BASIC CONCEPT OF PHYSICS

Measurements :

When we describe the things without using the numbers our description is unsatisfactory. For satisfactory description we use some quantitative explanation. The description of physics is quantitative and involves measurement.

Physical Quantities :

The building blocks of physics are physical quantities in term of which the laws of physics are expressed. "A physical quantity is a physical property that can be quantified by measurement. Time, mass, length, force, velocity, work, temperature etc. are Illustrations of physical quantities. On the basis of some arbitrary operations physical quantities can be divided into fundamental and derived quantities.

Fundamental or Base Quantities :

The quantity whose defining operations do not depend on other quantity called fundamental or base quantity e.g.. time, length, mass, temperature etc.

Derived Quantities :

The quantity whose defining operation depend on other quantities called derived quantity e.g., velocity, force, work etc.

Unit :

The measurement of any quantity is always multiple of some well defined standard measurement. This standard measurement is called unit.
Thus Measurement of a physical quantity = numerical value (n) × unit (u)
Ex. Height of Jitu = 1.2 m, here 1.2 is numerical value and metre is unit of height.

System Of Units

Commonly used systems of units are
(1) the FPS system (foot, pound and seconds system)
(2) the MKS system (metre, kilogram and second system)
(3) the CGS system (centimetre, gram and seconds system)
(4) the SI system (Systeme Intenational)
The first three systems have just got the three mechanical quantitites as the fundamental units (i.e. mass, length and time)

The fourth system, the SI system has seven fundamental units and two supplementary units

S.I. SYSTEM OF UNITS :

BASIC UNITS SYSTEMS

(I)

Basic Quantity	**Unit**	**Symbol**
Length	Meter	(m)
Mass	Kilogram	(kg)
Time	Second	(s)
Temperature	Kelvin	(K)
Electric Current	Ampere	(A)
Luminous Intensity	Candela	(cd)
Amount of Substance	mole	(mol)

(II)

Supplementary Quantity	**Unit**	**Symbol**
Plane angle	Radian	rad
Solid angle	Steradian	sr

S.I. Prefixes			
S. No.	**Perfix**	**Symbol**	**Power of 10**
1.	exa	E	18
2.	peta	P	15
3.	tera	T	12
4.	giga	G	9
5.	mega	M	6
6.	kilo	K	3
7.	hector	h	2
8.	deca	da	1
9.	deci	d	-1
10.	centi	c	-2
11.	milli	m	-3
12.	micro	μ	-6
13.	nano	n	-9
14.	pico	p	-12
15.	femto	f	-15
16.	atto	a	-18

Practical units of Length	
S. No.	
1.	Light year = 9.46×10^{15}m
2.	Parsec = 3.084×10^{16}m
3.	Angstrom(Å) = 10^{-10}m
4.	Micrometer = 10^{-6}m
5.	Astronomical Unit (AU) = 1.496×10^{11}m
6.	Otto meter = 10^{-21}m.

Derived Units: The units of derived quantities or the units that can be expressed in terms of the base units are called derived units. e.g. unit of speed
Some derived units are named in honour of great scientists.
e.g. unit of force - newton (N), unit of frequency - hertz (Hz) etc.

Illustration 1 : Unit of density in SI system is $1\frac{kg}{m^3}$ convert into C.G.S. System i.e., g/cm^3.

Solution $1\frac{kg}{m^3} = \frac{10^3 g}{10^2 \times 10^2 \times 10^2 cm^3}$ =10^{-3} g/cm^3

Thus 1000 kg/m^3 = 10^3 ´ 10^{-3} g/cm^3

⇒1 g/cm^3 = 10^3 kg/m^3 = 1 g/cm^3.

Illustration 2 : Practical unit of speed or velocity is, km/hr. Convert 1 km/hr into m/s.

Solution. $1\frac{km}{hr} = \frac{1000m}{60 \times 60S} = \frac{5}{18} m/s$

Thus 72 km/hr = $\left(72 \times \frac{5}{18}\right)$ m/s = 20 m/s

⇒ **1 m/s = 18/5 km/hr.**

Illustration 3 : In SI system unit of force is 1N = $1kg \times \frac{m}{s^2}$

Solution. In C.G.S. system unit of force is idyne = 1g ´ cm/s^2
convert 1N into 1 dyne.

1N = 1 kg ´ $1\frac{m}{s^2}$ =10^3 g ´ 10^2 cm/s^2

= 10^5 g cm/s^2 = 10^5 dyne.

⇒ 1dyne = 10^{-5}N.

Illustration 4 : The relation between velocity and time of body is given

$$v = A + \frac{B}{t} + Ct^2$$

the units of A, B and C will be

	A	B	C
(a)	m	m/s	m/s^2
(b)	m/s	m	m/s^3
(C)	m/s^2	m/s^3	m/s^4
(d)	m/s	m/s^2	m/s^3

Solution [Ans. (b)]

By the principle of homogeneity unit of A, $\frac{B}{t}$ and ct^2 must be equal to v.

v = A i.e. A = m/s

$v = \frac{B}{t}$ B = m

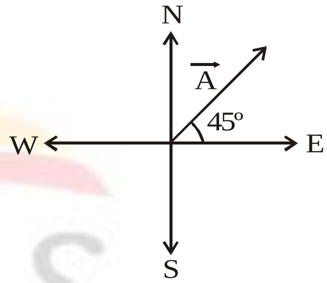
v = Ct^2 C = m/s^3

VECTORS

- **Vectors & Scalars Quantities**
- **Scalar:** A *scalar* quantity is defined as a quantity that has magnitude only. Typical Illustrations of scalar quantities are time, speed, temperature, and volume. A scalar quantity or parameter has no directional component, only magnitude. For Illustration, the units for time (minutes, days, hours, etc.) represent an amount of time only and tell nothing of direction. Additional Illustrations of scalar quantities are density, mass, and energy.

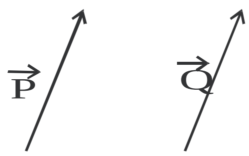
- **Vector:** A physical quantity which has magnitude and specific direction and which follows vector law of addition is called vector. For Illustration displacement, force, acceleration, electric field & torque etc.
- **Graphical Representation Of Vectors**

To work with vector quantities, one must know the method for representing these quantities. Magnitude, or "size" of a vector, is also referred to as the vector's "displacement." It can be thought of as the scalar portion of the vector and is represented by the length of the vector. By definition, a vector has both magnitude and direction. Direction indicates how the vector is oriented relative to some reference axis, as shown in Figure.

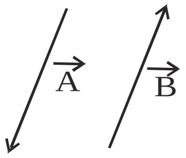


Using north/south and east/west reference axes, vector "A" is oriented in the NE quadrant with a direction of 45 north of the o EW axis. G iving direction to scalar "A" makes it a vector. The length of "A" is representative of its magnitude or displacement.

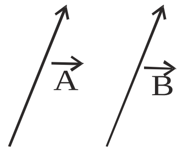
- **Different Kinds Of Vectors**
- **Parallel vectors:** If two vectors have the same direction called parallel vectors. In figure $\vec{P}$ and $\vec{Q}$ are parallel vectors.



- **Anti Parallel Vectors:** Two vectors have opposite direction called anti parallel vectors. In figure $\vec{A}$ and $\vec{B}$ are anti parallel vectors.

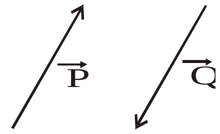


- **Equal vectors:** "If two or more vectors have equal magnitude and acting in the same direction, they are said to be equal vectors". The two vectors shown in figure have equal length and same orientation.



Hence they represent two equal vectors $\vec{A}\,\&\,\vec{B}$ even though they have at different initial points terminal points.

- **Negative vector:** If two vectors $\vec{P}$ and $\vec{Q}$ are such that they have equal magnitude but opposite directions, each vector is negative of the other.
Thus $\vec{P} = -\vec{Q}$ or $\vec{Q} = -\vec{P}$.



- **Null vector:** "A vector of zero magnitude is called zero vector or null vector". It is represented by $\vec{0}$. The initial point and terminal point of the null vector coincide. Its direction is indeterminate.

- **Unit Vector:** "A vector of unit magnitude is called unit vector". The unit vector in the direction of given vector is obtained by dividing the given vector with its magnitude. It is conventional to denote unit vector with a "cap" instead of "bar" over the symbol. Thus if $\vec{A}$ is a given vector, the unit vector in the direction of A is written as
$A = \frac{\vec{A}}{|\vec{A}|}$ (where A is read as A cap or A hat)

- **Note:** In the right handed Cartesian coordinate system, $\hat{i}, \hat{j}$ and $\hat{k}$ are choosen as unit vectors along, the X-axis, Y-axis and Z-axis respectively.

Co-planar Vector

Vectors, which are in the same plane are called co-planar vectors.

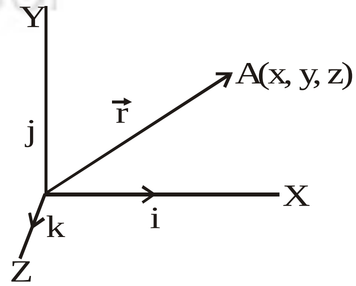
Non Co-planar Vector

Vectors, which are in different planes are called non-co-planar vectors.

- **Position Vector**

"The vector used to specify the position of a point with respect to some fixed point (say origin 'O') is called position vector". It is denoted as $\vec{r}$.

Consider a point 'A' with coordinates x, y, z in the Cartesian coordinate system. Thus the position of 'A' can be expressed in the vector form as $\overrightarrow{OA} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Here i, j and k are unit vectors along the X, Y and Z axes respectively. The distance of 'A' from the origin eventually becomes the magnitude of $\vec{r}$.



- **Displacement**

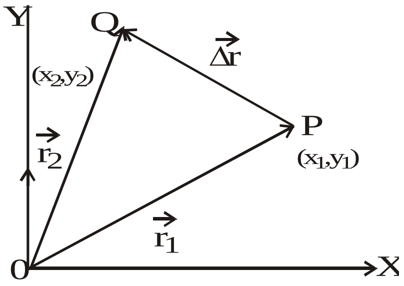
Displacement is a vector quantity have magnitude equal to shortest distance between two points and direction from initial point to final point.

