TRIGONOMETRIC RATIOS & IDENTITIES

LEARNING OBJECTIVES:

1. Systems of Angles
2. Trigonometric Ratios of various angles
3. Domain and Range of trigonometric functions
4. Graphs

SYNOPSIS

Angle: An angle is the union of two rays having a common end point in a plane.

Measurement of an angle :

Sexagesimal system:

(i) One right angle = $\frac{\pi}{2}$ radian = 90º.

(ii) π radian = 2 right angles = 180º.

(iii) $1^\circ = 60\,\text{minutes}(60')$

(iv) $1' = 60\,\text{seconds}(60'')$

(v) $1^o = 0.001745$ radian

(vi) $1\,\text{radian} \cong 57^0 17^1 45^{11}$ (approx)

Centisimal system:

(i) 1 right angle = 100 grades written as 100^g

(ii) 1 grade or 1^g = 100 minutes (100')

(iii) 1 minute or 1' = 100 seconds (100")

Circular system :

Radian: A radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle. The length of arc $l = r\theta$.

(i) 1 revolution = 2π radians $= 360^\circ$

(ii) π radians = 2right angles $= 2 \times 90^\circ = 180^\circ$

(iii) 1 degree $(1^\circ) = \frac{\pi}{180}$ rad $\cong 0.01745$ rad

(iv) 1 rad $(1^c) = \frac{180}{\pi}$ degrees $\cong 57^\circ 17' 46''$

Note : (i) Value of $\pi = \frac{22}{7} (or) \frac{355}{113} (or) 3.1416$

(ii) π is an irrational number

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(iii) $\pi = \dfrac{\text{circumference of the circle}}{\text{diameter of the circle}}$

Trigonometric Identities :

i) $(\sin\theta)(\cos ec\,\theta) = 1, \theta \neq n\pi, n \in Z$

ii) $(\cos\theta)(\sec\theta) = 1, \theta \neq (2n+1)\dfrac{\pi}{2}, n \in Z$

iii) $(\tan\theta)(\cot\theta) = 1, \theta \neq (2n+1)\dfrac{\pi}{2}, \theta \neq n\pi$

iv) $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$ v) $\sec^2\theta - \tan^2\theta = 1 \Rightarrow \sec^2\theta = 1 + \tan^2\theta$

$\left(\theta \neq (2n+1)\dfrac{\pi}{2}, n \in Z\right)$.

(vi) $\cos ec^2\theta - \cot^2\theta = 1$

$\Rightarrow \cos ec^2\theta = 1 + \cot^2\theta \ \ (\theta \neq n\pi, n \in Z)$

Note: (i) If $\theta \neq (2n+1)\dfrac{\pi}{2}, n \in Z$, then

$$\sec^2\theta - \tan^2\theta = 1 \Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$$

$$\Rightarrow \sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$$

(ii) If $\theta \neq n\pi, n \in Z$ then $\cos ec\theta - \cot\theta = \dfrac{1}{\cos ec\theta + \cot\theta}$

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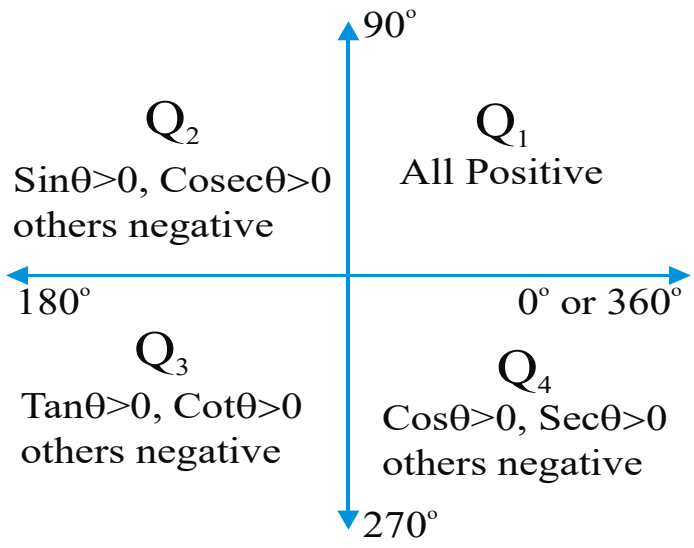
Trigonometric ratios of various angles:

Trig. Ratio	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\cos ec\,\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0



Note : i) $\sin n\pi = \tan n\pi = \cos(2n+1)\frac{\pi}{2} = 0, \forall n \in Z$

ii) $\sin(2n+1)\frac{\pi}{2} = (-1)^n \text{ and } \cos n\pi = (-1)^n, \forall n \in Z$

Domain and range of trigonometric functions :

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Trig. function	Domain	Range
sin	$\mathbb{R}$	[-1, 1]
cos	$\mathbb{R}$	[-1, 1]
tan	$\mathbb{R}-\left\{(2n+1)\frac{\pi}{2}, n \in z\right\}$	$\mathbb{R}$
cot	$\mathbb{R}-\{n\pi, n \in z\}$	$\mathbb{R}$
cosec	$\mathbb{R}-\{n\pi, n \in z\}$	$(-\infty,-1]\cup[1,\infty)$
sec	$\mathbb{R}-\left\{(2n+1)\frac{\pi}{2}, n \in z\right\}$	$(-\infty,-1]\cup[1,\infty)$

EXAMPLES

W.E-1: If $\sin\theta = \frac{a-4}{2}$ ***then a lies in***

Sol: $-1 \le \sin\theta \le 1 \Rightarrow -1 \le \frac{a-4}{2} \le 1$

$\Rightarrow -2 \le a-4 \le 2 \Rightarrow 2 \le a \le 6 \Rightarrow a \in [2,6]$

Some useful results :

(a) If $A+B = 90^\circ \text{ or } 270^\circ$, then

(i) $\sin^2 A + \sin^2 B = 1$ (ii) $\cos^2 A + \cos^2 B = 1$

(iii) $\tan A.\tan B = 1$ (iv) $\cot A.\cot B = 1$

(b) If $A+B = 180^\circ$, then

(i) $\cos A + \cos B = 0$

(ii) $\sin A - \sin B = 0$

(iii) $\tan A + \tan B = 0$

(c) If $A+B = 360^\circ$, then

(i) $\sin A + \sin B = 0$

(ii) $\cos A - \cos B = 0$

(iii) $\tan A + \tan B = 0$

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W.E-2: $\tan 130^\circ.\tan 140^\circ =$

