

WS-17 Circular motion & th foundation

Task

(1)

Given $v_{\text{car}} = v_{\text{scooter}}$

$$R_c = 2 R_s$$

From $v = R \omega \Rightarrow \omega = \frac{v}{R}$

$$\Rightarrow \frac{\omega_{\text{car}}}{\omega_{\text{scooter}}} = \frac{R_{\text{scooter}}}{R_{\text{car}}} = \frac{R_s}{2 R_s} = \frac{1}{2} \rightarrow A$$

(2)

we know Angular speed $\omega = \frac{2\pi}{t}$

For min hand $T_{\text{min}} = 60 \text{ min}$

For hr hand $T_{\text{hr}} = 12 \text{ hrs} = 12 \times 60 \text{ min}$

$$\frac{\omega_{\text{min}}}{\omega_{\text{hr}}} = \frac{T_{\text{hr}}}{T_{\text{min}}} = \frac{12 \times 60}{60} = \frac{12}{1} \rightarrow D$$

(3)

Given Diameter $= D = 2r = 1 \text{ m} \Rightarrow r = \frac{1}{2} \text{ m}$

No. of revolutions Per min = 1200

$$\omega = \frac{1200 \times 2\pi}{60} \text{ rad/s}$$

$$\omega = 40\pi \text{ rad/s}$$

Angular acceleration $= r \omega^2$

$$= \frac{1}{2} \times 1600 \pi^2$$

$$= 800 \pi^2 \text{ m/s}^2$$

(4)

Given Angular velocity $\omega_1 = 600 \text{ rpm} = 600 \times \frac{2\pi}{60} = 20\pi \text{ rad/s}$

$$\omega_2 = 1200 \text{ rpm} = 1200 \times \frac{2\pi}{60} = 40\pi \text{ rad/s}$$

Increase in Angular velocity $= \omega_2 - \omega_1 = 20\pi \text{ rad/s}$
 $\rightarrow B$

(5)

Given Diameter $D = 3 \text{ m} \Rightarrow r = \frac{3}{2} \text{ m}$

$$v = 18 \text{ m/s}$$

$$\text{Angular velocity } \omega = \frac{v}{r} = \frac{18}{\frac{3}{2}} = 12 \text{ rad/s} \rightarrow A$$

(6)

Given $r = 25 \text{ cm}$; $v = 5 \text{ m/s}$

$$\text{Angular velocity } \omega = \frac{v}{r} = \frac{5}{25 \times 10^{-2}} = \frac{500}{25}$$

$$\omega = 20 \text{ rad/s}$$

(7)

Given $m_1 = 70 \text{ kg}$; $m_2 = 100 \text{ gm} = 0.1 \text{ kg}$

Total length of the string $L = 1 \text{ m}$

length of hanging part $L_2 = 60 \text{ cm} = 0.6 \text{ m}$

The length of the string that rotates in a circle $= \text{radius} = L - L_2 = 1 - 0.6 = 0.4 \text{ m}$

The forces acting on m_2 are $w = m_2 g = 0.1 \times 10 = 1 \text{ N}$
 $= 1 \text{ N}$

Tension in the string $T = m_2 g = 0.1 \times 10 = 1 \text{ N}$

we know T provides centripetal force

$$T = \frac{m v^2}{r}$$

$$\Rightarrow \frac{0.07 v^2}{0.4} = 1 \Rightarrow v^2 = \frac{0.4}{0.07} \approx 5.7$$

$$\Rightarrow v = \sqrt{5.7} = 2.4 \text{ m/s}$$

From $v = r\omega$

$$\Rightarrow \omega = \frac{v}{r} = \frac{2.4}{0.4} \approx 5.975 \text{ rad/s}$$

frequency $f = \frac{\omega}{2\pi} = \frac{5.975}{2 \times 3.14} \approx 0.95 \text{ Hz}$

$$f = \frac{\sqrt{140}}{4\pi} = 0.95 \text{ Hz}$$

(8)

Given

Diameter = $D = 2 \text{ m} \Rightarrow r = \frac{D}{2} = 1 \text{ m}$

Angular velocity $\omega = 600 \times \frac{2\pi}{60} = 20\pi \text{ rad/s}$

Angular acceleration $= r\omega^2$
 $= 1^2 \times (20\pi)^2 = 400\pi^2$
 $\rightarrow C$

(9)

length of string = radius = $20 \text{ cm} = 20 \times 10^{-2} \text{ m}$

$a_c = 980 \text{ cm/s}^2$

$r\omega^2 = 980$

$\Rightarrow 20\omega^2 = 980 \Rightarrow \omega^2 = 49 \Rightarrow \omega = 7 \text{ rad/s}$
 $\rightarrow A$

(10)

Given Diameter $D = 2r = 400 \Rightarrow r = 200 \text{ cm}$

velocity $v = 1600 \text{ cm/s}$

Angular velocity $\omega = \frac{v}{r} = \frac{1600}{200} = 8 \text{ rad/s}$
 $\rightarrow A$

(11)

Here constant speed may not lead to constant velocity because in circular motion the direction of velocity changes from point to point, and it is always away from centre.

