



TRANSFORMATION OF ANGLES



The following eight formulae derived here are Called the trigonometric transformation formulae, and they can be used to transform the sum or difference of trigonometric to their products and products of trigonometric functions to their sum or difference.

Synopsis

$$\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$$

$$\sin(A + B) - \sin(A - B) = 2 \cos A \sin B$$

$$\cos(A + B) + \cos(A - B) = 2 \cos A \cos B$$

$$\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$$

$$\sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$$

$$\sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$$

$$\cos C + \cos D = 2 \cos \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$$

$$\cos C - \cos D = -2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right) = 2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{D-C}{2} \right)$$

Identities

If $A+B+C = 180^\circ$ then

i) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

ii) $\sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$

iii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$

iv) $\cos 2A + \cos 2B - \cos 2C = 1 - 4 \sin A \sin B \cos C$

v) $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

vi) $\sin A - \sin B + \sin C = 4 \sin \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$

vii) $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

viii) $\cos A + \cos B - \cos C = -1 + 4 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$

ix) $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$

x) $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2 \cos A \cos B \cos C$

$$\text{xi) } \sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1 - 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$\text{xii) } \cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \cos^2 \frac{C}{2} = 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$\begin{aligned} 1) \quad \sin 9^\circ &= \frac{1}{4} \left[\sqrt{3+\sqrt{5}} - \sqrt{5-\sqrt{5}} \right] \\ &= \sqrt{\frac{4 - \sqrt{10+2\sqrt{5}}}{8}} \\ &= \frac{1}{4} \sqrt{8 - 2\sqrt{10+2\sqrt{5}}} = \cos 81^\circ \end{aligned}$$

$$\begin{aligned} 2) \quad \cos 9^\circ &= \frac{1}{4} \left[\sqrt{3+\sqrt{5}} + \sqrt{5-\sqrt{5}} \right] \\ &= \sqrt{\frac{4 + \sqrt{10+2\sqrt{5}}}{8}} \\ &= \frac{1}{4} \sqrt{8 + 2\sqrt{10+2\sqrt{5}}} = \sin 81^\circ \end{aligned}$$

$$3) \cos x \cdot \cos 2x \cdot \cos 4x \cdots \cos(2^n x) = \frac{1}{2^{n+1}} \frac{\sin(2^{n+1} x)}{\sin x}$$

$$4) \sin \frac{\pi}{n} + \sin \frac{3\pi}{n} + \sin \frac{5\pi}{n} + \dots \text{ n terms} = 0$$

$$\begin{aligned} 5) \quad \forall x \in R, \tan x + \frac{1}{2} \tan \frac{x}{2} + \frac{1}{2^2} \tan \frac{x}{2^2} + \dots + \frac{1}{2^{n-1}} \tan \left(\frac{x}{2^{n-1}} \right) \\ = \frac{1}{2^{n-1}} \cot \left(\frac{x}{2^{n-1}} \right) - 2 \cot 2x \end{aligned}$$

$$6) \quad \sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots \text{ n terms} = \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \sin \left\{ \alpha + \frac{n-1}{2} \beta \right\}$$

$$7) \quad \cos \alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \dots \text{ n terms} = \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \cos \left\{ \alpha + \frac{n-1}{2} \beta \right\}$$

