

Solving Eqs. (iii) and (iv), we get

$F_1=6N$ and $F_2=10N$.

Resolution in 3D Space

A point in space can be specified by a position vector $\vec{R}=R_x\hat{i}+R_y\hat{j}+R_z\hat{k}$ where R_x, R_y *and* R_z being the coordinates of the point in Cartesian coordinate system.

Magnitude of position vector $\vec{R}$ is

$$R=\sqrt{R_x^2+R_y^2+R_z^2}$$

§§ **Laws of vector addition:**

Vector addition follows commutative,associative and distributive laws.

a) Commutative law : $\overline{A} + \overline{B} = \overline{B} + \overline{A}$

b) Associative law : $\overline{A} + (\overline{B} + \overline{C}) = (\overline{A} + \overline{B}) + \overline{C}$

c) Distributive law : m($\overline{A} + \overline{B}$)=m $\overline{A}$ +m $\overline{B}$ where m is a scalar

d) Vector subtraction doesnot follows commutative law and association.

e) Vector subtraction follows distributive law.

Resultant is a single vector that gives the total effect of number of vectors. Resultant can be found by using

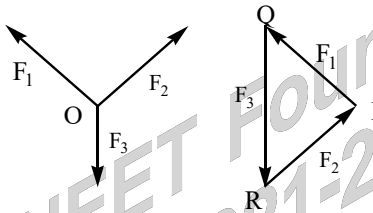
a) Triangle law of vectors

b) Polygon law of vectors

c) Parallelogram law of vectors

§§ **Triangle law of vector addition:**

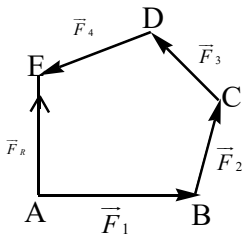
When three forces acting at a point can keep a particle in equilibrium the three forces can be represented as the sides of a triangle taken in order both in magnitude and direction.



Suppose three forece $\vec{F}_1, \vec{F}_2, \vec{F}_3$ are simultaneously acting at point O and the point is in equilibrium. Then the three forces can be represented as three sides of a triangle.The triangle PQR is constructed by drawing parallel lines to the directions in which he forces are applied. The magnitudes of the forces and the corresponding sides of the triangle have equal ratio i.e., $\frac{F_1}{PQ} = \frac{F_2}{PR} = \frac{F_3}{QR}$.

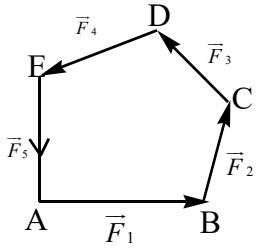
§§ **Polygon law of vector addition:**

If a number of vectors are represented by the sides of a polygon both in magnitude and direction taken in order, their resultant is represented by the closing side of the polygon taken in reverse order in magnitude and direction .



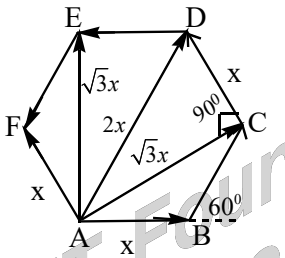
(or)

If a number of coplanar forces are acting simultaneously at a point keep the particle in equilibrium, these forces can be represented as the sides of a polygon taken in order both in magnitude and direction.



$$\therefore \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 = 0$$

Note : If x is the side of a hexagon ABCDEF as shown in figure



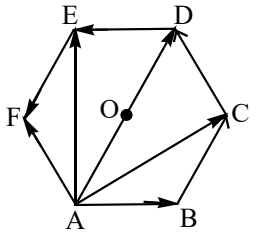
AB = x, AC = $\sqrt{3}$**x** , AD = 2x, AE = $\sqrt{3}$**x** , AF= x

EXAMPLE

√ **Example-4:**

ABCDEF is a regular hexagon with point O as centre. Find the value of

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$.



Sol. From the diagram $\left(\overrightarrow{AB} = -\overrightarrow{DE}\right)\left(\overrightarrow{BC} = -\overrightarrow{EF}\right)$

$$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = \cancel{\overrightarrow{AB}} + \left(\cancel{\overrightarrow{AB}} + \cancel{\overrightarrow{BC}}\right) + \overrightarrow{AD} + \left(\overrightarrow{AD} + \cancel{\overrightarrow{DE}}\right) + \left(\overrightarrow{AD} + \cancel{\overrightarrow{DE}} + \cancel{\overrightarrow{EF}}\right)$$

$$= 3\overrightarrow{AD} = 3\left(2\overrightarrow{AO}\right) = 6\left(\overrightarrow{AO}\right)$$

Note:

1. If n forcs each of same magnitude are acting at a point with the angle 360/n between them then the resultant is zero.
2. If n-1 forces of equal magnitude are acting at a point such that each vector makes an angle 360/n with the preceding one then the magnitude of resultant force is equal to the magnitude of force acting.

