7TH CLASS WS-4 SOLUTIONS

Logarithms

Teaching Task

1. $\log(ab)^m = m\log(ab) = m(\log a + \log b)$
2. $\log_{10}\sqrt{5} + \log_{10}\sqrt{7} = \log_{10}\sqrt{5}\sqrt{7} = \log_{10}\sqrt{35}$
3. $\log_{10} 2 = 0.3010$

   $\log_{10} 32^5 = 5\log_{10} 32 = 5\log_{10} 2^5 = 25\log_{10} 2$

   $= 25 \times 0.3010 = 7.525$
4. $\bar{6}\cdot 325 = -6 + 0.325$
5. $\log_{10} x = -3.5196 = -(4 - 0.4804)$

   $= -4 + 0.4804$

   $= -4.4804$

   $= \bar{4}\cdot 4804$
6. $\log_{10} 5^3 + \log_{10} 2^3 = \log_{10}\left(5^3 \cdot 2^3\right) = \log_{10} 10^3 = 3\log_{10} 10 = 3$
7. $2\log 5\sqrt{5} + 3\log 2 - \log 2 - \log 50 - 1$

   $= \log\left(5\sqrt{5}\right)^2 + \log 2^3 - \log 2 - \log 50 - \log 10$

   $= \log\left(\dfrac{5\sqrt{5}\times 5\sqrt{5}\times 2\times 2\times 2}{2\times 50\times 10}\right) = \log 1$

   Key – C
8. $\log_3 27 = a \Rightarrow \log_3 3^3 = a \Rightarrow 3\log_3 3 = a$

   $\Rightarrow a = 3$

   $\log_2 16 = \log_2 2^4 = 4\log_2 2 = 4 \rightarrow$ (1)

   From option (A)

   $6\times\left(\dfrac{3+a}{3a}\right) = 6\left(\dfrac{3+3}{3(3)}\right) = 6\left(\dfrac{6}{9}\right) = 4 \rightarrow$ (2)

   $\therefore \log_2 16 = 6\left(\dfrac{3+a}{3a}\right)$
9. $\dfrac{1}{\log_{x^n} xyz} + \dfrac{1}{\log_{y^n} xyz} + \dfrac{1}{\log_{z^n} xyz} =$

   $= \log_{xyz} x^n + \log_{xyz} y^n + \log_{xyz} z^n$

   $= \log_{xyz} x^n \cdot y^n \cdot z^n = \log_{xyz}(xyz)^n = n$
10. $\log_{c^2} a^b \cdot \log_{a^3} b^c \cdot \log_{b^4} c^a = \dfrac{b}{2}(\log_c a)\dfrac{c}{3}(\log_a b)\dfrac{a}{4}(\log_b c)$

    $= \dfrac{abc}{24}\cdot\dfrac{\log a}{\log c}\cdot\dfrac{\log b}{\log a}\cdot\dfrac{\log c}{\log b} = \dfrac{abc}{24}$

Multiple correct answer type

11. From definition for logarithm

Options A, B, C are correct.

12. $2^{\log_{\sqrt{2}} 15} = 2^{\left(\log_{2^{\frac{1}{2}}} 15\right)} = 2^{\frac{1}{\left(\frac{1}{2}\right)}\log_2 15}$

$= 2^{2\log_2 15} = 2^{\log_2 15^2} = 15^2 = 225 = \sqrt[3]{225^3}$

A and B are correct.

13. If $\log_{a^{\frac{1}{n}}} m^1 = \frac{n}{x}$

$\frac{1}{\frac{1}{n}}\log_a m^1 = \frac{n}{x}$

$n\log_a m = \frac{n}{x}$

$\log_a m^n = \frac{n}{x}$

$\therefore a^{\frac{n}{x}} = m^n$

$\left(a^{\frac{1}{x}}\right)^n = m^n$

$\therefore m = a^{\frac{1}{x}} \Rightarrow m^x = a$

$\Rightarrow \log_m a = x$ from definition

A, C are correct.

