Exercise 4.3

Q.1) Find the roots of the following quadratic equations if they exist, by the method of completing the square.

i) $2x^2-7x+3=0$.

Method of completing the square means, equation should contain something in the form of $(a+b)^2$ + something $=0$

or $(a-b)^2$ + something $=0$.

consider $2x^2-7x+3=0$.

(Here in place of this equation we make make a square term of type $(a-b)^2$, because of '−' sign in between $2x^2$ and $7x$)

$2x^2$ cannot be perfect square.

So, divide complete eqn by 2, then eqn becomes

$$x^2-\frac{7x}{2}+3/2=0.$$

In place of $(a-b)^2$

$=a^2-2ab+b^2$

$a=x$.

$$x^2-(2)\left(\frac{7}{4}\right)x+3/2=0$$

Here $b=7/4$.

So, we need b^2.

$$x^2-(2)(7/4)x+(7/4)^2-(7/4)^2+3/2=0$$

We have to add $(7/4)^2$ because we need b^2 and we have to subtract it to balance the eqn.

$$\underbrace{x^2 - 2\left(\tfrac{7}{4}\right)(x) + \left(\tfrac{7}{4}\right)^2} - \left(\tfrac{7}{4}\right)^2 + \tfrac{3}{2} = 0.$$

till here it is in the form of $(a-b)^2$
where $a = x$, $b = 7/4$. Now,

$$\left(x - \tfrac{7}{4}\right)^2 - \left(\tfrac{7}{4}\right)^2 + \tfrac{3}{2} = 0$$

$$\left(x - \tfrac{7}{4}\right)^2 - \frac{49}{16} + \tfrac{3}{2} = 0$$

$$\left(x - \tfrac{7}{4}\right)^2 = \tfrac{49}{16} - \tfrac{3}{2}$$

$$\left(x - \tfrac{7}{4}\right)^2 = \frac{49 - 24}{16}$$

$$\left(x - \tfrac{7}{4}\right)^2 = \frac{25}{16}$$

$$x - \tfrac{7}{4} = \sqrt{\tfrac{25}{16}}$$

$$x - \tfrac{7}{4} = \pm \tfrac{5}{4}$$

If $x - \tfrac{7}{4} = \tfrac{5}{4}$ (or) $x - \tfrac{7}{4} = -\tfrac{5}{4}$

$x = \tfrac{12}{4}$

$\boxed{x = 3}$ (or) $x = \dfrac{-5+7}{4} = \tfrac{2}{4} = \tfrac{1}{2}$

$\boxed{x = \tfrac{1}{2}}$

So, $\therefore x = 3$ or $\tfrac{1}{2}$

ii) $2x^2+x-4=0$

Here also divide eqn by 2.

$x^2+x/2-2=0$. Here x^2 is now a perfect square.

$x^2+(2)\left(\frac{x}{4}\right)-2=0$ we can make $(a+b)^2$ + something $=0$.

$x^2+2(x/4)+(1/4)^2-(1/4)^2-2=0$ where $a=x$.

$(a+b)^2=a^2+2ab+b^2$

$(x+1/4)^2-(1/4)^2-2=0$ Here $2ab=2(x/4)$

$b=1/4$.

$(x+1/4)^2-1/16-2=0$

$(x+1/4)^2=1/16+2$

$(x+1/4)^2=\frac{33}{16}$

$x+1/4=\frac{\pm\sqrt{33}}{4}$

$x=+\frac{\sqrt{33}}{4}-1/4$ or $x=-\frac{\sqrt{33}}{4}-1/4$

$x=\boxed{\frac{\sqrt{33}-1}{4}}$ or $x=\boxed{\frac{-\sqrt{33}-1}{4}}$

iii) $4x^2+4\sqrt{3}x+3=0$

Here $4x^2$ can be written as $(2x)^2$

$(2x)^2+4\sqrt{3}x+3=0.$

$(2x^2)+(2)(2x)(\sqrt{3})+3=0$

for b^2 add and subtract $(\sqrt{3})^2$

$(2x)^2+2(2x)(\sqrt{3})+(\sqrt{3})^2-(\sqrt{3})^2+3=0$

$\underbrace{(2x)^2+4\sqrt{3}x+(\sqrt{3})^2}-\cancel{3}+\cancel{3}=0$

Here for $(a+b)^2$
$=a^2+2ab+b^2$
we have
$a=2x$
$2ab=4\sqrt{3}x$
$2(2x)b=4\sqrt{3}x$
$4xb=4\sqrt{3}x$
$b=\sqrt{3}.$

