Wst-13 centripetal and centrifugal force

Teaching task

1. The primary force responsible for keeping an object moving in a circular path is the

centripetal force [1].

This force is always directed toward the center of the circular path and acts perpendicular to the object's velocity vector at any given moment [1, 2]. Without a net centripetal force, an object would obey Newton's first law and move in a straight line tangent to the circle

2. The force that appears to throw riders outward in a rotating amusement park ride is commonly referred to as

centrifugal force [1].

It is important to note that centrifugal force is a perceived, or "fictitious," force from the riders' perspective within the rotating frame of reference. The actual physical force at play is **centripetal force**.

Here is how it works:

1. **Inertia:** According to Newton's First Law of Motion, the riders' natural tendency (due to inertia) is to continue moving in a straight line
2. **Centripetal Force:** A real, inward force (the centripetal force) is exerted by the ride's structure (like the walls or seats) on the riders, constantly pulling them toward the center of rotation and forcing them to move in a circle
3. **Perception of Centrifugal Force:** As the riders resist this change in direction and push against the side of the ride due to their inertia, they feel an apparent outward push—this is the perceived centrifugal force [2]. The ride exerts an inward force on them, and they exert an equal and opposite outward force on the ride structure

- 3. Centripetal force is defined as a force that makes a body follow a curved path and is always directed towards the center of the circle.
- According to Newton's second law, a net force causes acceleration ($\vec{F}=m\vec{a}$). For an object to move in a circle, its velocity direction must constantly change, which requires an inward acceleration. The centripetal force is this net force that causes the necessary **inward acceleration**.

Why other options are incorrect

- **A) It acts outward from the center of rotation**: This is incorrect. Centripetal force acts *inward* towards the center. A force acting outward is called a centrifugal force, which is a fictitious (pseudo) force observed from a rotating frame of reference.
- **B) It is always equal to the centrifugal force**: This is misleading. While they have the same magnitude ($F=mv2/r$) and act in opposite directions, they are not an action-reaction pair as they typically act in different frames of reference or on different bodies, and centripetal force is a real force while centrifugal force is a fictitious force.
- **D) It is not related to the speed of the object**: This is incorrect. The magnitude of centripetal force is directly proportional to the square of the object's speed, as shown in the formula $Fc=mv^2/r$

. Higher speeds require greater centripetal force to maintain the same circular path

4. The centrifugal force acts in a direction **away from the center of the circular path**. It is an apparent or pseudo force that is perceived by an observer in a rotating (non-inertial) frame of reference and is equal in magnitude and opposite in direction to the real centripetal force, which pulls an object toward the center.

- **Direction**: Outward, away from the center of the circle.

5.

If the speed of an object in circular motion is doubled, the required centripetal force becomes four times greater, assuming the radius remains constant. This is because the centripetal force is proportional to the square of the velocity (v^2), so doubling the velocity ($2v$) results in $(2v)^2 = 4v^2$.

- **Original centripetal force**: $F_c = \frac{mv^2}{r}$
- **New centripetal force**: $F_{c,new} = \frac{m(2v)^2}{r} = \frac{m(4v^2)}{r} = 4 \times \frac{mv^2}{r}$

6.

8.

The centripetal force (F_c) acting on an object moving in a circle is given by the formula $F_c = \frac{mv^2}{r}$, where m is the mass, v is the speed, and r is the radius. We are given $m = 2\ \text{kg}$, $v = 3\ \text{m/s}$, and $r = 4\ \text{m}$.
Substituting the values into the formula yields:

$$F_c = \frac{2\ \text{kg} \times (3\ \text{m/s})^2}{4\ \text{m}}$$

$$F_c = \frac{2 \times 9}{4}\ \text{N}$$

$$F_c = 4.5\ \text{N}$$

7.

$$v = 72\ \text{km/h} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{h}}{3600\ \text{s}}$$

$$v = \frac{72000}{3600}\ \text{m/s}$$

$$v = 20\ \text{m/s}$$

The centripetal force (F_c) is calculated using the formula $F_c = \frac{mv^2}{r}$, where m is the mass (1000 kg), v is the speed (20 m/s), and r is the radius (40 m).
Substituting the values into the formula yields:

$$F_c = \frac{1000\ \text{kg} \times (20\ \text{m/s})^2}{40\ \text{m}}$$

$$F_c = \frac{1000 \times 400}{40}\ \text{N}$$

$$F_c = 10000\ \text{N}$$

8.