14. Statement – I – True

$5^{7\log_5 2} = 5^{\log_5 2^7} = 2^7 = 128$

Statement – II is false

15. From statement – I – false

$\log_x a + \log_y x + \log_b y \neq \log_b a$

Statement – II – True.

$\log_5 2 \times \log_7 5 \times \log_{10} 7 =$

$= \frac{\log 2}{\log 5} \times \frac{\log 5}{\log 7} \times \frac{\log 7}{\log 10} = \frac{\log 2}{\log 10} = \log_{10} 2$

Key – D

16. $\frac{1}{\log_{a^2} abc} + \frac{1}{\log_{b^2} abc} + \frac{1}{\log_{c^2} abc} =$

$\log_{abc} a^2 + \log_{abc} b^2 + \log_{abc} c^2 = \log_{abc} a^2 \cdot b^2 \cdot c^2$

$= \log_{abc}(abc)^2 = 2\log_{abc} abc = 2$

17. $\log_b a \cdot \log_c b \cdot \log_d c \cdot \log_{\frac{1}{a}} d =$

$= \frac{\log a}{\log b} \cdot \frac{\log b}{\log c} \cdot \frac{\log c}{\log d} \cdot \frac{\log d}{\log a^{-1}} = \frac{\log a}{-\log a} = -1$

18. $(\log_a x)(\log_b y) = \frac{\log x}{\log a} \cdot \frac{\log y}{\log b}$

$= \frac{\log x}{\log b} \cdot \frac{\log y}{\log a} = \log_b x \cdot \log_a y$

Comprehension If $\log_{10} x = p, \log_{10} y = q$

19. $\frac{p}{q} = \frac{\log_{10} x}{\log_{10} y} = \frac{\log x}{\log 10} \cdot \frac{\log 10}{\log y} = \frac{\log x}{\log y} = \log_y x$

20. $x = 10^p, y = 10^q$ from definition for logarithm

$xy = 10^p \cdot 10^q = 10^{p+q}$

21. $\frac{x+y+1}{x+y-1} = \frac{10^p + 10^q + 1}{10^p + 10^q - 1}$

22. $64^{(\log_4 2)} = 4^{3\log_4 2} = 4^{\log_4 2^3} = 2^3 = 8$

23. $\log_2\{\log_3(\log_2 x)\} = 1$

$\log_3(\log_2 x) = 2^1 = 2$

$\log_2 x = 3^2 = 9$

$x = 2^9$

Matrix matching type

24. Column I

a) $\log_{0.2} 0.008 = \log_{0.2}(0.2)^3 = 3\log_{0.2} 0.2 = 3 \rightarrow$ (r)

b) $\log_8 2 = \log_{2^3} 2 = \frac{1}{3}\log_2 2 = \frac{1}{3} \rightarrow$ (p)

c) $\frac{\log 10 \times \log 16}{\log 2} = \log 10 \times \frac{\log 2^4}{\log 2} = \log 10 \times 4 = 4\log 10 = 4 \rightarrow$ (q)

d) $\frac{\log 216}{\log \sqrt{6}} = \log_{\sqrt{6}} 216 = \log_{6^{\frac{1}{2}}} 6^3 = \frac{3}{\frac{1}{2}} = 3 \times 2 = 6 \rightarrow$ (s)

25. Apply formulae

$\log ab = \log a + \log b$

$\log \frac{a}{b} = \log a - \log b$

$\log a^n = n \log a$

$a - r, b - t, c - q, d - p$

LEARNER'S TASK

1. $2^4 = 16 \Rightarrow \log_2 16 = 4$ from definition
2. $\log_5 a^2 = 0 \Rightarrow a^2 = 5^{20} \Rightarrow a = 5^{10}$
3. Conceptual
4. $\frac{\log 8}{\log 2} = \log_2 2^3 = 3\log_2 2 = 3$
5. $\log 36 = \log(2 \times 18) = \log 2 + \log 18$

