SURFACE AREA AND VOLUME

Class: IX. Mathematics

BOARD INTEGRATED (TELANGANA BOARD)

TEACHING TASK

01. for cylinder

$r = \frac{2.1}{2}$ m, $h = 5$ mt

T.S.A $= 2\pi r(r+h)$

$= 2 \times \frac{22}{7} \times \frac{21}{20}\left(\frac{21}{20} + 5\right)$

$= 2 \times \frac{22}{7} \times \frac{21}{20} \times \frac{121}{20}$.

total cost @ Rs 10 per mt^2

$= 2 \times \frac{22}{7} \times \frac{21}{20} \times \frac{121}{20} \times 10 =$ Rs 399.30 (Approx

Ans: B

02 C.S.A. of cylinder $= 2\pi rh$

C.S.A of cone $= \pi rl$

$\frac{2\pi rh}{\pi rl} = \frac{6}{5}$

$\Rightarrow \frac{h}{\sqrt{r^2+h^2}} = \frac{3}{5}$

$\Rightarrow \frac{h^2}{r^2+h^2} = \frac{9}{25}$

$25h^2 = 9r^2 + 9h^2$

$16h^2 = 9r^2$

$\Rightarrow \frac{h^2}{r^2} = \frac{9}{16}$

$\Rightarrow h : r = 3 : 4$

03 Let Initial edge $= a$ | final edge $= a'$ (2)
Initial Surface Area $= A$ | final S.A $= A'$

Given $a' = a + 50\%$ of a

$= a + \frac{a}{2} = \frac{3a}{2}$

$A = 6a^2$ =

$A' = 6\left(\frac{3a}{2}\right)^2 = \frac{9}{4}(6a^2) = \frac{9}{4} \cdot A$

percentage Increase $= \frac{A' - A}{A} \times 100$

$= \frac{\frac{9}{4}A - A}{A} \times 100$

$= \frac{5}{4} \times 100$

$= 125\%$

Ans: A

04 Area of hall $= \frac{\text{Total cost of decoration}}{\text{cost of decoration per sq.mt}}$

$= \frac{4050}{4.50} = 900 \text{ sq.mt}$

$l : b = 3 : 2 \Rightarrow l = 3x, \; b = 2x$

~~$2(lb + bh + hl) = 900$~~

~~$2(3x \cdot 2x + 2x \cdot (4.5) + (4.5)$~~

L.S.A of hall $= 2h(l + b)$

$900 = 2 \times 4.5 \times (3x + 2x)$

$\Rightarrow x = 20$

$\therefore l = 60 \text{ mt}, \; b = 40 \text{ mt}$

Ans: C

05 C.S.A of cone $= A = \pi r l$

$l = p\%$ increased

New slant height $L = l + p\%$ of l

$= l + \frac{p}{100} l$

$L = l\left(\frac{100+p}{100}\right)$

~~S.A. increase (%) $= l\left(\frac{100+p}{100}\right) -$~~

New S.A $= \pi r L$

$= \pi \cdot r \cdot \left(\frac{l(100+p)}{100}\right)$

$\therefore$ % increase in C.S.A $= \frac{\text{Final} - \text{original}}{\text{original}} \times 100$

$= \frac{\pi r \left(\frac{l(100+p)}{100}\right) - \pi r l}{\pi r l} \times 100$

$= p\ \%$

06. Surface Area of sphere $= 4\pi r^2$

$r \longrightarrow p\%$ of r

$r \rightarrow \frac{p}{100} \times r$

New radius $= r + \frac{p}{100} r = r\left(\frac{100+p}{100}\right)$

New S.A $= 4\pi\left(r\left(\frac{100+p}{100}\right)\right)^2$

% change in S.A $= \frac{\text{Final} - \text{original}}{\text{original}} \times 100$

$= \frac{4\pi\left[r\left(\frac{100+p}{100}\right)\right]^2 - 4\pi r^2}{4\pi r^2} \times 100 = \left(2p + \frac{p^2}{100}\right)\%$