√ **Example 5 :**

Find the resultant of the following $\overline{F_1} = 5\bar{i} - 10\bar{i},\ \overline{F_2} = 15\bar{j} - 10\bar{j}$

Solution : $\overline{R} = \overline{F_1} + \overline{F_2}$

$\overline{R} = 5\bar{i} - 10\bar{i} + 15\bar{j} - 10\bar{j} \quad \overline{R} = -5\bar{i} + 15\bar{j}$

$\left|\overline{R}\right| = \sqrt{(5)^2 + (5)^2}$

$\left|\overline{R}\right| = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}N$

√ **Example 6:**

Find the unit vector along the resultant of $A = \bar{i} + 2\bar{j}$ *and* $B = 2\bar{i} + \bar{j}$

$\text{Resultant}\left(\overline{R}\right) = A + B = \bar{i} + 2\bar{j} + 2\bar{i} + \bar{j} = 3\bar{i} + 3\bar{j}$

Solution: $= \frac{\overline{R}}{R} = \frac{3\bar{i} + 3\bar{j}}{\sqrt{3^2 + 3^2}} = \frac{3\bar{i} + 3\bar{j}}{3\sqrt{2}} = \left(\frac{1}{\sqrt{2}}\right)\bar{i} + \left(\frac{1}{\sqrt{2}}\right)\bar{j}$

√ **Example 7:**

Find the unit vector along the resultant of the vectors $\bar{a} = \bar{i} - \bar{j},\ \bar{b} = \bar{i} + 2\bar{j},\ \bar{c} = 4\bar{i} - \bar{j}$

solution: $\overline{R} = a + b + c = \bar{i} - \bar{j} + \bar{i} + 2\bar{j} + 4\bar{i} - \bar{j} = 6\bar{i} \qquad \hat{\overline{R}} = \frac{\overline{R}}{R} = \frac{6\bar{i}}{\sqrt{6^2}} = \bar{i}$

√ **Example 8:**

Find the vector $\bar{a}$ if the resultant of $\bar{a}, \bar{b} = 4\bar{i} + 2\bar{j}$ and $\bar{c} = 2\bar{i} - \bar{j}$ is $\bar{0}$

solution: a+b+c=0

$\bar{a} + 4\bar{i} + 2\bar{j} + 2\bar{i} - \bar{j} = \bar{0} \Rightarrow \bar{a} = -6\bar{i} - \bar{j}$

√ **Example 9 :**

$\bar{a} = \bar{i} - 2\bar{j}, \bar{b} = 2\bar{i} + \bar{j}, \bar{c} = x\bar{i} + y\bar{j}$ find $\hat{\bar{c}}$ if the resultant of $\bar{a}, \bar{b}$ *and* $\bar{c}$ is $\bar{0}$

solution: $\bar{c} = -(\bar{a} + \bar{b}) \Rightarrow \quad \bar{c} = -\left(2\bar{i} + \bar{j} + \bar{i} - 2\bar{j}\right)$

$\Rightarrow \qquad \bar{c} = -3\bar{i} + \bar{j} \qquad \Rightarrow \hat{\bar{c}} = \frac{\bar{c}}{\left|\bar{c}\right|}$

$\hat{\bar{c}} = \frac{-3\bar{i} + \bar{j}}{\sqrt{3^2 + 1^2}} \Rightarrow \hat{\bar{c}} = \frac{-3\bar{i} + \bar{j}}{\sqrt{10}}$

TEACHING TASK

I) Choose the correct option:

1. Two vectors are given by $\vec{a} = 2\vec{i} + \vec{j} - 3\vec{k}$ and $\vec{b} = 5\vec{i} + 3\vec{j} - 2\vec{k}$. Find $\vec{c}$ such that $3\vec{a} + 2\vec{b} - \vec{c} = 0$

1) $-4\vec{i} + 9\vec{j} - 13\vec{k}$ 2) $4\vec{i} - 9\vec{j} - 13\vec{k}$ 3) $16\vec{i} + 9\vec{j} - 13\vec{k}$ 4) non of the above

2. If three forces $\vec{F_1} = -3\vec{i} - \vec{j} + 2\vec{k}$; $\vec{F_2} = -\vec{i} + 3\vec{j} + 4\vec{k}$ *and* $\vec{F_3} = 4\vec{i} - 2\vec{j} - 6\vec{k}$ act on a particle. Their resultant is

1) zero 2) $\vec{i} + \vec{j} + \vec{k}$ 3) $2\vec{i} - 3\vec{j} + \vec{k}$ 4) None

3. Three forces F_1= a(i–j+k)1 F_2= 2i-3j+4k and F_3= 8i-7j+6k act simultaneously on a particle. If the particle is in equilibrium, the value of a is

1) 10 2) -10 3) 8 4) 2

4. If $\bar{A} = 3\bar{i} + 4\bar{j}$ and $\bar{B} = 7\bar{i} + 24\bar{j}$, then the vector having the same magnitude as that of $\bar{B}$ and parallel to $\bar{A}$ is

1) $15\bar{i} + 20\bar{j}$ 2) $15\bar{i} - 20\bar{j}$ 3) $20\bar{i} + 15\bar{j}$ 4) $20\bar{i} - 15\bar{j}$

5. Two vectors $\bar{A}$ and $\bar{B}$ are related as $\bar{A} - 2\bar{B} = -3\left(\bar{A} + \bar{B}\right)$. If $\bar{A} = 6\bar{i} - 2\bar{k}$ then $\bar{B} =$