(12)

In uniform circular motion the magnitude of velocity is constant.

$$\text{Since } K.E = \frac{1}{2} m v^2 \therefore K.E = \text{constant}$$

→ C

But momentum changes with time because of change in direction of velocity from point to point.

(15)

In uniform circular motion direction of velocity is tangential to the circumference of a circle and it changes from point to point and hence direction of acceleration also changes

→ C

(16)

When the body moving in a circular path displacement and centripetal force are perpendicular i.e. $\theta = 90^\circ$

$$\begin{aligned} \text{Since } W &= \vec{F} \cdot \vec{s} \cos \theta \\ &= F s \cos 90^\circ \\ &= 0 \rightarrow A \end{aligned}$$

(5)

from $\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

The particle starts from rest $\omega_0 = 0$

$$\theta = 0 \times t + \frac{1}{2} \alpha t^2$$

$$\theta = \frac{1}{2} \alpha t^2 \Rightarrow \theta \propto t^2$$

if $\alpha = \text{constant}$

(9)

when a particle is moves in the plane of paper along clockwise direction, Angular velocity and acceleration are in same direction, so direction of acceleration is into the paper and perpendicular to plane of paper

(10)

The body will get an angular acceleration. This is because from $\tau = I\alpha$, a Torque acting on the body causes an angular acceleration whether or not the torque is steady. There will be some angular acceleration and the body won't rotate at a constant speed.

See main level

(11)

$$T_{\text{Mars}} = 12 \text{ hrs} ; T_{\text{Earth}} = 24 \text{ hrs}$$

$$\frac{\omega_{\text{Mars}}}{\omega_{\text{Earth}}} = \frac{T_{\text{Earth}}}{T_{\text{Mars}}} = \frac{24}{12} = \frac{2}{1} \rightarrow B$$

② Given $v = 45 \text{ kmph} = 45 \times \frac{5}{18} \text{ m/s}$
 Angular displacement $= \frac{25}{2} \text{ m/s}$
 Angular velocity $= 50 \text{ cm} = \frac{1}{2} \text{ m}$

Angular velocity $\omega = \frac{v}{r} = \frac{\frac{25}{2}}{\frac{1}{2}} = 25 \text{ rad/s} \rightarrow B$

③ Given velocity $v = 36 \text{ kmph} = 36 \times \frac{5}{18} \text{ m/s}$
 $= 10 \text{ m/s}$
 radius $r = 25 \text{ m}$

Acceleration $a = r \omega^2 = 25 \times (10)^2 = 2500 \text{ m/s}^2$
 $= \frac{v^2}{r} = \frac{(10)^2}{25} = 4 \text{ m/s}^2$

④ Given New speed $v' = 2v$
 New angular velocity $\omega' = \frac{\omega}{2}$
 Centripetal acceleration $= v' \omega'$
 $a' = 2v \frac{\omega}{2} = v\omega$
 $= a$

No change $\rightarrow D$

⑤ $\frac{\text{linear acceleration}}{\text{angular acceleration}} = \frac{a}{\frac{a}{r}} = r$
 $\frac{\text{linear velocity}}{\text{angular velocity}} = \frac{v}{\frac{v}{r}} = r$
 Both same $\rightarrow C$

(8)

$$\text{acc acceleration} = r\omega^2$$

$$a \propto r \quad \text{since } r_p > r_q$$

$$a_p > a_q$$

(9)

$$\text{Given radius } r = 4\text{m} ; \text{ time} = 2\text{sec}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rad/sec}$$

$$\text{Acceleration} = r\omega^2 = 4\pi^2 \rightarrow D$$

(10)

$$\omega = \text{Angular velocity of second hand}$$

$$= \frac{2\pi}{60} = \frac{\pi}{30} = \frac{3.14}{3 \times 10}$$

$$= \frac{0.314}{3} = 0.105 \text{ rad/sec}$$