Ans. C

07. $l = 6x$, $b = 5x$, $h = 4x$ (4)

$2(lb + bh + hl) = 3300$

$\Rightarrow 2(6x \cdot 5x + 5x \cdot 4x + 4x \cdot 6x) = 3300$

$\Rightarrow x = 15$

$\therefore l = 90$, $b = 75$, $h = 60$

Ans: D

08 $R = \frac{25}{2}$ cm, $r = \frac{24}{2}$ cm

T.S.A of hollow hemispherical vessel

$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= 2 \times \frac{22}{7} \times \left(\frac{25}{2}\right)^2 + 2 \times \frac{22}{7} \times (12)^2 + \frac{22}{7}\left(\left(\frac{25}{2}\right)^2 - (12)^2\right)$

$= 1925.78 \text{ cm}^2$

Total cost of painting $= 1925.78 \times 0.05$

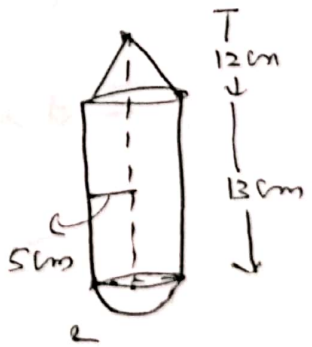
$=$ Rs 96.29

Ans: D

09 slant height of cone

$l^2 = h^2 + r^2 = 144 + 25 = 169$

$\therefore l = 13$ cm



S.A. of the toy

$= 2\pi rh + \pi r l + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 5 \times 13 + \frac{22}{7} \times 5 \times 13 + 2 \times \frac{22}{7} \times 5^2$

$= 770 \text{ cm}^2$

Ans: A

10

10 Given $l = 6m$, $b = 5m$, $h = 3.5m$ (5)

door → $l = 1.2mt$, $b = 2m$

window → $l = 1m$, $b = 1.9 mt$

C.S.A of cuboid $= 2h(l+b)$

Required Area

$= 2h(l+b) - 2 \times$ doors $-$ 3 windows

$= 2 \times 3.5 (6+5) - 2 \times 1.2 \times 2 - 3 \times 1 \times 1.9$

$= 7 \times 11 - 4.8 - 5.7$

$= 77 - 10.5$

$= 66.5$ sq. mt.

Given wall paper $b = 70 cm = 0.7 mt$

$l =$ [illegible] mt.

Cost per $1 mt^2$ → Rs. 6.50.

Total paper required $= \frac{66.5}{1 \times 0.7}$ mt

Cost of the paper $= \frac{66.5}{0.7} \times 6.5$

$=$ Rs 617.5

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11. $l = 4x$, $b = 3x$, $h = 5.5$

$2h(l+b) = 2 \times 5.5 \times (4x+3x) = 77x^2$

Total cost $\Rightarrow 77x^2 \times 6.6 = 5082$

$\Rightarrow 77x^2 = 5082$

11) $l = 4x$, $b = 3x$, $h = 5.5$ mt (6)

Cost of decorating = Rs 5082

~~$2h(l+b) =$~~

cost per mt^2 × Area = 5082

$\Rightarrow 6.60 \times$ Area = 5082

$\Rightarrow$ Area = 770

$\therefore 2h(l+b) = 770$

$\Rightarrow 2 \times 5.5 (4x+3x) = 770$

$\Rightarrow x = 10$

$\therefore l = 40$, $b = 30$

Ans: A, D

12. Given $a = 4$cm

Volume of 4cm cube $= a^3 = 4^3 = 64$.

Volume of cube having 1cm edge

$= 1^3 = 1$

No. of small cubes $= \frac{V_1}{V_2} = \frac{64}{1} = 64$.

