TRIGONOMETRIC EQUATIONS – I

Class: IX, Mathematics

IIT ADVANCED

TEACHING TASK

01. $\operatorname{cosec}\theta + 2 = 0$

$\Rightarrow \operatorname{cosec}\theta = -2$

$\Rightarrow \operatorname{cosec}\theta = \operatorname{cosec}(180^\circ + 60^\circ) = \operatorname{cosec} 240^\circ$

(OR) $= \operatorname{cosec}(360^\circ - 60^\circ) = \operatorname{cosec} 300^\circ$

$\therefore \theta = 240^\circ$ or 300° $\quad \therefore$ Ans: B

02. $7\sin^2 x + 3\cos^2 x = 4$

$\Rightarrow 7\sin^2 x + 3(1 - \sin^2 x) = 4$

$\Rightarrow 4\sin^2 x = 1$

$\Rightarrow \sin^2 x = \left(\frac{1}{2}\right)^2$

$\Rightarrow \sin^2 x = \sin^2 \frac{\pi}{6}$

$\Rightarrow x = n\pi \pm \frac{\pi}{6}, \; n \in \mathbb{Z}$

Ans: D

03. $\cos 2\theta = 2\sin^2\theta$

$\Rightarrow 1 - 2\sin^2\theta = 2\sin^2\theta$

$\Rightarrow 4\sin^2\theta = 1$

$\Rightarrow \sin^2\theta = \left(\frac{1}{2}\right)^2$

$\sin^2\theta = \sin^2\frac{\pi}{6}$

$\therefore \theta = n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z}$

Ans: C

(2)

04 $\cot\theta = -\sqrt{3}$, $\operatorname{cosec}\theta = 2$

$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$, $\sin\theta = \frac{1}{2}$

$\Rightarrow \tan\theta = \tan\frac{5\pi}{6}$, $\sin = \sin\frac{5\pi}{6}$

$\therefore \theta = 2n\pi + \frac{5\pi}{6}$

Ans: D

05 $\tan\theta = -1$, $\cos\theta = \frac{1}{\sqrt{2}}$

$\tan\theta = \tan\left(\frac{7\pi}{4}\right)$, $\cos\theta = \cos\frac{7\pi}{4}$

$\alpha = \frac{7\pi}{4}$

hence $\theta = 2n\pi + \frac{7\pi}{4}$

Ans: B

06 $2\sin x + \operatorname{cosec} x = 3$

$\Rightarrow 2\sin^2 x + 1 = 3\sin x$

$\Rightarrow 2\sin^2 x - 3\sin x + 1 = 0$

$\Rightarrow (\sin x - 1)(2\sin x - 1) = 0$

$\Rightarrow \sin x = 1$ or $\sin x = \frac{1}{2}$

$\Rightarrow \sin x = \sin\frac{\pi}{2}$ or $\sin x = \sin\frac{\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n\frac{\pi}{2}$ or $x = n\pi + (-1)^n\frac{\pi}{6}$

Ans: D

07 option varification

let $x = \frac{\pi}{4}$.

$LHS = \cos 2x = \cos 2\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{2} = 0$

$RHS = (\sqrt{2}+1)\left(\cos x - \frac{1}{\sqrt{2}}\right) = (\sqrt{2}+1)\left(\cos\frac{\pi}{4} - \frac{1}{\sqrt{2}}\right) = 0$

hence $x = \{2n\pi \pm \frac{\pi}{4}\}$

Ans: D

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08 $\sin^{10}x - \cos^{10}x = 1$

opt: A. $\sin^{10}\left(n\pi+\frac{\pi}{2}\right) - \cos^{10}\left(n\pi+\frac{\pi}{2}\right)$

let $n=1$

$= \sin^{10}\left(\pi+\frac{\pi}{2}\right) - \cos^{10}\left(\pi+\frac{\pi}{2}\right)$

$= \left[-\sin\frac{\pi}{2}\right]^{10} - \left[-\cos\frac{\pi}{2}\right]^{10}$

$= [-1]^{10} - [0]^{10} = 1 - 0 = 1$

Ans: A

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09 $r\sin\theta = \sqrt{3}$ | $r + 4\sin\theta = 2(\sqrt{3}+1)$

$\Rightarrow r = \dfrac{\sqrt{3}}{\sin\theta}$ | $\dfrac{\sqrt{3}}{\sin\theta} + 4\sin\theta = 2(\sqrt{3}+1)$

$\Rightarrow \sqrt{3} + 4\sin^2\theta = 2\sin\theta(\sqrt{3}+1)$

$\Rightarrow 4\sin^2\theta - 2\sin\theta + \sqrt{3} - 2\sqrt{3}\sin\theta = 0$

$\Rightarrow 2\sin\theta(2\sin\theta - 1) \cancel{+} \sqrt{3}(2\sin\theta - 1) = 0$

$\Rightarrow (2\sin\theta - 1)(2\sin\theta - \sqrt{3}) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}$ or $\sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin\theta = \sin\frac{\pi}{6}$ or $\sin\theta = \sin\frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{2\pi}{3}$ satisfies the above equations

10 $4\cos^2 x \sin x - 2\sin^2 x - 3\sin x = 0$

(4)