Sol: $130^\circ + 140^\circ = 270^\circ$

$\therefore \tan 130^\circ.\tan 140^\circ = 1$

W.E-3: $\sin^2 55^\circ + \sin^2 35^\circ =$

Sol: $55^\circ + 35^\circ = 90^\circ$

$\therefore \sin^2 55^\circ + \sin^2 35^\circ = 1$

(i) If $a\cos\theta + b\sin\theta = c$ and $a\sin\theta - b\cos\theta = K$ then $a^2 + b^2 = c^2 + k^2$

(ii) If $a\sec\theta + b\tan\theta = c$ and $a\tan\theta + b\sec\theta = k$ then $a^2 - b^2 = c^2 - k^2$

(iii) If $a\,\text{cosec}\,\theta + b\cot\theta = c$ and $a\cot\theta + b\,\text{cosec}\,\theta = k$ *then* $a^2 - b^2 = c^2 - k^2$

W.E-4: If $8\cos\theta + 6\sin\theta = 5$ ***then*** $8\sin\theta - 6\cos\theta =$

Sol: Let $8\sin\theta - 6\cos\theta = k$

$a^2 + b^2 = c^2 + k^2$

$\therefore 8^2 + 6^2 = 5^2 + k^2$

$\Rightarrow k^2 = 75 \Rightarrow k = \pm 5\sqrt{3}$

$$\sqrt{x^2}=|x|=\begin{cases} x & if \quad x>0 \\ -x & if \quad x<0 \\ 0 & if \quad x=0 \end{cases}$$

for example the value of

$\sqrt{\cos^2 100^o}=|\cos 100^o|$

$=-\cos 100^o\left(\because \cos 100^o<0\right)$

(i) $\sin\theta+\sin(\pi+\theta)+\sin(2\pi+\theta)+.....$

$....+\sin(n\pi+\theta)=\begin{cases} 0 \ if \ n \ is \ odd \\ \sin\theta \ if \ n \ is \ even \end{cases}$

(ii) $\cos\theta+\cos(\pi+\theta)+\cos(2\pi+\theta)+.....$

$....+\cos(n\pi+\theta)=\begin{cases} 0 \ if \ n \ is \ odd \\ \cos\theta \ if \ n \ is \ even \end{cases}$

W.E-5: $\sin\theta+\sin\left(\pi+\theta\right)+.....+\sin\left(100\pi+\theta\right)=$

Sol: $n=100$ is even

$\therefore$ the required value = $\sin\theta$

W.E-6: $\cos\theta+\cos\left(\pi+\theta\right)+.....+\cos\left(299\pi+\theta\right)=$

Sol: $n=299$ is odd

$\therefore$ required value = 0

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If a>0, b > 0 and $f\left(x\right)>0$ then $af\left(x\right)+\frac{b}{f\left(x\right)}\geq 2\sqrt{ab}$

Proof: We know that A.M. $\geq$ G.M.

$$\Rightarrow \frac{af\left(x\right)+\frac{b}{f\left(x\right)}}{2}\geq\sqrt{af\left(x\right)\frac{b}{f\left(x\right)}} \quad \Rightarrow af\left(x\right)+\frac{b}{f\left(x\right)}\geq 2\sqrt{ab}$$

Note: (i) $x+\frac{1}{x}\geq 2$, for x > 0

(ii) $a\tan x+b\cot x\geq 2\sqrt{ab}$

(iii) $25\cos x+16\sec x\geq 2\sqrt{25\times 16}=40$

(i) $\cot^2\theta-\cos^2\theta=\cot^2\theta\cos^2\theta$

(ii) $\tan^2\theta-\sin^2\theta=\tan^2\theta\sin^2\theta$

(iii) $\cos ec^2\theta+\sec^2\theta=\cos ec^2\theta.\sec^2\theta$

(i) $\sin^4\theta+\cos^4\theta=1-2\sin^2\theta\cos^2\theta$

(ii) $\sin^6\theta+\cos^6\theta=1-3\sin^2\theta\cos^2\theta$

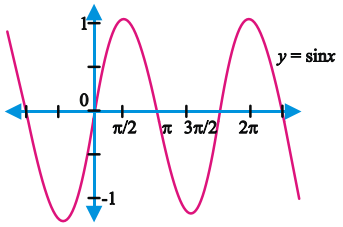
(iii) $\sin^2\theta+\cos^4\theta=1-\sin^2\theta\cos^2\theta$

(iv) $\cos^2\theta+\sin^4\theta=1-\sin^2\theta\cos^2\theta$

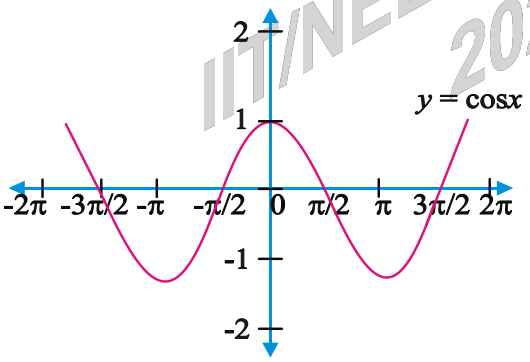
W.E-7: $\dfrac{2+\sin^6\theta+\cos^6\theta}{\sin^2\theta+\cos^4\theta}=$

Sol: $\dfrac{2+\left(1-3\sin^2\theta\cos^2\theta\right)}{1-\sin^2\theta\cos^2\theta}=\dfrac{3\left(1-\sin^2\theta\cos^2\theta\right)}{\left(1-\sin^2\theta\cos^2\theta\right)}=3$