TEACHING TASK

I. MCQ with single correct answer

1. $\sin A + \sin 3A + \sin 5A + \sin 7A =$
 A. $4 \sin A \cos 2A \cos 4A$ B. $4 \sin A \cos 2A \cos 2A$
 C. $4 \cos A \sin 2A \sin 4A$ D. $4 \cos A \cos 2A \sin 4A$
2. $4 \cos 6\theta \cos 4\theta \cos 2\theta =$
 A. $\cos 12\theta + \cos 8\theta + \cos 4\theta + 1$ B. $\cos 12\theta + \cos 8\theta - \cos 4\theta + 1$
 C. $\cos 12\theta - \cos 8\theta + \cos 4\theta + 1$ D. $\cos 12\theta - \cos 8\theta - \cos 4\theta - 1$
3. $\sin \frac{\theta}{2} \cdot \sin \frac{7\theta}{2} + \sin \frac{3\theta}{2} \cdot \sin \frac{11\theta}{2} - \sin 2\theta \cdot \sin 5\theta =$
 A. 0 B. 1 C. -1 D. 2
4. $2(1 - 2 \sin^2 \theta) \cos 4\theta =$
 A. $\sin 6\theta + \cos 2\theta$ B. $\sin 6\theta + \sin 2\theta$ C. $\cos 6\theta + \cos 2\theta$ D. $\cos 6\theta + \sin 2\theta$
5. $\cot 16^\circ \cdot \cot 44^\circ + \cot 44^\circ \cdot \cot 76^\circ - \cot 76^\circ \cdot \cot 16^\circ =$
 A. 3 B. 0 C. 1 D. 4
6. $\cos 48^\circ \cdot \cos 12^\circ =$
 A. $\frac{1 - \sqrt{5}}{8}$ B. $\frac{\sqrt{5} + 3}{8}$ C. $\frac{\sqrt{5} - 1}{8}$ D. $\frac{\sqrt{5} + 1}{8}$
7. $\cos 66^\circ + \sin 84^\circ =$
 A. $\frac{\sqrt{15} - \sqrt{3}}{4}$ B. $\frac{\sqrt{15} - 3}{4}$ C. $\frac{\sqrt{15} + \sqrt{3}}{4}$ D. $\frac{\sqrt{15} + 3}{4}$
8. $4 \sin(420^\circ - \alpha) \cos(60^\circ + \alpha) =$
 A. $\sqrt{3} - 2 \sin 2\alpha$ B. $\sqrt{3} + 2 \sin 2\alpha$ C. $\sqrt{3} - 2 \cos 2\alpha$ D. $\sqrt{3} + 2 \cos 2\alpha$
9. $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ =$
 A. 3/16 B. 1/32 C. 1/16 D. 1/8
10. $\cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cdot \cos 16^\circ =$
 A. 1/2 B. -1/4 C. 0 D. 3/4

11. $\frac{\cos^2 33^\circ - \cos^2 57^\circ}{\sin 21^\circ - \cos 21^\circ} =$
 A. $-1/\sqrt{2}$ B. $1/2$ C. $1/\sqrt{2}$ D. $-1/2$
12. $\sin \alpha + \sin \beta = a, \cos \alpha + \cos \beta = b \Rightarrow \cos(\alpha + \beta) =$
 A. $\frac{a^2 + b^2}{2ab}$ B. $\frac{2ab}{a^2 + b^2}$ C. $\frac{b^2 - a^2}{b^2 + a^2}$ D. $\frac{ab}{a^2 + b^2}$
13. $(1 + \sqrt{1+a}) \tan \alpha = 1 + \sqrt{1-a} \Rightarrow \sin 4\alpha =$
 A. -1 B. a C. 1 D. 0
14. $\pi < \alpha - \beta < 3\pi, \sin \alpha + \sin \beta = \frac{-21}{65}, \cos \alpha + \cos \beta = \frac{-27}{65} \Rightarrow \cos\left(\frac{\alpha - \beta}{2}\right) =$
 A. $\frac{-6}{65}$ B. $\frac{-3}{\sqrt{130}}$ C. $\frac{3}{\sqrt{130}}$ D. $\frac{6}{65}$
15. $\cos x + \cos y = \frac{4}{5}, \cos x - \cos y = \frac{2}{7} \Rightarrow 14 \tan\left(\frac{x-y}{2}\right) + 5 \cot\left(\frac{x+y}{2}\right) =$
 A. 0 B. $1/4$ C. $5/4$ D. $3/4$
16. If n is an odd integer then $\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n =$
 A. 0 B. $\cot^n\left(\frac{A+B}{2}\right)$ C. $\cot^n\left(\frac{A-B}{2}\right)$ D. $2 \tan^n\left(\frac{A+B}{2}\right)$
17. $\cos 2B = \frac{\cos(A+C)}{\cos(A-C)} \Rightarrow \tan A, \tan B, \tan C$ are in
 A. A.P. B. H.P. C. G.P. D. A.G.P.
18. $A + B + C = 180^\circ \Rightarrow \cos 2A + \cos 2B + \cos 2C =$
 A. $1 - 4 \sin A \sin B \sin C$ B. $1 + 4 \sin A \sin B \sin C$
 C. $1 + 4 \cos A \cos B \cos C$ D. $-1 - 4 \cos A \cos B \cos C$
19. $A + B + C = 180^\circ \Rightarrow \sin 2A - \sin 2B + \sin 2C =$
 A. $2 \sin A \cos B \sin C$ B. $2 \cos A \sin B \cos C$
 C. $4 \sin A \cos B \sin C$ D. $4 \cos A \sin B \cos C$