$= \log(3\times 12) = \log 3 + \log 12$

$= \log(9\times 4) = \log 9 + \log 4$

Key – D

6. $2^{\log_{10} 5} = 2^{(\log_2 5 \cdot \log_{10} 2)} = \left(2^{\log_2 5}\right)^{\log_{10} 2} = (5)^{\log_{10} 2}$

7. $\log_3 4 \times \log_2 3 = \frac{\log 4}{\log 3}\frac{\log 3}{\log 2} = \frac{\log 4}{\log 2} = \log_2 4$

8. $\log_{10} 20 - \log_{10} 2 = \log_{10}\left(\frac{20}{2}\right)$

9. $\log_{\sqrt[3]{7}} \sqrt[5]{6} = \log_{7^{\frac{1}{3}}} 6^{\frac{1}{5}} = \frac{\frac{1}{5}}{\frac{1}{3}}\log_7 6 = \frac{3}{5}\log_7 6$

10. $\log_{q^p} p^q = \frac{q}{p}\log_q p$ Conceptual

JEE MAIN LEVEL

1. $2^{3\log_2 2} + 3^{2\log_3 2} = 2^{\log_2 2^3} + 3^{\log_3 2^2} = 2^3 + 2^2 = 8 + 4 = 12$
2. $4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$

   $2^{2\log_{3^2} 3} + 3^{2\log_2 2^2} = 10^{\log_x 83}$

   $\Rightarrow 2^{2\times\frac{1}{2}\log_3 3} + 3^{2\times 2\log_2 2} = 10^{\log_x 83}$

   $2 + 3^4 = 10^{\log_x 83}$

   $83 = 10^{\log_x 83}$

   $\therefore x = 10$ where $10^{\log_{10} 83} = 83$
3. Let $a^x = b^y = c^z = d^w = k$

   $a^x = k, b^y = k, c^z = k, d^w = k$

   Apply log for all.

   $x\log a = \log k, y\log b = \log k, z\log c = \log k, w\log d = \log k$

   $\Rightarrow \log a = \frac{\log k}{x}, \log b = \frac{\log k}{y}$

   $\log c = \frac{\log k}{z}, \log d = \frac{\log k}{w}$

   $\therefore \log_a bcd = \frac{\log b + \log c + \log d}{\log a}$

   $= \frac{\frac{\log k}{y} + \frac{\log k}{z} + \frac{\log k}{w}}{\frac{\log k}{x}}$

   $= \frac{x\log k\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)}{\log k} = x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right)$

   Key – C
4. $\log_{10} 5 = 0.69897$

$$\log_{10} 50 = \log_{10}(5\times10) = \log_{10} 5 + \log_{10} 10$$
$$= 0.69897 + 1 = 1.69897$$

5. $a = 1+\log_x yz, b = 1+\log_y zx, c = 1+\log_z xy$

$$a = \log_x x + \log_x yz = \log_x xyz = \frac{\log xyz}{\log x}$$

$$b = \log_y y + \log_y zx = \log_y xyz = \frac{\log xyz}{\log y}$$

Similarly $c = \frac{\log xyz}{\log z}$

$$ab+bc+ca = \frac{\log xyz}{\log x}\cdot\frac{\log xyz}{\log y} + \frac{\log xyz}{\log y}\cdot\frac{\log xyz}{\log z} + \frac{\log xyz}{\log z}\cdot\frac{\log xyz}{\log x}$$

$$= (\log(xyz))^2 \left\{\frac{1}{\log x \log y} + \frac{1}{\log y \log z} + \frac{1}{\log z \log x}\right\}$$

$$= (\log(xyz))^2 \left(\frac{\log z + \log x + \log y}{\log x \cdot \log y \cdot \log z}\right)$$

$$= \log(xyz)\log(xyz)\frac{\log xyz}{\log x\cdot\log y\cdot\log z}$$

$$= \frac{\log xyz}{\log x}\cdot\frac{\log xyz}{\log y}\cdot\frac{\log xyz}{\log z} = a\cdot b\cdot c$$

6. $5^x = (0.5)^y = 1000 = 10^3$

$5^x = 10^3 \rightarrow$ (1) and $(0.5)^y = 10^3 \rightarrow$ (2)