$\sin x\,(4\cos^2 x - 2\sin x - 3) = 0$

$\sin x = 0$ or $4(1-\sin^2 x) - 2\sin x - 3 = 0$

$\Rightarrow 4 - 4\sin^2 x - 2\sin x - 3 = 0$

$\Rightarrow x = n\pi$

$\Rightarrow 4\sin^2 x + 2\sin x - 1 = 0$

$\Rightarrow \sin x = \dfrac{-1 \pm \sqrt{5}}{4}$

$\sin x = \dfrac{\sqrt{5}-1}{4}$ or $\sin x = \dfrac{-1-\sqrt{5}}{4}$

$\sin x = \sin\dfrac{\pi}{10}$ $\qquad$ $\sin x = \sin\left(-\dfrac{3\pi}{10}\right)$

$x = n\pi + (-1)^n \cdot \dfrac{\pi}{10}$ $\qquad$ $x = n\pi + (-1)^n \cdot \left(-\dfrac{3\pi}{10}\right)$.

Hence the required solution.

$\{n\pi, n\in Z\} \cup \left\{n\pi + (-1)^n \dfrac{\pi}{10}\right\} \cup \left\{n\pi + (-1)^n\left(-\dfrac{3\pi}{10}\right)\right\}$

(4)

10 $4\cos^2 x \sin x - 2\sin^2 x - 3\sin x = 0$

$\sin x\,(4\cos^2 x - 2\sin x - 3) = 0$

$\sin x = 0$ or $4(1-\sin^2 x) - 2\sin x - 3 = 0$

$\Rightarrow 4 - 4\sin^2 x - 2\sin x - 3 = 0$

$\Rightarrow x = n\pi$ $\quad\Rightarrow 4\sin^2 x + 2\sin x - 1 = 0$

$\Rightarrow \sin x = \dfrac{-1 \pm \sqrt{5}}{4}$

$\sin x = \dfrac{\sqrt{5}-1}{4}$ or $\sin x = \dfrac{-1-\sqrt{5}}{4}$

$\sin x = \sin\dfrac{\pi}{10}$ $\quad \sin x = \sin\left(-\dfrac{3\pi}{10}\right)$

$x = n\pi + (-1)^n \cdot \dfrac{\pi}{10}$ $\quad x = n\pi + (-1)^n \cdot \left(-\dfrac{3\pi}{10}\right)$.

Hence the required solution.

$\{n\pi, n\in Z\} \cup \{n\pi + (-1)^n \frac{\pi}{10}\} \cup \{n\pi + (-1)^n(-\frac{3\pi}{10})\}$

Ans: A

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11 $a\cos 2\theta + b\sin 2\theta = c$

$\Rightarrow a\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right) = c$.

$\Rightarrow a - a\tan^2\theta + 2b\tan\theta = c + c\tan^2\theta$

$\Rightarrow (a+c)\tan^2\theta - 2b\tan\theta + (c-a) = 0$

$\therefore \tan\alpha \cdot \tan\beta = \dfrac{c-a}{c+a}$ $\quad$ Ans: C

12

12. $\sin^3\theta + \sin\theta\cos\theta + \cos^3\theta = 1$ (5)

$(\sin^3\theta + \cos^3\theta) + \sin\theta\cos\theta = 1$

$(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) - (1 - \sin\theta\cos\theta) = 0$

$(1 - \sin\theta\cos\theta)\left[\sin\theta + \cos\theta - 1\right] = 0$

$\sin\theta\cos\theta = 1$ or $\sin\theta\cos\theta = 1$

$2\sin\theta\cos\theta = 2$

$\sin 2\theta = 2$

$\downarrow$

Not possible

$\frac{1}{\sqrt{2}}(\cancel{\sin}($

$\frac{1}{\sqrt{2}}\left(\cos\left(\theta - \frac{\pi}{4}\right)\right) = \frac{1}{\sqrt{2}}$

$\cos\left(\theta - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$\therefore \theta - \frac{\pi}{4} = 2n\pi \pm \pi/4$

$\theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$

$\theta = 2n\pi + \pi/2$ or $\theta = 2n\pi$

Ans. B

13. Statement I: $\cos\theta = \frac{3}{2} = 1.5 \rightarrow$ This is not possible. (False)

Statement II: Conceptual (True)

Ans: D

~~14 $\sin x$~~

6

14 $\sin x \cdot \cos 2y = (a^2-1)^2+1 \rightarrow (1)$

$\cos x \cdot \sin 2y = a+1 \rightarrow (2)$

Since the L.H.S parts of (1) and (2) does not exceed 1

We have $(a^2-1)^2+1 \le 1$ and $a+1 \le 1$

$a \le 0 \rightarrow (3)$

$(a^2-1)^2 \le 0$

$\Rightarrow (a^2-1)^2 = 0$

$\Rightarrow a = \pm 1 \rightarrow (4)$

from (3) and (4) $\Rightarrow$ $a=-1$ is the only ~~solution~~ value, which satisfies (1) & (2) Ans. A

15 from the solution of problem no: 14.

We get $a = -1$.