- **Initial Force:** $F_1 = \frac{mv_1^2}{r}$
- **Final Force:** The new force F_2 is 4 times the initial force, so $F_2 = 4F_1$.
- **New Velocity:** We want to find the new velocity v_2. The new force equation $F_2 = \frac{mv_2^2}{r}$.
- **Substitute and Solve:**
  $4F_1 = \frac{mv_2^2}{r}$
  $4\left(\frac{mv_1^2}{r}\right) = \frac{mv_2^2}{r}$
  $4v_1^2 = v_2^2$
  $v_2 = \sqrt{4v_1^2}$
  $v_2 = 2v_1$

9.

$$T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

Given values are $L = 1\ \text{m}$, $\theta = 60°$, and g is assumed to be $9.8\ \text{m/s}^2$.

Step 2: Substitute values and calculate

Substitute the given values into the formula:

$$T = 2\pi\sqrt{\frac{1\ \text{m} \times \cos(60°)}{9.8\ \text{m/s}^2}}$$

Since $\cos(60°) = 0.5$:

$$T = 2\pi\sqrt{\frac{1\ \text{m} \times 0.5}{9.8\ \text{m/s}^2}} = 2\pi\sqrt{\frac{0.5}{9.8}\ \text{s}^2}$$

$$T \approx 2\pi\sqrt{0.05102}\ \text{s} \approx 2\pi \times 0.2259\ \text{s} \approx 1.419\ \text{s}$$

10.

$$T = \frac{mg}{\cos(\theta)}$$

Step 3: Substitute values and calculate

Given values:

- Mass $m = 0.3\,\text{kg}$
- Angle $\theta = 45°$
- Acceleration due to gravity $g = 9.8\,\text{m/s}^2$

Substitute these values into the formula:

$$T = \frac{0.3\,\text{kg} \times 9.8\,\text{m/s}^2}{\cos(45°)}$$

$$T = \frac{2.94\,\text{N}}{\frac{1}{\sqrt{2}}}$$

$$T = 2.94\,\text{N} \times \sqrt{2} \approx 4.16\,\text{N}$$

11.

Both statements are correct.

 Velocity is a vector, and its direction changes continuously as the particle moves along the circular path, even if the magnitude (speed) is constant. Since the direction is changing, the velocity is changing.

 The acceleration is also changing because, while the magnitude of centripetal acceleration (v^2 /r) is constant, its direction is always towards the center of the circle. As the particle moves, the direction of this centripetal acceleration vector changes.

True. The formula for centripetal acceleration can be expressed in terms of angular velocity (ω) as $a_c = r\omega^2$, where r is the radius of the circle. This shows a direct dependence.

12.

Both statements are true, and Statement-II is the correct explanation for Statement-I. The work done by the centripetal force is zero because this force is always perpendicular to the object's instantaneous displacement in circular motion, and the work done is calculated as

$W = Fd\cos\theta$; $\cos(90°) = 0$

Statement-II is true: In circular motion, the centripetal force points toward the center of the circle, while the object's displacement is tangential to the circle. This means the force and displacement are always perpendicular

13.

- **A) In magnitude centripetal force = centrifugal force:** The centripetal force (a real force in an inertial frame of reference, directed towards the center) and the centrifugal force (a fictitious, or pseudo, force in a non-inertial rotating frame of reference, directed away from the center) have equal magnitudes but act in opposite directions. They are not an action-reaction pair because they do not act on the same body.
- **B) Work done by the centripetal force is zero:** Centripetal force is always directed perpendicular to the direction of motion (velocity vector) in uniform circular motion. The definition of work done in physics is force multiplied by the distance moved in the direction of the force ($W = F \cdot d \cdot \cos\theta$). Since the angle between the centripetal force and the direction of motion (θ) is 90 degrees, and $\cos(90^\circ) = 0$, the work done by the centripetal force is zero.
- **C) The gravitational force of attraction between earth and sun provides centripetal force:** The gravitational pull of the Sun on the Earth provides the necessary inward force to keep the Earth in its orbit, acting as the centripetal force. This force prevents the Earth from moving in a straight line, as predicted by Newton's first law of motion.

LEARNER TASK

CUQ'S

8.

Water does not fall from a bucket rotated in a vertical circle because

its inertia (tendency to continue in a straight line) keeps it moving in a circular path, and the weight of the water provides the necessary centripetal force to maintain this motion. At the top of the circle, the required centripetal force is directed downward towards the center of the circle, and the combined forces of gravity and the normal force from the bucket's bottom meet this requirement. As long as the speed is sufficient, the water's inertia keeps it pressed against the bottom of the bucket, preventing it from falling out.

- Inertia: Water naturally wants to move in a straight line, but the rotating bucket constantly forces it into a circular path.
- Centripetal Force: This is the inward force needed to keep the water moving in a circle. At the top of the circle, gravity is pulling the water down, and the normal force from the bucket (if sufficient) pushes it up, and the combination of these provides the necessary centripetal force.

9.

If the radius of the circular path is doubled while the speed and mass remain constant, the required centripetal force is **halved**. This is because the centripetal force formula is inversely proportional to the radius, so a larger radius requires less force to maintain the same speed.