1) $-24\bar{i} + 8\bar{k}$ 2) $8\bar{i} - 24\bar{k}$ 3) $2\bar{k} - 6\bar{i}$ 4) $2\bar{k} + 6\bar{i}$

6. A vector A has a negative X- component 3 units in length and a positive Y component 2 units in length. What vector B when added to A gives a resultant vector with no component X and a negative Y component 4 units in length

1) $3\vec{i} - 6\vec{j}$ 2) $6\vec{i} - 3\vec{j}$ 3) $6\vec{i} + 6\vec{j}$ 4) $5\vec{i} + 6\vec{j}$

7. 6 forces each of magnitude 10N are acting at a point. The angle between successive vectors is 60^0 Find the resultant force

1) zero 2) 6 N 3) 10 N 4) 60 N

II) Multiple option type:

8. The vector $a\bar{i} + b\bar{j} + c\bar{k}$ and $2\bar{i} + 3\bar{j} + \bar{k}$ are parallel to each other. Which of the following relation is a valid one

a) a=2,b=1,c=3 b) a=4,b=6,c=2 c) a=8,b=12,c=4 d) a=8,b=10,c=4

1) only a, b 2) only b, c 3) only a, c 4) all a, b, c

[Hint : If two vectors $\vec{A} = a\bar{i} + b\bar{j} + c\bar{k}, \vec{B} = p\bar{i} + q\bar{j} + r\bar{k}$ are parallel vectors, then $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$]

9. Three vectors are $\bar{a} = 6\bar{i} - 7\bar{j}$, $\bar{b} = 3\bar{i} - 2\bar{j}$, $\bar{c} = 5\bar{i} - 8\bar{j}$. Then choose the correct

a) $\vec{a} + \vec{b} = 9\bar{i} - 9\bar{j}$ b) $\vec{a} - \vec{b} = 3\bar{i} + \bar{j}$ c) $\vec{a} + \vec{b} + \vec{c} = 14\bar{i} - 17\bar{j}$

1) only a, b 2) only b, c 3) only a, c 4) all a, b, c

III) Fill in the blanks:

10. Addition of two vectors gives ..... quantity

11. Vector addition obeys ..... law

12. Vector subtraction doesn't obey ..... law

13. Multiplication of a scalar with a vector gives .....

IV) Match the following:

- *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

  *If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

14. For any three vectors $\vec{A},\vec{B}\,\&\,\vec{C}$ acting at a point keeps it in equilibrium

a) Commutative law	1) $\vec{A}+\vec{B}+\vec{C}=0$
b) Associative law	2) $(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$
c) Distributive law	3) $\vec{A}+\vec{B}=\vec{B}+\vec{A}$
d) Triangle law	4) $2(\vec{A}+\vec{B})=2\vec{B}+2\vec{A}$

1) a-1, b-2, c-3, d-4 2) a-3, b-2, c-4, d-1

3) a-1, b-3, c-2, d-4 4) a-3, b-3, c-2, d-1

V) Comprehention type:

- *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

15. $\bar{A}=\bar{i}+2\bar{j};B=2\bar{i}+\bar{j};\bar{C}=\bar{i}-\bar{j}$

i) $\bar{A}+\bar{B}+\bar{C}$

1) $4\bar{i}+2\bar{j}$ 2) $4\bar{i}-2\bar{j}$ 3) $2\bar{i}-2\bar{j}$ 4) $\bar{i}-\bar{j}$

ii) $\left|\bar{A}+\bar{B}+\bar{C}\right|=$.....

1) 4 2) 2 3) $\sqrt{6}$ 4) $2\sqrt{5}$

iii) $(\bar{A}+\bar{B})-\bar{C}=$

1) $4\bar{i}+2\bar{j}$ 2) $4\bar{i}-2\bar{j}$ 3) $2\bar{i}+4\bar{j}$ 4) $\bar{i}-\bar{j}$

iv) $\bar{A}-(\bar{B}+\bar{C})=$

1) $4\bar{i}+2\bar{j}$ 2) $-2\bar{i}+2\bar{j}$ 3) $2\bar{i}+4\bar{j}$ 4) $\bar{i}-\bar{j}$

VI) Higher order thinking skills (HOTS)

16. If $\vec{A}=3\hat{i}-4\hat{j}$ and $\vec{B}=-\hat{i}-4\hat{j}$, calculate the direction of $\vec{A}+\vec{B}$

1. $\tan^{-1}(4)$ with positive $X-$ axis in clock wise

2. $\tan^{-1}(4)$ with negative $X-$axis in clock wise

3. $\tan^{-1}(4)$ with positive $X-$axis in anticlock wise

4. $\tan^{-1}(4)$ with negative $X-$ axis in anticlock wise

17. Two vectors are given by $\vec{a}=-2\hat{i}+\hat{j}-3\hat{k}$ and $\vec{b}=5\hat{i}+3\hat{j}-2\hat{k}$. If $3\vec{a}+2\vec{b}-\vec{c}=0$ then third vector $\vec{c}$ is

1. $4\hat{i}+9\hat{j}-13\hat{k}$ 2. $-4\hat{i}-9\hat{j}+13\hat{k}$

3. $4\hat{i}-9\hat{j}-13\hat{k}$ 4. $2\hat{i}-3\hat{j}+13\hat{k}$

18. The vector sum of two vectors of magnitudes 10 units and 15 units can never be

1) 28 units 2) 22 units 3) 18 units 4) 8 units

19. The car makes a displacement of 100 m towards east and then 200 m towards north. Find the magnitude and direction of the resultant.