(1) $\Rightarrow$ $\cos x \cdot \cos 2y = \phi$

and (2) $\Rightarrow$ $\cos x \cdot \sin 2y = 0$

$\Rightarrow \sin 2y = 0$

$\Rightarrow 2y = 0, \pi, 2\pi, 3\pi, 4\pi$

$\Rightarrow y = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

When y ... $\cos x = 1$

$\cos x = \cos 0$

$x = 2n$...

⑦

15. From the solution of problem no: 14

We get ~~cos x~~. $\sin x \cdot \cos 2y = 1$ and

$\cos x \cdot \sin 2y = 0$

Case I: when $\sin x = \cos 2y = 1$

$\sin x = 1$

$\Rightarrow x = \frac{\pi}{2}$ [$\because$ in $[0, 2\pi]$]

for $\cos 2y = 1$

$\Rightarrow 2y = 2n\pi$, $n \in \mathbb{Z}$

$\Rightarrow y = n\pi$

$\Rightarrow \boxed{y = 0, \pi, 2\pi}$ [in $[0, 2\pi]$]

Case II: when $\sin x = \cos 2y = -1$

for $\sin x = -1$

$\Rightarrow x = \frac{3\pi}{2}$

for $\cos 2y = -1$

$\Rightarrow 2y = (2n+1)\frac{\pi}{2}$

$\Rightarrow \boxed{y = \frac{\pi}{2}, \frac{3\pi}{2}}$ in $[0, 2\pi]$

So, we total 5 values of y.

Ans: D

16. ~~$(2\sin x - \cos x)(1 + \cos x) = 1 - \cos^2 x$~~

~~$(2\sin x - \cos x)(1 + \cos x) = (1 + \cos x)(1 - \cos x)$~~

16. $(2\sin x - \cos x)(1+\cos x) = \sin^2 x$ (8)

$(2\sin x - \cos x)(1+\cos x) = 1-\cos^2 x$

$(2\sin x - \cos x)(1+\cos x) = (1+\cos x)(1-\cos x)$

$(1+\cos x)[2\sin x - \cos x - 1 + \cos x] = 0$

$1+\cos x = 0$ or $2\sin x - 1 = 0$

$\cos x = -1$ or $\sin x = \frac{1}{2}$

$x = \pi$ ⊗ $\quad \cos x = \frac{\sqrt{3}}{2}$

$\therefore \quad \boxed{\cos\alpha = \frac{\sqrt{3}}{2}}$

$3\cos^2 x - 10\cos x + 3 = 0$

$\Rightarrow 3\cos^2 x - 9\cos x - \cos x + 3 = 0$

$\Rightarrow 3\cos x[\cos x - 3] - [\cos x - 3] = 0$

$\Rightarrow \cos x - 3 = 0 \quad \cos x = \frac{1}{3}$

$\Rightarrow \cos x = 3$ ⊗ $\quad \boxed{\cos\beta = \frac{1}{3}}$

Again $1-\sin 2x = \cos x - \sin x$

$\Rightarrow (\sin x - \cos x)^2 + (\sin x - \cos x) = 0$

$\Rightarrow (\sin x - \cos x)[\sin x - \cos x + 1] = 0$

$\Rightarrow \sin x - \cos x = 0 \quad \therefore \boxed{\cos\gamma = \frac{1}{\sqrt{2}}}$

$\Rightarrow \tan x = 1$

$\Rightarrow \cos x = \frac{1}{\sqrt{2}}$

Now $\cos\alpha + \cos\beta + \cos\gamma$

$= \frac{\sqrt{3}}{2} + \frac{1}{3} + \frac{1}{\sqrt{2}}$

$= \frac{3\sqrt{6} + 2\sqrt{2} + 6}{6\sqrt{2}}$

opt Ans : A

17. from the solution of problem 16.

$\cos\alpha = \frac{\sqrt{3}}{2}$ | $\cos\beta = \frac{1}{3}$ | $\cos\gamma = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin\alpha = \frac{1}{2}$ | $\Rightarrow \sin\beta = \frac{2\sqrt{2}}{3}$ | $\sin\gamma = \frac{1}{\sqrt{2}}$

$\therefore \sin\alpha + \sin\beta + \sin\gamma$

$= \frac{1}{2} + \frac{2\sqrt{2}}{3} + \frac{1}{\sqrt{2}}$

$= \frac{14+3\sqrt{2}}{6\sqrt{2}}$

Ans: A

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18. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \frac{1}{2}\cdot\frac{1}{3} - \frac{\sqrt{3}}{2}\cdot\frac{2\sqrt{2}}{3}$

$= \frac{1-2\sqrt{6}}{6}$

Ans: C

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19 $|\sin x\cdot\cos x| + \sqrt{2+\tan^2 x+\cot^2 x} = \sqrt{3}$

$\Rightarrow |\sin x\cdot\cos x| + \sqrt{(\tan x+\cot x)^2} = \sqrt{3}$

$\Rightarrow |\sin x\cdot\cos x| + |\tan x+\cot x| = \sqrt{3}$

$\Rightarrow |\sin x\cdot\cos x| + \left|\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right| = \sqrt{3}$

