(1)

1. Differentiation

Task

① Given $y = x^{\frac{7}{2}}$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $n = \frac{7}{2}$

$\therefore \frac{dy}{dx} = \frac{d}{dx} [x^{\frac{7}{2}}]$

$\Rightarrow \frac{7}{2} x^{\frac{7}{2}-1} = \frac{7}{2} x^{\frac{5}{2}}$

② Given $a = x^{-3}$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $n = -3$

$\therefore \frac{da}{dx} = \frac{d}{dx} x^{-3}$

$= -3 (x^{-3-1})$

$= -3 x^{-4}$

③ Given $a = t$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $x = t$; $n = 1$

$\therefore \frac{da}{dt} = \frac{dt}{dt} = 1.$

④ Given $y = x^5 + x^3$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $n = 5$: $n = 3$.

$\therefore \frac{dy}{dx} = \frac{d}{dx} [x^5 + x^3]$

$\Rightarrow \frac{d}{dx} x^5 + \frac{d}{dx} x^3$

$\Rightarrow 5 x^{5-1} + 3 x^{3-1}$

$\Rightarrow 5x^4 + 3x^2$

⑤ Given

$P = 5t^4 + 6t^3 + 9t$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $n = 4, 3, 1$

$\therefore \frac{d}{dt} P = \frac{d}{dt} [5t^4 + 6t^3 + 9t]$

$\Rightarrow 5 \frac{d}{dt} t^4 + 6 \frac{d}{dt} t^3 + 9 \frac{d}{dt} t$

$\Rightarrow 5(4t^{4-1}) + 6(3t^{3-1}) + 9(1)$

$\Rightarrow 20t^3 + 18t^2 + 9$

⑥ Given $\int x^{15} dx$

We know that $\int x^n dx = \frac{x^{n+1}}{n+1}$

Here $n = 5$

$\int x^{15} dx = \frac{x^{15+1}}{15+1} + c$

$= \frac{x^{16}}{16} + c$

⑦ Given $\int x^{-\frac{3}{2}} dx$

we know that $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Here $n = -\frac{3}{2}$

$\int x^{-\frac{3}{2}} dx = \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}$

$\Rightarrow -2 x^{-\frac{1}{2}} + c$

⑧ Given $\int_0^{\frac{\pi}{2}} \sin x \, dx.$

we know that

$\int \sin x \, dx = -\cos x.$

$\Rightarrow \int_0^{\frac{\pi}{2}} \sin x \, dx = -[\cos x]_0^{\frac{\pi}{2}}$

$\Rightarrow -[\cos \frac{\pi}{2} - \cos 0]$

$= -[0 - 1]$

$= 1.$

⑨ Given $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$

we know that

$\int \cos x \, dx = \sin x$

$\Rightarrow \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx = [\sin x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$\Rightarrow [\sin \frac{\pi}{2} - \sin(-\frac{\pi}{2})]$

$\Rightarrow [1 - (-1)]$

$= 1 + 1 = 2$

②

⑩ Given $\int_0^4 x^{-\frac{1}{2}} dx$.

we know that $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

Here $n = -\frac{1}{2}$

$\Rightarrow \int_0^4 x^{-\frac{1}{2}} dx = \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_0^4$

$\Rightarrow 2 \left[x^{\frac{1}{2}} \right]_0^4$

$\Rightarrow 2 [4^{\frac{1}{2}} - 0^{\frac{1}{2}}]$

$\Rightarrow 2 [2 - 0]$

$\Rightarrow 2(2) = \underline{4}$.

⑪ For all options use $\frac{d}{dx} x^n = n x^{n-1}$

Ⓐ $n = 4$

$\frac{d}{dx} x^4 = 4 x^{4-1}$
$= 4x^3$

Ⓑ $n = 3$

$\frac{d}{dx} x^3 = 3x^{3-1}$
$= 3x^2$

Ⓒ $n = -5$

$\frac{d}{dy} y^{-5} = (-5 y^{-5-1})$
$= -5 y^{-6}$

⑬ Assertion $\frac{d}{dt} t^5 = 5 t^{5-1} = 5t^4$

is Based on $\frac{d}{dx} x^n = n x^{n-1}$

⑭ derivative of any constant is zero.

⑮ Given $y = \sqrt{x}$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $n = \frac{1}{2}$

$\therefore \frac{dy}{dx} = \frac{d}{dx} x^{1/2} = \frac{1}{2} x^{\frac{1}{2}-1}$

$= \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

at $x = 2$

$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{2}}$

⑯ Given $a = t^{-\frac{3}{2}}$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $n = -\frac{3}{2}$

$\frac{da}{dt} = \frac{d}{dt} [t^{-\frac{3}{2}}]$

$\Rightarrow -\frac{3}{2} t^{-\frac{3}{2}-1}$

$\Rightarrow -\frac{3}{2} t^{-\frac{5}{2}} = \cancel{\frac{-3}{2t}} \frac{-3}{2(t)^{\frac{5}{2}}}$

At $t = 4$

$\frac{da}{dt} = \cancel{\frac{-3}{2\sqrt{4}}} = \cancel{\frac{-3}{2\times 2}} = \cancel{\frac{-3}{4}}$

$= \frac{-3}{2(4)^{\frac{5}{2}}} = \frac{-3}{2(2^2)^{\frac{5}{2}}} = \frac{-3}{\underline{64}}$

⑰ $\frac{dy}{dx} \times \frac{da}{dt}$

substitute $\frac{dy}{dx}$ & $\frac{da}{dt}$ value from ⑮, ⑯

$\therefore \frac{dy}{dx} \times \frac{da}{dt} = \frac{1}{2\sqrt{x}} \times \frac{-3}{2} t^{-\frac{5}{2}}$