Surface area of Larger cube $= 6a^2$

$= 6 \times 4^2$

$= 96$

Surface area of small cube $= 6a^2 = 6 \times 1^2 = 6$

Ratio = 96 : 6 = 16 : 1

Ans: A, B, D

13. Inital diameter = d ⑦

decreased by 25% = d − 25% of d

$= d - \frac{25}{100} \times d$

$\therefore$ New diameter $= \frac{3d}{4}$. $\therefore$ radius $= \frac{3d}{2}$

% of decreased C.S.A of ~~Initial~~ Sphere

$= \frac{\text{Final} - \text{original}}{\text{original}} \times 100$

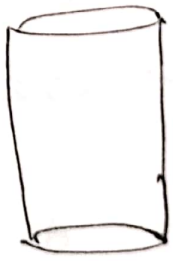
$= \frac{4\pi\left(\frac{3d}{4}\right)^2 - 4\pi\left(\frac{d}{2}\right)^2}{4\pi\left(\frac{d}{2}\right)^2} \times 100$

$= 43.75\%$ (True)

Statement II: C.S.A increases when diameter increases (False) Ans: C

14 Statement I: $l = 25\text{cm}$

$b = 3.5\text{cm}$



C.S.A $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 25 \times 3.5$

$= 550\text{cm}^2$ (True)

Statement II: Conceptual (True) Ans: A

15. for a cube.

original edge $\to a$

final edge $= a + 50\%$ of a

$$= a + \frac{50}{100} \times a$$

$$= \frac{3a}{2}.$$

$\therefore$ Increase in surface area

$$= \frac{\text{final} - \text{original}}{\text{original}} \times 100$$

$$= \frac{6\left(\frac{3a}{2}\right)^2 - 6a^2}{6a^2} \times 100$$

$$= 125\%.$$

16. $6a^2 = 150 \Rightarrow a^2 = 25 \Rightarrow a = 5$

Length of diagonal $= \sqrt{3}a = \sqrt{3} \times 5 = 5\sqrt{3}$

17. Surface area of sphere $= 616$

$\Rightarrow 4\pi r^2 = 616$

$\Rightarrow r = 7$

$\therefore$ diameter $= 14$ cm

18 ~~6 H~~ $4\pi r^2 = 616 \Rightarrow r = 7.$

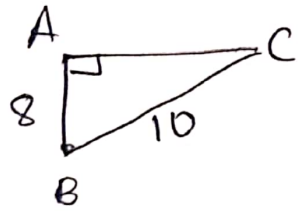
New area = ~~6 b~~ $616 - 75\%$ of 616

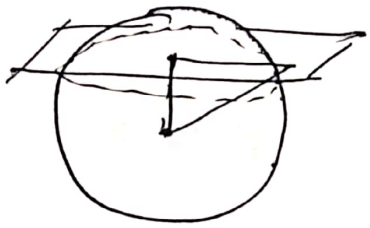
$= 616 - 462$

$= 154$ ~~cm~~ cm^2

$4\pi r^2 = 154 \Rightarrow r = 3.5$ ~~cm~~ cm

19





$\therefore \quad AC^2 + AB^2 = BC^2$

$\Rightarrow AC^2 + 64 = 100$

$\Rightarrow AC^2 = 36 \Rightarrow AC = 6$

20

T.S.A of each hemisphere $= 3\pi r^2$

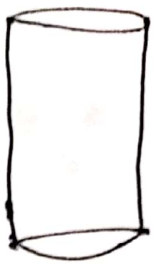
total T.S.A. of two hemisphere $= 6\pi r^2$

Rate of painting $= 6\pi r^2 \times 8$

$= 48\pi r^2 = a\pi r^2$

$\Rightarrow a = 48$

21 (i) <u>for sphere</u>



$r = 8$ cm

<u>for cylinder</u>

$R = 4$ cm, $h = h$ cm.

