SOLVED EXAMPLES

Ex. 1 If α and β are the roots of $ax^2 + bx + c = 0$, find the value of $(a\alpha + b)^{-2} + (a\beta + b)^{-2}$.

Sol. We know that $\alpha + \beta = -\frac{b}{a}$ & $\alpha\beta = \frac{c}{a}$

$$(a\alpha + b)^{-2} + (a\beta + b)^{-2} = \frac{1}{(a\alpha + b)^2} + \frac{1}{(a\beta + b)^2}$$

$$= \frac{a^2\beta^2 + b^2 + 2ab\beta + a^2\alpha^2 + b^2 + 2ab\alpha}{(a^2\alpha\beta + ba\beta + ba\alpha + b^2)^2} = \frac{a^2(\alpha^2 + \beta^2) + 2ab(\alpha + \beta) + 2b^2}{(a^2\alpha\beta + ab(\alpha + \beta) + b^2)^2}$$

($\alpha^2 + \beta^2$ can always be written as $(\alpha + \beta)^2 - 2\alpha\beta$)

$$= \frac{a^2\left[(\alpha+\beta)^2 - 2\alpha\beta\right] + 2ab(\alpha + \beta) + 2b^2}{(a^2\alpha\beta + ab(\alpha + \beta) + b^2)^2} = \frac{a^2\left[\frac{b^2 - 2ac}{a^2}\right] + 2ab\left(-\frac{b}{a}\right) + 2b^2}{\left(a^2\frac{c}{a} + ab\left(-\frac{b}{a}\right) + b^2\right)^2} = \frac{b^2 - 2ac}{a^2c^2}$$

Ex. 2 If the coefficient of the quadratic equation are rational & the coefficient of x^2 is 1, then find the equation one of whose roots is $\tan\frac{\pi}{8}$.

Sol. We know that $\tan\frac{\pi}{8} = \sqrt{2} - 1$

Irrational roots always occur in conjugational pairs.

Hence if one root is $(-1 + \sqrt{2})$, the other root will be $(-1 - \sqrt{2})$. Equation is

$(x-(-1+\sqrt{2}))(x-(-1-\sqrt{2}))=0 \quad \Rightarrow \quad x^2 + 2x - 1 = 0$

Ex. 3 If equation $\frac{x^2 - bx}{ax - c} = \frac{k-1}{k+1}$ has roots equal in magnitude & opposite in sign, then find the value of k

Sol. Let the roots are α & −α.

given equation is

$(x^2 - bx)(k + 1) = (k - 1)(ax - c)$ {Considering, $x \neq c/a$ & $k \neq -1$}

$\Rightarrow \quad x^2(k + 1) - bx(k + 1) = ax(k - 1) - c(k - 1)$

$\Rightarrow \quad x^2(k + 1) - bx(k + 1) - ax(k - 1) + c(k - 1) = 0$

Now sum of roots = 0 $\quad (\because \alpha - \alpha = 0)$

$\therefore \quad b(k + 1) + a(k - 1) = 0 \quad \Rightarrow \quad k = \frac{a-b}{a+b}$

Ex. 4 The coefficient of x in the quadratic equation $x^2 + px + q = 0$ was taken as 17 in place of 13, its roots were found to be − 2 and − 15. Find the roots of the original equation.

Sol. Here $q = (-2) \times (-15) = 30$, correct value of $p = 13$. Hence original equation is

$x^2 + 13x + 30 = 0$ as $(x + 10)(x + 3) = 0$

$\therefore$ roots are −10, −3

Ex. 5 Find all the integral values of a for which the quadratic equation $(x - a)(x - 10) + 1 = 0$ has integral roots.

Sol. Here the equation is $x^2 - (a + 10)x + 10a + 1 = 0$. Since integral roots will always be rational it means D should be a perfect square.

From (i) $D = a^2 - 20a + 96$.

$\Rightarrow \quad D = (a - 10)^2 - 4 \qquad \Rightarrow \quad 4 = (a - 10)^2 - D$

If D is a perfect square it means we want difference of two perfect square as 4 which is possible only when $(a - 10)^2 = 4$ and $D = 0$.

$\Rightarrow \quad (a - 10) = \pm 2 \qquad \Rightarrow \quad a = 12, 8$

Ex. 6 If the equation $(\lambda^2 - 5\lambda + 6)x^2 + (\lambda^2 - 3\lambda + 2)x + (\lambda^2 - 4) = 0$ has more than two roots, then find the value of λ ?