$\Rightarrow |\sin x\cdot\cos x| + \frac{1}{|\sin x\cdot\cos x|} = \sqrt{3}$

let $|\sin x\cdot\cos x| = a$

$\Rightarrow a + \frac{1}{a} = \sqrt{3}$

$\Rightarrow a^2 - \sqrt{3}a + 1 = 0$

This equation does not have any real roots since $\Delta < 0$

hence No. of solutions $= 0$

Ans: 0

20 $(\sqrt{3}+1)^{2x}+(\sqrt{3}-1)^{2x} = 2^{3x}$ (10)

$\Rightarrow (\sqrt{3}+1)^{2x}+(\sqrt{3}-1)^{2x} = (2\sqrt{2})^{2x}$

$\Rightarrow \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^{2x} + \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^{2x} = 1$

$\Rightarrow (\sin 75^\circ)^{2x} + (\cos 75^\circ)^{2x} = 1$

$\Rightarrow x = 1$ Ans: 1

No. of solutions : 1

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21 $\cos 4x = \cos 2.2x$

$= (2\cos^2 2x - 1)$

$= 2[2(2\cos^2 x - 1)^2 - 1]$

$= 2\{2(4\cos^4 x - 4\cos^2 x + 1) - 1$

$= 8\cos^4 x - 8\cos^2 x + 1$

$= 1 - 8\cos^2 x + 8\cos^4 x$

$= a_0 + a_1 \cos^2 x + a_2 \cos^4 x$

$\therefore a_0 = 1,\ a_1 = -8,\ a_2 = 8$

$5a_0 + a_1 + a_2 = 5(1) - 8 + 8 = 5$ Ans: 5

22 a) $4\sin^2 x + \sin^2 2x = 3$ (11)

$\Rightarrow 4\sin^2 x + 4\sin^2 x\cos^2 x = 3$

$\Rightarrow 4\sin^2 x + 4\sin^2 x(1-\sin^2 x) = 3$

$\Rightarrow 4\sin^4 x - 8\sin^2 x + 3 = 0$

$\Rightarrow (2\sin^2 x - 3)(2\sin^2 x - 1) = 0$

$\Rightarrow \sin^2 x = \frac{3}{2}$ or $\sin^2 x = \frac{1}{2}$

$\Rightarrow \sin x = \frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4}$

$\therefore \tan x = 1 \Rightarrow x = \tan^{-1}(1)$

b) $4\cos^2 2x + 8\cos^2 x = 7$

$\Rightarrow 4(2\cos^2 x - 1)^2 + 8\cos^2 x = 7$

$\Rightarrow 4(4\cos^4 x - 4\cos^2 x + 1) + 8\cos^2 x - 7 = 0$

$\Rightarrow 16\cos^4 x - 8\cos^2 x - 3 = 0$

$\Rightarrow (4\cos^2 x + 1)(4\cos^2 x - 3) = 0$

$\Rightarrow \cos x = \frac{\sqrt{3}}{2} \Rightarrow x = 30^\circ \Rightarrow \sin x = \frac{1}{2}$

$x = \sin^{-1}(\frac{1}{2})$

c) $3(1+\sin x) = 1 + \cos 2x$

$\Rightarrow 3 + 3\sin x = 1 + 1 - 2\sin^2 x$

$\Rightarrow 2\sin^2 x + 3\sin x + 1 = 0$

$\Rightarrow (\sin x + 1)(2\sin x + 1) = 0$

$\Rightarrow \sin x + 1 = 0 \Rightarrow x = \sin^{-1}(-1)$

d) $4\sin^3 x - 2\sin^2 x - 2\sin x + 1 = 0$

$= 2\sin^2 x(2\sin x - 1) - 1(2\sin x + 1) = 0$

$= (2\sin x - 1)(2\sin^2 x - 1) = 0$

$\Rightarrow \sin x = \frac{1}{2}$

$\Rightarrow x = \sin^{-1}(\frac{1}{2})$ Ans: P, S, Q, S

23 a) $\tan^2 x + \cot^2 x = 2$ (12)

$\Rightarrow (\tan x - \cot x)^2 = 0$

$\Rightarrow \tan x - \cot x = 0$

$\Rightarrow \tan^2 x = 1$

$\Rightarrow \tan^2 x = \tan^2 \frac{\pi}{4}$

$\Rightarrow x = n\pi \pm \frac{\pi}{4}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

No. of Solutions = 4

b) $\sin^2 x - \cos x = \frac{1}{2}$

$\Rightarrow 1 - \cos^2 x - \cos x = \frac{1}{2}$

$\Rightarrow 2 - 2\cos^2 x - 2\cos x = 1$

$\Rightarrow 2\cos^2 x + 2\cos x - 1 = 0$

$\Rightarrow (\cos x + 1)(2\cos x - 1) = 0$

$\Rightarrow \cos x = -1$ or $\cos x = \frac{1}{2}$

$x = \pi, 2\pi$ $\quad$ $x = \frac{\pi}{3}, \frac{5\pi}{3}$

No. of solutions = 4

c) $4\sin^2 x - 6\cos^2 x = 10$

$\Rightarrow 4\sin^2 x - 6(1 - \sin^2 x) = 10$

$\Rightarrow 10\sin^2 x - 16 = 0$

$\Rightarrow \sin^2 x = \frac{8}{5} \Rightarrow \sin x = \frac{2\sqrt{2}}{\sqrt{5}} \approx 1.26$