T.S.A of cylinder $= \frac{1}{2} \times$ S.A. of sphere

$\Rightarrow 2\pi R(R+h) = \frac{1}{2} \times 4\pi r^2$

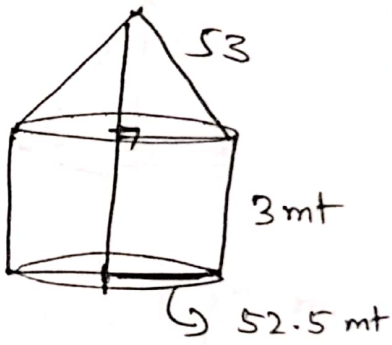
$\Rightarrow 4(4+h) = 8^2$

$\Rightarrow h = 12$ cm

(ii) Area of the canvas required =

C.S.A of conical part +

C.S.A of cylindrical part



$= \pi r l + 2\pi r h$

$= \frac{22}{7} \times 52.5 \times 53 + 2 \times \frac{22}{7} \times 52.5 \times 3$

$= 3097.5\pi$

(iii) T.S.A $= 1150\pi$

$\Rightarrow 2(7x \times 5x + 5x \cdot 3x + 3x \cdot 7x) = 1150\pi$

$\Rightarrow x = 9$

$\therefore$ breadth $= 5x = 5 \times 9 = 45$ cm.

(iv) $2\pi rh = 660$

$\Rightarrow 2 \times \frac{22}{7} \times r \times 15 = 660$

$\Rightarrow r = 7$ cm

Ans. (i), (ii), (iii), (iv)

22 (i) $2\pi r = 33$; $h = 16$ (11)

$\Rightarrow 2 \times \frac{22}{7} \times r = 33$

$\Rightarrow r = \frac{21}{4}$

$l^2 = h^2 + r^2 = 16^2 + \left(\frac{21}{4}\right)^2$

$\therefore l = \sqrt{16^2 + \left(\frac{21}{4}\right)^2} = \sqrt{256 + 27.56} = 16.84$

$S.A = \pi r l = \frac{22}{7} \times \frac{21}{7} \times 16.84$

$= 158.78$

(ii) $h = 12$ mt, $r = 16$ m

C.S.A $= l^2 = h^2 + r^2$

$= 144 + 256$

$\therefore l = 20$.

$C.S.A = \pi r l = \frac{22}{7} \times 16 \times 20 = \frac{7040}{7}$ mt

Total cloth required

Area of cloth = Area of Tent

$l \times 1.1 = \frac{7040}{7} \Rightarrow l = \frac{6400}{7}$ mt.

$= 914.29$ mt

$\therefore A = l \times b = \frac{6400}{7} \times 1 = 1$

(iii) Given for cones $d_1 = d_2 \Rightarrow r_1 = r_2$

$l_1 : l_2 = 5 : 4$

Ratio of Curved surface Area $= \pi r_1 l_1 : \pi r_2 l_2$

$= l_1 : l_2$

$= 5 : 4$

(iv) Case I Case II

$$(CSA)_1 = 2\,(ESA_2)$$

$$\Rightarrow \pi r_1 l_1 = 2 \cdot \pi r_2 l_2$$

$$\Rightarrow r_1 l_1 = 2 r_2 l_2$$

$$\Rightarrow r_1 l_1 = 2 r_2 (2 l_1) \qquad \text{Since } l_2 = 2 l_1$$

$$\Rightarrow r_1 = 4 r_2$$

$$\Rightarrow r_1 : r_2 = 4 : 1$$

(iv) Case I Case II

$$(CSA)_1 = 2\,(CSA_2)$$

$$\Rightarrow \pi r_1 l_1 = 2 \cdot \pi r_2 l_2$$

$$\Rightarrow r_1 l_1 = 2 r_2 l_2$$

$$\Rightarrow r_1 l_1 = 2 r_2 (2 l_1) \quad \text{Since } l_2 = 2 l_1$$

$$\Rightarrow r_1 = 4 r_2$$

$$\Rightarrow r_1 : r_2 = 4 : 1$$

LEARNERS TASK

CUQ's

01.	Conceptual	Ans: D
02	Conceptual	Ans: C
03	Conceptual	Ans: D
04	Conceptual	Ans: B
05	Conceptual	Ans: A
06	Conceptual	Ans: D
07.	Conceptual	Ans: D
08	Conceptual	Ans: D
09.	Conceptual	Ans: D
10.	Conceptual	Ans: B