Sol. As the equation has more than two roots so it becomes an identity. Hence

$\lambda^2 - 5\lambda + 6 = 0 \qquad \Rightarrow \quad \lambda = 2, 3$

and $\quad \lambda^2 - 3\lambda + 2 = 0 \quad \Rightarrow \quad \lambda = 1, 2$

and $\quad \lambda^2 - 4 = 0 \quad \Rightarrow \quad \lambda = 2, -2$

So $\quad \lambda = 2$

Ex. 7 The equations $5x^2 + 12x + 13 = 0$ and $ax^2 + bx + c = 0$ $(a,b,c \in R)$ have a common root, where a, b, c are the sides of the ΔABC. Then find $\angle C$.

Sol. As we can see discriminant of the equation $5x^2 + 12x + 13 = 0$ is negative so roots of the equation are imaginary. We know that imaginary roots always occurs in pair. So this equation can not have single common roots with any other equation having real coefficients. So both roots are common of the given equations.

Hence $\dfrac{a}{5} = \dfrac{b}{12} = \dfrac{c}{13} = \lambda$ (let)

then $\quad a = 5\lambda, b = 12\lambda, c = 13\lambda$

Now $\quad \cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{25\lambda^2 + 144\lambda^2 - 169\lambda^2}{2(5\lambda)(12\lambda)} = 0$

$\therefore \quad \angle C = 90°$

Ex. 8 If the roots of the equation $(x - a)(x - b) - k = 0$ be c and d, then prove that the roots of the equation $(x - c)(x - d) + k = 0$, are a and b.

Sol. By given condition

$(x - a)(x - b) - k \equiv (x - c)(x - d)$

or $\quad (x - c)(x - d) + k \equiv (x - a)(x - b)$

Above shows that the roots of $(x - c)(x - d) + k = 0$ are a and b.

Ex. 9 Show that the expression $x^2 + 2(a + b + c)x + 3(bc + ca + ab)$ will be a perfect square if $a = b = c$.

Sol. Given quadratic expression will be a perfect square if the discriminant of its corresponding equation is zero.

i.e. $\quad 4(a + b + c)^2 - 4.3\,(bc + ca + ab) = 0$

or $\quad (a + b + c)^2 - 3(bc + ca + ab) = 0$

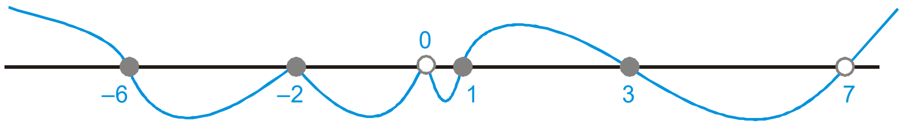
or $\quad \dfrac{1}{2}((a - b)^2 + (b - c)^2 + (c - a)^2) = 0$

which is possible only when $a = b = c$.

Ex. 10 Let $f(x)=\dfrac{(x-1)^3(x+2)^4(x-3)^5(x+6)}{x^2(x-7)^3}$. Solve the following inequality

(i) $f(x)>0$ **(ii)** $f(x)\ge 0$ **(iii)** $f(x)<0$ **(iv)** $f(x)\le 0$

Sol. We mark on the number line zeros of the function : 1, –2, 3 and –6 (with black circles) and the points of discontinuity 0 and 7 (with white circles), isolate the double points : –2 and 0 and draw the wavy curve :



From graph, we get

(i) $x\in(-\infty,-6)\cup(1,3)\cup(7,\infty)$ **(ii)** $x\in(-\infty,-6]\cup\{-2\}\cup[1,3]\cup(7,\infty)$

(iii) $x\in(-6,-2)\cup(-2,0)\cup(0,1)\cup(3,7)$ **(iv)** $x\in[-6,0)\cup(0,1]\cup[3,7)$

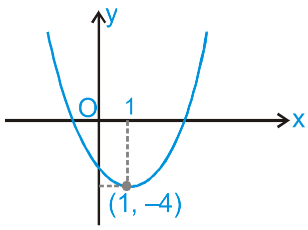
Ex. 11 If $y=x^2-2x-3$, then find the range of y when :

(i) $x\in R$ **(ii)** $x\in[0,3]$ **(iii)** $x\in[-2,0]$

Sol. We know that minimum value of y will occur at

$$x=-\frac{b}{2a}=-\frac{(-2)}{2\times 1}=1$$

$$y_{min}=-\frac{D}{4a}=\frac{-(4+3\times 4)}{4}=-4$$



(i) $x\in R$;

$y\in[-4,\infty)$ **Ans.**

(ii) $x\in[0,3]$

$f(0)=-3$, $f(1)=-4$, $f(3)=0$

$\because$ $f(3)>f(0)$

$\therefore$ y will take all the values from minimum to f(3).