Apply log on both sides

$x\log 5 = 3\log 10 = 3$ and $y\log(0.5) = 3\log 10$

$\frac{3}{x} = \log 5$ $\qquad$ $y\log(0.5) = 3$

$$\frac{3}{y} = \log\left(\frac{5}{10}\right)$$

$$\frac{3}{x} - \frac{3}{y} = \log 5 - \log\frac{5}{10}$$

$$= \log\frac{5}{\frac{5}{10}} = \log 10 = 1$$

$$3\left(\frac{1}{x} - \frac{1}{y}\right) = 1$$

$$\therefore \frac{1}{x} - \frac{1}{y} = \frac{1}{3}$$

7. $\log 2 = a, \log 3 = b, \log 7 = c$

$6^x = 7^{x+4} = 7^x \times 7^4$

Taking log on both sides

$$x\log(3\times2) = x\log 7 + 4\log 7$$

$$x\log(3\times2) - x\log 7 = 4\log 7$$

$$x[\log 3 + \log 2 - \log 7] = 4\log 7$$

$x[b+a-c]=4c$

$x=\dfrac{4c}{a+b-c}$

8. Let $x=3^{40}$, if $\log_{10}3=0.477$

$\log_{10}x=40\log_{10}3=40(0.477)=19.08$

$\therefore$ No. of digits in x = 19 + 1 = 20

9. $\log_{10}2=0.3010, \log_{10}3=0.4771$

Let $x=\left(2^{10}\cdot 3^{5}\right)^{20}=2^{200}\times 3^{100}$

Taking log on both sides

$\log_{10}x=200\log_{10}2+100\log_{10}3$

$=200(0.3010)+100(0.4771)$

$=60.20+47.71=107.91$

$\therefore$ No. of digits in $\left(3^{10}\cdot 3^{5}\right)^{20}$ is

107 + 1 = 108

10. $\log_3 4\times\log_4 5\times\log_5 6\times\log_6 7\times\log_7 8\times\log_8 9$

$=\dfrac{\log 4}{\log 3}\times\dfrac{\log 5}{\log 4}\times\dfrac{\log 6}{\log 5}\times\dfrac{\log 7}{\log 6}\times\dfrac{\log 8}{\log 7}\times\dfrac{\log 9}{\log 8}$

$=\dfrac{\log 9}{\log 3}=\dfrac{\log 3^2}{\log 3}=\dfrac{2\log 3}{\log 3}=2$

11. $\dfrac{\log x}{y-z}=\dfrac{\log y}{z-x}=\dfrac{\log z}{x-y}=k$

Let $\dfrac{x(\log x)}{x(y-z)}=\dfrac{y(\log y)}{y(z-x)}=\dfrac{z(\log z)}{z(x-y)}=k$

$\therefore \log x^x=kx(y-z)=k(xy-xz)$

Similarly $\log y^y=k(yz-yx)$

$\log z^z=k(zx-zy)$

$\therefore \log x^x+\log y^y+\log z^z=k(xy-xz+yz-yx+zx-zy)=k(o)=0$

$\log x^x\cdot y^y\cdot z^z=0$

$x^x\cdot y^y\cdot z^z=1$

12. $a=\log_{12}18 \quad =\dfrac{\log 18}{\log_2 12}=\dfrac{\log_2\left(2\times 3^2\right)}{\log_2\left(3\times 2^2\right)}$

$=\dfrac{1+2\log_2 3}{2+\log_2 3}$

$b=\log_{24}54=\dfrac{\log 54}{\log 24}=\dfrac{\log_2\left(2\times 3^3\right)}{\log_2\left(3\times 2^3\right)}=\dfrac{1+3\log_2 3}{3+\log_2 3}$

Let $x=\log_2 3$

$\therefore a=\frac{1+2x}{2+x}, b=\frac{1+3x}{3+x}$

$\therefore ab+5(a-b)=\frac{1+2x}{2+x}\cdot\frac{1+3x}{3+x}+5\left(\frac{1+2x}{2+x}-\frac{1+3x}{3+x}\right)$