1) 223.7m, $\tan^{-1}(2), N\ of\ E$

2) 223.7m, $\tan^{-1}(2), E\ of\ N$

3) 300m, $\tan^{-1}(2), N\ of\ E$

4) 100m, $\tan^{-1}(2), N\ of\ E$

20. If a vector has an x -component of -25.0units and a y- component of 40.0 units, then the magnitude and direction of this vector is

1) $5\sqrt{89}\ units; \sin^{-1}\frac{-5}{\sqrt{89}}$ with x-axis

2) $5\sqrt{89}\ units; \cos^{-1}\frac{-5}{\sqrt{89}}$ with x-axis

3) $45\ units; \cos^{-1}\frac{-5}{9}$ with x-axis

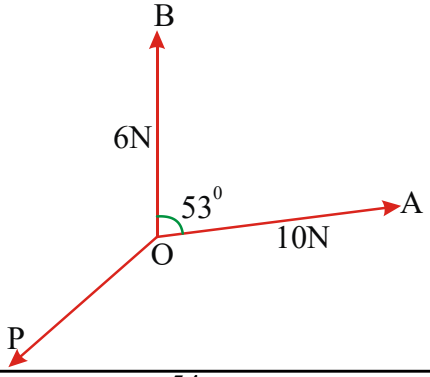
4) $45\ units; \sin^{-1}\frac{-5}{9}$ with x-axis

21. A force of 10N is resolved into the perpendicular components. If the first component makes 30^0 with the force, the magnitudes of the components are

1) 5N, 5N 2) $5\sqrt{2}$ N, 5 N

3) $5\sqrt{3}$ N, 5N 4) 10 N, $10\sqrt{3}$ N

22. If the system is in equilibrium (cos 53^0 = 3/5), then the value of 'P' is



1) 16N 2) 4N 3) $\sqrt{208}$N 4) $\sqrt{232}$N

23. Two billiard balls are moving on a table and the component velocities along the length and breadth are 5,5 ms^{-1} for one ball $2\sqrt{3}$, 2ms^{-1} for the other ball the angle between the motion of balls is

1) 30^0 2)60^0 3)40^0 4) 15^0

24. If $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, its components in $yz-$plane and $zx-$plane are respectively

1. $\sqrt{13}$ and 5 2. 5 and $2\sqrt{5}$

3. $2\sqrt{5}$ and $\sqrt{13}$ 4. $\sqrt{13}$ and $\sqrt{29}$

25. A car weighing 100kg is on a slope that makes an angle 30^0 with the horizontal. The component of car's weight parallel to the slope is $\left(g = 10ms^{-2}\right)$

1) 500N 2) 1000N 3) 15,000N 4) 20,000N

26. A room has dimensions $3m \times 4m \times 5m.$ A fly starting at one corner ends up at the diametrically opposite corner. The magnitude of the displacement of the fly is

1) 12m 2) 60 m 3) $2\sqrt{5}m$ 4) $5\sqrt{2}m$

KEY

ΦΦ TEACHING TASK :

1) 3, 2) 1, 3) 2, 4) 1, 5) 1, 6) 1, 7) 1, 8) B, 9) C,
10) vector, 11) commutative, 12) commutative, 13) vector,
14) 2, 15) i) 1, ii) 4, iii) 3, iv) 2, 16) 1, 17) 1, 18) 1 ,
19) 1, 20)2, 21)3, 22)3, 23)4, 24)2, 25)1, 26)4

LEARNER'S TASK

BEGINNERS (Level - I)

I) Choose the correct option:

1. The unit vector parallel to the resultant of vectors $\bar{A} = 4\bar{i} + 3\bar{j} + 6\bar{k}$ and $\bar{B} = -\bar{i} + 3\bar{j} - 8\bar{k}$ is

1) $\frac{1}{7}(3\bar{i} + 6\bar{j} - 2\bar{k})$ 2) $-\frac{1}{7}(3\bar{i} + 6\bar{j} - 2\bar{k})$ 3) $\frac{1}{49}(3\bar{i} + 6\bar{j} - 2\bar{k})$ 4) $\frac{1}{47}(3\bar{i} + 6\bar{j} - 2\bar{k})$

2. The unit vector in the direction of resultant of two vectors with components (2, 4, -5) and (1, 2, 3) is

1) $\frac{3\bar{i} + 6\bar{j} + 2\bar{k}}{49}$ 2) $\frac{3\bar{i} + 6\bar{j} + 2\bar{k}}{7}$ 3) $\frac{3\bar{i}}{7} + \frac{6\bar{j}}{7} - \frac{2\bar{k}}{7}$ 4) $\frac{3\bar{i} + 6\bar{j} - 2\bar{k}}{49}$