$y\in[-4,0]$ **Ans.**

(iii) $x\in[-2,0]$

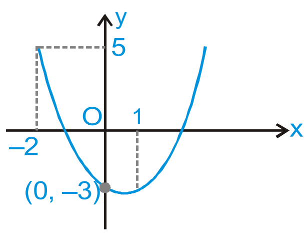
This interval does not contain the minimum value of y for $x\in R$.

y will take values from f(0) to f(–2)

$f(0)=-3$

$f(-2)=5$

$y\in[-3,5]$ **Ans.**



Ex. 12 If $ax^2+bx+10=0$ does not have real & distinct roots, find the minimum value of 5a–b.

Sol. Either $f(x)\ge 0\ \forall\ x\in R$ or $f(x)\le 0\ \forall\ x\in R$

$\because$ $f(0)=10>0\Rightarrow f(x)\ge 0\ \forall\ x\in R$

$\Rightarrow$ $f(-5)=25a-5b+10\ge 0$

$\Rightarrow$ $5a-b\ge -2$ **Ans.**

Ex. 13 Find the maximum and minimum values of $f(x) = x^2 - 5x + 6$.

Sol. minimum of f(x) $= -\frac{D}{4a}$ at $x = -\frac{b}{2a} = -\left(\frac{25-24}{4}\right)$ at $x = \frac{5}{2} = -\frac{1}{4}$

maximum of $f(x) \to \infty$

Hence range is $\left[-\frac{1}{4}, \infty\right)$.

Ex. 14 Find the values of the parameter 'a' for which the roots of the quadratic equation $x^2 + 2(a-1)x + a + 5 = 0$ are

(i) real and distinct **(ii)** equal **(iii)** opposite in sign

(iv) equal in magnitude but opposite in sign **(v)** positive

(vi) negative **(vii)** greater than 3 **(viii)** smaller than 3

(ix) such that both the roots lie in the interval (1, 3)

Sol. Let $f(x) = x^2 + 2(a-1)x + a + 5 = Ax^2 + Bx + C$ (say)

$\Rightarrow$ $A = 1, B = 2(a-1), C = a + 5$.

Also $D = B^2 - 4AC = 4(a-1)^2 - 4(a+5) = 4(a+1)(a-4)$

(i) $D > 0$

$\Rightarrow$ $(a+1)(a-4) > 0$ $\Rightarrow$ $a \in (-\infty, -1) \cup (4, \infty)$.

(ii) $D = 0$

$\Rightarrow$ $(a+1)(a-4) = 0$ $\Rightarrow$ $a = -1, 4$.

(iii) This means that 0 lies between the roots of the given equation.

$\Rightarrow$ $f(0) < 0$ and $D > 0$ i.e. $a \in (-\infty, -1) \cup (4, \infty)$

$\Rightarrow$ $a + 5 < 0$ $\Rightarrow$ $a < -5$ $\Rightarrow$ $a \in (-\infty, -5)$.

(iv) This means that the sum of the roots is zero

$\Rightarrow$ $-2(a-1) = 0$ and $D > 0$ i.e. $a \in -(-\infty, -1) \cup (4, \infty)$ $\Rightarrow$ $a = 1$

which does not belong to $(-\infty, -1) \cup (4, \infty)$

$\Rightarrow$ $a \in \phi$.

(v) This implies that both the roots are greater than zero

$\Rightarrow$ $-\frac{B}{A} > 0, \frac{C}{A} > 0, D \geq 0.$ $\Rightarrow$ $-(a-1) > 0, a + 5 > 0, a \in (-\infty, -1] \cup [4, \infty)$

$\Rightarrow$ $a < 1, -5 < a, a \in (-\infty, -1] \cup [4, \infty)$ $\Rightarrow$ $a \in (-5, -1]$.

(vi) This implies that both the roots are less than zero

$\Rightarrow$ $-\frac{B}{A} < 0, \frac{C}{A} > 0, D \geq 0.$ $\Rightarrow$ $-(a-1) < 0, a + 5 > 0, a \in (-\infty, -1] \cup [4, \infty)$

$\Rightarrow$ $a > 1, a > -5, a \in (-\infty, -1] \cup [4, \infty)$ $\Rightarrow$ $a \in [4, \infty)$.