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20. $A + B + C = 0^\circ \Rightarrow \sin A + \sin B + \sin C =$
- A. $2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$ B. $-2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
- C. $4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$ D. $-4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
21. $A + B + C = 2S \Rightarrow \sin S + \sin(S - A) + \sin(S - B) - \sin(S - C) =$
- A. $4 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2}$ B. $4 \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \sin \frac{C}{2}$
- C. $4 \cos \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \cos \frac{C}{2}$ D. $4 \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2}$
22. $\frac{1 - \cos A + \cos B - \cos(A + B)}{1 + \cos A - \cos B - \cos(A + B)} =$
- A. $\sin \frac{A}{2} \cdot \cos \frac{B}{2}$ B. $\sec \frac{A}{2} \cos \frac{B}{2}$ C. $\tan \frac{A}{2} \cdot \cot \frac{B}{2}$ D. $2 \sin \frac{A}{2} \cdot \cos \frac{B}{2}$
23. $\frac{\sin A + \sin 5A + \sin 9A}{\cos A + \cos 5A + \cos 9A} =$
- A. $\tan 3A$ B. $\tan 5A$ C. $\tan 4A$ D. $\tan 2A$
24. $\sin x + \sin y = \frac{1}{4}$, $\sin x - \sin y = \frac{1}{5} \Rightarrow 4 \cot \left(\frac{x - y}{2} \right) =$
- A. $5 \cot \left(\frac{x - y}{2} \right)$ B. $5 \tan \left(\frac{x - y}{2} \right)$ C. $5 \cot \left(\frac{x + y}{2} \right)$ D. $5 \tan \left(\frac{x + y}{2} \right)$
25. $\cos x + \cos y = 1/3$, $\sin x + \sin y = 1/4 \Rightarrow \sin(x + y)$
- A. $7/25$ B. $25/24$ C. $25/7$ D. $24/25$
26. $\sin \theta = n \sin(\theta + 2\alpha) \Rightarrow (1 - n) \tan(\theta + \alpha) =$
- A. $(n + 1) \tan \alpha$ B. $(n + 1) \tan \beta$ C. $(n - 1) \tan \alpha$ D. $(n - 1) \tan \beta$
27. $x = \cos 55^\circ$, $y = \cos 65^\circ$, $z = \cos 175^\circ \Rightarrow xy + yz + zx =$
- A. $-3/4$ B. $3/4$ C. $3/2$ D. $1/2$

II. MCQ with one or more than one correct answer :

1. Which of the following are true ?
- A) $\cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cos 16^\circ = 3/4$
- B) $\cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cos 16^\circ = 1/4$

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- C) In $\triangle ABC$ $\sin^2 A + \sin^2 B + \sin^2 C = 4 \sin A \cdot \sin B \cdot \sin C$
 D) In $\triangle ABC$ $\sin^2 A - \sin^2 B + \sin^2 C = 4 \cos A \cdot \sin B \cdot \cos C$
2. Which of the following are true?
 A) $\sin 6q + \sin 4q = 2 \sin 5q \cos q$ B) $\cos 3q - \cos 7q = 2 \sin 5q \sin 2q$
 C) $\sin(2n+1)A \cdot \sin A = \sin^2(n+1)A - \sin^2 nA$
 D) $2 \cos 11q \cdot \cos 3q = \cos 14q + \cos 8q$