- **Formula:** The formula for centripetal force (F_c) is $F_c = \frac{mv^2}{r}$, where m is mass, v is velocity, and r is the radius.
- **New radius:** If the radius is doubled, the new radius is $2r$.
- **New force:** The new centripetal force (F_c') is calculated as $F_c' = \frac{mv^2}{2r}$.
- **Comparison:** You can rewrite the new force as $F_c' = \frac{1}{2} \times \frac{mv^2}{r}$.
- **Conclusion:** Since the original force is $F_c = \frac{mv^2}{r}$, the new force is exactly half of the original force, or $F_c' = \frac{1}{2} F_c$.

10.

The centripetal force for a car making a turn is provided by the **force of friction between the tires and the road surface**. This friction acts sideways, pulling the car towards the center of the circular path and preventing it from skidding outwards.

- **How it works:** When you turn the steering wheel, the tires are angled, but the car's inertia wants to continue moving in a straight line. The friction between the tires and the road is the force that counteracts this inertia, pushing the car inward to follow the curved path.

- **Why it's friction:** On a level road, gravity and the normal force from the road are vertical and cancel each other out. There are no other significant forces pulling the car toward the center of the turn, so friction is the only force available to provide the necessary centripetal force.

JEE MAINS LEVEL

1.

$$F_c = \frac{mv^2}{r}$$

Step 2: Calculate the Centripetal Force

Substitute the provided values into the formula to calculate the magnitude of the centripetal force:

$$F_c = \frac{3 \times 4^2}{2}$$

$$F_c = \frac{3 \times 16}{2}$$

$$F_c = \frac{48}{2}$$

$$F_c = 24$$

2.

$$v = 72\,\text{km/h} \times \frac{1000\,\text{m}}{3600\,\text{s}} = 20\,\text{m/s}$$

Step 2: Calculate Centripetal Force

Next, use the formula for centripetal force, $F_c = \frac{mv^2}{r}$, where m is mass, v is velocity, and r is the radius of the bend.

$$F_c = \frac{1500\,\text{kg} \times (20\,\text{m/s})^2}{50\,\text{m}}$$

$$F_c = \frac{1500\,\text{kg} \times 400\,\text{m}^2/\text{s}^2}{50\,\text{m}}$$

$$F_c = \frac{600000\,\text{N} \cdot \text{m}}{50\,\text{m}}$$

$$F_c = 12000\,\text{N}$$

3.

If the centripetal force quadruples, the velocity of the body **doubles**. This is because centripetal force is proportional to the square of the velocity, so if the force increases by a factor of four, the velocity must have increased by a factor of two ($\sqrt{4} = 2$).

- The formula for centripetal force (F_c) is: $F_c = \frac{mv^2}{r}$, where m is mass, v is velocity, and r is the radius.
- If the force is quadrupled, the new force (F_c') is $4 \times F_c$.
- So, $4 \times \frac{mv^2}{r} = \frac{m(v')^2}{r}$, where v' is the new velocity.
- This simplifies to $4v^2 = (v')^2$.
- Taking the square root of both sides gives $2v = v'$.

4.

The formula relating the radius, length, and angle is $\sin(\theta) = \frac{R}{L}$. We are given $R = 0.4 \text{ m}$ and $L = 0.8 \text{ m}$.

$$\sin(\theta) = \frac{0.4 \text{ m}}{0.8 \text{ m}} = 0.5$$

Step 3: Calculate the angle

To find the angle θ, we take the inverse sine (arcsin) of the value calculated in Step 2.

$$\theta = \arcsin(0.5)$$

This value corresponds to an angle of **30 degrees**.

5.

- Angle with the vertical, $\theta = 60^\circ$
- Acceleration due to gravity (assumed), $g = 9.8\ \text{m/s}^2$

The formula for the period of a conical pendulum is $T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$.

Step 2: Substitute the values and calculate the period

Substitute the values into the formula:

$$T = 2\pi\sqrt{\frac{1\ \text{m} \times \cos(60^\circ)}{9.8\ \text{m/s}^2}}$$

Since $\cos(60^\circ) = 0.5$:

$$T = 2\pi\sqrt{\frac{1 \times 0.5}{9.8}}$$

$$T = 2\pi\sqrt{\frac{0.5}{9.8}}$$

$$T = 2\pi\sqrt{0.05102}$$

$$T \approx 2\pi \times 0.2259$$

$$T \approx 1.42\ \text{s}$$

6.

If the speed of an object in circular motion is doubled, the required centripetal force increases by a factor of four, assuming the radius of the circle remains constant. This is because centripetal force is directly proportional to the square of the velocity ($F \propto v^2$).

