WS- 19th advanced (1)

T task

triangle & Parallelogram law

(1)

Given $m = \sqrt{3}$ kg ; $\theta = 60^\circ$

Horizontal force $F = mg \tan\theta$

$\Rightarrow F = \sqrt{3} \times 9.8 \times \tan 60$

$\Rightarrow F = \sqrt{3} \times 9.8 \times \sqrt{3}$

$= 3 \times 9.8$

$= 29.4$ N

(2)

Given $m = 10$ kg

$F = 10\sqrt{3}$ kg wt $= 10\sqrt{3}\, g$

Horizontal force $F = mg \tan\theta$

$\Rightarrow 10\sqrt{3} g = 10 \times 10 \tan\theta$

$\Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$

(3)

let 'x' be a unit vector along x-axis.

Given $\vec{A} = 2\hat{i} - 4\hat{j} + 7\hat{k}$; $\vec{B} = 7\hat{i} + 2\hat{j} - 5\hat{k}$; $\vec{C} = -4\hat{i} + 7\hat{j} + 3\hat{k}$

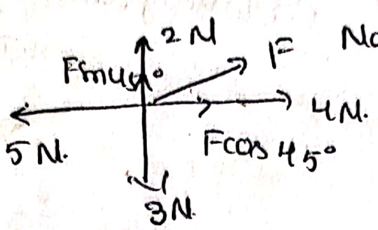
$\therefore \vec{A} + \vec{B} + \vec{C} + x = \hat{i}$

$\Rightarrow 2\hat{i} - 4\hat{j} + 7\hat{k} + 7\hat{i} + 2\hat{j} - 5\hat{k} + 4\hat{i} + 7\hat{j} + 3\hat{k} + x = \hat{i}$

$\Rightarrow 5\hat{i} + 5\hat{j} + 5\hat{k} + x = \hat{i}$

$\Rightarrow x = \hat{i} - (5\hat{i} + 5\hat{j} + 5\hat{k}) = -4\hat{i} - 5\hat{j} - 5\hat{k}$.

(4)



under equilibrium $F\cos 45 + 4 - 5 = 0$

$\Rightarrow F\left(\frac{1}{\sqrt{2}}\right) = 1$

$\Rightarrow F = \sqrt{2}\ N$

(6)

$R = 20\ N.$; $F_1 = 20\sqrt{3}\ N$; $\theta = 30°$

According to Parallelogram law of vectors

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\Rightarrow (20)^2 = (20\sqrt{3})^2 + F_2^2 + 2(20\sqrt{3})F_2\cos 30°$$

$$\Rightarrow 400 = 1200 + F_2^2 + \cancel{2}\, 20\sqrt{3}\, F_2 \frac{\sqrt{3}}{\cancel{2}}$$

$$\Rightarrow -800 = 60F_2 + F_2^2$$

$$\Rightarrow F_2^2 + 60F_2 + 800 = 0$$

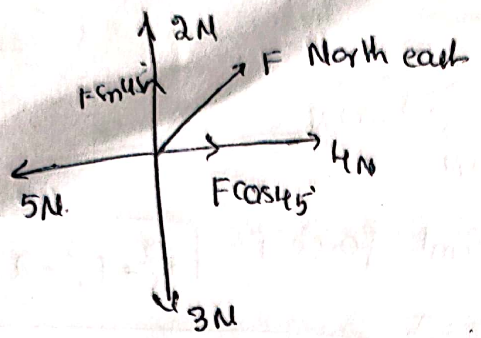
$$\Rightarrow F_2^2 + 40F_2 + 20F_2 + 800 = 0$$

$$\Rightarrow F_2(F_2 + 40) + 20(F_2 + 40) = 0$$

$$(F_2 + 40)(F_2 + 20) = 0$$

$$F_2 = -40\ N \text{ (or) } -20\ N$$

(4)



under equilibrium

$F\cos 45 + 4 - 5 = 0$

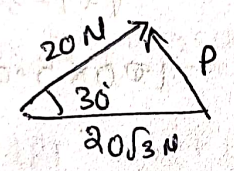
$\Rightarrow F(\frac{1}{\sqrt{2}}) - 1 = 0$

$\Rightarrow F = \sqrt{2}$ N.