No. of solutions = 0

d) $\sin x = 1$

$\Rightarrow \sin x = \sin \frac{\pi}{2}$

$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{2}$

$\Rightarrow x = \frac{\pi}{2}$

No. of solutions = 1 $\quad$ Ans: S, S, Q, R

LEARNERS TASK

CUQ's

01. $\sin\theta = \frac{1}{2}$ — Ans: C

$\Rightarrow \theta = \frac{\pi}{6}$

02. $\sin\theta = 1$ — Ans: A

$\Rightarrow \theta = \frac{\pi}{2}$

03. $\sin\theta = 0$ — Ans: B

$\Rightarrow \theta = n\pi, n \in Z$

04. $\sec\theta = \sqrt{2}$

$\Rightarrow \theta = \frac{\pi}{4}$ — Ans: B

05. $\tan\theta = -\sqrt{3}$

$\Rightarrow \tan\theta = \tan\left(-\frac{\pi}{3}\right)$ — Ans: D

$\Rightarrow \theta = -\frac{\pi}{3}$

06. $\cos\theta = 0$ — Ans: C

$\Rightarrow \theta = (2n+1)\frac{\pi}{2}, n \in Z$

07. $\tan\theta = 0$ — Ans: A

$\Rightarrow \theta = n\pi, n \in Z$

08. $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ — Ans: C

09. $\alpha \in [0, \pi]$ — Ans: B

10. $\tan\theta = \tan\alpha \Rightarrow \alpha = R$ — Ans: D

JEE MAINS LEVEL

01. $\cos\theta = \cos\frac{5\pi}{4}$

$\Rightarrow \theta = 2n\pi \pm \frac{5\pi}{4}$ — Ans: C

02. $\sqrt{3}\cos\theta = \sin\theta$

$\Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \tan\frac{\pi}{3}$

$\Rightarrow \theta = n\pi + \frac{\pi}{3}, n \in \mathbb{Z}$

Ans: A

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03 $\sqrt{\sin x} + \cos x = 0$

$\Rightarrow \sqrt{\sin x} = -\cos x$

$\Rightarrow \sin x = \cos^2 x$

$\Rightarrow \sin x = 1 - \sin^2 x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{-1+\sqrt{5}}{2} = \frac{\sqrt{5}-1}{2}$

Ans: D

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04 $4\cos\theta - \sec\theta = 4\tan\theta$

$\Rightarrow 4\cos\theta - \frac{1}{\cos\theta} = \frac{4\sin\theta}{\cos\theta}$

$\Rightarrow 4\cos^2\theta - 1 = 4\sin\theta$

$\Rightarrow 4(1-\sin^2\theta) - 1 = 4\sin\theta$

$\Rightarrow 4\sin^2\theta + 4\sin\theta - 3 = 0$

$\Rightarrow (2\sin\theta + 3)(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \sin\theta = \sin\frac{\pi}{6}$

$\Rightarrow \theta = n\pi + (-1)^n \cdot \frac{\pi}{6}, n \in \mathbb{Z}$

Ans. A

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05 $3(1-2\sin^2\theta) + 2 = 7\sin\theta$

$\Rightarrow 6\sin^2\theta + 7\sin\theta - 5 = 0$

$\Rightarrow (2\sin\theta - 1)(3\sin\theta + 5) = 0$

$\Rightarrow \sin\theta = 1/2$ $\quad \sin\theta = \frac{-5}{3}$ not possible

$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}, n \in \mathbb{Z}$

Ans: B

15

06 $\frac{\cos 3\theta}{2\cos 2\theta - 1} = \frac{1}{2}$

$\Rightarrow \frac{4\cos^3\theta - 3\cos\theta}{2(2\cos^2\theta - 1) - 1} = \frac{1}{2}$

$\Rightarrow \frac{4\cos^3\theta - 3\cos\theta}{4\cos^2\theta - 3} = \frac{1}{2}$

$\Rightarrow \frac{\cos\theta(4\cos^2\theta - 3)}{4\cos^2\theta - 3} = \frac{1}{2}$

$\Rightarrow 2\cos\theta(4\cos^2\theta - 3) - (4\cos^2\theta - 3) = 0$

$\Rightarrow (4\cos^2\theta - 3)(2\cos\theta - 1) = 0$

$\Rightarrow \cos^2\theta = \frac{\sqrt{3}}{2}$ $\therefore \cos\theta = \frac{1}{2}$

$\Rightarrow \cos^2\theta = \cos^2\frac{\pi}{3}$, $\cos\theta = \cos\frac{\pi}{3}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$, $\theta = 2n\pi \pm \frac{\pi}{3}$ Ans: ~~5~~ B

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07 $\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0$

$\Rightarrow \tan^2\theta - \tan\theta - \sqrt{3}\tan\theta + \sqrt{3} = 0$

$\Rightarrow (\tan\theta - 1)(\tan\theta - \sqrt{3}) = 0$

$\Rightarrow \tan\theta = 1$ or $\tan\theta = \sqrt{3}$

$\Rightarrow \tan\theta = \tan\frac{\pi}{4}$ or $\tan\theta = \tan\frac{\pi}{3}$

$\Rightarrow \theta = n\pi + \frac{\pi}{4}$ $\theta = n\pi + \frac{\pi}{3}$ Ans: C