- **Displacement Vector**

The position of the point Q with reference to the origin is represented by the position vector $\vec{r}_2$. Let the coordinates of the point Q are (x_2, y_2).

Similarly $\overrightarrow{OP}$ represented by a position vector $\vec{r}_1$, let the coordinates of the point P are (x_1, y_1).

As the displacement vector is the difference of two position vectors $\vec{r}_1 = x_1\hat{i} + y_1\hat{j}$ and $\vec{r}_2 = x_2\hat{i} + y_2\hat{j}$, where $\hat{i}, \hat{j}$ are unit vectors along X, Y axis respectively. Thus, the displacement vector $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{i}$.



Angle between two vectors :

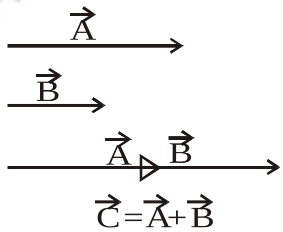
To find angle between two vectors, the two vectors from a point are drawn such that their arrow heads should be away from that point. The angle obtained in this way, is the angle between the vectors.

- **Addition Of Vectors**

Scalars can be added or subtracted following the simple rules of algebra or arithmetic. But vectors do not follow the same simple rules, because while adding or subtracting vectors, their direction also has to be considered. **For Illustration,** when a mass of 5 kg is added to another mass of 5 kg, the result is exactly 10 kg. But when a vector of magnitude 5 units is added to another vector of magnitude 5 units (of course of same physical quantity), the result may have a magnitude from zero to a maximum of 10 units, depending on relative orientations of the two vectors. The vector sum is also called **resultant**.

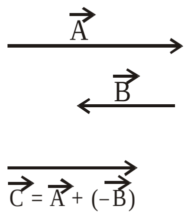
- **Addition Of Two Vectors In Same Direction**

If two vectors are in the same direction, their resultant (sum) is obtained by adding their vector lengths as shown in the figure. The direction of resultant is same as the individual vectors. Figure – Addition of vectors in same direction.



- **Addition Of Two Vectors In Opposite Direction**

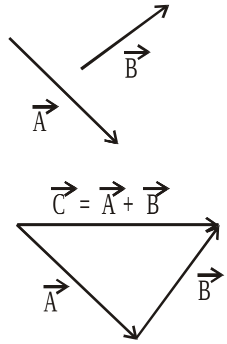
If the vectors are mutually opposite, their resultant is obtained by subtracting the length of smaller vector.



Addition of vectors in opposite from that of larger vector as shown in figure. The direction of resultant is same as that of larger vector. (Triangle law)

➢ Addition Of Two Vectors Inclined Mutually

If two vectors are mutually inclined, the following procedure is adopted to find their sum. A and B are the given vectors. B is slides parallel to itself, such that its "tail" coincides with the head of A as shown in figure. Then the directed line segment drawn from the tail of A to the head of B represents the addition of A and B.

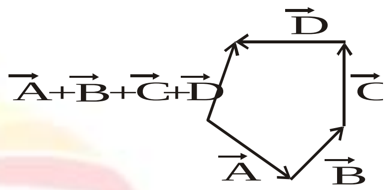


➢ Triangle Law Of Vectors

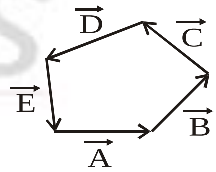
If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, the third side of the triangle taken in reverse order represents their resultant in magnitude and direction.

➢ Polygon Law of Vectors

When a number of vectors are represented both in magnitude and direction by the sides of a polygon taken in an order, then their resultant is given by the closing side of that polygon taken in the reverse order both in magnitude and direction.



When a number of vectors simultaneously acting at a point have zero resultant, then these vectors can be represented in magnitude and direction by the sides of a polygon taken in an order.

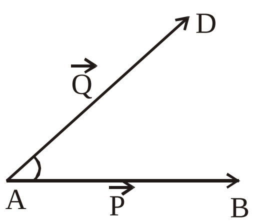


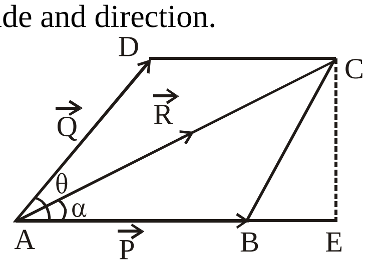
In the above figure $\vec{A}+\vec{B}+\vec{C}+\vec{D}+\vec{E}=0$.

➢ Parallelogram Law Of Vectors

Two vector quantities (Ex- velocity, acceleration, force, et(C) can be added using parallelogram law. This law is useful to find both magnitude and direction of resultant.

- **Statement:** If two vectors are represented in magnitude and direction by the adjacent sides of a parallelogram drawn from a point, the diagonal passing through that point represents their resultant both in magnitude and direction.





- **Explanation:** $\vec{P}$ *and* $\vec{Q}$ are two vectors represented by $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Both vectors act at the common point A and mutually inclined at angle 'θ' as shown in figure. If the parallelogram ABCD is completed taking AB and AD as adjacent sides, then the diagonal $\vec{AC}$ represents their resultant $\left(\vec{R}\right)$ both in magnitude and direction.

Magnitude of the Resultant : The line of action of $\vec{P}$ is extended. The perpendicular drawn from 'C' meets the extension of AB at E.

From the figure, it is obvious that $\vec{BC} = \vec{AD} = \vec{Q}$ and $\angle \vec{CBE} = \theta$

$\therefore$ Length of AB = magnitude of $\vec{P} = P$ Length of BC = magnitude of $\vec{Q} = Q$

Length of AC = magnitude of $\vec{R} = R$

From triangle CBE, $BE = BC\cos\theta \Rightarrow BE = Q\cos\theta$ .... (i)

$\frac{EC}{BC} = \sin\theta$ and $EC = Q\sin\theta$ .... (ii)

$(AC)^2 = (AE)^2 + (EC)^2$

$\Rightarrow$ $(AC)^2 = (AB + BE)^2 + (EC)^2$ $\Rightarrow$ $(AC)^2 = (AB)^2 + (BE)^2 + 2AB.BE + (EC)^2$

$\Rightarrow$ $R^2 = P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta \Rightarrow R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\Rightarrow$ $\boldsymbol{R = \sqrt{P^2 + Q^2 + 2PQcos\theta}}$ .... (iii)

- **Direction of the Resultant:** The resultant makes angle 'α' with $\vec{A}$ (say).

From triangle CAE: $\tan\alpha = \frac{EC}{AE} = \frac{EC}{AB + BE} \Rightarrow \tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$

$\Rightarrow$ $\boldsymbol{\alpha = tan^{-1}\left[\frac{Qsin\theta}{P + Qcos\theta}\right]}$ .... (iv)

The expression (iii) and (iv) gives the magnitude and direction of the resultant of $\vec{P}$ and $\vec{Q}$.

➢ **Special Cases :**

- If $\vec{P}$ and $\vec{Q}$ are in the same direction, then $\theta = 0^\circ$, and $\cos\theta = 1$

  $\therefore$ From equation (iii) and (iv), R = P + Q and $\alpha = 0$

  Hence the magnitude of resultant is sum of the magnitude of individual vectors. The direction of resultant is same as that of individual vectors.

- If $\vec{P}$ and $\vec{Q}$ are opposite, then $\theta = 180°$ and $\cos\theta = -1$

  $\therefore$ $R = P - Q$ i.e. $R = P - Q$ or $Q - P$ and $\alpha = 0^\circ$ or 180°.

  Thus the magnitude of resultant is equal to difference of magnitudes of individual vectors and the direction of resultant is same as that of the vector or larger magnitude.

- If $\vec{P}$ and $\vec{Q}$ are perpendicular, then $\theta = 90°$ and $\cos\theta = 0$

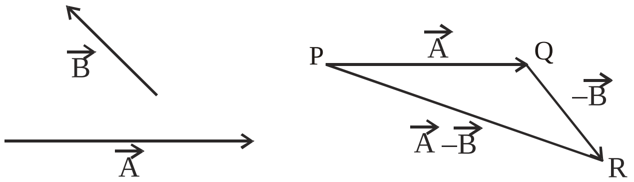
  $\therefore$ $R = \sqrt{P^2 + Q^2}$ and $\alpha = \tan^{-1}(Q/P)$

- If $\left|\vec{P}\right| = \left|\vec{Q}\right|$, then $R = 2P\cos\theta/2$ and $\alpha = \theta/2$

  $\therefore$ If the vectors have equal magnitude, then the resultant will bisect the angle between them.

Subtraction Of Vectors

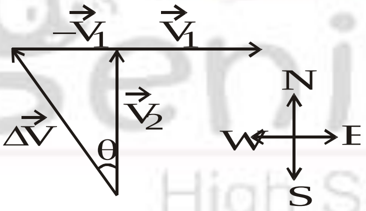
The subtracting of vector $\vec{B}$ from the vector $\vec{A}$, is same as addition of $-\vec{B}$ to $\vec{A}$ as shown in figure i..e., $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$



The vector $\vec{B}$ is reversed to get negative vector of $\vec{B}$. Then reversed vector $-\vec{B}$ is shifted parallel to itself such that tail of $-\vec{B}$ coincides with head of $\vec{A}$. The directed line segment $\overrightarrow{PR}$ represents the subtracting of $\vec{B}$ from $\vec{A}$.
Vector subtracting does not obey commutative law. i.e., $\vec{A}-\vec{B}\neq\vec{B}-\vec{A}$.

Change In Velocity

- Consider the body moving with velocity $\vec{V_1}$ due to east. After a time interval 't' its final velocity is $\vec{V_2}$ due to north.



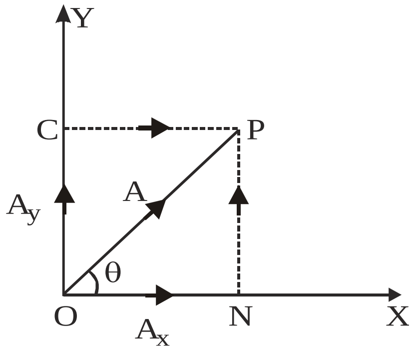
- The change in velocity is obtained by vector subtracting method. The initial velocity vector $\vec{V_1}$ is reversed to get $-\vec{V_1}$ and $\vec{V_2}$ following the usual procedure for vector addition i.e., the tail of $-\vec{V_1}$ coincides with head of $\vec{V_2}$.