3. If $\bar{A} = 3\bar{i} + 5\bar{j} - 2k$, and vector $\bar{B} = -3\bar{j} + 6\bar{k}$. Find a vector $\bar{C}$ such that $2\bar{A} + 7\bar{B} + 4\bar{C} = 0$

1) $-6\bar{i} + 11\bar{j} - 38\bar{k}$ 2) $11\bar{j} - 4\bar{k}$ 3) $-1.5\bar{i} + 2.75\bar{j} - 9.5\bar{k}$ 4) $4\bar{j} - 6\bar{k}$

4. Given the vectors $\vec{a} = 3\vec{i} - 4\vec{j} - 2\vec{k}$ and $\vec{b} = 2\vec{i} + 4\vec{j} - 5\vec{k}$. Then $\vec{a} + \vec{b}$ is

1) $7\vec{i}-5\vec{k}$ 2) $5\vec{i}-7\vec{k}$ 3) $6\vec{i}-16\vec{j}+10\vec{k}$ 4) none of the above

5. If $\vec{a}=\vec{i}-2\vec{j}+3\vec{k}$; $\vec{b}=4\vec{i}+5\vec{j}-6\vec{k}$ and $\vec{c}=-7\vec{i}-8\vec{j}+9\vec{k}$ then $\left|\vec{a}+\vec{b}+\vec{c}\right|$ is

1) $\sqrt{140}$ 2) $\sqrt{150}$ 3) $\sqrt{160}$ 4) $\sqrt{65}$

6. Three forces are given $\vec{F_1}=6\vec{i}$ newton, $\vec{F_2}=9\vec{j}$ newton and $\vec{F_3}=(-3\vec{i}+4\vec{j})$ newton. Find the force $\vec{F}$ that must be added these forces to make resultant force zero.

1) 3i+13j 2) $-3\vec{i}-13\vec{j}$ 3) $\vec{i}-\vec{j}$ 4) none

7. Find the resulant $\left|\vec{F}\right|$ of the forces $\vec{F_1}=2\vec{i}+3\vec{j}-5\vec{k}, \vec{F_2}=\vec{i}-\vec{j}+\vec{k}$ *and* $\vec{F_3}=3\vec{i}+4\vec{j}-2\vec{k}$ act on a particle

1) 6 2) $6\sqrt{3}$ 3) $7\sqrt{3}$ 4) $8\sqrt{3}$

8. If three forces $\vec{F_1}=7\vec{i}+2\vec{j}-3\vec{k}$ and $\vec{F_2}=-2\vec{i}+2\vec{j}-6\vec{k}$ and $\vec{F_3}$ keep a particle in equilibrium. Then $\left|\vec{F_3}\right|$ =

1) $\sqrt{120}$ 2) $\sqrt{122}$ 3) $\sqrt{125}$ 4) none

9. Five equal forces each of 20N are acting at a point in the same plane. If the angles between them are same, the resultant of these forces is

1) 0 2) 40N 3) 20N 4) $20\sqrt{2}$

10. 4 forces each of magnitude 10N are acting at a point. The angle between successive vectors is 90^0. Find the resultant force

1) 0 2) 10 N 3) $10\sqrt{2}$ 4) none

11. Vector $\overline{A}$ has an x and y components of -8.70 cm and 15.0 cm respectively. $\overline{B}$ vector has an x and y components of 13.2cm and -6.60cm respectively. If $\overline{A}-\overline{B}+3\overline{C}=0$ then x and y components of $\overline{C}$

1) -7.20cm and 7.30cm 2) 7.30cm and -7.20cm
3) -7.30cm and 7.20cm 4) -7.30cm and -7.20cm

12. Find the vector and its magnitude with initial point P (2, 4, 6) and terminal point Q (1, 2, 3)

1) $-\vec{i}-2\vec{j}-3\vec{k};\sqrt{14}$ 2) $\vec{i}+2\vec{j}+3\vec{k};\sqrt{14}$ 3) $3\vec{i}+2\vec{j}-\vec{k};\sqrt{14}$ 4) none

13. A vector has co-ordinates (1,1,2) for initial point and (2,1,3) as the terminal point. The length of the vector is

1) $\sqrt{2}$ 2) $\sqrt{34}$ 3) 2 4) $\sqrt{6}$

14. If $\vec{A}=3\hat{i}-4\hat{j}$ and $\vec{B}=-\hat{i}-4\hat{j}$, calculate the direction of $\vec{A}-\vec{B}$.

1) along positive x- axis
2) along negative x-axis
3) along positive y- axis
4) along negative y -axis

15. The resultant of the forces $\vec{F_1}=4\hat{i}-3\hat{j}$ *and* $\vec{F_2}=6\hat{i}+8\hat{j}$ is

1) $5\sqrt{5}$ 2) $10\hat{i}-5\hat{j}$ 3) 125 4) $-2\hat{i}-3\hat{j}$

16. The vector sum of two vectors of magnitudes 10 units and 15 units can never be
1) 20 units 2) 22 units 3) 18 units 4) 3 units