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08 $\sqrt{1-\cos x} = \sin x \Rightarrow 1 - \cos x = 1 - \cos^2 x$

$\Rightarrow \cos^2 x - \cos x = 0$

$\Rightarrow \cos x(\cos x - 1) = 0$

$\Rightarrow \cos x = 0$ or $\cos x = 1$

$x = (2n+1)\frac{\pi}{2}$

$\cos x = \cos 0$

$x = (2n+1$

$x = 2n\pi \pm 0$

$x = 2n\pi$

Ans: A

(16)

09 $\tan\theta - \cot\theta = 0$

$\Rightarrow \tan\theta - \dfrac{1}{\tan\theta} = 0$

$\Rightarrow \tan^2\theta - 1 = 0$

$\Rightarrow \tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \tan^2\dfrac{\pi}{4}$

$\Rightarrow \theta = n\pi \pm \dfrac{\pi}{4}$

Ans : D

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10 ~~$\sin 6\theta = \sin 4\theta - \sin 2\theta$~~

~~$= 2\cos\left(\dfrac{4\theta+2\theta}{2}\right)\cdot \sin\left(\dfrac{4\theta-2\theta}{2}\right)$~~

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10 $\sin 6\theta = \sin 4\theta - \sin 2\theta$

$\Rightarrow \sin 6\theta + \sin 2\theta = \sin 4\theta$

$\Rightarrow 2\sin 4\theta \cdot \cos 2\theta - \sin 4\theta = 0$

$\Rightarrow \sin 4\theta\,(2\cos 2\theta - 1) = 0$

$\Rightarrow \sin 4\theta = 0$ or $\cos 2\theta = \dfrac{1}{2}$

$\Rightarrow 4\theta = n\pi$ or $\cos 2\theta = \cos\dfrac{\pi}{3}$

$\Rightarrow \theta = \dfrac{n\pi}{4}$ $\qquad 2\theta = 2n\pi \pm \dfrac{\pi}{3}$

$\theta = n\pi \pm \dfrac{\pi}{6}$

Ans : B

17

11. $a\cos x + b\sin x = c$

$\Rightarrow a\cos x - c = -b\sin x$

$\Rightarrow a^2\cos^2 x + c^2 - 2ac\cos x = b^2(1-\cos^2 x)$

$\Rightarrow a^2\cos^2 x + c^2 - 2ac\cos x - b^2 + b^2\cos^2 x = 0$

$\Rightarrow a^2\cos^2 x + c^2 - 2ac\cos x$

$\Rightarrow (a^2+b^2)\cos^2 x - 2ac\cos x + c^2 - b^2 = 0$

$$\cos\alpha + \cos\beta = \frac{2ac}{a^2+b^2}$$ Ans: B

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12. $a\tan\theta + b\sec\theta = c$

$\Rightarrow a\tan\theta - c = -b\sec\theta$

$\Rightarrow a^2\tan^2\theta - 2ac\tan\theta + c^2 = b^2(1+\tan^2\theta)$

$\Rightarrow a^2\tan^2\theta - 2ac\tan\theta + c^2 - b^2 - b^2\tan^2\theta = 0$

$\Rightarrow a^2\tan^2\theta - 2ac\tan\theta + c^2 - b^2 = 0$

$\Rightarrow (a^2-b^2)\tan^2\theta - 2ac$

$$\tan\alpha + \tan\beta = \frac{2ac}{a^2+b^2}$$

$$\tan\alpha \cdot \tan\beta = \frac{c^2-b^2}{a^2-b^2}$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\cdot\tan\beta}$$

$$= \frac{\left(\frac{2ac}{a^2-b^2}\right)}{1-\left(\frac{c^2-b^2}{a^2-b^2}\right)} =$$

$$= \frac{2ac}{a^2-c^2}$$ Ans: A

13 **Statement I:** $\sin x = -1$

$$\sin x = \sin\left(-\frac{\pi}{2}\right)$$

$$x = n\pi + (-1)^n\left(-\frac{\pi}{2}\right) \quad \text{(False)}$$

Statement II: Conceptual (True)

Ans : D

14. **Statement I:** $3\sin x + 4\cos x = 7$

$$7 > \sqrt{3^2+4^2} \quad (\cancel{\text{False}})\ (\text{True})$$

Statement II: Conceptual (True)

Ans : A

15 **Statement I:** $\dfrac{\tan 3x - \tan 2x}{1+\tan 3x \cdot \tan 2x} = 1$

$$\Rightarrow \tan(3x-2x) = 1$$

$$\Rightarrow \tan x = 1$$

$$\Rightarrow \tan x = \tan\frac{\pi}{4}$$

$$\Rightarrow x = n\pi + \frac{\pi}{4} \quad \text{(False)}$$

Statement II: Conceptual (True) Ans :D

16. $\cos\theta \cdot \cos 2\theta \cdot \cos 3\theta = \frac{1}{4}$

$\Rightarrow 2\cos 2\theta \left[2\cos\theta \cdot \cos 3\theta\right] = 1$

$\Rightarrow 2\cos 2\theta \cdot \left[\cos 4\theta + \cos 2\theta\right] = 1$

$\Rightarrow 2\cos 2\theta \cdot \cos 4\theta + 2\cos^2 2\theta - 1 = 0$

$\Rightarrow 2\cos 2\theta \cdot \cos 4\theta + \cos 4\theta = 0$

$\Rightarrow \cos 4\theta \left(2\cos 2\theta + 1\right) = 0$

$\Rightarrow \cos 4\theta = 0$ or $\cos 2\theta = -\frac{1}{2}$

$4\theta = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (2n+1)\frac{\pi}{8}$