$= \frac{1}{4\sqrt{x}} \frac{-3}{t^{\frac{5}{2}}}$

$\Rightarrow \frac{-3}{4\sqrt{x}(\sqrt{t})^5}$

L Task

Single ans type :-

① Given $y = x^{15}$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $n = 15$

$\frac{d}{dx} x^{15} = 15 x^{15-1}$
$= 15 x^{14}$

② Given $a = t^{\frac{5}{2}}$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $n = \frac{5}{2}$

$\frac{da}{dx} = \frac{d}{dx} t^{\frac{5}{2}} \Rightarrow \frac{5}{2} t^{\frac{5}{2}-1}$

$= \frac{5}{2} t^{\frac{3}{2}}$

③ Given $y = 5t^2 + 3t$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $n = 2$ $\quad n = 1$

$\frac{dy}{dt} = \frac{d}{dt} (5t^2 + 3t)$

$\Rightarrow 5 \frac{d}{dt} t^2 + 3 \frac{d}{dt} t$

$\Rightarrow 5(2t^{2-1}) + 3(1)$

$\Rightarrow 10t + 3$

④ Given $y = -3t^{-3}$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $n = -3$

$\therefore \frac{d}{dt} y = \frac{d}{dt}(-3t^{-3})$

$= -3 \frac{d}{dt}(t)^{-3}$

$\Rightarrow -3(-3t^{-3-1}]$

$\Rightarrow 9t^{-4}$

⑤ $a = bx^3 + cx^4$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

Here $n = 3$, $n = 4$

$\therefore \frac{da}{dx} = \frac{d}{dx}[bx^3 + cx^4]$

$\Rightarrow b \frac{d}{dx} x^3 + c \frac{d}{dx} x^4$

$\Rightarrow b[3x^{3-1} + c4x^{4-1}]$

$\Rightarrow 3bx^2 + 4cx^3$

⑥ Given $v = 9t^2$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

$n = 2$

$\therefore \frac{dv}{dt} = \frac{d}{dt} 9t^2$

$\Rightarrow 9 \frac{d}{dt} t^2$

$\Rightarrow 9[2t^{2-1}]$

$\Rightarrow 18t$

⑦ Given $\int_1^5 x^2 dx$

we know that $\int x^n dx = \frac{x^{n+1}}{n+1}$

Here $n = 2$

$\int_1^5 x^2 dx = \left[\frac{x^{2+1}}{2+1}\right]_1^5$

$\Rightarrow \left[\frac{x^3}{3}\right]_1^5$

$\Rightarrow \left[\frac{5^3 - 1^3}{3}\right]$

$\Rightarrow \frac{124}{3}$

⑧ Given $\int_0^{\frac{\pi}{2}} \cos x \, dx$

we know that $\int \cos x \, dx = \sin x$

$\therefore \int_0^{\frac{\pi}{2}} \cos x \, dx = [\sin x]_0^{\frac{\pi}{2}}$

$\Rightarrow [\sin \frac{\pi}{2} - \sin 0]$

$= [1 - 0]$

$= 1.$

⑨ Given $\int x^{-\frac{5}{3}} dx$

we know that $\int x^n dx = \frac{x^{n+1}}{n+1}$

Here $n = -\frac{5}{3}$

$\int x^{-\frac{5}{3}} dx = \frac{x^{-\frac{5}{3}+1}}{-\frac{5}{3}+1}$

$\Rightarrow -\frac{3}{2} x^{-\frac{2}{3}}$

⑮ Given $\int x^{\frac{p}{q}} dx$

we know that $\int x^n dx = \frac{x^{n+1}}{n+1}$

$\therefore$ Here $n = \frac{p}{q}$

$\therefore \int x^{\frac{p}{q}} dx = \frac{x^{\frac{p}{q}+1}}{\frac{p}{q}+1} = \frac{x^{\frac{p+q}{q}}}{\frac{p+q}{q}} = \frac{q}{p+q} x^{\frac{p+q}{q}}$

⑩ Given $y = 4x^3 + 2x^2$

$\frac{d}{dx} x^n = n x^{n-1}$

Here $n = 3, 2$

$\frac{d}{dx} y = \frac{d}{dx}[4x^3 + 2x^2]$

$\Rightarrow 4 \frac{d}{dx} x^3 + 2 \frac{d}{dx} x^2$

$\Rightarrow 4(3x^{3-1}) + 2(2x^{2-1}]$

$\Rightarrow 12x^2 + 4x$

at $x = 1$

$\frac{dy}{dx} = 12(1)^2 + 4(1)$

$\Rightarrow 12 + 4 = 16$

⑪ at $x = 3$

$\frac{dy}{dx} = 12(3)^2 + 4(3)$

$= 12 \times 9 + 12$

$\Rightarrow 120$

⑫ at $x = \frac{1}{2}$

$\frac{dy}{dx} = 12(\frac{1}{2})^2 + 4(\frac{1}{2})$

$= \cancel{12}^3 \times \frac{1}{\cancel{4}} + 2 = 3 + 2$

$= 5$

⑬ Given $\int x^{\frac{11}{2}} dx$

we know that $\int x^n dx = \frac{x^{n+1}}{n+1}$

Here $n = \frac{11}{2}$

$\int x^{\frac{11}{2}} dx = \frac{x^{\frac{11}{2}+1}}{\frac{11}{2}+1} = \frac{2}{13} x^{\frac{13}{2}} + C$

⑭ Given $\int x^{-7} dx$

we know that $\int x^n dx = \frac{x^{n+1}}{n+1}$

Here $n = -7$

$\int x^{-7} dx = \frac{x^{-7+1}}{-7+1}$

$= -\frac{1}{6} x^{-6}$