17. A car moves 40m due east and turns towards north and moves 30m then turns 45^0 east of north and moves $20\sqrt{2}m.$ The net displacement of car is (east is taken positive x - axis, North as positive y - axis)

1) $50\hat{i}+60\hat{j}$ 2) $60\hat{i}+50\hat{j}$ 3) $30\hat{i}+40\hat{j}$ 4) $40\hat{i}+30\hat{j}$

18. A bird moves in such a way that it has a displacement of 12 m towards east, 5 m towards north and 9 m vertically upwards. Find the magnitude of its displacement

1) $5\sqrt{2}m$ 2) $5\sqrt{10}m$ 3) $5\sqrt{5}m$ 4) 5m

19. An aeroplane is heading north east at a speed of 141.4 ms^{-1}. The northward component of its velocity is

1) 141.4 ms^{-1} 2) 100 ms^{-1} 3) zero 4) 50 ms^{-1}

20. The unit vector parallel to the resultant of the vectors $\vec{A}=4\hat{i}+3\hat{j}+6\hat{k}$ and $\vec{B}=-\hat{i}+3\hat{j}-8\hat{k}$ is

1) $\frac{1}{7}\left(3\hat{i}+6\hat{j}-2\hat{k}\right)$ 2) $\frac{1}{7}\left(3\hat{i}+6\hat{j}+2\hat{k}\right)$ 3) $\frac{1}{49}\left(3\hat{i}+6\hat{j}-2\hat{k}\right)$ 4) $\frac{1}{49}\left(3\hat{i}-6\hat{j}+2\hat{k}\right)$

21. The vector parallel to $4\hat{i}-3\hat{j}+5\hat{k}$ and whose length is the arithmetic mean of lengths of two vectors $2\hat{i}-4\hat{j}+4\hat{k}$ and $\hat{i}+\sqrt{6}\hat{j}+3\hat{k}$ is

1) $4\hat{i}-3\hat{j}+5\hat{k}$ 2) $(4\hat{i}-3\hat{j}+5\hat{k})/\sqrt{3}$ 3) $(4\hat{i}-3\hat{j}+5\hat{k})/\sqrt{2}$ 4) $(4\hat{i}-3\hat{j}+5\hat{k})/\sqrt{5}$

22. Given two vectors $\vec{A}=\hat{i}-2\hat{j}-3\hat{k}$ and $\vec{B}=4\hat{i}-2\hat{j}+6\hat{k}$. The angle made by $\left(\vec{A}+\vec{B}\right)$ with the X - axis is

1) 30^0 2) 45^0 3) 60^0 4) 90^0

23. To go from town A to town B a plane must fly about 1780 km at an angle of 30^0 West of north. How far West of A is B ?

1) 1542km 2) 1452 km 3) 1254 km 4) 890 km

24. A vector $\hat{i}+\sqrt{3}\hat{j}$ rotates about its tail through an angle 60^0 in clockwise direction then the new vector is

1) $\hat{i}+\sqrt{3}\hat{j}$ 2) $3\hat{i}-4\hat{j}$ 3) $2\hat{j}$ 4) $2\hat{i}$

ACHIEVERS (Level - II)

I) Solve the following:

1. Find the unit vector along the resultant of $8\bar{i}-3\bar{j}$ *and* $4\bar{i}-2\bar{j}$

2. Find the value of x if the $0.4\hat{i}+x\hat{j}$ represents a unit vector

3. Find the unit vector along the direction $3\bar{i}-3\sqrt{3}\bar{j}$

4. Find the vector $\bar{a}$ if the resultant vector of $\bar{a}$, $\bar{b}=6\bar{i}-9\bar{j}$ *and* $\bar{c}=3\bar{i}+7\bar{j}$ *is* $\bar{0}$

5. $\bar{a} = 4\bar{i} + 3\bar{j}$, $\bar{b} = -5\bar{i} + 2\bar{j}$, $\bar{c} = x\bar{i} + y\bar{j}$ find $\hat{c}$ if the resultant of $\bar{a}, \bar{b}, and\ \bar{c}\ is\ \bar{0}$

6. Find the unit vector along the resultant of the vectors $\bar{a} = 6\bar{i} - 7\bar{j}$, $\bar{b} = 3\bar{i} - 2\bar{j}$, $\bar{c} = 5\bar{i} - 8\bar{j}$

7. The lengths of the vectors $8\bar{i} + 6\bar{j}$ and $5\sqrt{2}\,\bar{i} + a\bar{j}$ are equal Find the value of 'a'

8. If $a\bar{i} + 2\bar{j} + c\bar{k}$ & $d\bar{i} + 4\bar{j} - e\bar{k}$ are parallel and $\frac{d}{c} = k\frac{a}{e}$ what is the value of k?

9. Three forces $\bar{A} = \left\{\hat{i} + \hat{j} + \hat{k}\right\}$, $\bar{B} = \left\{2\hat{i} - \hat{j} + 3\hat{k}\right\}$ and $\bar{C}$ acting on a body.To keep it in equilibrium value of $\bar{C}$ is

10. The vectors $\bar{a} = 2\hat{i} + \hat{j} + \hat{k}$ & $\bar{b} = -4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\bar{c}$ are represented as the adjacent sides of a triangle taken in order.Then find $\bar{c}$.