- **Formula:** The centripetal force (F) is calculated using the formula $F = \frac{mv^2}{r}$, where m is the mass, v is the velocity, and r is the radius of the circular path.
- **Doubling the speed:** When the speed (v) is doubled, it becomes $2v$.
- **New force:** The new force (F_{new}) can be calculated by substituting $2v$ into the formula: $F_{new} = \frac{m(2v)^2}{r}$.
- **Result:** Squaring the term ($2v$) gives $4v^2$, so the formula becomes $F_{new} = \frac{m(4v^2)}{r}$. This can be rewritten as $F_{new} = 4 \times \frac{mv^2}{r}$.
- **Conclusion:** Since $\frac{mv^2}{r}$ is the original centripetal force, the new force is four times the original force.

7.

 Centrifugal force is classified as a **fictitious** or **pseudo-force**.

 It is not a true force that arises from a physical interaction (like gravity or electromagnetism) but rather an apparent force experienced by an observer in a non-inertial (accelerating or rotating) frame of reference.

 The effects of centrifugal force (such as an object seeming to be pushed outward when a car turns) are real, but the force itself is a consequence of inertia and the observer's accelerating frame of reference.

-

Why other options are incorrect

- **A) It is a real force acting outward from the center of rotation.** This is incorrect because centrifugal force is a pseudo-force, not a real force. While it appears to act outward, it is a perceived effect in a rotating frame.
- **B) It is a force acting inward toward the center of rotation.** This describes the **centripetal force**, which is a real force necessary to maintain circular motion, directed toward the center of the circle.

- **D) It is independent of the speed of the object.** This is incorrect. The magnitude of the centrifugal force is dependent on the speed (or angular velocity) of the object and the radius of the circular path, specifically proportional to the square of the speed or angular velocity

8.

The formula shows that the force is **inversely proportional** to the radius (are kept constant.
Therefore, if the radius of the circular path increases, the centrifugal force decreases.

Why other options are incorrect

- **A) It increases**: This would be the case if the angular velocity (RPM) were constant, but not if the linear speed is constant.
- **C) It remains the same**: The force changes as the radius changes, so it cannot remain the same.
- **D) It depends on the speed of the object**: While the force does depend on speed (it's proportional to the square of the speed), the question asks what happens when the radius changes. Assuming other variables (like speed) are held constant, a clear relationship between force and radius exists.
- **9**.

An object in motion naturally tends to move in a straight line at a constant velocity, as stated by Newton's first law of motion. For it to follow a curved or circular path, a net force must continuously act upon it to change its direction.

- This center-seeking force, directed toward the center of the curve or circle, is called the **centripetal force** (from Latin *centrum* "center" and *petere* "to seek").
- The centripetal force is not a new, fundamental type of force itself, but rather a role that an existing, real force (such as gravity, tension, or friction) can play in causing circular motion. For example, the tension in a string provides the centripetal force when swinging a ball in a circle, and the Earth's gravity provides the centripetal force that keeps the Moon in orbit.

10.

Centrifugal force is classified as a **fictitious** or **pseudo-force**. It is not a real force arising from a physical interaction (like gravity or tension) but rather an apparent force that appears to act on an object when it is viewed from a **non-inertial (accelerating or rotating) frame of reference**. It results from an object's **inertia**, its natural tendency to continue moving in a straight line at a constant speed, as described by Newton's first law.

11.

 Statement I: False. The centripetal force is proportional to the square of the speed, so doubling the speed quadruples the force. Doubling the radius doubles the force. The combined effect of doubling both is 8 times the original force, not double.

 Statement II: False. Centripetal force is directly proportional to the square of the speed, not directly proportional to the speed itself

12.

 Statement I: True

- In uniform circular motion, the acceleration is always directed towards the center of the circle, which is known as centripetal acceleration.
- This is because the body's velocity is constantly changing direction as it moves along the circle, and a force (and therefore acceleration) is needed to continuously change the direction of the velocity vector.

 Statement II: False

- The acceleration in uniform circular motion is *only* directed towards the center; there is no component of acceleration that changes the speed.
- If an acceleration were to change the speed of the body, it would mean the motion was no longer uniform, as "uniform" implies constant speed.
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    -

13. **Option A is correct**: The process used to separate cream from milk is called **centrifugation**, which uses a centrifuge machine. This process works because the denser skim milk particles are forced outward, while the lighter cream particles gather near the center.

 Option C is correct: Centrifugal force is considered a **pseudo force** (or fictitious force) because it arises from the inertia of an object in a rotating, non-inertial frame of reference, rather than a real physical interaction.

-

Why other options are incorrect

- **B) centrifugal force acts towards the centre of circular path**: This statement is incorrect. Centrifugal force acts *outward*, away from the center of rotation. The force that acts *towards* the center is called **centripetal force**, which is the real force responsible for maintaining circular motion.
- **D) None of the above**: This option is incorrect because both options A and C are correct statements