$(2x+\sqrt{3})^2=0.$

$2x+\sqrt{3}=0$

$2x=-\sqrt{3}$

$\boxed{x=-\sqrt{3}/2}$

So, $x=-\sqrt{3}/2, -\sqrt{3}/2$

Both the roots are same.

iv. $2x^2+x+4=0$

To obtain $2x^2$ as perfect square multiply the complete eqn. by 2.

$4x^2+2x+8=0$.

$(2x)^2+2x+8=0$

$\cancel{(2x)^2+(2)\frac{(2x)}{2}\left(\frac{1}{2}\right)}+$

Here the square can be obtained in the form $(a+b)^2$
$=a^2+2ab+b^2$.
where $a=2x$
$2ab = 2x$
$2(2x)b=2x$
$b=\frac{2x}{4x}=\frac{1}{2}$.

$(2x)^2+(2)(2x)(1/2)+8=0$.

We need b^2 also.

So, we need $(1/2)^2$

So, add and subtract $(1/2)^2$, to balance the equation.

$$\underbrace{(2x)^2+(2)(2x)(1/2)+(1/2)^2}-(1/2)^2+8=0$$

$$(2x+1/2)^2 - 1/4+8=0$$

$$(2x+1/2)^2 = 1/4-8$$

$$(2x+1/2)^2 = -31/4$$

$$2x+\frac{1}{2}=\sqrt{-31/4}$$

• Here root of a negative number does not exist.

So, no real root for this equation.

Question-2

Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

A) i, $2x^2 - 7x + 3 = 0$.

compare this with $ax^2 + bx + c = 0$

So, $a = 2, b = -7, c = 3$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\Rightarrow \frac{7 \pm \sqrt{25}}{4} \Rightarrow \frac{7 \pm 5}{2}$$

$$x = \frac{7+5}{4}, \frac{7-5}{4}$$

$$x = \frac{12}{4}, \frac{2}{4}$$

$\boxed{x = 3, \frac{1}{2}}$ Roots of this equation.

ii, $2x^2+x-4=0$

Here if you compare this equation with $ax^2+bx+c=0$.

$$a=2,\ b=1,\ c=-4$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-1\pm\sqrt{(1)^2-4(2)(-4)}}{2a}$$

$$x=\frac{-1\pm\sqrt{1+32}}{2(2)}$$

$$x=\frac{-1\pm\sqrt{33}}{4}$$

So, $x=\boxed{\frac{-1+\sqrt{33}}{4}}$ or $\boxed{\frac{-1-\sqrt{33}}{4}}$

roots of this equation.

iii, $4x^2 + 4\sqrt{3}x + 3 = 0$

compare with $ax^2 + bx + c = 0$.

$a = 4,\ b = 4\sqrt{3},\ c = 3.$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4\sqrt{3} \pm \sqrt{(4\sqrt{3})^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{-4\sqrt{3} \pm \sqrt{48 - 48}}{8}$$

$$x = \frac{-4\sqrt{3} \pm \sqrt{0}}{8}$$

$$x = -\frac{\cancel{4}\sqrt{3}}{\cancel{8}\ 2}$$

$x = -\sqrt{3}/2,\ -\sqrt{3}/2$ Equal roots for this equation.

iv) $2x^2+x+4=0$.

compare with $ax^2+bx+c=0$

here $a=2,\ b=1,\ c=4$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2-4(2)(4)}}{2a}$$

$$x\ \frac{-1 \pm \sqrt{1-32}}{2(2)}$$

$$x \Rightarrow \frac{-1 \pm \sqrt{-31}}{4}$$

Here $\sqrt{-31}$ does not exist, square root of negative number does not exist.