(5)

let the two forces be $\vec{a}$ & $\vec{p}$ and the resultant by $\vec{r}$

$|\vec{r}| = 20$; $|\vec{a}| = 20\sqrt{3}$.



$\vec{a} + \vec{p} = \vec{r}$

$\Rightarrow p^2 = a^2 + r^2 - 2ra\cos\theta$

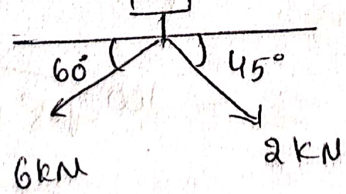
$= (20\sqrt{3})^2 + 20^2 - 2\times 20\times 20\sqrt{3}\times\cos 30$

$p^2 = 1200 + 400 - 80\sqrt{3}\times\frac{\sqrt{3}}{2}$

$\Rightarrow p^2 = 1200 + 400 - 1200$

$\Rightarrow p^2 = 400 \Rightarrow P = 20$ N.

(8)



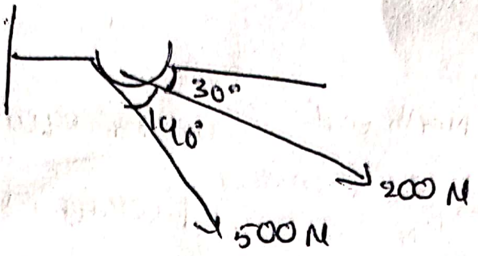
The resultant force $= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$= \sqrt{(2)^2 + (6)^2 + 2(2)(6)\cos 75}$

$= \sqrt{4 + 36 + 24\times 0.921}$

$= \sqrt{61.64} = 7.8$ KN

9



The resultant force $F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$$F = \sqrt{(500)^2 + (200)^2 + 2(500)(200)\cos 140^\circ}$$

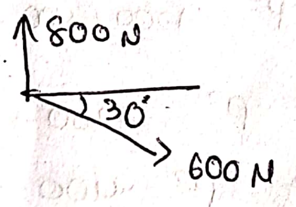
$$= 100\sqrt{5^2 + 2^2 + 2\times5\times2\,(-0.197)}$$

$$= 100\sqrt{25 + 9 + 20(-0.197)}$$

$$= 100\sqrt{29 - 3.94} = 100\sqrt{25.26}$$

$$= 100 \times 5.02 = 502\ N.$$

10



Angle between two forces is $= 120^\circ$

The resultant force $= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$$= \sqrt{(600)^2 + (800)^2 + 2(800)(600)\cos 120^\circ}$$

$$= 100\sqrt{8^2 + 6^2 + 2\times8\times6\left(-\tfrac{1}{2}\right)}$$

$$= 100\sqrt{64 + 36 - 48} = 100\sqrt{52} =$$

$$= 100 \times 7.21$$

$$= 721\ N.$$

7

R is the resultant of P and Q.

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$\Rightarrow R^2 = P^2+Q^2+2PQ\cos\theta \rightarrow (1)$$

$$\Rightarrow R^2-2PQ\cos\theta = P^2+Q^2 \rightarrow (2)$$

when Q is doubled, R is also doubled

$$(2R)^2 = P^2+(2Q)^2+2P(2Q)\cos\theta$$

$$\Rightarrow 4R^2 = P^2+4Q^2+4PQ\cos\theta \rightarrow (3)$$

when 'Q' is reversed, R is doubled

$$(2R)^2 = P^2+(-Q)^2+2P(-Q)\cos\theta$$

$$\Rightarrow 4R^2 = P^2+Q^2-2PQ\cos\theta \rightarrow (4)$$

From (2) & (4) $\quad 4R^2 = R^2-2PQ\cos\theta-2PQ\cos\theta$

$$\Rightarrow 3R^2 = -4PQ\cos\theta \rightarrow (5)$$

sub (5) in (3) we get $\quad 4R^2 = P^2+4Q^2-3R^2$

$$\Rightarrow P^2+4Q^2 = 7R^2 \rightarrow (6)$$

Also sub (5) in (2) we get $\quad R^2-\left(-\frac{3R^2}{2}\right) = P^2+Q^2$

$$\Rightarrow R^2+\frac{3R^2}{2} = P^2+Q^2$$

$$\Rightarrow P^2+Q^2 = \frac{5R^2}{2} \rightarrow (7)$$

By multiplying solving (6) & (7)