$\cos 2\theta = \cos\frac{2\pi}{3}$

$2\theta = 2n\pi \pm \frac{2\pi}{3}$

$\theta = n\pi \pm \frac{\pi}{3}$.

here $0 \le \theta \le \pi$

$\therefore \theta = \frac{\pi}{8}, \frac{3\pi}{8}, \cancel{\frac{4\pi}{8}}, \frac{5\pi}{8}, \cancel{\frac{6\pi}{8}}, \frac{7\pi}{8}$. Also

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

$\text{Sum} = \frac{\pi}{8} + \frac{3\pi}{8} + \frac{5\pi}{8} + \frac{7\pi}{8} + \frac{\pi}{3} + \frac{2\pi}{3}$

$= \frac{16\pi}{8} + \frac{3\pi}{3} = 2\pi + \pi = 3\pi$

Ans: C

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17 $3\sec\theta = 4\tan\theta + 5$

$\Rightarrow 9(1+\tan^2\theta) = 16\tan^2\theta + 40\tan\theta + 25$

$\Rightarrow 7\tan^2\theta + 40\tan\theta + 16 = 0$

17. $3\sec\theta - 5 = 4\tan\theta$ (20)

$$\frac{3}{\cos\theta} - 5 = \frac{4\sin\theta}{\cos\theta}$$

$\Rightarrow 3 - 5\cos\theta = 4\sin\theta$ since $\cos\theta \neq 0$

$\Rightarrow 5\cos\theta + 4\sin\theta = 3$

$\Rightarrow \frac{5}{\sqrt{41}}\cos\theta + \frac{4}{\sqrt{41}}\sin\theta = \frac{3}{\sqrt{41}}$

$\Rightarrow \cos\alpha\cos\theta + \sin\alpha\sin\theta = \frac{3}{\sqrt{41}}$

$\Rightarrow \cos(\theta - \alpha) = \frac{3}{\sqrt{41}}$ $\cos\alpha = \frac{5}{\sqrt{41}}$

$\Rightarrow \theta - \alpha = 2n\pi \pm \beta$ $\sin\alpha = \frac{3}{\sqrt{41}}$

$\Rightarrow \theta = 2n\pi \pm \beta + \alpha$ $\beta = \cos^{-1}\left(\frac{3}{\sqrt{41}}\right)$

$\therefore \theta = 2n\pi + \alpha + \beta$ (or)

$\theta = 2n\pi + \alpha - \beta$

Now in $[0, 4\pi]$ $\theta = 2n\pi + (\alpha - \beta)$ gives two solutions

Also $\theta = 2n\pi + (\alpha + \beta)$ gives two more solutions

$\therefore$ total no. of solutions $= 2 + 2$
$= 4$

Ans. 4

18. a) $\tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \tan^2\frac{\pi}{4}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$

b) $\cos^2\theta = \left(\frac{1}{2}\right)^2$

$\Rightarrow \cos^2\theta = \cos^2\frac{\pi}{3}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$

c) $\sin^2\theta = \left(\frac{1}{2}\right)^2$

$\Rightarrow \sin^2\theta = \sin^2\left(\frac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$

d) $\csc^2\theta = 1$

$\Rightarrow \sin^2\theta = 1$

$\Rightarrow \sin^2\theta = \sin^2\frac{\pi}{2}$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{2}$

Ans: q, r, p, s

19 a) $1+\sin 2x = \sin x + \cos x$ (22)

$1+2\sin x \cos x = \sin x + \cos x.$

Let $\cos x = 0 \Rightarrow x = \frac{\pi}{2}$

$1+2\sin x(0) = \sin\frac{\pi}{2} + 0$

$1 = \sin\frac{\pi}{2}$ (True)

b) $(2\sin x - \cos x)(1+\cos x) = \sin^2 x$

Let $\sin x = \frac{1}{2} \Rightarrow \cos x = \frac{\sqrt{3}}{2}$

$LHS = \left(2\times\frac{1}{2} - \frac{\sqrt{3}}{2}\right)\left(1+\frac{\sqrt{3}}{2}\right)$

$= \left(1-\frac{\sqrt{3}}{2}\right)\left(1+\frac{\sqrt{3}}{2}\right) = 1-\frac{3}{4} = \frac{1}{4}$