So, ~~for~~ this equation, there are no real roots.

Scanned with CamScanner

Question-3

Find the roots of the following equations:-

i) $x - 1/x = 3, x \neq 0$

A) You can do this by any Method.
first form a quadratic equation.

$$x - 1/x = 3.$$

$$\frac{x^2-1}{x} = 3 \Rightarrow x^2 - 1 = 3x \Rightarrow \boxed{x^2 - 3x - 1 = 0}$$

this quadratic equation is difficult to be solved by the method of factorisation.
So, it is better, if we use the formula.

$x^2 - 3x - 1 = 0$ compare to $ax^2 + bx + c = 0$

$$a = 1, b = -3, c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - (4)(1)(-1)}}{2(1)}$$

$$\Rightarrow \frac{3 \pm \sqrt{9+4}}{2}$$

$$\Rightarrow \frac{3 \pm \sqrt{13}}{2}$$

So, $x = \boxed{\frac{3+\sqrt{13}}{2}}$ or $\boxed{\frac{3-\sqrt{13}}{2}}$

ii, $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

first form a quadratic equation.

$$\frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\frac{x-7-x-4}{x^2+4x-7x-28} = 11/30$$

$$\frac{-\cancel{11}\,1}{x^2-3x-28} = \cancel{11}/30$$

$$-30 = x^2-3x-28$$

$$0 = x^2-3x-28+30$$

$$0 = \boxed{x^2-3x+2}$$

for this quadratic equation, factorisation method can be used

$$x^2-3x+2=0. \qquad 2=-2\times-1$$

$$x^2-2x-x+2=0$$

$$x(x-2)-1(x-2)=0$$

$$(x-1)(x-2)=0$$

$$x-1=0 \text{ or } x-2=0$$

So, $x=1$ or $x=2$

$x=1,2$ are the roots of this eqn.

Question-4 :-

The sum of reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now

A) Let the present age of Rehman be $\boxed{x}$ years.

Rehman's age three years ago, $\boxed{x-3}$ yrs.

Rehman's age, five years hence, $\boxed{x+5}$ yrs.

Here, sum

Question-4 :-

The sum of reciprocals of Rehman's ages (in yrs) 3 years ago and 5 years from now is $1/3$.

Find his present age.

A) Assume Rehman's present age $= x$ yrs.

3 years ago, Rehman's age $= x-3$ yrs.

5 years hence, Rehman's age $= x+5$ yrs.

given $\frac{1}{x-3} + \frac{1}{x+5} = 1/3$ (form a quadratic equation here)

$$\frac{x+5+x-3}{(x-3)(x+5)} = 1/3 \Rightarrow \frac{2x+2}{x^2+5x-3x-15} = 1/3$$

$$\Rightarrow \frac{2x+2}{x^2+2x-15} = 1/3$$

$$\Rightarrow 6x+6 = x^2+2x-15$$

$$\Rightarrow 0 = x^2-4x-21$$

$$\Rightarrow x^2-4x-21=0.$$

You can factorise this equation.

$x^2 - 4x - 21 = 0$

$-21 = -3 \times 7$

$= \boxed{-7 \times 3}$

$x^2 - 7x + 3x - 21 = 0$

$x(x-7) + 3(x-7) = 0$

$(x+3)(x-7) = 0$

$x = -3$ or 7

age cannot be ~~3~~ negative

So, (x cannot be -3)

$\therefore x = 7.$

Rehman's present age = 7 years.

Question-5

In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in two subjects.

A) Assume Shefali's Maths marks as x.

So, English marks would be $30-x$.

Because, their sum is 30.