$=\frac{6x^2+5x+1+5((3+x)(1+2x)-(1+3x)(2+x))}{(x+2)(x+3)}$

$=\frac{6x^2+5x+1+5(1-x^2)}{(x+2)(x+3)}=\frac{x^2+5x+6}{x^2+5x+6}=1$

13. $\log_3 x+\log_3 y=2+\log_3 2$

$\therefore \log_3 xy=2(1)+\log_3 2=2\log_3 3+\log_3 2=\log_3 3^2+\log_3 2$

$=\log_3(9\times 2)=\log 18$

$\therefore xy=18\rightarrow$ (1)

Given $\log_3(x+y)=2\Rightarrow x+y=3^2=9\rightarrow$ (2)

And $(x-y)^2=(x+y)^2-4xy=9^2-4(18)=9$

$\therefore x-y=3\rightarrow$ 3 by verification, we get option (C)

14. $\log_{30} 3=c, \log_{10} 5=d$

$\log_{30} 8=\log_{30} 2^3=3\log_{30} 2=3\log_3\left(\frac{30}{15}\right)=3\log_{30}\left(\frac{30}{3\times 5}\right)$

$=3[\log_{30} 30-\log_{30} 3-\log_{30} 5]$

= 3 (1 – c – d)

15. $a^2+b^2=7ab$

Adding 2ab on both sides

$a^2+b^2+2ab=9ab$

$(a+b)^2=9ab$

$a+b=3\sqrt{ab}$

$\frac{a+b}{3}=\sqrt{ab}=(ab)^{\frac{1}{2}}$

$\log\left(\frac{a+b}{3}\right)=\frac{1}{2}\log(ab)$

ADVANCED LEVEL

Multiple correct answer type

1. $\log_{q^b} p^a=\frac{b^2}{a^2}$

$\Rightarrow \frac{a}{b}\log_q p=\frac{b^2}{a^2}\Rightarrow a^3\log_q p=b^3$

$\Rightarrow \log_q p^{a^3}=b^3$

$\Rightarrow p^{a^3}=q^{b^3}$

And $\log_q p=\frac{b^3}{a^3}\Rightarrow p=(q)^{\frac{b^3}{a^3}}$

Key – B

2. $a^{\log_{m^3} b} = b^{\log_{n^3} a}$

$\Rightarrow a^{\frac{1}{3}\log_m b} = b^{\frac{1}{3}\log_n a}$

$\Rightarrow a^{\log_m (b)^{\frac{1}{3}}} = b^{\log_n (a)^{\frac{1}{3}}}$

It is true when $m = n \neq 0$ only.

Key – A

3. Conceptual

4. Statement – I True

Statement – II True

$$\frac{1}{\log_x xy} + \frac{1}{\log_y xy} = \log_{xy} x + \log_{xy} y = \log_{xy} xy = 1$$

Key – A

5. Statement – I True

$\log x_1 + \log x_2 + ... + \log x_n = \log\left(x_1 \cdot x_2 \cdot x_3 ...... x_n\right)$

Statement II – False

$$\log\frac{a^2}{b} + \log\frac{b^2}{c} + \log\frac{c^2}{a} - \log abc$$

$$= \log\left(\frac{\frac{a^2}{b}\cdot\frac{b^2}{c}\cdot\frac{c^2}{a}}{abc}\right) = \log\left(\frac{a^2b^2c^2}{a^2b^2c^2}\right) = \log 1 = 0$$