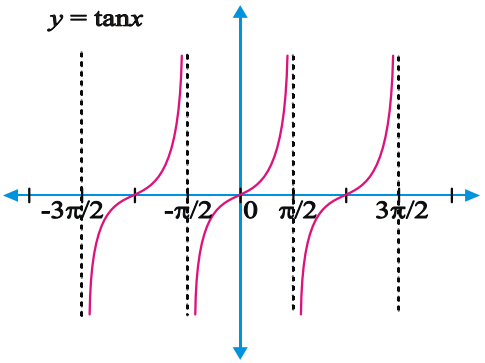
Graph of $\sin x$ **:**



Graph of $\cos x$ **:**



Graph of $\tan x$ **:**



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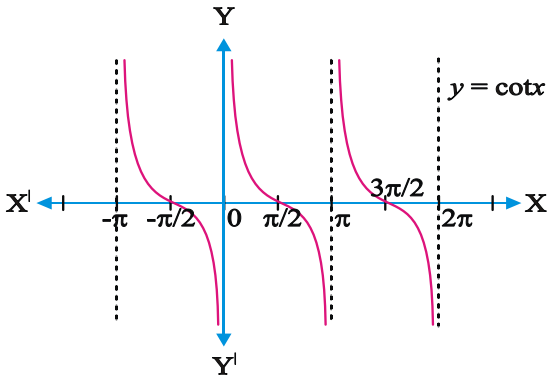
Graph of $\cot x$:



TEACHING TASK

LEVEL - I

1. $-13\pi/6$ radians =

A) -390^o B) -490^o C) -410^o D) -30^o

2. $Cot(1358^o)+\tan(3608^o)=$

A) -1 B)0 C)1 D)2

3. $\dfrac{\sin(-660^o)\tan(1050^o)\sec(-420^o)}{\cos(225^o)\,\mathrm{cosec}(315^o)\cos(510^o)}=$

A) $\dfrac{\sqrt{3}}{4}$ B) $\dfrac{\sqrt{3}}{2}$ C) $\dfrac{2}{\sqrt{3}}$ D) $\dfrac{4}{\sqrt{3}}$

4. $\log\tan 17^0+\log\tan 37^0+\log\tan 53^0+\log\tan 73^0=$

A) 0 B) 1 C) 2 D) 3

5. $\cos 24^o+\cos 55^o+\cos 125^o+\cos 204^o=$

A) –1 B) 0 C) 1 D) 2

6. $\sec\theta-\tan\theta=3\Rightarrow\theta$ lies in the quadrant

A) I B) II C) III D) IV

7. $3[\sin x-\cos x]^4+6[\sin x+\cos x]^2+4[\sin^6 x+\cos^6 x]=$

A) 3 B) 6 C) 4 D) 13

8. $\dfrac{\sin^2\alpha}{1+\cot^2\alpha}+\dfrac{\tan^2\alpha}{(1+\tan^2\alpha)^2}+\cos^2\alpha=$

A) – 1 B) 0 C) 1 D) 2

9. If A, B, C are the angles of a triangle ABC then $\cos\left(\dfrac{3A+2B+C}{2}\right)+\cos\left(\dfrac{A-C}{2}\right)=$

A) 0 B) 1 C) cosA D) cosC

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10. $\frac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}=$

A) $\frac{\sin\theta}{1+\cos\theta}$ B) $\frac{\sin\theta}{1-\cos\theta}$ C) $\frac{1-\cos\theta}{\sin\theta}$ D) $\frac{\cos\theta}{1-\sin\theta}$

11. $\sin^2(51^\circ-x)+\sin^2(39^\circ+x)=$

A) –1 B) 0 C) 1 D) 2

12. $\left[\sin\left(\frac{\pi}{2}-x\right)+\sin(\pi-x)\right]^2+\left[\cos\left(\frac{3\pi}{2}-x\right)+\cos(2\pi-x)\right]^2=$

A) 0 B) 2 C) 4 D) 8

13. $\sin\theta_1+\sin\theta_2+\sin\theta_3=3\Rightarrow\ \cos\theta_1+\cos\theta_2+\cos\theta_3=$

A) 0 B) 1 C) 2 D) 3

14. If $\sin x+\cos x=a$ then $|\sin x-\cos x|=$

A) $\sqrt{1-a^2}$ B) $\sqrt{a^2-1}$ C) $\sqrt{2-a^2}$ D) $\sqrt{a^2-2}$

15. $\operatorname{cosec}^2 A\cot^2 A-\sec^2 A\tan^2 A-\left(Cot^2 A-\tan^2 A\right)\left(\sec^2 A\operatorname{cosec}^2 A-1\right)=$

A) 0 B) 1 C) -1 D) 2

16. $\tan^2\theta+\sec\theta=5\Rightarrow\sec\theta=$

A) 3 B) 2 C) 1 D) -1

17. If $\tan(\alpha+\beta)=\sqrt{3},\ \tan(\alpha-\beta)=1$ then $\tan 6\beta=$

A) –1 B) 0 C) 1 D) 2

18. If $x=r\cos\theta\cos\phi,\ y=r\cos\theta\sin\phi,\ z=r\sin\theta$ then $x^2+y^2+z^2=$

A) r^2 B) y^2 C) x^2 D) z^2

19. If $x\tan^2 120^\circ+4\cos^2 150^\circ=9$ then $x=$

A) 3 B) 1 C) 2 D) 4

20. In ΔABC, right angled at C, then tan A + tanB =

A) $\frac{a^2}{bc}$ B) $\frac{b^2}{ac}$ C) $\frac{c^2}{ab}$ D) $\frac{ab}{c}$

21. If $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1,\ \frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta=1$ then $\frac{x^2}{a^2}+\frac{y^2}{b^2}=$

A) 1 B) -1 C) 2 D) 3

22. If α,β are complementary angles and $\sin\alpha=\frac{3}{5}$, then $\sin\alpha\cos\beta-\cos\alpha\sin\beta=$

A) $\frac{7}{25}$ B) $-\frac{7}{25}$ C) $\frac{25}{7}$ D) $-\frac{25}{7}$

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23. If θ lies in the first quadrant and $5\tan\theta = 4$, then $\frac{5\sin\theta - 3\cos\theta}{\sin\theta + 2\cos\theta} =$

A) 5/14 B) 3/14 C) 1/14 D) 0

24. If $a\sin^2\theta + b\cos^2\theta = c$, then $\tan^2\theta =$

A) $\frac{b-c}{a-c}$ B) $\frac{c-b}{a-c}$ C) $\frac{a-c}{b-c}$ D) $\frac{a-c}{c-b}$

25. If θ is not in 4th quadrant and $\tan\theta = -\frac{4}{3}$ then

$5\sin\theta + 10\cos\theta + 9\sec\theta + 16\operatorname{cosec}\theta + 4\cot\theta =$