JEE MAINS LEVEL

01 $d = 7$ mt, $r = \frac{7}{2}$ mt

$S.A = 4\pi r^2$

$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 154$ sq.mt

Ans: A

02 $l : b : h = 2 : 3 : 4$

$\Rightarrow l = 2x,\ b = 3x,\ h = 4x$

$\therefore C.S.A = 2(lb + bh + hl)$

$= 2(2x \cdot 3x + 3x \cdot 4x + 4x \cdot 2x)$

$= 52x^2$ sq.mt

Total cost = Area × cost per 1 sq.mt

$= 52x^2 \times (4.50) - 52x^2 \cdot (4)$

$= 26x^2 = 416$

$\Rightarrow x = 4.$

$\therefore$ The dimensions are 8, 12, 16

Ans: C

03 $2\pi r = 17.6$

$\Rightarrow 2 \times \frac{22}{7} \times r = 17.6$

$\Rightarrow r = 2.8$ mt

$C.S.A = 2\pi r^2$

$= 2 \times \frac{22}{7} \times (2.8)^2$

$= 49.28.$

Cost of painting → per 100 cm^2 → ₹ 5.

Now C.S.A = 49.28 mt^2

$= 49.28 \times 100 \times 100\ cm^2$

$= 492800\ cm^2$

$\therefore$ cost of painting $= \frac{5}{100} \times 492800$

$=$ ₹ 24,640.

Ans: D

04 original S.A $= 6a^2$

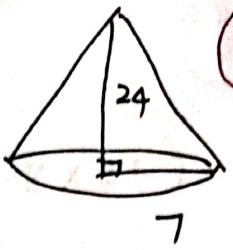
New S.A $= 6(2a)^2 = 24a^2$

% increa $= \frac{\text{New} - \text{original}}{\text{original}} \times 100$

$= \frac{18a^2}{6a^2} \times 100 = 300\ \%$

Ans: D

05 **for cone**



$r = 7, \quad h = 24$

$l^2 = h^2 + r^2$

$= 24^2 + 7^2$

$\therefore l^2 = 625 \Rightarrow l = 25$

C.S.A $= \pi r l$

$= \frac{22}{7} \times 7 \times 25 = 550 \text{ mt}^2$

Now Area $= l \times b$

$= l \times 5$

$\therefore 5l = 550$

$\Rightarrow l = 110$ mt

Ans: B

06 $l = 20, \quad b = 16, \quad h = 12$

Length of the rod $= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{20^2 + 16^2 + 12^2}$

$= \sqrt{400 + 256 + 144}$

$= \sqrt{800}$

$= 10\sqrt{8} = 10 \times 2\sqrt{2}$

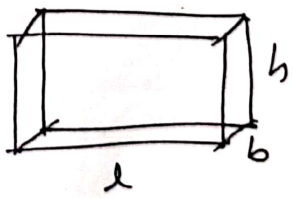
$= 20 \times 1.414$

$= 28.28$

Ans: D

07. diagonal of floor $= \sqrt{l^2 + b^2}$

$\therefore \sqrt{l^2 + b^2} = 10 \Rightarrow l^2 + b^2 = 100 \rightarrow (1)$



diagonal $= \sqrt{l^2 + b^2 + h^2} = 10\sqrt{2}$

$\Rightarrow l^2 + b^2 + h^2 = 200$

$\Rightarrow 100 + h^2 = 200$

$\Rightarrow h = 10$ mt

Ans: D

08 $l = 25cm,\ b = 15cm,\ h = 8cm$ (15)