$$\begin{array}{r} \cancel{P^2}+4Q^2 = 7R^2 \\ \cancel{P^2}+Q^2 = \frac{5R^2}{2} \\ \hline 3Q^2 = 7R^2-\frac{5R^2}{2} \end{array}$$

$$\Rightarrow 3Q^2 = \frac{9R^2}{2}$$

$$\Rightarrow \boxed{Q = \sqrt{\frac{3}{2}}\,R}$$

7th continuation

sub Q in (7) we get

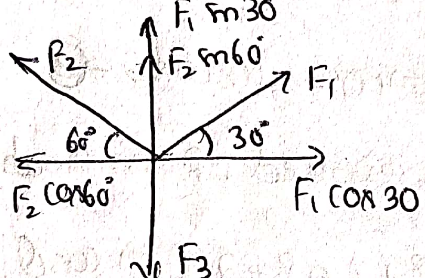
$$p^2 + \frac{3R^2}{2} = \frac{5R^2}{2}$$

$$\Rightarrow p^2 = \frac{5R^2 - 3R^2}{2} = \frac{2R^2}{2}$$

$$\Rightarrow p^2 = R^2 \Rightarrow P = R$$

$\therefore P : Q : R = R : \frac{\sqrt{3}}{\sqrt{2}} R : R \Rightarrow \sqrt{2} : \sqrt{3} : \sqrt{2}$

(12)



Given $F_2 = 5\sqrt{3}$ N

under equilibrium $F_1 \cos 30 = F_2 \cos 60$

$$\Rightarrow F_1 \frac{\sqrt{3}}{2} = 5\sqrt{3}\ \frac{1}{2}$$

$$\Rightarrow F_1 = 5N.$$

$F_3 = F_1 \sin 30 + F_2 \sin 60$

$$= 5 \times \frac{1}{2} + 5\sqrt{3}\ \frac{\sqrt{3}}{2} = \frac{5}{2} + \frac{15}{2} = \frac{20}{2} = 10N$$

(15)

let $F_1 = 5N$; $F_2 = 3N$ are in same direction

ie) $\theta = 0°$

Then resultant $R = F_1 + F_2$

$= 5 + 3 = 8N$

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(16)

let $F_1 = 20N$; $F_2 = 25N$. ; $R = 39N$.

According to Parallelogram law of vectors

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\Rightarrow 39 = \sqrt{(20)^2 + (25)^2 + 2(20)(25)\cos\theta}$$

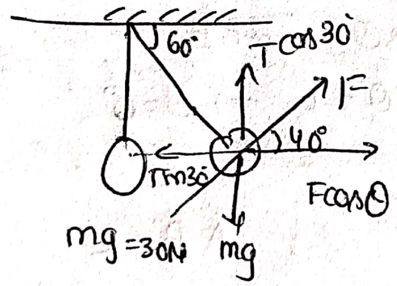
$$\Rightarrow (39)^2 = 400 + 625 + 1000\cos\theta$$

$$\Rightarrow 1521 = 1025 + 1000\cos\theta$$

$$\Rightarrow 496 = 1000\cos\theta \Rightarrow \cos\theta = \frac{496}{1000} \simeq \frac{1}{2}$$

$$\Rightarrow \theta = 60^\circ$$

(17) (18)



$T\cos 30 = mg \rightarrow (1)$

$T\sin 30^\circ = F\cos\theta \rightarrow (2)$

By dividing $\frac{(2)}{(1)}$

$$\frac{F\cos\theta}{mg} = \tan 30^\circ$$

$$\Rightarrow F\cos 40 = 30 \times \tan 30^\circ$$

$$\Rightarrow F \times 0.766 = 30 \times \frac{1}{\sqrt{3}}$$

$$\Rightarrow F = 22.61\,N.$$

From (2) $T\sin 30 = 22.61 \times \cos 40^\circ$

$$\Rightarrow T \times \frac{1}{2} = 22.61 \times 0.766$$

$$\Rightarrow T = 18.93\,N.$$

(19)

let the two force are $F_1 = 4x$
$F_2 = 3x$

Given $F_1 + F_2 = 7N$
$F_1 - F_2 = 1N$

$\Rightarrow 2F_1 = 8N$

$F_1 = 4N$

$\Rightarrow 4x = 4$

$\Rightarrow x = 1$

(20)