A) –1 B) 2/5 C) 4/5 D) 0

26. $f(x) = x^3 - 3x + 5$ *then* $f\left(\sin\frac{3\pi}{2}\right) + f\left(\cos\frac{3\pi}{2}\right) =$

A) 10 B) 12 C) 14 D) 16

27. If $\tan\theta, 2\tan\theta + 2, 3\tan\theta + 3$ are in G.P then the value of $\frac{7 - 5\cot\theta}{9 - 4\sqrt{\sec^2\theta - 1}}$ is

A) $\frac{12}{5}$ B) $\frac{-33}{28}$ C) $\frac{33}{100}$ D) $\frac{12}{13}$

28. Which of the following is correct?

A) $\sin 1^0 > \sin 1$ B) $\sin 1^0 < \sin 1$ C) $\sin 1^0 = \sin 1$ D) $\sin 1^0 = \frac{\pi}{180}\sin 1$

29. $\operatorname{cosec} A = 4p + \frac{1}{16p} \Rightarrow \operatorname{cosec} A + \cot A =$

A) 8p B) $\frac{1}{8p}$ C) $-8p$ (or) $\frac{1}{8p}$ D) $8p$ (or) $\frac{1}{8p}$

30. If $\sqrt{\frac{1-\sin A}{1+\sin A}} = \sec A - \tan A$ then A lies in the quadrants

A) I, II B) II, III C) I, IV D) I, III

31. If $\sin\theta + \cos\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$, then n $(m+1)(m-1) =$

A) m B) n C) 2m D) 2n

32. $\cos^2 5^o + \cos^2 10^o + \cos^2 15^o + ... + \cos^2 360^o =$

A) 18 B) 27 C) 36 D) 45

33. If $x = a\cos^2\theta\sin\theta$ and $y = a\sin^2\theta\cos\theta$, then $\frac{(x^2 + y^2)^3}{x^2y^2} =$

A) a B) a^3 C) a^2 D) a^5

34. $a\sec\theta + b\tan\theta = 1, a\sec\theta - b\tan\theta = 5 \Rightarrow a^2(b^2 + 4) =$

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A) $3b^2$ B) $9b^2$ C) b^2 D) $4b^2$

35. tanA = a tanB, sinA = b sinB $\Rightarrow \frac{b^2-1}{a^2-1} =$

A) $\sin^2 A$ B) $\sin^3 A$ C) $\cos^2 A$ D) $\cos^3 A$

36. $If\ a = x\cos^2 A + y\sin^2 A\ \text{then}(x-a)(y-a) + (x-y)^2 \sin^2 A\cos^2 A =$

A) 0 B) 1 C) $xy + a^2$ D) $xy - a^2$

37. If $a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha = m$ and $a\sin^3\alpha + 3a\cos^2\alpha\sin\alpha = n$ then $(m+n)^{2/3} + (m-n)^{2/3}$ =

A) $2a^2$ B) $2a^{1/3}$ C) $2a^{2/3}$ D) $2a^3$

38. $a = \frac{1+\sin x}{1-\cos x+\sin x} \Rightarrow \frac{1+\cos x+\sin x}{2\sin x} =$

A) a B) $\frac{1}{a}$ C) a^2 D) $\frac{1}{a^2}$

39. Eliminate θ from $x = 1+\tan\theta, y = 2+\cot\theta$

A) $xy+1 = x+y$ B) $xy+2 = 2x+y$

C) $xy+1 = 2x+y$ D) $xy+1 = 2y+x$

40. If $\theta = \frac{11\pi}{6}$, then $\cos\theta + \sin\theta =$

A) $\frac{\sqrt{3}+1}{\sqrt{2}}$ B) $\frac{\sqrt{3}-1}{\sqrt{2}}$ C) $\frac{\sqrt{3}-1}{2}$ D) $\frac{\sqrt{3}+1}{2}$

41. If $\cos x = \tan y, \cot y = \tan z$ and $\cot z = \tan x$, then $\sin x =$

A) $\frac{\sqrt{5}+1}{4}$ B) $\frac{\sqrt{5}-1}{4}$ C) $\frac{\sqrt{5}+1}{2}$ D) $\frac{\sqrt{5}-1}{2}$

42. If $\sin\theta + \cos\theta = p$ and $\sin^3\theta + \cos^3\theta = q$, then $p\left(p^2-3\right) =$

A) q B) 2q C) -q D) -2q

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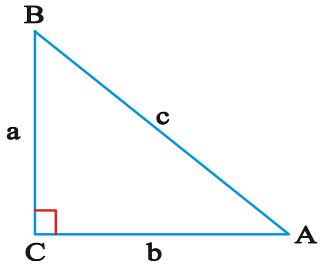
KEY

01) A	02) B	03) C	04) A	05)B	06) D
07) D	08) C	09) A	10) B	11) C	12) B
13) A	14) C	15) A	16) B	17) C	18) A
19) C	20) C	21) C	22) B	23) A	24) B
25) D	26) B	27) B	28) B	29) D	30) C
31) C	32)C	33)C	34)B	35) C	36) A
37)C	38)A	39)C	40)C	41) D	42) D

HINTS

1. $-13\times\frac{\pi}{6}\times\frac{180}{\pi}=-390^0$
2. $1358^0 = 8\,(180^0)-82^0$,
   $3608^0 = 20(180^0)+8^0$
3. Convert the corresponding values.
4. $\because$ *If* $A+B=90^0 \Rightarrow \tan A \tan B = 1$
5. $\cos\left(204^0\right)=\cos\left(180^0+24^0\right)$
   $\cos\left(125^0\right)=\cos\left(180^0-55^0\right)$
6. Use $\left(\sec^2 A - Tan^2 A\right)=1$
7. Put $x=90^o$
8. Put $\alpha=45^o$
9. A = B = C = 60^0
10. $1=\cos ec^2\theta-\cot^2\theta$
11. $A+B=90^0 \Rightarrow \sin^2 A+\sin^2 B=1$
12. Simplify
13. Put $\theta_1=\theta_2=\theta_3=90^\circ$
14. $\alpha=\frac{\pi}{4}$
15. $Put\ A=45^0$
16. $\tan^2\theta+\sec\theta=5$, $\Rightarrow\left(Sec^2\theta-1\right)+\sec\theta=5$,
    $\Rightarrow Sec^2\theta+\sec\theta-6=0$, $\sec\theta=2\ or\ -3$
17. Let $(\alpha+\beta)=60^0$
    $(\alpha-\beta)=45^0 \Rightarrow 2\beta=15^0 \Rightarrow 6\beta=45^0$
18. Square and add
19. Substitute the corresponding values.
20. 