EXPLORERS (Level - III)

I) Multiple option type:

- *This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options*

1. If the vectors $\vec{A} = a\bar{i} + b\bar{j} + 1.5\bar{k}, and\ \vec{B} = 2\bar{i} + \bar{j} + 3\bar{k}$ are parallel to each other, then

a) the values of a 1 b) the value of b is 0.5 c) both are unit vectors

A) only a, b B) only b, c C) only a, c D) all a, b, c

2. Choose the correct

a) If three vectors can be represented by the adjacent sides of a triangle taken in order, then their resultant is zero

b) If 4 vectors acting at a point can be represented by the adjacent sides of a quadrilateral then their resultant is zero

c) If n vectors acting at point can be represented as the sides of a polygon taken in order then their resultant is zero.

A) only a, b B) only b, c C) only a, c D) all a, b, c

II) Fill in the blanks:

3. Sum of three vectors is zero. Then three vectors can be represented as sides of ....., takn in order

4. When three forces are acting at a same point and it is under balenced condition . so that resultant force is.....

5. Time is a ..... quantity.

6. If several vectors acting at a point can be represented both in direction and magnitude as the side of a polygon , thire resultant is represented by the .....

7. If a particle possesses three velocities simultaneously, represented by the three sides of a triangle taken in order, then what is velocity of the particle is .....

III) Match the following:

- *This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column–I** have*

*to be matched with statements (p, q, r, s) in **Column–II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.*

*If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:*

8. a) Addition of vectors — 1) gives another parallel vector

b) subtraction of vectors — 2) anti commutative

c) multiplication of vector by a scalar — 3) obeys commutative

d) division of vector with another vector — 4) not possible

A) a-3, b-2, c-1, d-4 B) a-2, b-3, c-1, d-4

C) a-1, b-2, c-3, d-4 D) a-4, b-3, c-2, d-1

9.

Vectors	**Resultant**
a) $F_1 = 3\vec{i}$, $F_2 = 5\vec{j}$, $F_3 = -4\vec{k}$	1) zero
b) $\vec{A} = 3\vec{i} + 7\vec{j}$, $\vec{B} = -7\vec{i} - 3\vec{j}, \vec{C} = \vec{i}$	2) $\vec{i}$
c) three forces are in equilibrium	3) $5\vec{i} + 2\vec{j}$
d) $\vec{A} = 7\vec{i} + 2\vec{j}, \vec{B} = 5\vec{i} - 7\vec{k}$	4) $3\vec{i} + 5\vec{j} - 4\vec{k}$

A) a - 1, b - , c - 3, d - 4 B) a - 4, b - 3, c -1, d - 2

C) a - 1, b - 4, c - 3, d - 2 D) a - 4, b - 2, c - 1, d - 3

IV) Comprehention type:

- *This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE i**s correct. Choose the correct option.*

10. There is a sense of time dustigushin past, present and future telling us about the time that passed, that is going on and the time going to come. Time is the fourth dimention in which events can be ordered from the past through present into future.

i) time is the property of universe which has

A) both magnitude and direction B) only magnitude and no direction

C) only direction and no magnitude D) none

ii) time is a

A) vector quantity B) scalar quantity C)some time vector D) none

11. If $\vec{A} = 4\vec{i} - 3\vec{j}$ and $\vec{B} = 8\vec{i} - 6\vec{k}$ are two vectors , so that

i) Find the magnitude of $\vec{A}$

A) 4 B) 12 C)5 D) -3

ii) Find the magnitude of $\vec{B}$

A) 8 B) 10 C)-6 D) 2

iii) Find the magnitude of their resultant

A) 15 B) 12 C)-9 D) 8

12. Two vectors are given by $\vec{A} = 3\vec{i} - 2\vec{j}$ and $\vec{B} = -\vec{i} - 4\vec{j}$.

i) Find $\left|\vec{A} + \vec{B}\right|$

A) $\sqrt{20}$ B) $\sqrt{40}$ C) $\sqrt{10}$ D) none

ii) Find $\left|\vec{A}-\vec{B}\right|$

A) $\sqrt{40}$ B) $\sqrt{20}$ C) $\sqrt{10}$ D) none

RESEAR CHERS (Level - IV)

I) Choose the correct option:

1. If $0.5\bar{i}+0.8\bar{j}+C\bar{k}$ is a unit vector, then C is (1994E)

1) $\sqrt{89}$ 2) 0.2 3) 0.3 4) $\sqrt{0.11}$

2. If the magnitudes of $\bar{A}$ and $\bar{B}$ are a and b respectively, the magnitude of the resultant vector is always (1993E)

1) equal to $(a+b)$ 2) less than $(a+b)$

3) greater than $(a+b)$ 4)not greater than $(a+b)$

3. If $\bar{A}=3\bar{i}+4\bar{j}$ and $\bar{B}=7\bar{i}+24\bar{j}$, then the vector having the same magnitude as that of $\bar{B}$ and parallel to $\bar{A}$ is (1989E)

1) $15\bar{i}+20\bar{j}$ 2) $15\bar{i}-20\bar{j}$ 3) $20\bar{i}+15\bar{j}$ 4) $20\bar{i}-15\bar{j}$