(a) $\vec{A} - \vec{B} = \vec{C}$; $A - B = C$

$\Rightarrow A^2 + B^2 - 2AB\cos\theta = C^2$; $\Rightarrow A^2 + B^2 - 2AB = C^2$

$\therefore A^2 + \cancel{B^2} - 2AB\cos\theta = \cancel{A^2 + B^2} - 2AB$

$\Rightarrow -\cancel{2AB}\cos\theta = -\cancel{2AB}$

$\Rightarrow \cos\theta = 1 \Rightarrow \theta = 0^\circ$

(b)

$\vec{A} + \vec{B} = \vec{C}$, $A - B = C$

$\Rightarrow A^2 + B^2 + 2AB\cos\theta = C^2$; $A^2 + B^2 - 2AB = C^2$

$\Rightarrow A^2 + \cancel{B^2} + 2AB\cos\theta = A^2 + \cancel{B^2} - 2AB$

$\Rightarrow \cancel{2AB}\cos\theta = -\cancel{2AB} \Rightarrow \cos\theta = -1$

$\theta = 180^\circ = \pi$

(c) $\vec{A} - \vec{B} = \vec{C}$; $A^2 + B^2 = C^2$

$\Rightarrow A^2 + B^2 - 2AB\cos\theta = C^2$; $\Rightarrow A^2 + B^2 = C^2$

$\Rightarrow A^2 + \cancel{B^2} - 2AB\cos\theta = A^2 + \cancel{B^2}$

$\Rightarrow -2AB\cos\theta = 0$

$\cos\theta = 0 \Rightarrow \theta = 90^\circ = \frac{\pi}{2}$

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(d) $\vec{A} + \vec{B} = \vec{C}$

$\Rightarrow A = B = C$

$\Rightarrow A^2 + B^2 + 2AB\cos\theta = C^2$

$\Rightarrow \cancel{A^2} + A^2 + 2A(A)\cos\theta = A^2$

$\Rightarrow 2A^2\cos\theta = -A^2$

$\Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120^\circ = \frac{2\pi}{3}$

L Task

Jee main level

(1) $F_1 = F_2 = 5N$

$\theta = 60^\circ$

The resultant of two forces

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$R = \sqrt{5^2 + 5^2 + 2(5)(5)\cos 60}$

$= \sqrt{5^2 + 5^2 + \cancel{2}(5)^2 \frac{1}{\cancel{2}}}$

$= \sqrt{5^2 + 5^2 + 5^2} = 5\sqrt{3}$

$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$

$= \frac{5\sin 60}{5 + 5\cos 60}$

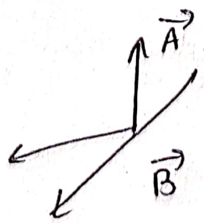
$= \frac{\cancel{5}\frac{\sqrt{3}}{2}}{\cancel{5}(1+\frac{1}{2})} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = 30^\circ$

(2) Here direction of $\vec{A}$ ↑ and $\vec{B}$ ↙

ie)



so the resultant must be in between them.

It should not be horizontal because the angle between $\vec{A}$ and $\vec{B}$ is $> 90^\circ$

(3)

let the two forces are F_1 & F_2

Given maximum resultant ie $F_1+F_2=17$

minimum resultant $F_1-F_2=7N$

Here F_1 & F_2 are orthogonal to each other [$\theta=90^\circ$]

$\therefore$ Their resultant $R=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$

$\Rightarrow R=\sqrt{(12)^2+5^2+2(12)(5)\cos 90^\circ}$

$=\sqrt{144+25}$

$=\sqrt{169}$

$=13N$

$F_1+F_2=17$
$F_1-F_2=7$
$2F_1=24$
$F_1=12N$
$\therefore F_2=5N$

(4)

Given $|\vec{A}|=10$

$|\vec{B}|=5$; $\theta=120^\circ$

$|\vec{A}+\vec{B}|=\sqrt{A^2+B^2+2AB\cos\theta}$

$=\sqrt{10^2+5^2+2\times10\times5\cos120}$

$=\sqrt{100+25+2\times50\times(-\frac{1}{2})}$

$=\sqrt{75}=5\sqrt{3}$

(5)