- The vector drawn from tail of $\vec{V_2}$ to the head of $-\vec{V_1}$ represents change of velocity $\Delta\vec{V}$ both in magnitude and direction. $\vec{V_1}$ and $\vec{V_2}$ are mutually perpendicular as per their directions mentioned.

- Magnitude of change of velocity $\Delta V=\left|\vec{V_2}-\vec{V_1}\right|=\sqrt{V_1^2+V_2^2-2V_1V_2\cos 90^\circ}$
$\Rightarrow \Delta V=\sqrt{V_1^2+V_2^2}$.

Resolution Of A Vector Into Two Rectangular Components

- The process of splitting a vector is called resolution of a vector. The parts obtained after resolution are known as components of the given vector.



- If the components of a given vector are perpendicular to each other, then they are called **rectangular components.** These are the most important components of a vector.

  Let, $\vec{A}_x$ and $\vec{A}_y$ are the rectangular components of $\vec{A}$.

  Applying triangle law of vectors to the vector triangle ONP, we get

  $\vec{A}_x + \vec{A}_y = \vec{A}$

- This equation confirms that $\vec{A}_x$ and $\vec{A}_y$ are the components of $\vec{A}$.

  In right angled triangle ONP,

  $\cos\theta = \frac{A_x}{A}$ or $A_x = A\cos\theta$ (i)

  $\sin\theta = \frac{A_y}{A}$ or $A_y = A\sin\theta$ (ii)

- Squaring and adding (1) and (2), we get

  $A_x^2 + A_y^2 = A^2\cos^2\theta + A^2\sin^2\theta$

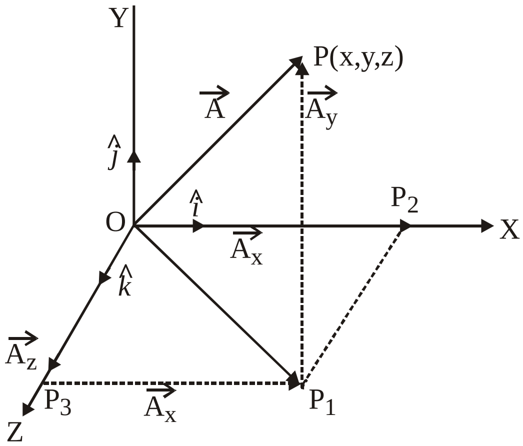
  or $A_x^2 + A_y^2 = A^2(\cos^2\theta + A^2\sin^2\theta)$ $\therefore A_x^2 + A_y^2 = A^2$

  $[\because \cos^2\theta + \sin^2\theta = 1]$

  or $A = \sqrt{A_x^2 + A_y^2}$ (iii)

➢ **Resolution Of A Vector Into Three Rectangular Components**

- Consider a right handed three dimensional coordinate system. Let $\vec{A}(=\overrightarrow{OP})$ be a vector drawn through the origin O. The three rectangular components of $\vec{A}$ can be determined as follows:



- Draw PP_1 perpendicular from P upon X-Z plane. From P_1 draw P_1P_2 and P_1P_3 perpendiculars to X-axis and Z-axis respectively.

- $\overrightarrow{OP_2}$, $\overrightarrow{P_1P_2}$ and $\overrightarrow{OP_3}$ are known as the x-component, y-component and the z-component, respectively, of $\vec{A}$.

- In ΔOP_1P_3, $\vec{A}_z + \vec{A}_x = \overrightarrow{OP_1}$ ... (i)

- In ΔOP_1P, $\overrightarrow{OP_1} + \overrightarrow{P_1P} = \overrightarrow{OP} = \vec{A}$ $\therefore \left(\vec{A}_z + \vec{A}_x\right) + \vec{A}_y = \vec{A}$

  or $\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$ or $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ ... (2)

- According to the Pythagoras theorem in three dimensions,

  $A^2 = A_x^2 + A_y^2 + A_z^2$ or $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ ... (3)

  This gives the magnitudes of vector $\vec{A}$.

- So, the magnitude of any vector is equal to the square root of the sum of the square, if the magnitude of its three rectangular components.

- Again, $\hat{A}$ = unit vector in the direction of $\vec{A} = \frac{\vec{A}}{|\vec{A}|}$.

- Substituting the value of $\vec{A}$ and $|\vec{A}|$ from equations (2) and (3), we get

$$\hat{A} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

➢ **Equilibrium**

- When a body does not change its state of rest or uniform motion on the application of one or more external forces, the body is said to be in equilibrium.

- A single force acting on a body or point cannot keep the body in equilibrium.

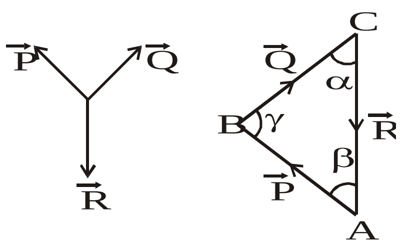
- The minimum number of forces which can keep a point in equilibrium is 2. These two forces are equal in magnitude and opposite in direction.

➢ **Application Of Triangle Law Of Vectors (Lami's Theorem)**

If three vectors, simultaneously acting at a point, have zero resultant, then these three vectors can be represented in magnitude and direction by the three sides of a triangle taken in order.

➢ **Explanation:**

- $\vec{P}, \vec{Q}$ and $\vec{R}$ are three forces (vectors) which simultaneously act at the point 'O' and keep it in equilibrium. Hence they represented, in magnitude and direction, by the sides AB, BC and CA of the triangle ABC taken in order.



- The magnitude of $\vec{P}, \vec{Q}$ and $\vec{R}$ are proportional to the lengths of AB, BC, CA respectively. i.e., $\frac{P}{AB} = \frac{Q}{BC} = \frac{R}{CA} = K$ (constant).

- The orientation of $\vec{P}, \vec{Q}$ and $\vec{R}$ are parallel to the sides AB, BC and CA.
  Whenever a triangle is formed to represent three forces keeping a body in equilibrium, then the lengths of each side of the triangle is proportional to the sine of the angle opposite it.
  i.e., $\frac{AB}{\sin\angle ACB} = \frac{BC}{\sin\angle BAC} = \frac{CA}{\sin\angle ABC}$ $\Rightarrow$ $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$.

➢ **Relative Velocity**

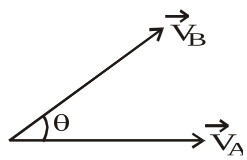
- The concept of relative velocity is useful to understand the motion of one body in relation to any other moving body.

- The relative velocity is obtained by vector subtraction method, whether the bodies are moving in same direction or opposite direction or inclined direction.

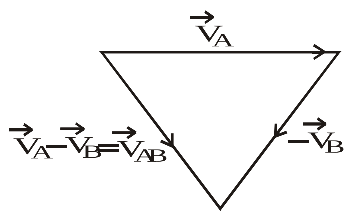
- Let two bodies A and B be moving with the velocities V_A and V_B in the same direction.
  Velocity of 'A" relative to 'B' = $\vec{V}_A - \vec{V}_B$
  Velocity of 'B' relative to 'A' = $\vec{V}_B - \vec{V}_A$

- If two bodies 'A' and 'B' are moving in opposite directions the velocity of 'A' relative to 'B' is $\vec{V}_A + \vec{V}_B$ and it is same as the velocity of 'B' relative to 'A'.

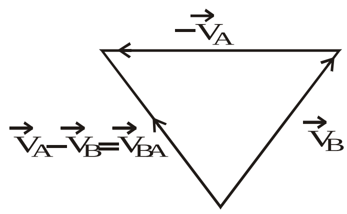
- Consider two bodies 'A' and 'B' moving with velocities $\vec{V}_A + \vec{V}_B$ are inclined at an angle 'θ' as shown in the figure.



- The velocity vector $\vec{V}_B$ is reversed to get $-\vec{V}_B$ and then added to $\vec{V}_A$ following the usual procedure for addition of vectors as shown in the figure. Thus the resultant $\vec{V}_A - \vec{V}_B$ is the relative velocity of 'A' w.r.t. 'B' i.e., $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$.



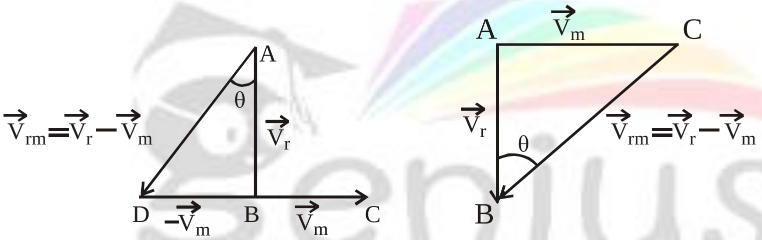
- Similarly the velocity vector $\vec{V}_A$ is reversed to get $-\vec{V}_A$ and then added to $\vec{V}_B$ following the usual procedure for addition of vectors as shown in the figure.



- The resultant $\vec{V}_B - \vec{V}_A$ is the relative velocity of 'B' w.r.t. 'A' i.e., $\vec{V}_{AB} = \vec{V}_B - \vec{V}_A$. It may be noted that $\vec{V}_{AB}$ and $\vec{V}_{BA}$ are equal in magnitude of relative velocity is obtained from parallelogram law of vector.
  $\therefore \quad \left|\vec{V}_{AB}\right| = \left|\vec{V}_{BA}\right| = \sqrt{V_A^2 + V_B^2 + 2V_A V_B \cos(180-\theta)}$
  $\Rightarrow \quad \left|\vec{V}_{AB}\right| = \left|\vec{V}_{BA}\right| = \sqrt{V_A^2 + V_B^2 - 2V_A V_B \cos\theta}$.

➢ Man Walking In Rain

- Consider rain falling vertically in the absence of wind. The velocity of rain is $\vec{V}_r$ vertically downwards.
- A man stands at rest in the rain and observes that the rain falls vertically down. He holds the umbrella upright to protect himself from the rain. If the man walks horizontally, with certain velocity $\vec{V}_m$, he observes the direction of rain inclined to the vertical. Hence he holds the umbrella in a direction inclined to the vertical.