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$\angle C = 90^o$

$\therefore \tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab} = \frac{c^2}{ab}$

21. by squaring and adding $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

22. $\sin\alpha = \frac{3}{5}, \cos\beta = \frac{3}{5}, \ \cos\alpha = \frac{4}{5}, \sin\beta = \frac{4}{5}$

23. Divide the Nr. and Dr. with $\cos\theta$

24. Divide throughout with $\cos^2\theta$

$\Rightarrow a\tan^2\theta + b = c\sec^2\theta \qquad \Rightarrow (a-c)\tan^2\theta = (c-b)$

25. θ lies in II quadrant, substitute the values

26. $\sin 270^0 = -1 \ \& \cos 270^0 = 0$

27. $(2\tan\theta + 2)^2 = \tan\theta(3\tan\theta + 3)$

$\Rightarrow \tan^2\theta + 5\tan\theta + 4 = 0$

$\Rightarrow \tan\theta = -1, -4$

but $\tan\theta = -4$ only satisfies above condition

28. $1 > 1^0$

29. Use $\left(\cos ec^2 A - \cot^2 A\right) = 1$

If $\cos ecA = \left(4P + \frac{1}{16P}\right)$

then $\cot A = \pm\left(4P - \frac{1}{16P}\right)$

30. $\sqrt{\cos^2 A} = \cos A$ then $A \in Q_1, Q_4$

31. Find m^2 and multiply

32. If (A+B)=360^0 or (A+B)=180^0 then $\cos^2 A = \cos^2 B$ and (A+B)=90^0 then

$\left(\cos^2 A + \cos^2 B\right) = 1$

33. θ can be eliminated by substitution.

34 Given $\left(a\sec\theta + b\tan\theta\right) = 1$

$\Rightarrow \left(a\sec\theta - b\tan\theta\right) = 5$, eliminate θ

35. Put A=30^0 and B=45^0 and verify

36. Put A=90^0

37. $m + n = a(\cos\alpha + \sin\alpha)^3$

$m - n = a(\cos\alpha - \sin\alpha)^3$

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38. In R.H.S. of given equation multiply and divide by $(1+\cos x+\sin x)$

39. $x-1=\tan\theta, y-2=\cot\theta$ Multiply two equations

40. Put $\theta=330^0$ substitution

41. $\left.\begin{array}{l}\cos x=\tan y\\ \tan z=\cot y\\ \cot z=\tan x\end{array}\right\}$ *multiplying we get* $\cos x=\tan x$

$\Rightarrow \cos^2 x=\sin x$

$\Rightarrow \sin^2 x+\sin x-1=0 \Rightarrow \sin x=\dfrac{-1\pm\sqrt{5}}{2}$

42. $\cos\theta+\sin\theta=p\Rightarrow p^2=1+2\sin\theta\cos\theta\Rightarrow\sin\theta\cos\theta=\dfrac{p^2-1}{2}$

also

$p^3=\cos^3\theta+\sin^3\theta+3\cos\theta\sin\theta(\cos\theta+\sin\theta)$

$\Rightarrow p^3=q+3\left(\dfrac{p^2-1}{2}\right)p$

$\Rightarrow 2p^3=2q+3p^3-3p\Rightarrow p^3-3p=-2q$

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LEARNER'S TASK

BEGINNERS (Level - I)

1. $-11\pi/3$ radians =

A) -390º B) -620º C) -610º D) -660º

2. Sin4530^0 =

A) $\dfrac{1}{2}$ B) $-\dfrac{1}{2}$ C) $\dfrac{\sqrt{3}}{2}$ D) $-\dfrac{\sqrt{3}}{2}$

3. $\dfrac{\sin 150^\circ-5\cos 300^\circ+7\tan 225^\circ}{\tan 135^\circ+3\sin 210^\circ}=$

A) 10 B) –5 C) –2 D) –3/2

4. $\log\tan 18^\circ+\log\tan 36^\circ+\log\tan 54^\circ+\log\tan 72^\circ=$

A) log4 B) log3 c C) log2 D) 0

5. $\tan 20^0+\tan 40^0+\tan 60^0+.....+\tan 180^0=$

A) 0 B) 1 C) 2 D) 3

6. $(\sin\alpha+\operatorname{cosec}\alpha)^2+(\sec\alpha+\cos\alpha)^2 = k+\tan^2\alpha+\cot^2\alpha \Rightarrow k=$

A) 9 B) 7 C) 5 D) 3

7. $\dfrac{1}{(1+\cot^2\alpha)^2}+\dfrac{\tan^2\alpha}{(1+\tan^2\alpha)^2}+\dfrac{1}{1+\tan^2\alpha}=$

A) -1 B) 0 C) 1 D) 2

8. $\dfrac{\cos^3 A+\sin^3 A}{\cos A+\sin A}+\dfrac{\cos^3 A-\sin^3 A}{\cos A-\sin A}=k$ then k =

A) 0 B) 1 C) 2 D) -1

9. $\dfrac{1+\cot\alpha+\operatorname{cosec}\alpha}{1-\cot\alpha+\operatorname{cosec}\alpha}=$

A) $\dfrac{\sin\alpha}{1+\cos\alpha}$ B) $\dfrac{\sin\alpha}{1-\cos\alpha}$ C) $\dfrac{1+\cos\alpha}{\sin\alpha}$ D) $\dfrac{1-\sin\alpha}{\cos\alpha}$

10. $\cos^2(125^\circ-x)-\cos^2(55^\circ+x)=$

A) -1 B) 0 C) 1 D) 2

11. $\sin^2\dfrac{\pi}{18}+\sin^2\dfrac{2\pi}{18}+\sin^2\dfrac{4\pi}{18}+\sin^2\dfrac{8\pi}{18}+\sin^2\dfrac{7\pi}{18}+\sin^2\dfrac{5\pi}{18}=$