III. Assertion and Reasoning type questions

- A) A is true, R is true and 'R' is correct explanation of A.
 B) A is true, R is true and 'R' is not correct explanation of A.
 C) A is true, R is false D) A is false, R is true
1. A: If $A + B + C = 180^\circ$ then $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$
 R: If $A + B + C = 180^\circ$ then $\cos 2A + \cos 2B + \cos 2C = 1 - 4 \cos A \cos B \cos C$
2. A: If $x + y + z = xyz$ then $\sum \left(\frac{2x}{1-x^2} \right) = \pi \left(\frac{2x}{1-x^2} \right)$
 R: If $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ then $A+B+C = n\pi, n \in \mathbb{N}$
3. A: If $x = \sin(a-b) \sin(g-d)$, $y = \sin(b-g) \sin(a-d)$, $z = \sin(g-a) \sin(b-d)$
 then $x + y + z = 0$
 R: $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$
4. A: $a = \tan q$, $b = \tan 2q$, $a \neq 0$, $b \neq 0$ and $\tan q + \tan 2q = \tan 3q$ then $a + b = 0$
 R: If $A - B = C$, then $\tan A - \tan B - \tan C = \tan A \tan B \tan C$
5. A: In $\triangle ABC$, $\sum \frac{\cos A}{\sin B \sin C} = 2$
 R: In $\triangle ABC$, $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

IV Match the following

1. If $A + B + C = 180^\circ$ then match the following
- | | |
|---------------------------------------|---|
| List - I | List - II |
| 1) $\cos A + \cos B + \cos C =$ | a) $2 + 2 \cos A \cos B \cos C$ |
| 2) $\sin^2 A + \sin^2 B + \sin^2 C =$ | b) $4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$ |
| 3) $\sin 2A + \sin 2B + \sin 2C =$ | c) $1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$ |
| 4) $\sin A + \sin B + \sin C =$ | d) $4 \sin A \sin B \sin C$ |
| A) c, a, d, b B) d, b, a, c | C) a, c, b, d D) b, d, c, a |

V. Comprehension type questions

1. If $A + B + C = 180^\circ$ then
- a) $\cos^2 A + \cos^2 B + \cos^2 C =$
- | | |
|---------------------------------|---------------------------------|
| A) $1 + 2 \cos A \cos B \cos C$ | B) $1 + 2 \sin A \sin B \sin C$ |
| C) $1 - 2 \cos A \cos B \cos C$ | D) $1 - 2 \sin A \sin B \sin C$ |

b) $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} =$

A) $1 + 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

B) $1 + 2 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

C) $1 - 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

D) $1 - 2 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

c) $\cos^2 2A + \cos^2 2B + \cos^2 2C =$

A) $1 + 2 \sin A \sin B \sin C$

B) $1 + 2 \cos A \cos B \cos C$

C) $1 + 2 \sin 2A \sin 2B \sin 2C$

D) $1 + 2 \cos 2A \cos 2B \cos 2C$

2. If $A + B + C = 90^\circ$ then

a) $\Rightarrow \cos^2 A + \cos^2 B + \cos^2 C =$

A) $1 + 2 \sin A \sin B \sin C$

B) $1 + 2 \cos A \cos B \cos C$

c) $2 + 2 \sin A \sin B \sin C$

D) $2 + 2 \cos A \cos B \cos C$

b). $\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C =$

A) $1 + 2 \cos A \cos B \cos C$

B) $1 + 2 \sin A \sin B \sin C$

C) $1 - 2 \sin A \sin B \sin C$

D) $1 - 2 \cos A \cos B \cos C$

VI. Descriptive type questions

1. Prove that $\frac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A} = \tan 4A$

2. Prove that $\frac{\sin A + \sin B}{\sin A - \sin B} = \tan \left(\frac{A+B}{2} \right) \cot \left(\frac{A-B}{2} \right)$

3. In $\triangle ABC$ prove the following identities

a) $\sin^2 2A + \sin^2 2B + \sin^2 2C = 2(1 - \cos 2A \cos 2B \cos 2C)$

b) $\sin 3A + \sin 3B + \sin 3C = -4 \cos \frac{3A}{2} \cos \frac{3B}{2} \cos \frac{3C}{2}$

c) $\cos^2 \frac{A}{2} \cos^2 \frac{B}{2} \cos^2 \frac{C}{2} = 2 + 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

4. Prove that

i) If $a+b+g = p$ then $\sin^2 a + \sin^2 b - \sin^2 g = 2 \sin a \sin b \cos g$

ii) If $a+b+g+d = 2p$ then $\cos 2a + \cos 2b + \cos 2g + \cos 2d = 4 \cos(a+b) \cos(a+g) \cos(a+d)$

5. If $A+B+C = 2S$ then prove that

$\cos(S-A) + \cos(S-B) + \cos(S-C) - \sin S = 4 \cos \left(\frac{S-A}{2} \right) \cos \left(\frac{S-B}{2} \right) \cos \frac{C}{2} - 1$



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I. MCQ'S with single correct answer.