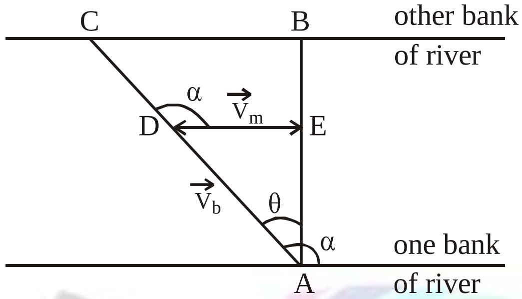
- The direction in which the umbrella is held, indicates the direction of rain fall in relation to the walking man.
- AB represents the velocity of rain $\vec{V}_r$ falling vertically. BC represents the velocity of man $\vec{V}_m$ walking horizontally. The direction of $\vec{V}_m$ is reversed to get $-\vec{V}_m$ and added to $\vec{V}_r$ following the procedure for vector addition. The line segment $\overrightarrow{AD}$ represents the value relative velocity of rain w.r.t. man in both magnitude and direction.
- In the other way, both velocity vectors are drawn such that they have common origin as shown. The line segment directed from the head $\vec{V}_m$ to the head of $\vec{V}_r$ represents the velocity of rain in relation to man both in magnitude and direction. In both figures, $\tan\theta = \frac{V_m}{V_r} \Rightarrow \theta = \tan^{-1}\left(\frac{V_m}{V_r}\right)$. And the magnitude of relative velocity $V = \sqrt{V_r^2 + V_m^2}$. The umbrella is held at an angle $\theta = \tan^{-1}\left(\frac{V_m}{V_r}\right)$ w.r.t. vertical.

➢ Motion Of Boat In River (Application Of Triangle Law)

- **Crossing river in shortest path:**

(i) Let $\vec{V}_w$ is the velocity of water current in a river. $\vec{V}_b$ is velocity of boat in still water. A person is on one bank of the river. He wishes to cross the river in the boat and reach exactly opposite point on the other bank i.e., he wants to cross the river in the shortest path. When the boat moves in river it is simultaneously subjected to two velocity vectors viz, $\vec{V}_w$ and $\vec{V}_b$.

(ii) In the figure the direction of river flow is from left to right. The person is at position 'A" on one bank. He wishes to reach exactly the opposite position 'B' on the other bank. He directs his boat upstream along AC, which makes angle 'θ' with normal line to the water current. The vectors $\vec{V}_b$ and $\vec{V}_w$ are respectively represented in magnitude and direction by the sides AD and DE of the triangle ADE taken in order. According to the triangle law, their resultant is represented in magnitude and direction by the third side AE of the triangle taken in reverse order.



(iii) From figure $\sin\theta = \dfrac{DE}{AD} = \dfrac{V_m}{V_b} \Rightarrow \theta = \sin^{-1}\left[\dfrac{V_m}{V_b}\right]$ ... (i)

$\therefore$ The boat should be directed at angle $\theta = \sin^{-1}\left(\dfrac{V_m}{V_b}\right)$ with normal line to water current.

Time of travel to cross the river:

(iv) The magnitude of resultant velocity $|\vec{V}| = \sqrt{|V_b|^2 - |V_w|^2} = \sqrt{V_b^2 - V_w^2}$

Width of river AB = d (say)

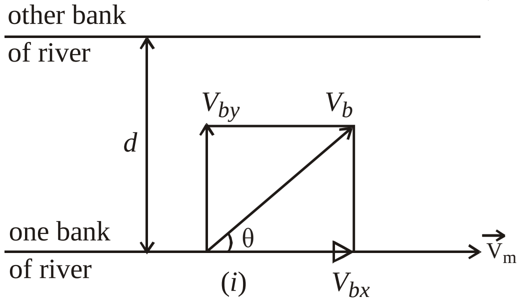
(v) Displacement of boat = d (to reach the other bank)

Time taken by the boat to cross the river $= \dfrac{\text{displacement AB}}{\text{velocity V}} = \dfrac{d}{\sqrt{V_b^2 - V_w^2}}$ ... (ii)

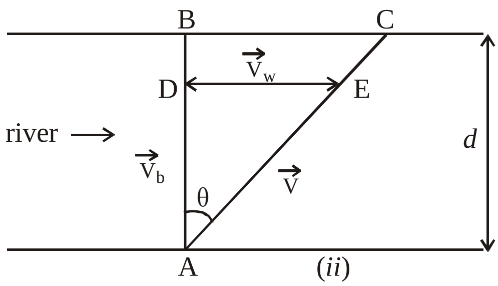
- **Crossing the river in shortest time**

(i) The boat is directed making an angle 'θ' with direction of water current. The angle between $\vec{V}_b$ and $\vec{V}_w$ is θ as shown in figure. The component of $\vec{V}_b$ across the river is given as $V_{by} = V_b \sin\theta$.

When the boat reaches the other bank,



(*i*)



(*ii*)

Displacement of boat = width of the river = d (say)

(ii) Time taken to cross the river = $\dfrac{\text{displacement of boat across the river}}{\text{component of velocity of boat across the river}}$

$$\Rightarrow \quad t = \frac{d}{V_{by}} \Rightarrow t = \frac{d}{V_b \sin\theta} \qquad \text{... (iii)}$$

(iii) For a given velocity of boat, the time of travel becomes minimum, when $\sin\theta$ is maximum i.e., $\sin\theta = 1 \Rightarrow \theta = 90^\circ$.

(iv) Hence the boat should be directed exactly towards the opposite point on the other bank i.e., making 90° with direction of water current so that the time becomes minimum.

Minimum time taken to cross the river, $t = \dfrac{d}{V_b}$

(v) The displacement of the boat parallel to the water current = BC = x (say)

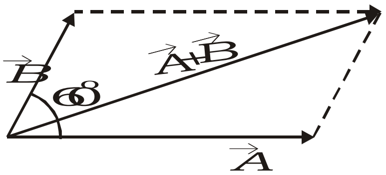
From triangle ADE and ACB, $\tan\theta = \dfrac{BC}{AB} = \dfrac{DE}{AE} \Rightarrow \dfrac{x}{d} = \dfrac{V_w}{V_B} \Rightarrow$

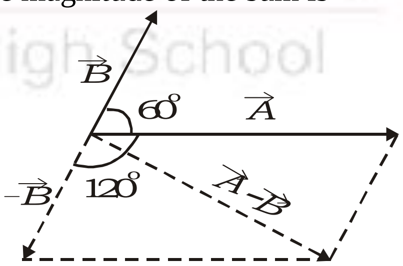
Shift of boat downstream, $x = d\left(\dfrac{V_w}{V_b}\right)$.

Illustration 5 : Two vectors of equal magnitude 5 unit have an angle 60° between them. Find the magnitude of (a) of the sum of the vectors and (b) the difference of the vectors.

Solution : Figure shows the construction of the sum $\vec{A}+\vec{B}$ and the difference $\vec{A}-\vec{B}$.

(a) $\vec{A}+\vec{B}$ is the sum of $\vec{A}$ and $\vec{B}$. Both have a magnitude of 5 unit and the angle between them is 60°. Thus the magnitude of the sum is





$$\left|\vec{A}+\vec{B}\right| = \sqrt{5^2+5^2+2\times5\times5\cos60^\circ} = 5\sqrt{3}\text{ unit}$$

(b) $\vec{A}-\vec{B}$ is the sum of $\vec{A}$ and $(-\vec{B})$. As shown in the figure, the angle between $\vec{A}$ and $(-\vec{B})$ is 120°. The magnitudes of both $\vec{A}$ and $(-\vec{B})$ is 5 unit, so,

$$\left|\vec{A}-\vec{B}\right| = \sqrt{5^2+5^2+2\times5\times5\cos120^\circ} = 5\text{ unit.}$$

Illustration 6 : The forces of 10 N and 6 N act upon a body. The directions of the forces are unknown. The resultant force on the body may lie between which limits ?

Solution : Let θ is the angle between the two forces P = 10 N and Q = 6 N. Then, the resultant force is given by

$$R = \sqrt{10^2+5^2+2\times10\times6\cos\theta}$$

The minimum value of cos θ is –1, while, the maximum values is +1.

Thus,

$R_{\min} = \sqrt{10^2 + 6^2 + 2 \times 10 \times 6 \times (-1)} = \sqrt{(10-6)^2} = 4N$

$R_{\max} = \sqrt{10^2 + 6^2 + 2 \times 10 \times 6 \times (+1)} = \sqrt{(10+6)^2} = 16N$

Thus, the resultant of the given two forces lies between 4 N and 16 N.

Note : The maximum and minimum magnitude of the resultant of two vectors is always $|\vec{P}| + |\vec{Q}|$ and $|\vec{P}| - |\vec{Q}|$. So, in this case, it will be 10 + 6 = 16 N and 10 – 6 = 4 N.

Illustration 7 : The vector sum of two vectors $\vec{P}$ and $\vec{Q}$ is R. If vector $\vec{Q}$ is reversed, the resultant becomes $\vec{S}$. Then, prove that $R^2 + S^2 = 2(P^2 + Q^2)$

Solution : Let θ be the angle between vectors $\vec{P}$ and $\vec{Q}$.

Then, $R^2 = P^2 + Q^2 + PQ \cos\theta$ ... (i)

When vector $\vec{Q}$ is reversed, angle between the vector $\vec{P}$ and $-\vec{Q}$ will become 180°–θ.

Thus, $S^2 = P^2 + Q^2 + 2PQ\cos(180° - \theta)$

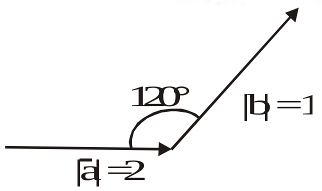
$= P^2 + Q^2 + 2PQ \times (-\cos\theta)$

or $S^2 = P^2 + Q^2 - 2PQ\cos\theta$ ... (ii)

Adding the equations (i) and (ii), we have

$R^2 + S^2 = 2(P^2 + Q^2)$

Illustration 8 : Find the magnitude and direction of resultant $\vec{a}$ and $\vec{b}$.