Key – C

6. $\log_{10}\frac{x^2}{2x} = \log_{10} 5 - 1 = \log_{10} 5 - \log_{10} 10 = \log_{10}\left(\frac{5}{10}\right) = \log_{10}\left(\frac{1}{2}\right)$

$$\therefore \frac{x^2}{2x} = \frac{1}{2} \Rightarrow x^2 = \frac{2x}{2} = x \Rightarrow x^2 - x = 0$$

$$x(x-1) = 0$$

$$x = 0, x = 1$$

7. $\log_a\left(\frac{a^2}{2}\right) = \log_a 4 - \log_a 2 + 1 = \log_a\left(\frac{4}{2}\right) + \log_a a$

$$= \log_a 2 + \log_a a = \log_a (2a)$$

$$\therefore \frac{a^2}{2} = 2a \Rightarrow a^2 = 4a \Rightarrow a^2 - 4a = 0$$

$$a(a-4) = 0$$

$$a = 0, a = 4$$

8. $\log_{x^{\frac{3}{2}}} x^3 = \log_x a^2 + 1 = \log_x a^2 + \log_x x = \log_x a^2 x$

$$\frac{3}{\frac{3}{2}}\log_x x = \log_x a^2 x$$

$$\log_x\left(a^2 x\right) = 2$$

$$a^2x = x^2$$
$$a^2 = x$$
$$a = \pm\sqrt{x}$$

9. $x^{\log_{x^2+2x} y} = y^{\log_{y^2+2y} x}$

Bases are equal $\therefore x = y$

And $x^2 + 2x = y^2 + 2y$ It is true when $x = 1, y = 1$

10. $2^{\log_{x^2+5x} 3} = 3^{\log_{25} 2}$, if $2 = 3^{\log_{25} 2}$

$$\therefore 2^{\log_{x^2+5x} 3} = 2 = 2^1$$
$$\therefore \log_{x^2+5x}(3) = 1$$
$$(x^2 + 5x)^1 = 3$$
$$x^2 + 5x - 3 = 0$$

Solving for 'x' , we get an irrational number because we can't factorise the above equation.

$\therefore$ Key – C

11. $5^{\log_{10}(-x)} = (-x)^{\log_{10} 5}$

By comparing we will get x = -5

-x = 5

Key – D

ADDITIONAL PRACTICE QUESTIONS

Comprehension : $\log_a m + \log_a n = \log_a mn$ & $\log_a m - \log_a n = \log_a\left(\frac{m}{n}\right)$

12. $\log_{10} 5 + \log_{10} 5x + 1 = \log_{10}(x+5) + \log_{10} 10$

$$\log_{10} 5(5x+1) = \log_{10}(x+5)10$$
$$\therefore 5(5x+1) = (x+5)10 = 2x + 10$$
$$5x + 1 - 2x - 10 = 0$$
$$3x - 9 = 0$$
$$3x = 9 \Rightarrow x = \frac{9}{3} = 3$$

13. $\log_5(x^2 + x) - \log_5(x+1) = 2 = \log_5 25$

$$\log_5\left(\frac{x^2+x}{x+1}\right) = \log_5(25)$$
$$\therefore \frac{x^2+x}{x+1} = 25$$
$$x^2 + x = 25(x+1) = 25x + 25$$
$$\therefore x^2 - 24x + x - 25 = 0$$
$$x^2 - 25x + x - 25 = 0$$

$$x(x-25)+1(x-25)=0$$

$$(x+1)(x-25)=0$$

$x+1=0$ or $x-25=0$

$x=-1$ $x=25$

14. $\log_4 x+\log_2 x=6$ - True

From option D) x = 16, we can check,

$$\log_4 4^2+\log_2 2^4=2\log_4 4+4\log_2 2=2+4=6$$

15. Conceptual
16. Conceptual
17. Conceptual
18. $\log_{2\sqrt{3}} 1728=\log_{\sqrt{12}} 12^3=\log_{(12)^{\frac{1}{3}}} 12^3$

$$=\frac{3}{\left(\frac{1}{3}\right)}\log_{12} 12=9\log_{12} 12=9$$

19. $\log_{\sqrt{8}} x=3\frac{1}{3}$ then x = ?

$\log_{\sqrt{8}} x=\frac{10}{3}$ $\because \sqrt{8}=\sqrt{2^3}=2^{\frac{3}{2}}$

$$x=\left(\sqrt{8}\right)^{\frac{10}{3}}=\left(2^{\frac{3}{2}}\right)^{\frac{10}{3}}=2^5=32$$

20. Conceptual
21. Conceptual
22. Conceptual