A) 1 B) 2 C) 3 D) 4

12. If A B C D is a quadrilateral then $\tan\left(\dfrac{A+B}{4}\right)=$

A) $\cos\left(\dfrac{C-D}{4}\right)$ B) $\cot\left(\dfrac{C-D}{4}\right)$ C) $\cos\left(\dfrac{C+D}{4}\right)$ D) $\cot\left(\dfrac{C+D}{4}\right)$

13. If $\cos\theta_1+\cos\theta_2+\cos\theta_3+\cos\theta_4+\cos\theta_5=5$ then $\sin\theta_1+\sin\theta_2+\sin\theta_3+\sin\theta_4+\sin\theta_5=$

A) 3 B) 2 C) 1 D) 0

14. If $\cot(\alpha+\beta)=0$ then $\sin(\alpha+2\beta)=$

A) $\sin\alpha$ B) $\cos\alpha$ C) $\sin\beta$ D) $\cos\beta$

15. $5\sin x+4\cos x=3 \Rightarrow 4\sin x-5\cos x=$

A) 4 B) $4\sqrt{2}$ C) $3\sqrt{2}$ D) $\sqrt{2}$

16. If $k=(\sec A+\tan A)(\sec B+\tan B)(\sec C+\tan C)=$

$(\sec A-\tan A)(\sec B-\tan B)(\sec C-\tan C)$ then k =

A) 0 B) ±1 C) ±3 D) ±4

17. If $\sin(\alpha+\beta)=1,\ \sin(\alpha-\beta)=\dfrac{1}{2}$ then $\tan(\alpha+2\beta)\tan(2\alpha+\beta)=$

A) 1 B) -1 C) 0 D) 2

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18. If $x = r\cos\alpha\cos\beta\cos\gamma;\ y = r\cos\alpha\cos\beta\sin\gamma;$
$z = r\sin\alpha\cos\beta;\ \mu = r\sin\beta$ then $x^2 + y^2 + z^2 + \mu^2 =$

A) r B) 2r C) r^2 D) $4r^2$

19. $\sin^4\theta + 2\sin^2\theta\left(1 - \frac{1}{\cos ec^2\theta}\right) + \cos^4\theta =$

A) 1 B) 0 C) $1/2$ D) -1

20. If $x\cot^2 120^o + 4\sin^2 150^o = 3$, then $x =$

A) 2 B) 7 C) 6 D) 5

21. If $x = \sin 130^o + \cos 130^o$ then

A) x <0 B) x = 0 C) x > 0 D) $x \geq 0$

22. If $x = a\sec^n\theta; y = b\tan^n\theta$ then $\left(\frac{x}{a}\right)^{2/n} - \left(\frac{y}{b}\right)^{2/n} =$

A) 0 B) -1 C) 1 D) 2

23. If $\tan\theta = \frac{p}{q}$ then $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} =$

A) $\frac{2p}{p^2+q^2}$ B) $\frac{2pq}{p^2+q^2}$ C) $\frac{p^2-q^2}{p^2+q^2}$ D) $\frac{q^2-p^2}{p^2+q^2}$

24 If $a\sec\theta + b\tan\theta = c$ then $(a\tan\theta + b\sec\theta)^2 =$

A) $a^2 + b^2 + c^2$ B) $-a^2 + b^2 + c^2$ C) $a^2 - b^2 + c^2$ D) $a^2 + b^2 - c^2$.

25. If $\cos\theta = \frac{3}{5}$ and θ is not in the first quadrant, then $\frac{5\tan(\pi+\theta) + 4\cos(\pi+\theta)}{5\sec(2\pi-\theta) - 4\cot(2\pi+\theta)} =$

A) $\frac{4}{5}$ B) $-\frac{4}{5}$ C) $\frac{5}{4}$ D) $-\frac{5}{4}$

26. $f(x) = x^3 - 2x^2 + 3x - 5 \Rightarrow f\left[\sin\left(\frac{5\pi}{2}\right)\right] + f\left[\sin\left(\frac{3\pi}{2}\right)\right] =$

A) 10 B) -10 C) 14 D) -14

27. $x = \cos 1^o, y = \cos 1 \Rightarrow$

A) x = y B) x > y C) x < y D) 2x = y

28. $a = \sec 2^o, b = \sec 2 \Rightarrow$

A) a = b B) a < b C) b < a D) 2a = b

29. $\tan\theta = P - \frac{1}{4p} \Rightarrow \sec\theta - \tan\theta =$

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A) $2p \text{ (or) } \frac{1}{2p}$ B) $\frac{1}{2p} \text{ (or) } -2p$

C) $-\frac{1}{2p} \text{ (or) } 2p$ D) $-\frac{1}{2p} \text{ (or) } -2p$

30. $\sin^2 5^o + \sin^2 10^o + ... + \sin^2 180^o =$

A) 18 B) 27 C) 1 D) 0

31. $a = \sec\theta - \tan\theta, b = \cos ec\theta + \cot\theta \Rightarrow a =$

A) $\frac{b+1}{b-1}$ B) $\frac{1+b}{1-b}$ C) $\frac{b-1}{b+1}$ D) $\frac{1-b}{1+b}$

32. $\cos A = a\cos B, \sin A = b\sin B \Rightarrow \left(b^2 - a^2\right)\sin^2 B =$

A) $1+a^2$ B) $2+a^2$ C) $1-a^2$ D) $2-a^2$

33. If $\frac{\sin x}{a} = \frac{\cos x}{b} = \frac{\tan x}{c} = k$ then $bc + \frac{1}{ck} + \frac{ak}{1+bk}$ is

A) $k\left(a + \frac{1}{a}\right)$ B) $\frac{1}{k}\left(a + \frac{1}{a}\right)$ C) $\frac{1}{k^2}$ D) $\frac{a}{k}$

34. If $m\cos^2 A + n\sin^2 A = p$, then $\cot^2 A =$

A) $\frac{p+n}{m+p}$ B) $\frac{p-n}{p-m}$ C) $\frac{p-n}{m-p}$ D) $\frac{n+1}{m+p}$

35. If $a\sin^3 x + b\cos^3 x = \sin x \cos x$ and $a\sin x = b\cos x$ then $a^2 + b^2 =$

A) 0 B) 1 C) 2 D) 3

36. If $\sin\theta + \cos\theta = a$, then $\sin^4\theta + \cos^4\theta =$

A) $1 - \frac{1}{2}\left(a^2+1\right)^2$ B) $1 - \frac{1}{2}\left(a^2-1\right)^2$ C) $1 + \frac{1}{2}\left(a^2+1\right)^2$ D) $1 + \frac{1}{2}\left(a^2-1\right)^2$