Solution : $R^2 = \sqrt{2^2 + 1^2 + 2 \times 2 \times 1 \cos 60°} = \sqrt{4 + 1 + 4 \times \frac{1}{2}}$

$R = \sqrt{5 + 2}$

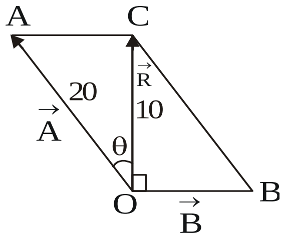
$R = \sqrt{7}$

$\tan\beta = \dfrac{a \sin\theta}{b + a\cos\theta}$

$\beta = \tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$.

Illustration 9 : The resultant of two vectors A and B where B > A is of magnitude 10 and perpendicular to B. If the magnitude A is 20. Find the magnitude of B and angle between A and B.

Solution : On Δ AOC



$$\cos\theta = \frac{10}{20} = \frac{1}{2}$$

$\therefore\ \theta = 60^\circ$

$\therefore$ Angle between $\vec{A}$ and $\vec{B} = 150^\circ$

$$|\vec{B}| = \sqrt{20^2 - 10^2} = \sqrt{400 - 100} = \sqrt{300}\,.$$

Illustration 10: Find the direction cosines of vector.

$$\vec{r} = 3\vec{a} - 2\vec{b} + \vec{c}$$

where, $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$; $\vec{c} = -\hat{i} - \hat{j} + \hat{k}$

Solution : $\vec{r} = 3(\hat{i} - 2\hat{j} + \hat{k}) - 2(2\hat{i} + \hat{j} + \hat{k}) + (-\hat{i} - \hat{j} + \hat{k})$

$$\vec{r} = -2\hat{i} - 9\hat{j} + 2\hat{k}$$

$$|\vec{r}| = \sqrt{89}$$

$$l = \frac{-2}{\sqrt{89}}$$

$$m = \frac{-9}{\sqrt{89}}$$

$$n = \frac{2}{\sqrt{89}}\,.$$

Illustration 11 : Three dimensional vector of magnitude 9 has direction cosines $\left(\frac{1}{3}, \frac{-2}{3}, n\right)$. Find all such vectors.

Solution : $l^2 + m^2 + n^2 = 1$

$$\Rightarrow \left(\frac{1}{3}\right)^2 + \left(\frac{-2}{3}\right)^2 + n^2 = 1 = \frac{1}{9} + \frac{4}{9} + n^2 = 1$$

$$\Rightarrow n^2 = 1 - \frac{1}{9} - \frac{4}{9} = \frac{4}{9}$$

$$n = \pm\frac{2}{3}$$

$$\vec{r} = r\,(l\hat{i} + n\hat{j} + n\hat{k})$$

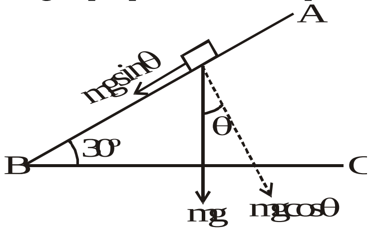
$$\bar{r} = 3[\hat{i} - 2\hat{j} \pm 2\hat{k}]\,.$$

Illustration 12: A mass of 2 kg lies on a plane making an angle 30° to the horizontal. Resolve its weight along and perpendicular to the plane. Assume $g = 10\ ms^{-2}$.

Solution In Figure, the component of weight along the plane

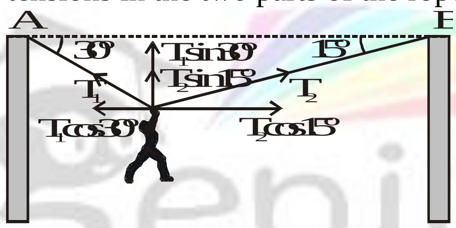
$= 2 \times 10 \times \sin 30 = 10$ N

The component of weight perpendicular to plane = mg cos 30°



$= 2 \times 10 \times \sqrt{\frac{3}{2}} = 17.3$ N.

Illustration 13: A rope is stretched between two poles. A 50N boy hangs from it, as shown in Figure. Find the tensions in the two parts of the rope.



Solution Resolving tension T_1 and T_2 into two rectangular components we have

$T_1 \cos 30°$ acts horizontally and $T_1 \sin 30°$ vertically upwards

$T_2 \cos 15°$ acts horizontally and $T_2 \sin 15°$ vertically upwards.

As the system is in equilibrium, so

$T_1 \cos 30° = T_2 \cos 15°$

or $T_1 \times 0.8660 = T_2 \times 0.9659$

or $T_1 = \frac{0.9659}{0.8660} T_2 = 1.11\, T_2$

and $T_1 \sin 30° + T_2 \sin 15° = W$

or $1.11\ T_2 \times \frac{1}{2} + T_2 + 0.2588\ T_2 = 50$

or $0.55\ T_2 + 0.2588\ T_2 = 50$

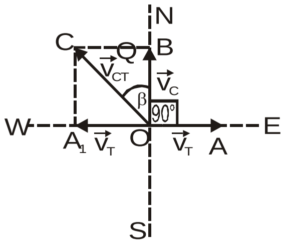
or $0.8\ T_2 = 50$ or $T_2 = 50/0.8 = 62.50$ N

$T_1 = 1.11 \times 62.50 = 69.37$ N.

Illustration 14: A train is moving due east and a car is moving due north, both with the same speed 30 km h^{-1}. What is the observed speed and direction of motion of car to the passenger in the train ?

Solution. Velocity of train,

$\vec{v}_T = (\overrightarrow{OA})$ and velocity of car, $\vec{v}_C = (\overrightarrow{OB})$, where $v_T = v_C = 30$ km h^{-1}.



The observed speed and direction of car to the passenger in the train will be the relative velocity of car with respect to train. To find the relative velocity of car w.r.t. train, bring the train at rest by applying equal and opposite velocity of train on it as well as on car i.e. $\vec{v}_T = (\overrightarrow{OA_1})$. Complete the parallelogram OA_1CB. Now the diagonal $(\overrightarrow{OC})$ will represent the relative velocity of car w.r.t. train i.e. $\vec{v}_{CT}$

Here, $v_{CT} = \sqrt{v_2^C + v_T^2 - 2v_Cv_T\cos 90°}$

$= \sqrt{v_2^C + v_T^2} = \sqrt{30^2 + 30^2} = 30\sqrt{2}$ km/h.

Illustration 15 : Rain is falling vertically with a speed of 35 ms^{-1}. Winds starts blowing after sometime with a speed of 12 ms^{-1} in East to West direction. In which direction should a boy waiting at a bus stop hold his umbrella.

Solution $\vec{v}_w = (\overrightarrow{OB}) = 12\ ms^{-1}$, along West.

$\vec{v}_r = (\overrightarrow{OA}) = 35\ ms^{-1}$, along vertical downward. The body can protect himself from the rain if he holds his umbrella in the direction of resultant velocity $\vec{R}$, i.e., along the direction OC as shown in Figure. If θ is the angle which $\vec{R}$ makes with the vertical direction, then

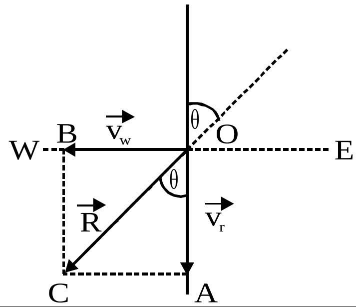
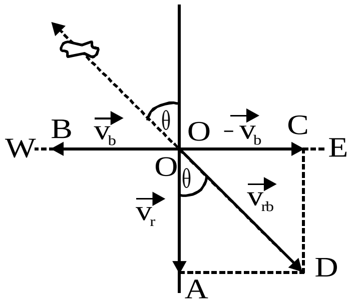


Illustration 16 : Rain is falling vertically with a speed of 35 ms^{-1}. A woman rides a bicycle with a speed of 12 ms^{-1} in East to West direction. What is the direction in which she should hold her umbrella?

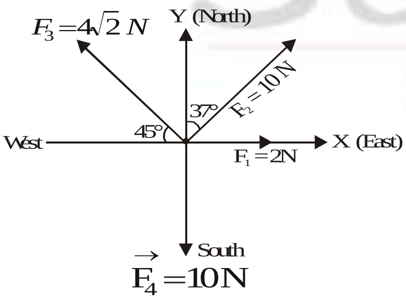
Solution



$\vec{v}_r = (\vec{OA}) = 35\,ms^{-1}$, along vertical

$\vec{v}_b = (\vec{OB}) = 12\,ms^{-1}$, along West. The woman can protect herself form the rain if she holds her umbrella in the direction of relative velocity of rain w.r.t. bicycle. To find the relative velocity of rain w.r.t. bicycle, bring the bicycle at rest by imposing a velocity $-\vec{v}_b (= \vec{OC})$ on bicycle and apply this velocity on rain also. Now the relative velocity of rain w.r.t. bicycle $(\vec{v}_{rb})$ will be the resultant of $\vec{v}_r\ (= \vec{OA})$ and $-\vec{v}_b\ (= \vec{OC})$; which will be represented by diagonal $\vec{OD}$ of rectangle OADC. $\tan\theta = \frac{AD}{OA} = \frac{v_b}{v_r} = \frac{12}{35} = 0.3429$, $\theta = 18°56'$ with the vertical towards the West.

Illustration 17 :



(Given sin 37° = cos 53° = 3/5, cos 37° = sin 53° = 4/5)
Find the resultant force and its direction.

Solution :

$\vec{F}_1 = 2\hat{i}$

$F_2 = 10\cos 37\hat{j} + 10\sin 37\hat{i} = 6\hat{i} + 8\hat{j}$

$\vec{F}_3 = -4\hat{i} + 4\hat{j}$

$\vec{F}_4 = -10\hat{j}$

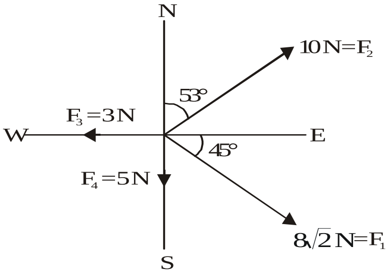
$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 4\hat{i} + 2\hat{j}$

$F = \sqrt{4^2 + 2^2} = \sqrt{20}\ N$

$\alpha = \tan^{-1}(F_y / F_x) = \tan^{-1}(1/2)$

North of East $\beta = \tan^{-1}\left(\frac{F_x}{F_y}\right) = \tan^{-1}(2)$East of North .