2 marks more in maths $\Rightarrow (x+2)$

3 marks less in English $\Rightarrow (30-x-3) \Rightarrow (27-x)$

product now = 210.

$$(x+2)(27-x)=210$$

$$27x+54-x^2-2x=210$$

$$-x^2+25x+54-210=0$$

$$-x^2+25x-156=0$$

$$-(x^2-25x+156)=0$$

$$x^2-25x+156=0.$$

$$x^2-12x-13x+156=0.$$

$$x(x-12)-13(x-12)=0$$

$$(x-12)(x-13)=0$$

$$x=12, 13.$$

156

$= -12 \times -13.$

If $x=12$,

Maths $=12$

English $=18$.

If $x=13$,

Maths $=13$

English $=17$

Question-6 :-

The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

A) Let the shorter side, breadth be x m.

larger side or length = $(x+30)$ m.

Given diagonal is 60 metres more than shorter side $\Rightarrow x+60$ m.

diagonal of a rectangle formula,

$$\sqrt{(x)^2+(x+30)^2} = x+60.$$

$$\sqrt{x^2+x^2+60x+900} = x+60$$

$$\sqrt{2x^2+60x+900} = x+60.$$

$$2x^2+60x+900 = (x+60)^2$$

$$2x^2+60x+900 = x^2+120x+3600$$

$$x^2-60x-2700=0.$$

$$x^2-90x+30x-2700=0.$$

$$x(x-90)+30(x-90)=0$$

$$(x-90)(x+30)=0.$$

$$x=90 \text{ or } -30.$$

breadth or length cannot be negative
So, take $x = 90$.

$\therefore$ Breadth $= 90$ m

length, $x+30 = 90+30 = 120$ m

Question-7:-

The difference of squares of two numbers is 180. The square of the smaller number is 8 time times the larger number. Find the two numbers.

A) let the larger number be x.
Smaller number be y.

given difference in squares of these numbers is 180. So, $\boxed{x^2 - y^2 = 180}$ —①

given square of smaller number is 8 times the larger number. So, $\boxed{y^2 = 8x}$ —②

~~put~~ substitute y^2 value in eqn ②, in eqn ①.

So, $x^2 - 8x = 180$

$x^2 - 8x - 180 = 0$

$x^2 - 18x + 10x - 180 = 0$

$x(x-18) + 10(x-18) = 0$.

$(x-18)(x+10) = 0$.

$x = 18$ or -10.

x cannot be negative because, if x is negative, $8x$ will also be negative, So, y^2 will also be negative, which is not possible. So, $\boxed{x = 18}$ → larger number.

$y^2 = 8x$

$y^2 = 8(18)$

$y^2 = 144 \quad y = \sqrt{144} = \pm 12$

So, larger number is 18, Smaller number = 12

or

larger number = 18, Smaller number = -12

Question-8:-

A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

A) let speed be s km/hr., distance = 360 km.

~~Origin~~ Actual time, $t_1 = \frac{\text{distance}}{\text{speed}} = \frac{360}{s}$

If speed is 5 km/h more, $(s+5)$,

then time $t_2 = \frac{360}{s+5}$

given difference is 1 hour between these times. $t_1 - t_2 = 1$ hr.

$$\frac{360}{s} - \frac{360}{s+5} = 1$$

$$\frac{\cancel{360s} + 1800 - \cancel{360s}}{s^2 + 5s} = 1$$

$$1800 = s^2 + 5s$$

$s^2+5s=1800$

$s^2+5s-1800=0$

$s^2+45s-40s-1800=0$

$s(s+45)-40(s+45)=0$

$(s-40)(s+45)=0$

$s=40$ or $\cancel{-45}$

(speed cannot be negative)

So, speed of the train, $s=40$ Kmph.

Question-9 :-

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Assume time taken by smaller one as x ~~hours~~ hours.

time taken by larger one $\Rightarrow x-10$ hours.

In 1 hr, part filled by smaller one $= 1/x$

In 1 hr, part filled by larger one $= 1/(x-10)$

Both together take $9\frac{3}{8}$ hrs $= 75/8$ hours.

In 1 hr, part filled by both $\Rightarrow \dfrac{1}{75/8} = 8/75$

Part filled by Smaller pipe in 1 hr + Part filled by larger pipe in 1 hr = Part filled by both in 1 hr.