37. If $\tan\theta + \sin\theta = m$, $\tan\theta - \sin\theta = n$, then $\left(m^2 - n^2\right)^2 =$

A) 16 mn B) 4 mn C) 32 mn D) 8 mn

38. If $\sec\theta + \cos\theta = 2$ then $\sin^2\theta + \tan^2\theta =$

A) 0 B) 1 C) 2 D) 4

39. Which of the following possible?

A) $\sin\theta = 2$ B) $\cos\theta = \frac{17}{5}$ C) $\tan\theta = 2$ D) $\sec\theta = \frac{4}{5}$

40. If $\theta = \frac{\pi}{21}$, then $\frac{\sin 23\theta - \sin 7\theta}{\sin 2\theta + \sin 14\theta} =$

A) 0 B) 1 C) -1 D) 2

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41. If $2\sin x+5\cos y+7\sin z=14$ then $7\tan\frac{x}{2}+4\cos y-6\cos z=$

A) 4 B) -3 C) 11 D) 5

42. If $\sin\theta$ and $\cos\theta$ are the roots of $px^2+qx+r=0$ then $q^2-p^2=$

A) 0 B) –2pr C) 2qr D) 2rp

KEY

01) D	2) B	03)C	04) D	05) A	6) B	7) C
08) C	9) C	10)B	11) C	12) D	13) D	14) D
15) B	16) B	17) A	18) C	19) A	20) C	21) C
22) C	23) C	24) B	25) B	26) D	27) B	28) C
29) B	30) A	31) C	32) C	33) B	34) C	35) B
36) B	37) A	38) A	39) C	40) C		
41)C	42)D					

HINTS

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1. $-11\frac{\pi}{3}\times\frac{180}{\pi}=-660^o$

2. $\sin(4530^o)=\sin((12)360^o+210^o)$

$=\sin 210^o=-\sin 30^o=\frac{1}{2}$

3. Convert the corresponding values.

4. $\log(\tan 18^o\tan 36^o\tan 54^o\tan 72^o)=\log 1=0$

$[\because If\ A+B=90^o\Rightarrow\tan A\tan B=1]$

5. $A+B=180^o\Rightarrow\tan A+\tan B=0$

6. Square and simplyfy

7. Put $\alpha=45^o$

8. Put A = 0^0

9. Use $1=(coese^2\alpha-\cot^2\alpha)$

10. $(125^o-x)+(55^o+x)=180^0$

11. If $(A+B)=\frac{\pi}{2}$ then $\sin^2 A+\sin^2 B=1$

12. $(A+B+C+D)=360^o\Rightarrow\frac{A+B}{4}=\frac{\pi}{2}-\frac{C+D}{4}$

13. Put $\theta_1 = \theta_2 = \theta_3 = 0^o$

14. $\cot(\alpha+\beta) = 0 \Rightarrow \alpha+\beta = \frac{\pi}{2}$

$\Rightarrow \sin(\alpha+2\beta) = \sin(90^o+\beta) = \cos\beta$

15. $a\sin x + b\cos x = c \Rightarrow b\sin x - a\cos x = \pm\sqrt{a^2+b^2-c^2}$

16. $K^2 = (1+\sin A)(1+\sin B)(1+\sin C)(1-\sin A)(1-\sin B)(1-\sin C)$

$K^2 = (1-\sin^2 A)(1-\sin^2 B)(1-\sin^2 C)$

$K^2 = \cos^2 A.\cos^2 B.\cos^2 C$

$K = \pm\cos A\cos B\cos C$

17. $\alpha+\beta = 90^o,\ \alpha-\beta = 30^o \Rightarrow \alpha = 60^o, \beta = 30^o$

18. Squaring and adding

19. $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$

20. $x.\frac{1}{3} + 4.\frac{1}{4} = 3 \Rightarrow x = 6$

21. $x = \sin 50^o - \cos 50^o$

22. $\sec^2\theta - \tan^2\theta = 1$

23. Divide the Nr and Dr. with $\cos\theta$

24. Let $a\tan\theta + b\sec\theta = k$

$\therefore a^2 - b^2 = c^2 - k^2 \Rightarrow k^2 = c^2 - a^2 + b^2$

25. θ lies in fourth quandrant substitute the cor-responding values.

26. $\sin\frac{3\pi}{2} = -1, \sin\frac{5\pi}{2} = 1$

27. $1 > 1^0 \Rightarrow \cos 1 < \cos 1^0$

28. $2 = 2^c = 114^0$ (approx)

29. Use $(\sec^2\theta - \tan^2\theta) = 1$

30. $A+B = 180^o \Rightarrow \sin^2 A = \sin^2 B$ and $A+B = 90^o \Rightarrow \sin^2 A + \sin^2 B = 1$

31. Put $\theta = 45^0$ and verify the options

32. squaring and adding

33. $bc + \frac{1}{ck} + \frac{ak}{1+bk} = \frac{\sin x}{k^2} + \frac{\cos x(1+\cos x)+\sin^2 x}{\sin x(1+\cos x)} = \frac{a}{k} + \frac{1}{\sin x} = \frac{a}{k} + \frac{1}{ak} = \frac{1}{k}\left(a+\frac{1}{a}\right)$

34. divided with $\sin^2 A$

weget $\cot^2 A = \frac{p-n}{m-p}$

35. Put $\sin x = b$ and $\cos x = a$

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$\therefore a^2+b^2=\cos^2 x+\sin^2 x=1$

36. Use $\left(a^4+b^4\right)=\left(a^2+b^2\right)^2-2a^2b^2$

37. $2\tan\theta=(m+n)$; $2\sin\theta=(m-n)$

Eliminate θ and use $\left(\tan^2\theta-\sin^2\theta\right)=\tan^2\theta.\sin^2\theta$

38. Put θ=0

39 Range of tan θ is R

40. Use $21\theta=\pi$

41. put x = $\frac{\pi}{2}$, y = 0, z = $\frac{\pi}{2}$.