$T\text{-}S.A = 2(lb + bh + hl)$

$= 2(25 \times 15 + 15 \times 8 + 8 \times 25)$

$= 1390\ cm^2$ Ans: C

09 Let base = ~~a~~radius = r

height = $h = 2r$

10 $r = 14cm$

$C.S.A = 2\pi r^2$

$= 2 \times \frac{22}{7} \times 14 \times 14$

$= 1232\ sq.mt$

Total cost = Rs 1232×3

= Rs 3696 Ans: D

11. $r = 14m,\quad d = 7cm$

$\therefore R = r + d = 14 + 7 = 21cm.$

$R = r + d$

T.S.A. of hollow hemisphere

$= 2\pi r^2 + 2\pi R^2 + \pi R^2 - \pi r^2$

$= \pi r^2 + 3\pi R^2$

$= \frac{22}{7} \times 14^2 + 3 \times \frac{22}{7} \times (21)^2$

$= 4774$ Ans: A

12. $l+b+h = 5\sqrt{3}$ (16)

$\sqrt{l^2+b^2+h^2} = 3\sqrt{5}$

$\Rightarrow l^2+b^2+h^2 = 45$

$(l+b+h)^2 - 2(lb+bh+hl) = 45$

$\Rightarrow lb+bh+hl = 15$

$\therefore$ T.S.A $= 2(lb+bh+hl)$

$= 2 \times 15$

$= 30$ sq. cm

Ans A

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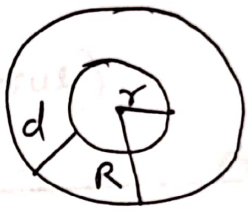
13 Statement I: $r = 15m$

$d = 7m$



$\therefore R = r+d$

$= 15+7$

$= 22$ mt

Area of path $= \pi R^2 - \pi r^2$

$= \frac{22}{7} \times 22^2 - \frac{22}{7} \times 15^2$

$= \frac{22}{7}(22+15)(22-15)$

$= \frac{22}{7} \times 37 \times 7$

$= \frac{814}{7}$ .814

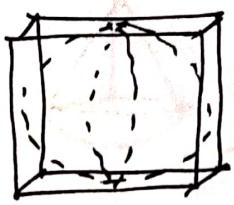
Total cost $= \frac{814}{10} \times 10$

$= Rs.\ 8140$ (True)

Statement II: Conceptual (True)

Ans: A

14 **Statement I:**



$r = 3.5 = \frac{7}{2}$ cm

C.S.A of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$.

C.S.A of cube $= 6a^2$

$= 6 \times (7)^2$ since $a = 2r$

$= 2 \times \frac{7}{2}$

$= 7$

Difference in S.A $= 6 \times 7^2 - 4 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

$= 140$ cm^2

Statement II: Conceptual (True)

Ans. A

15 $l = 3$mt, $b = 2$mt, $h = 1$mt

T.S.A $= 2(lb + bh + hl)$

$= 2(3 \times 2 + 2 \times 1 + 1 \times 3)$

$= 22$ mt^2

Total cost = Rs 22×10

= Rs 220.

Ans. B

16 S.A. of Remaing box = T.S.A − Area of side removed

$= 2(lb + bh + hl) - l \times b$

$= lb + 2bh + 2hl$

$= 3 \times 2 + 2 \times 2 \times 1 + 2 \times 1 \times 3$

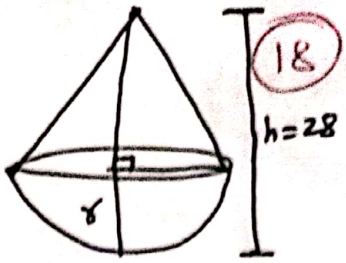
$= 6 + 4 + 6 =$

$= 22 - 2 \times 1$

$= 20$

Ans. C

17. ~~for Cone~~ hemisphere



$r = 14$ mt

for cone

$r = 14$ mt, $h = 28 - 14$
$= 14$ mt

~~Or.~~ $l^2 = h^2 + r^2$
$= 14^2 + 14^2$
$= 2 \times 14^2$

$\therefore l = \sqrt{2 \times 14^2}$
$= 14\sqrt{2}$

Area of canvas required = ~~Area~~ C.S.A.

of cone $= \pi r l$
$= \frac{22}{7} \times 14 \times 14\sqrt{2}$
$= 616\sqrt{2}$.