Illustration 18 :



Find the resultant force and its magnitude.

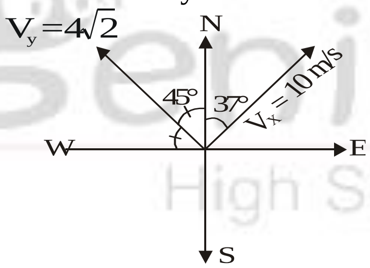
Solution : $\vec{F}_1 = 8\hat{i} - 8\hat{j}$ $\vec{F}_2 = 8\hat{i} + 6\hat{j}$

$\vec{F}_3 = -3\hat{i}$ $\vec{F}_4 = -5\hat{j}$

Adding all

$\therefore \quad \vec{F} = 13\hat{i} - 7\hat{j} \;\; \alpha = \tan^{-1}\left(\frac{7}{13}\right)$ South of East .

Illustration 19 : A car x is moving with a speed of 10 m/s. 37° E of N. Another car Y moves NW at $4\sqrt{2}$ m/s. Find the velocity of Y w.r.t. X and its direction.



Solution :

Resolving

S $\vec{V}_x$ and $\vec{V}_y$, where $\vec{V}_x = 6\hat{i} + 8\hat{j}$

and $\vec{V}_y = -4\hat{i} + 4\hat{j}$ $\therefore \quad \vec{V}_y - \vec{V}_x = -10\hat{i} - 4\hat{j}$

i.e., $\vec{V}_{yx} = \sqrt{116}$ m/s

and $\alpha = \tan^{-1}\left(\frac{2}{5}\right)$ South of West .

• BASIC MATHEMATICS

Differentiation

➢ Differential Co-Efficient, Or Derivative Of A Function

The differential co-efficient or derivative of variable y with respect to variable x is defined as the instantaneous rate of change of y w.r.t. x.

- It is denoted by $\frac{dy}{dx}$. i.e. y be a function of x i.e. $y = f(x)$ here, x is the independent variable and y is the dependent variable.
- The symbol $\frac{d(y)}{dx}$ represent the rate of change of y w.r.t. x, or the derivative of y w.r.t. x.

➢ Physical Illustrations Of Differential Co-efficient as rate Of Measurement

- Instantaneous speed of a body, v = rate of distance 's' with time t at the given instant.

i.e. $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

- Force is equal to rate of change of momentum

i.e. $F = \frac{dp}{dt}$

➢ Fundamental Formulae of Differentiation

- If 'c' is some constant then $\frac{d}{dx}(c) = 0$
- If $y = cx$ where c = constant

  then $= \frac{dy}{dx} = \frac{d(cx)}{dx} = c\frac{dx}{dx} = c$
- If $y = cu$, where c is a constant and u is function of x, then

  $\frac{dy}{dx} = \frac{d(cu)}{dx} = c\frac{du}{dx}$
- If $y = x^n$, where n is a real number, then

  $\frac{dy}{dx} = nx^{n-1}$
- If $y = u^n$, where n is the real number and u is the function of x.

  then $\frac{dy}{dx} = \frac{d(u^n)}{dx}$

  $\frac{dy}{dx} = n.u^{n-1}\left(\frac{du}{dx}\right)$
- If $y = (u + v)$

  where u and v are the functions of x

  $\frac{dy}{dx} = \frac{d(u+v)}{dx}$

  $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$
- If $y = (u - v)$

then $\frac{dy}{dx} = \frac{d(u-v)}{dx}$

$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

- If y = u.v, where u and v are function of *x*

  $\frac{dy}{dx} = \frac{d(u.v)}{dx} = \frac{vdu}{dx} + \frac{udv}{dx}$

- If $y = \frac{u}{v}$

Then, $\frac{dy}{dx} = \frac{d(u/v)}{dx}$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

➢ **Derivatives of Trigonometric Functions**

- If $y = \sin x$ then

  $\frac{dy}{dx} = \frac{d(\sin x)}{dx} = \cos x$
- If y = cos x then

  $\frac{dy}{dx} = \frac{d(\cos x)}{dx} = -\sin x$
- If $y = \tan x$ then

  $\frac{dy}{dx} = \frac{d(\tan x)}{dx} = \sec^2 x$
- If y = cot x ; then

  $\frac{dy}{dx} = \frac{d(\cot x)}{dx} = -\mathrm{cosec}^2 x$
- If y = sec x, then

  $\frac{dy}{dx} = \frac{d(\sec x)}{dx} = \tan x.\sec x$
- If y = cosec x then

  $\frac{dy}{dx} = \frac{d(\mathrm{cosec}\, x)}{dx} = -\cot x.\mathrm{cosec}\, x$

➢ **Derivative of Logarithmic**

- If $y = \log_e x$

  Then $\frac{dy}{dx} = \frac{1}{x}\log_e e = \frac{1}{x}$
- If $y = \log_e u$;

  then $\frac{dy}{dx} = \frac{1}{u} \times \frac{d(u)}{dx}$

- **Derivative of exponential**
  - If $y = e^x$

    $$\frac{dy}{dx} = \frac{d}{dx}\left(e^x\right) = e^x$$
  - If $y = a^x$

    $$\frac{dy}{dx} = \frac{d}{dx}(a^x) = a^x \log_e a$$

- **Function of Function – Chain Rule**

  If y = f(x) and x is a function of some othervariable z. Then $\frac{dy}{dx}$ can be written as the product of two derivatives.

  $$\frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx}$$

- **Double differentiation or second derivative**

  The second derivative of y with respect to x is defined as the derivative of the function $\frac{dy}{dx}$ (or the derivative of the derivative). It is usually written

  $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

- **Integration**
  - The process of integration is just the reverse of differentiation
  - The symbol $\int$ is used to represent the integration
  - $f(x)$ is the differential co-efficient of a function f(x) with respect to x.

- **Fundamental Formulae of Integration**
  - $\int x^n dx = \frac{x^{n+1}}{n+1}$, provided $n \neq -1$
  - $\int dx = \int x^{\circ} dx = \frac{x^{0+1}}{0+1} = x$
  - $\int (u+v)dx = \int udx + \int vdx$
  - $\int cudx = c\int udx$, where c is a constant and u is a function of x
  - $\int cx^n dx = c\left(\frac{x^{n+1}}{n+1}\right)$
  - $\int x^{-1} dx = \int \frac{dx}{x} = \log_e x$
  - (i) $\int \sin x dx = -\cos x$ (ii) $\int \sin(nx) = \frac{-\cos nx}{n}$

- (i) $\int \cos x.dx = \sin x$ (ii) $\int \cos nx.dx = \frac{\sin x}{n}$
- $\int \sec^2 x dx = \tan x$
- $\int \mathrm{cosec}^2 x.dx = -\cot x$
- $\int \sec x.\tan x.dx = \sec x$
- $\int \mathrm{cosec}\, x.\cot x.dx = -\mathrm{cosec}\, x$
- $\int \frac{dx}{x} = \log_e x$
- $\int e^x dx = e^x$
- $\int a^x dx = \frac{a^x}{\log_e a}$

➢ **Definite Integration**

- $\int_a^b x^n dx$, provided $n \neq -1 \Rightarrow \left[\frac{x^{n+1}}{n+1}\right]_a^b = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1}$
- $\int_{\theta_1}^{\theta_2} \cos\theta d\theta = [\sin\theta]_{\theta_1}^{\theta_2} = \sin\theta_2 - \sin\theta_1$.

Illustration 20 : Differentiate the following with respect to x

(i) $y = (4x + 2)$

(ii) $y = (2x^2 + 4)$

(iii) $y = \left(\frac{3}{2}x^2 + x\right)$

(iv) $= (ax + b)^n$.

Solution :

(i) $\frac{dy}{dx} = \frac{d(4x+2)}{dx}$

$\frac{dy}{dx} = \frac{4dx}{dx} + \frac{d(2)}{dx}$

$= 4 + 0$

$\frac{dy}{dx} = 4$

(ii) $\frac{dy}{dx} = \frac{d(2x^2+4)}{dx}$

$= \frac{d(2x^2)}{dx} + \frac{d(4)}{dx}$

$$= \frac{2d(x^2)}{dx} + \frac{d(4)}{dx}$$

$= 4x + 0 = 4x$

(iii) $y = \left(\frac{3}{2}x^2 + x\right)$

$$\frac{dy}{dx} = \frac{d\left(\frac{3}{2}x^2 + x\right)}{dx}$$

$$= \frac{3}{2}\frac{dx^2}{dx} + \frac{dx}{dx}$$

$$= \frac{3}{2} \times 2x + 1 = 3x + 1$$

(iv) $\frac{dy}{dx} = \frac{d\left(ax + b\right)^n}{dx}$

$$= n\left(ax + b\right)^{n-1} \times \frac{d\left(ax + b\right)}{dx} = n(ax + b)^{n-1}a$$

Illustration 21 : Differentiate the following with respect to x

(i) y = (1+sinx)

(ii) y = 2 + 2 cos x

(iii) y = (1+sin2x)

Solution :

(i) $\frac{dy}{dx} = \frac{d\left(1 + \sin x\right)}{dx}$

$$= \frac{d(1)}{dx} + \frac{d\sin x}{dx} = 0 + \cos x$$

$$\frac{dy}{dx} = \cos x$$

(ii) $\frac{dy}{dx} = \frac{d\left(2 + 2\cos x\right)}{dk} = \frac{d(2)}{dx} + \frac{2d\cos x}{dx}$

$$\frac{dy}{dx} = 0 + 2\left(-\sin x\right)$$

$$\frac{dy}{dx} = -2\sin x$$

(iii) $y = \left(1 + \sin 2x\right)$

$$\frac{dy}{dx} = \frac{d\left(1 + \sin 2x\right)}{dx} = \frac{d(1)}{dx} + \frac{d\sin 2x}{dx}$$

$$\frac{dy}{dx} = 0 + 2\cos 2x$$

$$\frac{dy}{dx} = 2\cos 2x$$

Illustrations 22: Find out the differentiation co–efficient of the following functions w.r.t. x.