42. $\sin\theta+\cos\theta=\frac{-q}{p},\sin\theta\cos\theta=\frac{r}{p}$

now $\left(\sin\theta+\cos\theta\right)^2=1+2\sin\theta\cos\theta$

ACHIEVERS (Level - II)

II. MCQ's with one or more than one correct answer

1. $\frac{\prod}{2}<\alpha<\prod\Rightarrow\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}+\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}=$?

$A)\ 2\sec\alpha$ $B)\frac{2}{\sin\alpha}$ $C)\ 2\cos ec\alpha$ $D)\ -2\cos ec\,\alpha$

2). $\sqrt{\frac{1+\cos\theta}{1+\cos\theta}}=$?

$A)\cos\sec\theta+\cot\theta$ $B)\cos ec\theta-\cot\theta$ $C)\frac{\sin\theta}{1-\cos\theta}$ $D)\frac{1+\cos\theta}{\sin\theta}$

3). $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=$?

$A)\ \cos ec\theta+\cot\theta$ $B)\ \cos\sec\theta-\cot\theta$ $C)\ \frac{\sin\theta}{1-\cos\theta}$ $D)\frac{\sin\theta}{1+\cos\theta}$

4). $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=$?

$A)\ \frac{1-\sin\theta}{\cos\theta}$ $B)\ \sec\theta-\tan\theta$ $C)\ \frac{co\,s\theta}{1+\sin\theta}$ $D)none$

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5). $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} =$?

$A)\ \frac{1+\sin\theta}{\cos\theta}$ $B)\ \frac{\cos\theta}{1-\sin\theta}$ $C)\sec\theta+\tan\theta$ $D)none$

6).1 right angle = ?

$A)90^0$ $B)\ \frac{\prod^C}{2}$ $C)\ 100^g$ $D)\ 2\prod^C$

II Assertion and reasoning type.

options: A) Both A and R are correct R is correct explanation of A
B) Both A and R are correct Ris not a correct explantion of A
C) A is true R false D) Ais false R is true

1. A:In a right angled triangles $\sin^2 A+\sin^2 B+\sec^2 C=2$
R:If $\alpha\ \beta$ are complementary angles then $\sin^2\alpha+\sin^2\beta=1$
2. A: The number of values of θ satisfying $\sec\theta+\cos\theta=1$ *is* 2
R: If $7\sin^2\alpha+3\cos^2\alpha=3$ *then* $\tan\alpha=0$
3. A: If $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$ *then* $\cos\theta_1+\cos\theta_2+\cos\theta_3=0$
R: The value of $\prod=\frac{355}{113}$
4. A: $\log\tan 17^0+log\tan 37^0+\log\tan 53^0+\log\tan 73^0=0$
R: $\log a+\log b+\log c=\log[abc]$
5. A: If x=r $\cos\theta\cos\phi,\ y=r\cos\theta\sin\phi,\ z=r\sin\theta$ *then* $x^2+y^2+z^2=r^2$
R: If $\sec^2\theta-\tan^2\theta=1$ *then* $\sec\theta+\tan\theta=\frac{1}{\sec\theta-\tan\theta}$

III. Match the following

1. Let $\frac{\sin x}{a}=\frac{\cos x}{b}=\frac{\tan x}{c}=k$ then

list-I			**list-II**
1.bc	[	]	a. $\frac{1}{b^2k^4}$
2. a^2+b^2	[	]	b. $\frac{1}{ak}$
3. $\frac{1}{ck}+\frac{ak}{1+bk}$	[	]	c. $\frac{a}{k}$
4. $a^2+b^2+c^2$	[	]	d. $\frac{1}{k^2}$

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A. 1-c , 2-d , 3-b , 4-a B. 1-d , 2-a , 3-c , 4-b
C. 1-a , 2-b , 3-d , 4-c D. 1-b , 2-c , 3-a , 4-d

2.

list-I	list-II
1) $\sin^2\frac{2\Pi}{3}+\cos^2\frac{5\Pi}{6}-\tan^2\frac{3\Pi}{4}$	*a*. $\frac{15}{2}$
2. $\sin^2\frac{11\Pi}{6}+\cos^2\frac{7\Pi}{6}-\tan^2\frac{5\Pi}{4}$	*b*. $\frac{-1}{2}$
3. $\sin^2\frac{3\Pi}{4}+\sec^2\frac{5\Pi}{3}+\tan^2\frac{2\Pi}{3}$	c) 0
4. $\cos^2\frac{2\Pi}{3}+\sin^2\frac{5\Pi}{3}-\frac{3}{2}\tan^2\frac{3\Pi}{4}$	*d*. $\frac{1}{2}$

A. 1-d , 2-c , 3-a , 4-b B. 1-a , 2-b , 3-c , 4-d
C. 1-b , 2-a , 3-d , 4-c D. 1-c , 2-d , 3-b , 4-a

3.

list-I	list-II
1. 1 right angle	*a*. $2\Pi^c$
2. 1 revolution	*b*. 0.01745^c
3. 1 radian	*c*. 100^g
4. 1 degree	*d*. $57^0.17^1 46^{11}$

A. 1-a , 2-c , 3-b , 4-d B.1-c , 2-a , 3-d , 4-b
C. 1-a , 2-c , 3-d , 4-b D.1-b , 2-a , 3-d ,4-c

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4.

list-I	list-II
1. $3\tan x+27\cot x\geq (XEQ_1)$	a. 24
2. $5\sec^2 x+125\cos^2 x\geq$	b.18
3. $16\cos ec^2 x+9\sin^2 x\geq$	c.50
4. $\sec^2 x-\tan^2 x=$	d.1

A. 1-a , 2-b , 3-c , 4-d B.1-c , 2-a , 3-b , 4-d
C. 1-b , 2-c , 3-a , 4-d D.1-c , 2-b , 3-a , 4-d

5.

list-I	list-II
1. $\tan 130^0.\tan 140^0=$	*a*) $\sqrt{3}$
2. $\sin^2 55^0+\sin^2 35^0-1=$	*b*) 0
3. *If* $8\cos\theta+6\sin\theta=5$ *then* $8\sin\theta-6\cos\theta=$	*c*) 1
4. $\cot 30^0=$	*d*) $\pm 5\sqrt{3}$

A. 1-b , 2-c , 3-d , 4-a B. 1-c , 2-b , 3-a , 4-d
C. 1-c , 2-b , 3-d , 4-a D.1-a , 2-b , 3-d , 4-c