1. $\sin 6\theta - \sin 2\theta =$
 A. $2 \sin 6\theta \cos 4\theta$ B. $2 \cos 4\theta \sin 2\theta$ C. $2 \sin 4\theta \cos 6\theta$ D. $2 \sin 4\theta \cos 2\theta$
2. $(2 \cos^2 3\theta - 1) \cos 5\theta =$
 A. $\frac{1}{2} [\cos 11\theta + \cos \theta]$ B. $\frac{1}{2} [\sin 11\theta + \sin \theta]$
 C. $\frac{1}{2} [\sin 11\theta + \cos \theta]$ D. $\frac{1}{2} [\cos 11\theta + \sin \theta]$
3. $\cos 25^\circ - \cos 65^\circ =$
 A. $\sqrt{2} \cos 20^\circ$ B. $\sqrt{2} \sin 20^\circ$ C. $\sqrt{3} \cos 20^\circ$ D. $\sqrt{3} \sin 20^\circ$
4. $\sin 65^\circ + \sin 25^\circ =$
 A. $\sqrt{2} \cos 20^\circ$ B. $\sqrt{2} \sin 20^\circ$ C. $\sqrt{3} \cos 20^\circ$ D. $\sqrt{3} \sin 20^\circ$
5. $\cos 40^\circ + \cos 80^\circ + \cos 160^\circ =$
 A. 2 B. 1 C. 0 D. 3
6. $\sin 47^\circ - \sin 25^\circ + \sin 61^\circ - \sin 11^\circ =$
 A. $\sin 7^\circ$ B. $2 \cos 7^\circ$ C. $2 \sin 7^\circ$ D. $\cos 7^\circ$
7. $\sin 10^\circ + \sin 20^\circ + \sin 40^\circ + \sin 50^\circ - \sin 70^\circ - \sin 80^\circ =$
 A. $1/2$ B. 0 C. $-1/2$ D. 1
8. $\sin 48^\circ \cdot \sin 12^\circ =$
 A. $\frac{\sqrt{5}+1}{8}$ B. $\frac{1+\sqrt{5}}{8}$ C. $\frac{1-\sqrt{5}}{8}$ D. $\frac{\sqrt{5}-1}{8}$
9. $\cos 66^\circ + \cos 6^\circ =$
 A. $\frac{\sqrt{3}(\sqrt{5}-1)}{4}$ B. $\frac{\sqrt{2}(\sqrt{5}+1)}{4}$ C. $\frac{\sqrt{2}(\sqrt{5}-1)}{4}$ D. $\frac{\sqrt{3}(\sqrt{5}+1)}{4}$