$RHS = \sin^2 x = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ (True)

c) $\tan\theta + \cot\theta = 2$

$\therefore \theta = \frac{\pi}{4}$

d) $\sin 7\theta = \sin 4\theta - \sin\theta$

$\Rightarrow \sin 7\theta + \sin\theta = \sin 4\theta$

$\Rightarrow 2\sin 4\theta \cdot \cos 3\theta = \sin 4\theta$

Let $\theta = \frac{\pi}{9}$

$LHS = 2\sin 4\theta \cdot \cos 3\frac{\pi}{9}$

$= 2\sin 4\theta \cdot \cos\frac{\pi}{3}$

$= 2\sin 4\theta \cdot \frac{1}{2} = \sin 4\theta$

Ans q, p, s, t

ADDITIONAL PRACTICE QUESTIONS

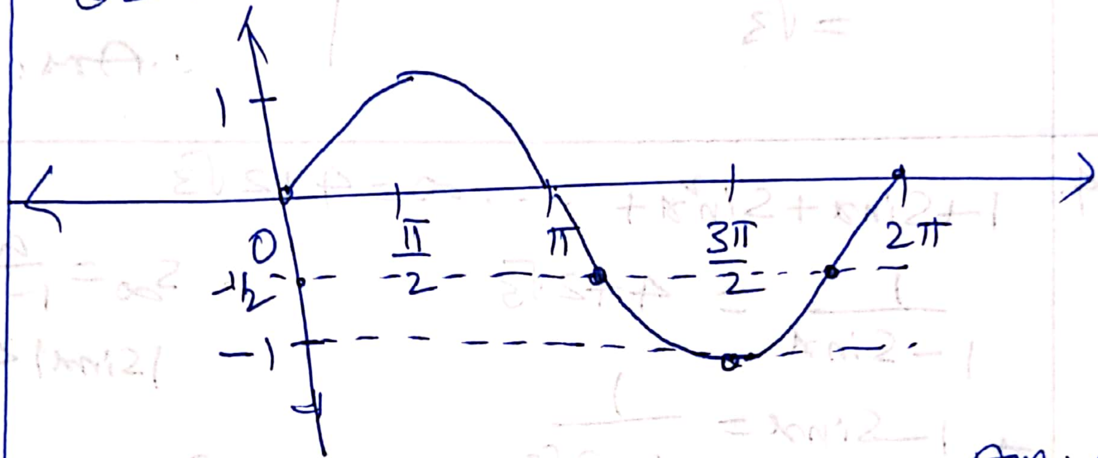
01. $2\sin^2\theta + 3\sin\theta + 1 = 0$

$(\sin\theta + 1)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = -1$ or $\sin\theta = -\frac{1}{2}$

$\Rightarrow \sin\theta = \sin\left(-\frac{\pi}{2}\right)$ $\quad$ $\sin\theta = \sin\left(-\frac{\pi}{6}\right)$

$\theta = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$ $\quad$ $\theta = n\pi + (-1)^n\cdot\left(-\frac{\pi}{6}\right)$



Number of solutions = 3

Ans : C

$\cancel{02}$ $\cancel{\sin(x + 28^\circ) = \cos(3x - 78^\circ)}$

$\cancel{x + 28^\circ = 3x - 78^\circ}$

$\cancel{\Rightarrow 2x = 106^\circ}$

$\cancel{\Rightarrow x = 53^\circ}$

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02 $\sin(x + 28^\circ) = \cos(3x - 78^\circ)$

$\Rightarrow x + 28^\circ + 3x - 78^\circ = 90^\circ$

$\Rightarrow x = 35^\circ$

Also $x = 8^\circ \Rightarrow \sin 36^\circ = \cos 54^\circ$ (true)

Ans: B

03 $3\tan(\theta-15^\circ)=\tan(\theta+15^\circ)$ (24)

Let $\theta=45^\circ$

$LHS = 3\tan(45^\circ-15^\circ)$

$= 3\tan 30^\circ$

$= 3\times\frac{1}{\sqrt{3}}$

$=\sqrt{3}$

$RHS = \tan(\theta+15^\circ)$

$= \tan(45^\circ+15^\circ)$

$= \tan 60^\circ$

$= \sqrt{3}$

$\therefore$ Ans: B

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04. $1+\sin\alpha+\sin^2\alpha+\cdots\infty = 4+2\sqrt{3}$

$S_\infty = \frac{a}{1-r}$

$|\sin\alpha|<1$

$\frac{1}{1-\sin\alpha} = 4+2\sqrt{3}$

$\Rightarrow 1-\sin\alpha = \frac{1}{4+2\sqrt{3}}$

$\Rightarrow \sin\alpha = 1-\frac{1}{4+2\sqrt{3}} = \frac{3+2\sqrt{3}}{4+2\sqrt{3}}$

$= \frac{\sqrt{3}(\cancel{\sqrt{3}+2})}{2(\cancel{2+\sqrt{3}})} = \frac{\sqrt{3}}{2}$

$\therefore \alpha = \frac{\pi}{3}, \frac{2\pi}{3}$

Ans: D

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05 $(1+\tan\alpha)(1+\tan 4\alpha) = 2$

$\Rightarrow 1+\tan 4\alpha+\tan\alpha+\tan\alpha\cdot\tan 4\alpha = 2$

$\Rightarrow \tan\alpha+\tan 4\alpha = 1-\tan\alpha\cdot\tan 4\alpha$

$\Rightarrow \frac{\tan\alpha+\tan 4\alpha}{1-\tan\alpha\cdot\tan 4\alpha} = 1$

$\Rightarrow \tan(\alpha+4\alpha) = \tan\frac{\pi}{4}$

$\Rightarrow 5\alpha = \frac{\pi}{4}$

$\alpha = \frac{\pi}{20}$

Ans: A