4. Three forces $\vec{A}=\{\hat{i}+\hat{j}+\hat{k}\}$, $\vec{B}=\{2\hat{i}-\hat{j}-3\hat{k}\}$ and $\vec{C}$ acting on a body to keep it in equilibrium is (2008M)

1) $-\{3\hat{i}+4\hat{k}\}$ 2) $-\{4\hat{i}+3\hat{k}\}$ 3) $3\hat{i}+4\hat{j}$ 4) $2\hat{i}-3\hat{k}$

5. Of the vectors given below, the parallel vectors are, (2006M)

$\vec{A}=6\hat{i}+8\hat{j}$ $\vec{B}=210\hat{i}+280\hat{k}$ $\vec{C}=5.1\hat{i}+6.8\hat{j}$ $\vec{D}=3.6\hat{i}+8\hat{J}+48\hat{k}$

1) $\vec{A}$ and $\vec{B}$ 2) $\vec{A}$ and $\vec{C}$ 3) $\vec{A}$ and $\vec{D}$ 4) $\vec{C}$ and $\vec{D}$

6. The unit vector parallel to the resultant of vectors $\bar{A}=4\bar{i}+3\bar{j}+6\bar{k}$ and $\bar{B}=-\bar{i}+3\bar{j}-8\bar{k}$ is (2000M)

1) $\frac{1}{7}(3\bar{i}+6\bar{j}-2\bar{k})$ 2) $-\frac{1}{7}(3\bar{i}+6\bar{j}-2\bar{k})$ 3) $\frac{1}{49}(3\bar{i}+6\bar{j}-2\bar{k})$ 4) $\frac{1}{47}(3\bar{i}+6\bar{j}-2\bar{k})$

II) Additional worksheet for practice :

1. A car travels 60km towards north and then 80km towards west. Find the displacement of the car.

2. Find the magnitude of the vector $3\bar{i}+4\bar{j}$

3. A vector has rectangular components 5 units along +ve X-axis and 3 units along -ve Y-axis

a) express the vector as the sum of rectangular components vectors

b) the modulus of the vector is ....

4. The magnitude of the vector $2\bar{i} + k\bar{j}$ is 3 units. Find the value of k.

5. Two vectors $\bar{a} = 2\hat{i} + \hat{j} - \hat{k}$ & $\bar{b} = 3\hat{i} + 2\hat{j} + \hat{k}$ are represented as the adjacent sides of a triangle taken in order. Write the closing side taken in reverse order

6. 11 forces each of magnitude 5N are acting on a particle simultaneously. If each force makes an angle 30^0 with the previous one, find the resultant of all the forces.

7. 6 forces each of magnitude 10N are acting at a point. The angle between successive vectors is 60^0 Find the resultant force

8. 3 forces each of magnitude 5N are acting on a particle simultaneously. If each force makes an angle 90^0 with the previous one, find the resultant of all the forces Ans:

9. 4 forces each of magnitude 10N are acting at a point. The angle between successive vectors is 90^0. Find the resultant force

10. The unit vector along the resultant of $\bar{A} = \bar{i} + \bar{j} + \bar{k}$ and $\bar{B} = 2\bar{i} + 3\bar{j} + \bar{k}$ is ....

11. The unit vector parallel to the resultant of vectors $\bar{A} = 4\bar{i} + 3\bar{j} + 6\bar{k}$ and $\bar{B} = -\bar{i} + 3\bar{j} - 8\bar{j}$ is....

12. The minimum number of equal forces in a plane that can keep a particle in equilibrium is

KEY

ΦΦ LEARNER'S TASK :

☐ **BEGINNERS :**

1) 2, 2) 1, 3) 1, 4) 2, 5) 3, 6) 2, 7) 2, 8) 2, 9) 1, 10) 2,
11) 2, 12) 1, 13) 1 14) 1 15) 1 16) 4 17) 2 18) 2 19) 2 20) 1
21) 3 22) 2 23)4 24)4

☐ **ACHIEVERS :** 1)(12i-5j)/13 2) $\sqrt{0.84}$ 3)(3i-3$\sqrt{3}$ j)/6 4)-9i+2j

5)(i-5j)/$\sqrt{26}$ 6)(14i -17j)/$\sqrt{485}$ 7)50 9)-$\left\{3\hat{i} + 4\hat{k}\right\}$ 10)-2i+-j+4k

☐ **EXPLORERS** :1) A, 2) D, 3) Triangle, 4) zero, 5) scalar, 6) closing side, 7) zero, 8) A, 9) D, 10) i) B, ii) B, 11) i) C, ii) B, iii) A, 12) i) B, ii) B

☐ **RESEARCHERS** :**I)** 1) 4, 2) 4, 3) 2, 4) 1, 5) 2, 6) 1 **II)** 1)100 km 2)5
3)a:3i+4j , b: 5 4) $\sqrt{5}$ 5)5i+3j 6)5N
7)zero 8)5N 9)zero 10)29
11) $(1/7)$ $(3\bar{i} + 6\bar{j} - 2\bar{k})$ 12)2