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10. $\sin 24^\circ + \cos 6^\circ =$
 A. $\frac{\sqrt{15} + \sqrt{3}}{4}$ B. $\frac{\sqrt{15} + 3}{4}$ C. $\frac{\sqrt{15} - 3}{4}$ D. $\frac{\sqrt{15} - \sqrt{3}}{4}$
11. $2 \cos \theta - \cos 3\theta - \cos 5\theta - 16 \cos^3 \theta \cdot \sin^2 \theta =$
 A. 2 B. 0 C. 1 D. -1
12. $\cos 10^\circ \cdot \cos 30^\circ \cdot \cos 50^\circ \cdot \cos 70^\circ =$
 A. $3/16$ B. $1/16$ C. $1/8$ D. $1/32$
13. $4(\cos 66^\circ + \sin 84^\circ) =$
 A. $\sqrt{5} - 1$ B. $\sqrt{3}(\sqrt{5} - 1)$ C. $(\sqrt{5} + 1)$ D. $\sqrt{3}(\sqrt{5} + 1)$
14. $\frac{\cos(45^\circ + A) - \cos(45^\circ - A)}{\sin(120^\circ + A) - \sin(120^\circ - A)} =$
 A. 2 B. $\sqrt{2}$ C. $2\sqrt{2}$ D. $\pm\sqrt{2}$
15. $\sin 85^\circ - \sin 35^\circ - \cos 65^\circ$
 A. 0 B. 1 C. 2 D. 3
16. $m \cdot \tan(\theta - 30^\circ) = n \cdot \tan(\theta + 120^\circ), \Rightarrow \cos 2\theta =$
 A. $\frac{m-n}{2(m-n)}$ B. $\frac{m+n}{2(m-n)}$ C. $\frac{m-n}{2(m+n)}$ D. $\frac{m-n}{(m+n)}$
17. $\tan(\alpha + \theta) = n \cdot \tan(\alpha - \theta), \Rightarrow (n+1) \sin 2\theta =$
 A. $(n+1) \sin 2\alpha$ B. $(n+1) \sin 2\beta$ C. $(n-1) \sin 2\alpha$ D. $(n-1) \sin 2\beta$
18. $\cot(15^\circ - A) + \tan(15^\circ + A) =$
 A. $\frac{4 \cos 2A}{1 + 2 \cos 2A}$ B. $\frac{4 \cos 2A}{1 - 2 \sin 2A}$ C. $\frac{4 \cos 2A}{1 + 2 \sin 2A}$ D. $\frac{4 \cos 2A}{1 - 2 \cos 2A}$
19. $A + B + C = 180^\circ \Rightarrow \cos A - \cos B + \cos C =$
 A. $1 + 4 \cos \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$ B. $1 + 4 \sin \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$
 C. $-1 + 4 \cos \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$ D. $-1 - 4 \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$
20. $A + B + C = 180^\circ \Rightarrow \sin A + \sin B + \sin C =$
 A. $2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$ B. $2 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

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- C. $4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$ D. $4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$
21. $A+B+C=90^\circ \Rightarrow \cos 2A + \cos 2B + \cos 2C =$
 A. $1 - 4 \cos A \cos B \cos C$ B. $1 - 4 \sin A \sin B \sin C$
 C. $1 + 4 \cos A \cos B \cos C$ D. $1 + 4 \sin A \sin B \sin C$
22. $A+B+C=90^\circ \Rightarrow \sin 2A + \sin 2B - \sin 2C =$
 A. $2 \cos A \cos B \sin C$ B. $2 \sin A \sin B \sin C$
 C. $4 \cos A \cos B \sin C$ D. $4 \sin A \sin B \cos C$

◆ ■ ◆ ACHIEVERS (Level - II) ◆ ■ ◆

II. Descriptive type questions

- Prove that $\frac{\cos A + \cos B}{\cos B - \cos A} = \cot \frac{A+B}{2} \cot \frac{A-B}{2}$
- If $A+B+C=0^\circ$ then prove that $\sin A + \sin B + \sin C = -4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
- If $A+B+C=2S$ then prove that $\sin(S-A) + \sin(S-B) + \sin(S-C) - \sin S = 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
- If $A+B+C+D=2p$ then prove that $\sin A - \sin B + \sin C - \sin D$
 $= -4 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A+D}{2} \right) \cos \left(\frac{A+D}{2} \right)$
- If $A+B+C=180^\circ$ then prove that $\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1 - 2 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$.

◆ ■ ◆ EXPLORERS (Level - III) ◆ ■ ◆

III. MCQs with one or more than one correct answers

- ◆ This section contains multiple choice questions. Each question has 4 choices (A), (B), (C), (D), out of which **ONE or MORE** is correct. Choose the correct options

- If $x/y = \frac{\cos A}{\cos B}$ where $A \neq B$ then.....
 A) $\tan \left(\frac{A+B}{2} \right) = \frac{x \tan A + y \tan B}{x+y}$ B) $\tan \left(\frac{A-B}{2} \right) = \frac{x \tan A - y \tan B}{x+y}$
 C) $\frac{\sin(A+B)}{\sin(A-B)} = \frac{y \sin A + x \sin B}{y \sin A - x \sin B}$ D) $x \cos A + y \cos B = 0$