Scanned with CamScanner

$$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\frac{x + -10 + x}{x(x-10)} = 8/75$$

$$\frac{2x-10}{x^2-10x} = 8/75$$

$$150x - 750 = 8x^2 - 80x$$

$$-8x^2 + 230x - 750 = 0$$

6000

$$-(8x^2 - 230x + 750) = 0.$$

$\Rightarrow -200x - 30$

$$8x^2 - 230x + 750 = 0$$

$$8x^2 - 200x - 30x + 750 = 0$$

$$8x(x-25) - 30(x-25) = 0$$

$$(8x-30)(x-25)$$

$x = 25$ or $x = 30/8$

If x is $30/8 = 3.75$

time taken by larger pipe $x-10$ hrs

$= 3.75 - 10$ will be negative.

So, x cannot be $30/8$.

$\therefore x = 25$ (Time taken by smaller pipe to fill the tank, separately)

$x - 10 = 15$ (Time taken by larger pipe to fill the tank, separately).

Question-10 :-

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the avg. speeds of the express train is 11 km/h more than that of the passenger train, find the average speed of two trains.

~~Yott~~ Ans) You know Average speed $= \frac{\text{Total dist.}}{\text{Total time}}$

let Average speed of passenger train be x km/hr.

Average speed of express train $\Rightarrow (x+11)$ km/hr.

Total time $= \frac{\text{Total dist}}{\text{Avg. speed}}$

$t_{\text{passenger train}} = \frac{132}{x}$, $t_{\text{express train}} = \frac{132}{x+11}$.

given $\boxed{\frac{132}{x} - \frac{132}{x+11} = 1}$

Because time taken by express train is 1 hr less than that of passenger train.

$$\frac{132(x+11) - 132x}{x(x+11)} = 1$$

$$\cancel{132x} + 1452 - \cancel{132x} = x^2 + 11x$$

$$1452 = x^2 + 11x.$$

$$-x^2 - 11x + 1452 = 0$$

$$-(x^2 + 11x - 1452) = 0$$

$$\underbrace{x^2 + 11x - 1452}_{} = 0$$

-1452

$\Rightarrow -44 \times 33$

or -33×44.

$x^2 - 33x + 44x - 1452 = 0$

$x(x-33) + 44(x-33) = 0$

$(x-33)(x+44) = 0$

$x = 33$ or -44 (Avg speed can't be negative)

→ Avg. speed of passenger train, x $= 33$ kmph.

Avg. speed of express train, $x+11$ $= 33 + 11 = 44$ kmph.

Question-11) Sum of areas of two squares is $468\ m^2$. If the difference of their perimeters is 24 m, find the sides of two squares.

A) let side of 1^{st} square be x m, other be y m.

Perimeters will be $4x$ and $4y$ respectively.

given, $4x - 4y = 24$ [If I consider $x > y$]

$4(x-y) = 24$

$x - y = 6$, $x = (6+y)$ m.

given sum of areas of squares $= 468\ m^2$.

$x^2 + y^2 = 468$

$(6+y)^2 + y^2 = 468$

$36 + y^2 + 12y + y^2 = 468$.

$2y^2 + 12y - 432 = 0$

$2(y^2 + 6y - 216) = 0 \Rightarrow y^2 + 6y - 216 = 0$

$y^2 + 6y - 216 = 0$

-216

$\Rightarrow -18 \times 12$

$\cancel{\Rightarrow 12 \times -18}$

$\Rightarrow 18 \times -12$

$y^2 + 18y - 12y - 216 = 0$

$y(y+18) - 12(y+18) = 0$

$(y-12)(y+18) = 0$

$y = 12$ or -18 — side of a square cannot be negative.

So, $y = \underline{12\,m}$ (side of 2nd square)

$x = 6 + y = 6 + 12 = \underline{18\,m}$ (side of 1st square)

<u>Sides of the squares are 18 m, 12 m respectively.</u>