Given $F_1=100N$

$F_2=150N$

$\theta=45^\circ$

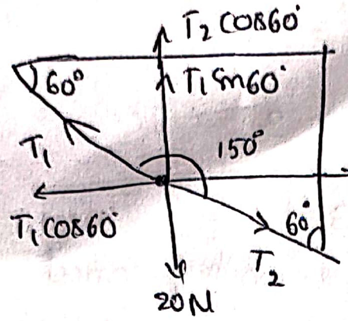
$R=\sqrt{F_1^2+F_2^2+2F_1F_2\cos45}$

$=\sqrt{(100)^2+(150)^2+2(100)(150)\frac{1}{\sqrt{2}}}$

$=231.76N$

(6)

(6)



under equilibrium

$T_1 \cos 60 = T_2 \sin 60$

$\Rightarrow T_1 \left(\frac{1}{2}\right) = T_2\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \frac{T_1}{T_2} = \sqrt{3}$

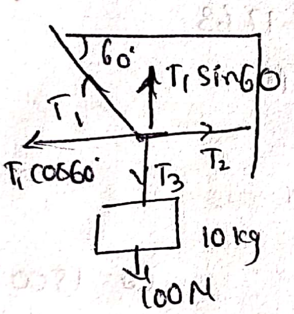
Also $T_1 \sin 60 + T_2 \cos 60 = 20$

$\Rightarrow \sqrt{3}T_2 \times \frac{\sqrt{3}}{2} + \frac{T_2}{2} = 20 \quad \Rightarrow 2T_2 = 20$

$\Rightarrow T_2 = 10N$

$T_1 = \sqrt{3}\,T_2 = 10\sqrt{3}N$

(7)



under equilibrium

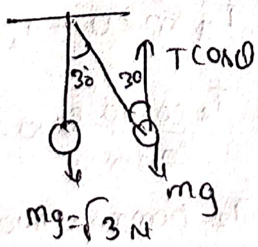
$T_2 = T_1 \cos 60°$

$T_1 \sin 60 = 100$

$\Rightarrow T_1 \frac{\sqrt{3}}{2} = 100 \Rightarrow T_1 = \frac{200}{\sqrt{3}}\,N = \frac{20}{\sqrt{3}}$ kg wt

$\therefore T_2 = \frac{200}{\sqrt{3}} \times \frac{1}{2} = \frac{100}{\sqrt{3}} = \frac{10}{\sqrt{3}}$ kg wt

(8)

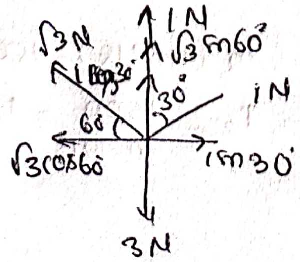


$T \cos\theta = \sqrt{3}$

$\Rightarrow T \cos 30 = \sqrt{3} \qquad T = 2N$

$\Rightarrow T \frac{\sqrt{3}}{2} = \sqrt{3}$

(9)



under equilibrium

Along x-axis $\sqrt{3}\cos 60 = 1 \times \cos 30$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ they cancel with each other

Along Y-axis

$$1+\sqrt{3}\sin 60+1\sin 30 = 3$$

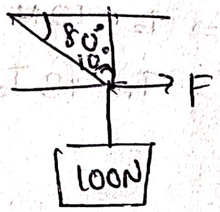
$$\Rightarrow 1+\sqrt{3}\times\frac{\sqrt{3}}{2}+\frac{1}{2} = 3$$

$$\Rightarrow 1+\frac{3}{2}+\frac{1}{2} = 3$$

$1+2=3 \rightarrow$ so they cancel

$\therefore$ Net force $=0$

(10)



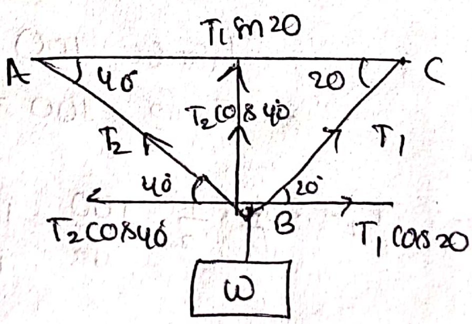
$$F = mg\tan\theta$$

$$F = 100\times\tan 10°$$

$$= 100\times 0.1763$$

$$= 17.63\ N$$

(15), (16)