IV. Assertion and Reason type Questions

- A : $\frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)} = \tan A$
 R : $\sin(A+B) + \sin(A-B) = \sin A$ and $\cos(A+B) + \cos(A-B) = \cos A$
- A : If $x = \sin(a-b) \sin(g-d)$, $y = \sin(b-g) \sin(a-d)$, $z = \sin(g-a) \sin(b-d)$ then $x+y+z=0$
 R : $2 \sin A \sin B = \cos(A-B) + \cos(A+B)$

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3. A : If $A+B+C = 180^\circ$ then $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$
 R : If $A+B+C = 180^\circ$ then $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$

4. A : If $x + y + z = xyz$ then $\sum \left(\frac{2x}{1-x^2} \right) = \pi \left(\frac{2x}{1-x^2} \right)$

R : If $\tan A + \tan B + \tan C = \tan A \tan B \tan C$, then $A + B + C = n\pi$, $n \in \mathbb{Z}$

V. Match the following:

- ◆ This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column-I** have to be matched with statements (p, q, r, s) in **Column-II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If $\cos a + \cos b = 1/2$ and $\sin a + \sin b = 1/3$

i) $\cos \left(\frac{\alpha + \beta}{2} \right) =$ a) $\pm \frac{\sqrt{13}}{12}$

ii) $\cos \left(\frac{\alpha - \beta}{2} \right) =$ b) $2/3$

iii) $\tan \left(\frac{\alpha + \beta}{2} \right) =$ c) $\pm 3 / \sqrt{13}$

iv) $\tan \left(\frac{\alpha - \beta}{2} \right) =$ d) $\pm \sqrt{\frac{131}{13}}$

A) d, b, a, c B) d, b, c, a C) c, a, b, d D) c, a, d, b

VI. Comprehensive type

- ◆ This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. Choose the correct option.

1. In $\triangle ABC$ if $A+B+C = 180^\circ$ then

- a) $\cos A + \cos B + \cos C =$
 A) $1 + 4 \sin A/2 \sin B/2 \sin C/2$ B) $4 \sin A \sin B \sin C$
 C) $-4 \sin A \sin B \sin C$ D) $1 - 4 \sin A/2 \sin B/2 \sin C/2$
 b) $\sin^2 A + \sin^2 B + \sin^2 C =$
 A) $4 \cos A/2 \cos B/2 \cos C/2$ B) $-4 \cos A/2 \cos B/2 \cos C/2$
 C) $4 \sin A/2 \sin B/2 \sin C/2$ D) $2 + 2 \cos A \cos B \cos C$
 c) $\sin 2A + \sin 2B + \sin 2C$
 A) $2 - 2 \cos A \cos B \cos C$ B) $2 + 2 \cos A \cos B \cos C$
 C) $2 - 2 \sin A \sin B \sin C$ D) $4 \sin A \sin B \sin C$

2. If $A+B+C=270^\circ$ then

- a) $\Rightarrow \cos^2 A + \cos^2 B - \cos^2 C =$
 A) $-2 \sin A \sin B \cos C$ B) $-2 \cos A \cos B \sin C$
 C) $2 \sin A \sin B \cos C$ D) $2 \cos A \cos B \sin C$
 b) $\Rightarrow \sin 2A - \sin 2B + \sin 2C =$
 A) $4 \sin A \cos B \sin C$ B) $4 \cos A \sin B \cos C$
 C) $-4 \sin A \cos B \sin C$ D) $-4 \cos A \sin B \cos C$



RESEARCHERS (Level - IV)