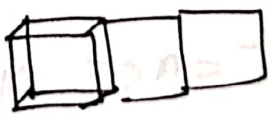
Ans: D

18. Area of metal sheet required
$=$ C.S.A. of hemisphere
$= 2\pi r^2$
$= 2 \times \frac{22}{7} \times 14^2$
$= 1232$

Ans. D

19. for cube

$a = 4$



for cuboid

$l = 12$, $b = 4$, $h = 4$

T.S.A $= 2(lb + bh + hl)$
$= 2(12 \times 4 + 4 \times 4 + 12 \times 4) = 224$.

T.S.A of 3 cubes $= 3 \times 6a^2 = 18 \times 4^2$
$= 288$

$\therefore$ Ratio $=$ ~~288 : 224~~ $= 224 : 288 = 7 : 9$

Ans: 7

20. for cylinder

$h = 8$ cm

C.S.A $= 2\pi rh = 704$

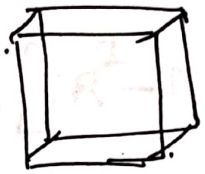
$2 \times \frac{22}{7} \times r \times 8 = 704$

$r = 14$

Sum of the digits $= 1 + 4 = 5$

Ans: 5

21 i) Let the side of cube be a



Now, diagonal $= \sqrt{3}\,a$

Side of the square $= s = \sqrt{3}\,a$

Area $= s^2 = (\sqrt{3}a)^2$

$= 3a^2 = 75$

$\Rightarrow a = 5$

(ii) $\pi r l = 154$

$\Rightarrow \frac{22}{7} \times r \times 7 = 154$

$\Rightarrow r = 7$

(iii) C.S.A of cylinder $= 968$

$\Rightarrow 2\pi rh = 968$

$\Rightarrow 2 \times \frac{22}{7} \times r \times 11 = 968$

$\Rightarrow r = 14$ $\quad \therefore 2r = 14 \Rightarrow r = 7$

(iv) Area of 4 walls $=$ L.S.A $= 2h(l + b) = 350$

New area $= 2 \times 3h[3l + 3b]$

$= 9 \times 2h(l + b)$

$= 9 \times 350$

$= ₹\,3150$

Units digit $= 0$ Ans: q, r, s, p

22 (i) $2\pi rh = 88$ (20)

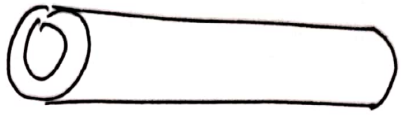
$\Rightarrow 2 \times \frac{22}{7} \times r \times 14 = 88$

$\Rightarrow r = 1 \quad \therefore$ diameter $= 2$

(ii) $h = 77$ cm

$d = 4$ cm | $D = 4.4$ cm

$\therefore r = 2$ cm | $R = 2.2$ cm.



Total S.A $= 2\pi Rh + 2\pi rh + 2\left[\pi R^2 - \pi r^2\right]$

$= 2 \times \frac{22}{7} \times 2.2 \times 77 + 2 \times \frac{22}{7} \times 2 \times 77 +$

$2\left[\frac{22}{7} \times (2.2)^2 - \frac{22}{7} \times (2)^2\right]$

$= 2038.08 \text{ cm}^2$

(iii) $d = 84 \Rightarrow r = 42$ cm

$h = 120$ cm

C.S.A $= 2 \cdot \pi r h$

$= 2 \cdot \frac{22}{7} \times 42 \times 120$

Total area covered for 500 complete revolu

$= 2 \cdot \frac{22}{7} \times 42 \times 120 \times 500$

$= 15840000$ sq.cm $= 1584$ sq.mt

(iv) $6a^2 = \frac{50}{3}$

$\Rightarrow a^2 = \frac{50}{3} \times \frac{1}{6} = \frac{25}{9} \quad \therefore a = \frac{5}{3}$

$\Rightarrow$ THE END $\in$ Ans: t, s, r, p