(i) $y = \cos\{\log(x^n)\}$

(ii) $y = \sqrt{\sin(\log x)}$

(iii) $y = e^x - \cos x + \log x$

(iv) $y = e^x + \tan x + \log_e x$

(v) $y = \dfrac{\log x}{\tan x}$

Solution :

(i) $\dfrac{dy}{dx} = \dfrac{-\sin[\log x^n]}{x^n} \times nx^{n-1}$.

(ii) $\dfrac{dy}{dx} = \dfrac{1\cos(\log x)}{2\sqrt{\sin\log x}} \times \dfrac{1}{x}$.

(iii) $\dfrac{dy}{dx} = e^x + \sin x + \dfrac{1}{x}$.

(iv) $\dfrac{dy}{dx} = e^x - \sec^2 x + \dfrac{1}{x}$.

(iv) $\dfrac{dy}{dx} = \dfrac{(\tan x)\dfrac{1}{x} - \log x(\sec^2 x)}{\tan^2 x}$.

Illustration 23: If x = $t^3 + t^2 + t + 1$, then find $\dfrac{d^2x}{dt^2}$ and $\dfrac{dx}{dt}$.

Solution : $\dfrac{dx}{dt} = 3t^2 + 2t + 1; \dfrac{d^2x}{dt^2} = 6t + 2$.

Illustration 24: The instantaneous power delivered by an engine is given by

P = (2t + 1) (t + 1) Watts

Find work done by the engine between

t = 1s and t = 2s.

Solution :

$$W = \int_1^2 P dt = \int_1^2 (2t+1)(t+1)dt$$

$$= \int_1^2 (2t^2 + 2t + t + 1)dt = \int_1^2 (2t^2 + 3t + 1)dt$$

$$\Rightarrow \left[\frac{2}{3}t^3 + \frac{3}{2}t^2 + t\right]_1^2$$

$$= \left(\frac{2}{3}\times 8 + \frac{3}{2}\times 4 + 2\right) - \left(\frac{2}{3} + \frac{3}{2} + 1\right) = \frac{61}{6} J.$$

• WORK–ENERGY THEOREM

Work done on a body under the action of all forces is equal to change in kinetic energy.

i.e., $\boxed{W = \Delta K = K_f - K_i}$

Conservation of Energy :

The energy of universe neither be created nor destroyed but converted from one form to another form. This is called principle of conservation of enrgy.

If the forces in action all are conservative then mechanical energy is conserved.

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KEY POINTS

- If units of two physical quantities are same, these need not represent the same physical characteristics. For example torque and work have the same units but their physical characteristics are different.
- S I System or MKS System of measurement is the internationally accepted system measurement includes seven fundamental quantities and their units.
- 1 inch = 2.54 cm
- 1 foot = 12 inches = 30.48 cm = 0.3048 m
- 1 mile = 5280 ft = 1.609 km
- 1 yard = 0.9144 m
- 1 slug = 14.59 kg
- 1 barn = 10^{-28} m^2
- 1 liter = 10^3 cm^3 = 10^{-3} m^3
- 1 km/ h = 5/18 m/s
- 1 m/s = 3.6 km / h
- 1g/cm^3 =1000 kg/m^3
- The magnitude of the resultant vector is maximum if the two vectors are acting in the same direction and is minimum if the two vectors are acting in the opposite direction.
- Vector addition is commutative i.e. $\vec{A}+\vec{B}=\vec{B}+\vec{A}$
- Vector addition is associative i.e. $(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{A})$
- **Rectangular components of a vector in a plane.** If $\vec{A}$ makes an angle θ with x-axis and $\vec{A}_x$ and $\vec{A}_y$ be the rectangular components of $\vec{A}$ along x-axis and y-axis respectively,

  than $\vec{A}=\vec{A}_x+\vec{A}_y=A_x\hat{i}+A_y\hat{j}$

  $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

  $A^2(\cos^2\theta+\sin^2\theta) = A_x^2+A_y^2$ or $A = (A_x^2+A_y^2)^{1/2}$ and $\tan\theta = A_y / A_x$

- **Addition of vector in rectangular coordinates.**

  If $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ and $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$

  Then $\vec{A}+\vec{B}=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})+(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})=(A_x+B_x)\hat{i}+(A_y+B_y)\hat{j}+(A_z+B_z)\hat{k}$.

- We use calculus when a quantity varies with another quantity non uniformly.
- dy/dx is instantaneous slop of a graph between y and x varying non uniformly.
- Integration is just a opposite method of differentiation.
- Integration of a function (1) is the function (2) whose differentiation is the function (1).
- Differentiation of any constant is always 0.
- The slop is positive when a function y increases with respect to another function x.
- The slop is negative when a function y decrease with respect to another function x.
- Work done on a body under the action of all forces is equal to change in kinetic energy.

  i.e., $\boxed{W = \Delta K = K_f - K_i}$

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ASSIGNMENT–1

1. Light year is the unit of
(A) Energy (B) Intensity of light
(C) Time (D) Distance

2. The SI unit of pressure is
(A) Atmosphere (B) Bar
(C) Pascal (D) mm of Hg

3. Value of acceleration due to gravity is 9.8 m/s^2. Find its value in km/hr^2.
(A) 127008 (B) 137008
(C) 12000 (D) 14000.

4. Candela is the unit of
(A) Electricity intensity (B) Luminous intensity
(C) Sound intensity (D) None of these

5. Which relation is wrong
(A) 1 Calorie = 4.18 Joules (B) $1Å = 10^{-10}$ m
(C) $1MeV = 1.6\times10^{-13}$ Joules (D) 1 Newton = 10^{-5} Dynes

6. The equation $\left(P+\frac{a}{V^2}\right)(V-b)$ constant where P is pressure and v is the velocity. The units of a is
(A) Dyne×cm^5 (B) Dyne×cm^4
(C) Dyne×cm^3 (D) Dyne×cm^2.

7. A physical quantity is measured and its value is found to be nu where n = numerical value and u = unit. Then which of the following relations is true.
(A) $n \propto u^2$ (B) $n \propto u$
(C) $n \propto \sqrt{u}$ (D) $n \propto \frac{1}{u}$.

8. Due to which of the following reasons the statement, "length of the rod is 10 cms" is not correct ?
(A) The length should be expressed in metres
(B) The procedure for measuring length is not mentioned
(C) The magnitude of the length is not correct
(D) Correct symbol for the unit of length has not been used

9. The unit of Planck's constant is
(A) Joule (B) Joule/ s
(C) Joule/m (D) Joule-s

10. The magnitude of $3\hat{i}+2\hat{j}-\hat{k}$ is,

(A) $\sqrt{5}$ (B) $\sqrt{6}$

(C) $\sqrt{14}$ (D) $\sqrt{24}$

11. If $\hat{n}$ is a unit vector in the direction of the vector $\vec{P}$, then $\hat{n}$ =

(A) $\dfrac{\vec{P}}{|\vec{P}|}$ (B) $\vec{P}|\vec{P}|$

(C) $\dfrac{|\vec{P}|}{\vec{P}}$ (D) $\vec{P}$

12. The maximum value of resultant of two vectors $\vec{P}$ and $\vec{Q}$ is

(A) P + Q (B) P – Q

(C) $\sqrt{P^2+Q^2}$ (D) $\sqrt{P^2-Q^2}$

13. If the angle between two forces increases, the magnitude of their resultant

(A) decreases (B) increases

(C) remains unchanged (D) decreases and increases

14. What is the angle between the forces (x + y) and (x – y) if their resultant is $\sqrt{2(x^2+y^2)}$?

(A) 0° (B) 30°

(C) 60° (D) 90°

15. Five forces, 9.81 N each, acting at a point are coplanar. If the angles between neighboring forces are equal, then the resultant is

(A) 0 N (B) 9.81 N

(C) 2 × 9.81 N (D) 3 × 9.81 N

16. The minimum number of vectors in different planes which can be added to give zero resultant is

(A) 5 (B) 4

(C) 3 (D) 2.

17. Two perpendicular forces of 8N and 6N can produce the effect of a single force equal to

(A) 5 N (B) 6 N

(C) 8 N (D) 10 N

18. Angular momentum is

(A) Scalar (B) an axial vector

(C) a polar vector (D) none of these.

19. Which of the following is a scalar quantity ?

(A) electric current (B) strain

(C) specific gravity (D) none of these.

20. Which of the following is not a vector quantity ?

(A) density (B) displacement

(C) electric field intensity (D) angular momentum.

ASSIGNMENT–2

1. Out of the following pairs, the resultant of which cannot be 4 newton
(A) 2 N and 2 N (B) 2 N and 6 N
(C) 2 N and 4 N (D) 2 N and 8 N.

2. If $|\vec{V}_1 + \vec{V}_2| = |\vec{V}_1 - \vec{V}_2|$ and $|\vec{V}_2|$ is finite, then
(A) $\vec{V}_1$ is parallel to $\vec{V}_2$ (B) $\vec{V}_1$ is perpendicular to $\vec{V}_2$
(C) $|\vec{V}_1| = |\vec{V}_2|$ (D) $V_1 = V_2$.

3. A particle is moving eastwards with a velocity of 5 m/s. In 10 s the velocity changes to 5 m/s north–wards. The average acceleration in this time is
(A) zero (B) $\frac{1}{\sqrt{2}} m/s^2$ towards north–west
(C) 30º west of north (D) 60º east of north.

4. The speed of a boat is 5 km/hr is still water. If it crosses a river of width 1 km along the shortest possible path in 15 minutes, then velocity of the river is
(A) 4 km/hr (B) 3 km/hr
(C) 2 km/hr (D) 1 km/hr.

5. A boat a sent across is river with a velocity of 8 km/h. If the resultant velocity of the boat with respect to ground is 10 km/h, the river is flowing with a velocity of
(A) 12.8 km/h (B) 6 km/h
(C) 8 km/h (D) 10 km/h.

6. A bird is flying towards south with a velocity 40 km/hr and a train is moving with a velocity 40 km/hr towards east. What is the velocity of the bird w.r.t. an observer in train
(A) $40\sqrt{2}$ km/hr north–east (B) $40\sqrt{2}$ km/hr south–east
(C) $40\sqrt{2}$ km/hr south–west (D) $40\sqrt{2}$ km/hr north–west.