1. $\sin \alpha + \sin \beta = a, \cos \alpha + \cos \beta = b \Rightarrow \sin(\alpha + \beta) =$ **(EAM-2010)**
 1. ab 2. $a+b$ 3. $\frac{2ab}{a^2 - b^2}$ 4. $\frac{2ab}{a^2 + b^2}$
2. $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b} \Rightarrow \frac{\tan x}{\tan y} =$ **(EAM-2011)**
 1. b/a 2. a/b 3. 1 4. 0
3. If $A+B+C = 270^\circ$, then $\cos 2A + \cos 2B + \cos 2C + 4 \sin A \sin B \sin C =$ **(EAM-2013)**
 1) 0 2) 1 3) 2 4) 3
4. $\tan 7 \frac{1}{2}^\circ =$ **(AIEEE-2005)**
 1. $\frac{2\sqrt{2}-1-\sqrt{3}}{\sqrt{3}-1}$ 2. $\frac{1+\sqrt{3}}{1-\sqrt{3}}$ 3. $\frac{1}{\sqrt{3}} + \sqrt{3}$ 4. $\frac{\sqrt{2}+1}{\sqrt{2}}$
5. $A+C = 2B \Rightarrow \frac{\cos C - \cos A}{\sin A - \sin C} =$ **(AIEEE-2010)**
 1) $\cot B$ 2) $\cot 2B$ 3) $\tan 2B$ 4) $\tan B$
6. $\cos \alpha \cdot \sin(\beta - \gamma) + \cos \beta \cdot \sin(\gamma - \alpha) + \cos \gamma \cdot \sin(\alpha - \beta) =$ **(AIEEE-2013)**
 1) 0 2) $1/2$ 3) 1 4) $4 \cos \alpha \cos \beta \cos \gamma$
7. $\cos 6^\circ \cdot \sin 24^\circ \cdot \cos 72^\circ =$ **(EAM-2014)**
 1. $-1/8$ 2. $1/8$ 3. $-1/4$ 4. $1/4$
8. The value of $\cos\left(\frac{2\pi}{15}\right) \cos\left(\frac{4\pi}{15}\right) \cos\left(\frac{8\pi}{15}\right) \cos\left(\frac{14\pi}{15}\right) =$ **(EAM-2006)**
 1) $1/16$ 2) $1/8$ 3) $3/4$ 4) $1/12$
9. $\cos 255^\circ + \sin 165^\circ =$ **(EAM-2005)**
 1. 0 2. $\frac{\sqrt{3}-1}{\sqrt{2}}$ 3. $\frac{\sqrt{3}-1}{2\sqrt{2}}$ 4. $\frac{\sqrt{2}+1}{\sqrt{2}}$
10. $A+B+C = 180^\circ \Rightarrow \sin^2 A + \sin^2 B + \sin^2 C =$ **(EAM-1998)**
 1. $1 + \cos A \cos B \cos C$ 2. $1 + \sin A \sin B \sin C$
 3. $2(1 + \cos A \cos B \cos C)$ 4. $2(1 + \sin A \sin B \sin C)$
11. $\cos(a+b+g) + \cos(a-b-g) + \cos(b-g-a) + \cos(g-a-b) =$ **(EAM-1996)**

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- 1) $2\cos a \cos b \cos c$ 2) $3\cos a \cos b \cos c$
 3) $4\cos a \cos b \cos c$ 4) $6\cos a \cos b \cos c$

12. $\cos A = \frac{3}{4} \Rightarrow 32 \sin\left(\frac{A}{2}\right) \sin\left(\frac{5A}{2}\right) =$
 1) 7 2) 8 3) 13 4) 11

(EAM-2011)



ΦΦ TEACHING TASK

I. MCQ with single correct answer

- I. 1.D 2.A 3.A 4.C 5.A 6.B 7.C 8.A 9.C 10.D 11.A
 12.C 13.D 14.B 15.A 16.A 17.C 18.D 19.D 20.D 21.B 22.C
 23.B 24.C 25.D 26.A 27.A

II. MCQ with one or more than one correct answer :

- II. 1.A,C,D 2.A,B,C,D

III. Assertion and Resoning type questions

- III. 1.A 2.A 3.C 4.A 5.B

IV Match the following

- IV. 1.A

V. Comprehension type questions

- V. 1) a)C b)C c)D
 2) a)C b)C

ΦΦ STUDENT TASK

I. MCQ'S with single correct answer.

- 1.B 2.A 3.B 4.A 5.C 6.D 7.B 8.D 9.D 10.A 11.B
 12.A 13.D 14.B 15.A 16.B 17.C 18.C 19.C 20.D 21.D 22.D

III. MCQs with one or more than one correct answers

- I) 1.A,C

IV. Assertion and Resoning :

- 1.C 2.C 3.A 4.A

III. Match the following :

1. C

V. Comprehensive type:

- 1) a)A b)D c)D 2.a)B b)A

VI. Researchers:

- 1.4 2.2 3.2 4.1 5.4 6.1 7.4
 8.1 9.1 10.3 11.3 12.4