$W = 1500\ N$

under equilibrium

$$T_1\cos 20 = T_2\cos 40$$

$$\Rightarrow T_1 = T_2\frac{\cos 40}{\cos 20} \quad (1)$$

$$T_1\sin 20 + T_2\sin 40 = W$$

$$\Rightarrow T_2\frac{\cos 40}{\cos 20}\sin 20 + T_2\times 0.64 = 1500$$

$$\Rightarrow T_2\tan 20\times\cos 40 + 0.64T_2 = 1500$$

$$\Rightarrow T_2\times 0.364\times 0.766 + T_2\times 0.64 = 1500$$

$$\Rightarrow T_2(0.918) = 1500$$

$$\Rightarrow T_2 = \frac{1500}{0.918} = 1630\ N$$

$\therefore$ From (1)

$$T_1 = \frac{1630\times 0.766}{0.939}$$

$$= 1630\times 0.81$$

$$= 1329\ N$$

(17)

$F_1 = F_2 = 15$ N. $\quad R = 5x \quad \theta = 120°$

∴ According to Parallelogram law of vectors

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\Rightarrow 5x = \sqrt{(15)^2 + (15)^2 + 2(15)(15)\cos 120°}$$

$$\Rightarrow (5x)^2 = \left(\sqrt{(15)^2 + (15)^2 + \cancel{2}(15)^2\left(-\frac{1}{\cancel{2}}\right)}\right)^{\cancel{2}}$$

$$\Rightarrow (5x)^2 = (15)^2 + \cancel{(15)^2} - \cancel{(15)^2}$$

$$\Rightarrow (5x)^2 = (15)^2 \Rightarrow 5x = 15 \Rightarrow x = 3$$

(18)

$\alpha = 90° \quad Q = 2P$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\Rightarrow P + Q\cos\theta = \frac{Q\sin\theta}{\tan 90} = 0$$

$$\Rightarrow Q\cos\theta = -P$$

$$\Rightarrow 2P\cos\theta = -P \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$$

$$\Rightarrow \frac{2\pi}{n} = \frac{2\pi}{3} \Rightarrow n = 3$$

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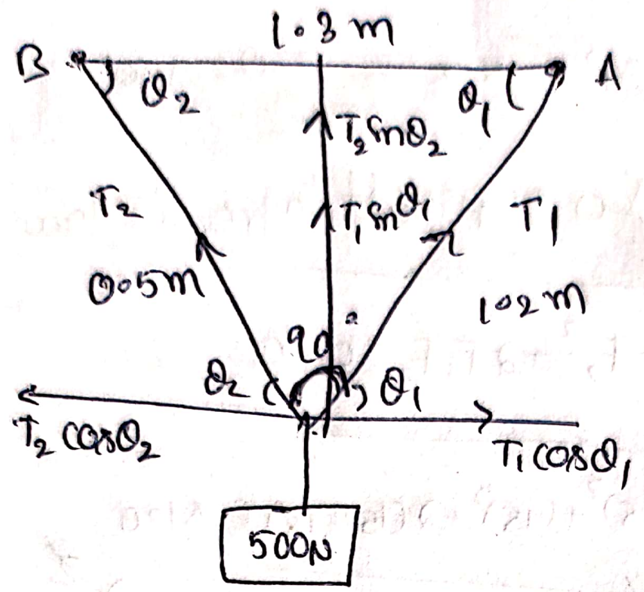
(19)



According to lami's theorem

$$\frac{500}{\sin 90^\circ} = \frac{T_2}{\sin(90+\theta)} = \frac{T_1}{\sin(180-\theta)}$$

$$\Rightarrow 500 = \frac{T_2}{\cos\theta} = \frac{T_1}{\sin\theta}$$

$$\Rightarrow T_1 = 500 \sin\theta \quad = \quad T_2 = 500 \cos\theta$$

$$= 500 \times \frac{0.5}{1.3} \qquad T_2 = 500 \times \frac{1.2}{1.3}$$

$$T_1 = 192.3\,N \qquad T_2 = 461.5\,N$$

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