7. To a person going east in a car with a velocity of 25 km/hr a train appears to move towards north with a velocity of $25\sqrt{3}$ km/hr. The actual velocity of the train will be
(A) 25 km/hr (B) 50 km/hr
(C) 5 km/hr (D) $5\sqrt{3}$ km/hr.

8. Person aiming to reach the exactly opposite point on the bank of a stream is swimming with a speed of 0.5 m/s at an angle of 120º with the direction of flow of water. The speed of water in the stream is
(A) 1 m/s (B) 0.67 m/s
(C) 0.433 m/s (D) 0.25 m/s.

9. A boy is running on the plane road with velocity v with a long hollow tube in his hand. The water is falling vertically downwards with velocity u. At what angle to the vertical, he must incline the tube so that the water drops enter it without touching its sides ?
(A) $\tan^{-1}\left(\frac{v}{u}\right)$ (B) $\sin^{-1}\left(\frac{v}{u}\right)$
(C) $\tan^{-1}\left(\frac{u}{v}\right)$ (D) $\cos^{-1}\left(\frac{v}{u}\right)$.

10. A swimmer can swim in still water with a speed v and the river flowing with velocity v/2. To cross the river in shortest time, he should swim at an angle θ with the upstream. The ratio of time taken by the swimmer to swim across in shortest time and shortest distance is
(A) cos θ (B) sin θ
(C) tan θ (D) cot θ.

11. A helicopter is flying at $\frac{72}{\sqrt{2}}$ km/hr towards north–east. The wind is blowing at 36 km/hr towards south. The displacement of the helicopter is 2 hours is
(A) 36 km (B) 72 km
(C) 84 km (D) 96 km.

12. If one of the rectangular components of force 50N along the horizontal is 30N, the other component is
(A) 20 N (B) 40 N
(C) 80 N (D) $\left(\frac{50}{\sqrt{2}} N\right)$

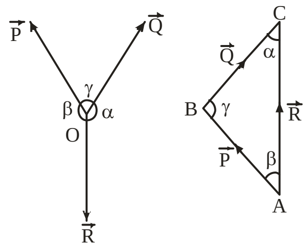
13. If one of the rectangular components of a force P is 3P/5, the other component is
(A) 4P / 5 (B) P / 5
(C) 3P / 5 (D) P

14. The components of a force of 100 N acting at an angle of 30° with the X - axis respectively are
(A) 70.72 N, 70.72 N (B) 50 N, 50 N
(C) 50 N, 25 $\sqrt{3}$ N (D) 50 $\sqrt{3}$ N , 50 N

15. Find the rectangular components (along X and Y axis) of a velocity vector $10\sqrt{3}$ m/s, making an angle of 60° to the X - axis
(A) 5 $\sqrt{3}$ m/s, 15 m/s (B) 5 $\sqrt{3}$ m/s, 20 m/s
(C) 10 $\sqrt{3}$ m/s, 20 m/s (D) 10 $\sqrt{3}$ 3 m/s, 15 m/s.

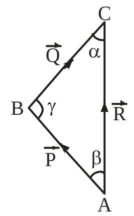
16. One of the two rectangular components of a force is 25 N and it makes an angle of 60° with the force. The magnitude of the other component is
(A) 25 N (B) 50 $\sqrt{3}$ N
(C) 25 $\sqrt{3}$ N (D) 25 $\sqrt{2}$ N.

17. If $\vec{P}, \vec{Q}$ and $\vec{R}$ are three forces (vectors) which simultaneously act at the point 'O' and keep it in equilibrium. Choose the correct relation from the figure shown



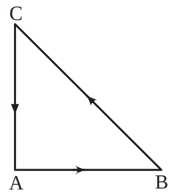
(A) $\frac{P}{BC} = \frac{Q}{AB} = \frac{R}{CA} = K\,(\text{constant})$
(B) $\frac{P}{AB} = \frac{Q}{BC} = \frac{R}{CA} = K\,(\text{constant})$
(C) $\frac{P}{BC} = \frac{R}{AB} = \frac{Q}{CA} = K\,(\text{constant})$
(D) $\frac{2P}{BC} = \frac{R}{AB} = \frac{Q}{CA} = K\,(\text{constant})$

18. If $\vec{P}, \vec{Q}$ and $\vec{R}$ are three forces (vectors) which simultaneously act at the point 'O' and keep it in equilibrium. Choose the correct relation from the figure shown



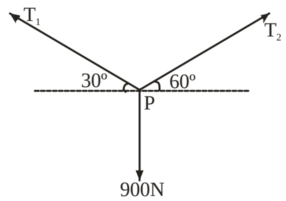
(A) $\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$
(B) $\frac{P}{\sin\beta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\gamma}$
(C) $\frac{Q}{\sin\alpha} = \frac{P}{\sin\beta} = \frac{R}{\sin\gamma}$
(D) $\frac{P}{\sin\alpha} = \frac{R}{\sin\beta} = \frac{Q}{\sin\gamma}$

19. Three forces start acting simultaneously on a particle moving with velocity $\vec{V}$. The forces are represented in magnitude and direction by the three sides of a triangle ABC (as shown). The particle will now move with velocity



(A) Less than $\vec{V}$
(B) Greater than $\vec{V}$
(C) both (A) and (B)
(D) $\vec{V}$ remaining unchanged

20. If P is in equilibrium, then tensions $\frac{T_1}{T_2} =$



(A) $\sqrt{3}$
(B) 2
(C) $\frac{1}{\sqrt{3}}$
(D) $\frac{1}{2}$

ASSIGNMENT–3

1. The displacement of a particle is given by $y = x^3 + 6x^2$ then, the velocity of the particle at $x = 1$
(A) 10 (B) 13
(C) 15 (D) 17.

2. $y = 3\cos^2 t$, find $\frac{dy}{dt}$
(A) $6\cos^2 t$ (B) $6 \sin t \cos t$
(C) $-6\sin^2 t$ (D) $-6 \sin t \cos t$.

3. Find $\frac{dy}{dx}$, if $y = \sin 3x$ at $x = \frac{\pi}{9}$
(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{3}{2}$
(C) $\frac{3\sqrt{3}}{2}$ (D) $\frac{9}{2}$.

4. If $y = \sin x/(1 + \cos x)$ then dy/dx at $x = \pi/2$ is,
(A) 1 (B) 0
(C) 1/2 (D) $\frac{\sqrt{3}}{2}$

5. $y = \int x^6 \cdot dx$ is,
(A) x^7 (B) $x^7/7$
(C) x^5 (D) 2x.

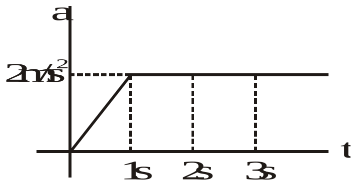
6. Evaluate $\int_1^4 \sqrt{x} \cdot dx$
(A) $\frac{10}{3}$ (B) $\frac{14}{3}$
(C) $\frac{5}{3}$ (D) $\frac{13}{3}$

7. The radius of a circle is increasing at a rate of 1 cm/s. The rate of increase of its area, when its radius is 10 cm is,
(A) 20 cm^2/s (B) 15 cm^2/s
(C) 20 π cm^2/s (D) 15 π cm^2/s.

8. Two cars A and B start from O simultaneously along OX and OY, with constant speeds 3 m/s and 4 m/s. Find their velocity of separation. (given OX $\perp$ OY).
(A) 3 m/s (B) 4 m/s
(C) 5 m/s (D) 6 m/s.

9. Acceleration particle traveling along x–axis is given by
$a = (2t + 1)\ m/s^2$
if velocity at t = 1 s is 5 m/s, then velocity of the particle at t = 2 sec. will be
(A) 8 m/s (B) 9 m/s
(C) 10 m/s (D) 11 m/s.

10. If initial velocity is 5 m/s. The velocity of the particle at t = 2s be
(A) 10 m/s (B) 5 m/s
(C) 8 m/s (D) 3 m/s.



11. A heavy stone is thrown from a cliff of height h with a speed v. The stone will hit the ground with maximum speed if it is thrown
(A) vertically downward
(B) vertically upward
(C) horizontally
(D) the speed does not depend on the initial direction

12. The negative of the work done by the conservative internal forces on a system equals the change in
(A) total energy (B) kinetic energy
(C) potential energy (D) None of these

13. The work done by the external forces on a system equals the change in
(A) total energy (B) kinetic energy
(C) potential energy (D) None of these

14. The work done by all the forces (external and internal) on a system equals the change in
(A) total energy (B) kinetic energy
(C) potential energy (D) None of these

15. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is
(A) 16 J (B) 8 J
(C) 32 J (D) 24 J

16. When a spring is stretched by 2 cm, it stores 100 J of energy. If it is stretched further by 2 cm, the stored energy will be increased by
(A) 100 J (B) 200 J
(C) 300 J (D) 400 J

17. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is
(A) –0.5 J (B) –1.25 J
(C) 1.25 J (D) 0.5 J

KEYS

ASSIGNEMENT – 1

1.	(D)	2.	(C)
3.	(A)	4.	(B)
5.	(D)	6.	(B)
7.	(D)	8.	(D)
9.	(D)	10.	(C)
11.	(A)	12.	(A)
13.	(A)	14.	(D)
15.	(A)	16.	(B)
17.	(D)	18.	(B)
19.	(A)	20.	(A)

ASSIGNEMENT – 2

1.	(D)	2.	(B)
3.	(B)	4.	(B)
5.	(B)	6.	(C)
7.	(D)	8.	(A)
9.	(B)	10.	(B)
11.	(B)	12.	(B)
13.	(A)	14.	(D)
15.	(A)	16.	(C)
17.	(B)	18.	(A)
19.	(B)	20.	(C)

ASSIGNEMENT – 3

1.	(C)	2.	(D)
3.	(B)	4.	(A)
5.	(B)	6.	(B)
7.	(C)	8.	(C)
9.	(B)	10.	(C)
11.	(D)	12.	(C)
13.	(A)	14.	(B)
15.	(B)	16.	(C)
17.	(B)		

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