(vii) In this case

$-\frac{B}{2a} > 3$, $A.f(3) > 0$ and $D \geq 0$.

$\Rightarrow$ $-(a-1) > 3, 7a + 8 > 0$ and $a \in (-\infty, -1] \cup [4, \infty)$

$\Rightarrow$ $a < -2, a > -8/7$ and $a \in (-\infty, -1] \cup [4, \infty)$

Since no value of 'a' can satisfy these conditions simultaneously, there can be no value of a for which both the roots will be greater than 3.

(viii) In this case

$-\frac{B}{2a} < 3$, A.f(3) > 0 and D ≥ 0.

$\Rightarrow$ $a > -2, a > -8/7$ and $a \in (-\infty, -1] \cup [4, \infty)$ $\Rightarrow$ $a \in (-8/7, -1] \cup [4, \infty)$

(ix) In this case

$1 < -\frac{B}{2A} < 3, A.f(1) > 0, A.f(3) > 0, D \geq 0.$

$\Rightarrow$ $1 < -1(a-1) < 3, 3a + 4 > 0, 7a + 8 > 0, a \in (-\infty, -1] \cup [4, \infty)$

$\Rightarrow$ $-2 < a < 0, a > -4/3, a > -8/7, a \in (-\infty, -1] \cup [4, \infty)$ $\Rightarrow$ $a \in \left(-\frac{8}{7}, -1\right]$

Ex. 15 If α is a root of the equation $ax^2 + bx + c = 0$ and β is a root of the equation $-ax^2 + bx + c = 0$, then prove that there will be a root of the equation $\frac{a}{2}x^2 + bx + c = 0$ lying between α and β.

Sol. Let $f(x) = \frac{a}{2}x^2 + bx + c$

$f(\alpha) = \frac{a}{2}\alpha^2 + b\alpha + c = a\alpha^2 + b\alpha + c - \frac{a}{2}\alpha^2$ $\Rightarrow$ $-\frac{a}{2}\alpha^2$ (As α is a root of $ax^2 + bx + c = 0$)

And $f(\beta) = \frac{a}{2}\beta^2 + b\beta + c = -a\beta^2 + b\beta + c + \frac{3a}{2}\beta^2$ $\Rightarrow$ $\frac{3a}{2}\beta^2$ (As β is a root of $-ax^2 + bx + c = 0$)

Now $f(\alpha) . f(\beta) = \frac{-3}{4}a^2\alpha^2\beta^2 < 0$

$\Rightarrow$ f(x) = 0 has one real root between α and β.

Ex. 16 Solve for x : **(a)** $||x-1|+2| \leq 4.$ **(b)** $\frac{x-4}{x+2} \leq \left|\frac{x-2}{x-1}\right|$

Sol. **(a)** $||x-1|+2| \leq 4$ $\Rightarrow$ $-4 \leq |x-1| + 2 \leq 4$

$\Rightarrow$ $-6 \leq |x-1| \leq 2$

$\Rightarrow$ $|x-1| \leq 2$ $\Rightarrow$ $-2 \leq x-1 \leq 2$

$\Rightarrow$ $-1 \leq x \leq 3$ $\Rightarrow$ $x \in [-1, 3]$

(b) **Case 1 :** Given inequation will be statisfied for all x such that

$\frac{x-4}{x+2} \leq 0$ $\Rightarrow$ $x \in (-2, 4] - \{1\}$ .....**(i)**

(Note : {1} is not in domain of RHS)

Case 2 : $\frac{x-4}{x+2} > 0$ $\Rightarrow$ $x \in (-\infty, -2) \cup (4, \infty)$ .....**(ii)**

Given inequation becomes

$\frac{x-2}{x-1} \geq \frac{x-4}{x+2}$ or $\frac{x-2}{x-1} \leq -\frac{x-4}{x+2}$

on solving we get | on solving we get

$x \in (-2, 4/5) \cup (1, \infty)$ | $x \in (-2, 0] \cup (1, 5/2]$

taking intersection with **(ii)** we get | taking intersection with **(ii)** we get

$x \in (4, \infty)$ .....**(iii)** | $x \in \phi$

Hence, solution of the original inequation : $x \in (-2,\infty) - \{1\}$ (taking union of **(i)** **(iii)**)

Ex. 17 Solve for x : **(a)** $\log_{0.5}(x^2-5x+6)\geq -1$ **(b)** $\log_{1/3}(\log_4(x^2-5))>0$

Sol. **(a)** $\log_{0.5}(x^2-5x+6)\geq -1 \Rightarrow 0<x^2-5x+6\leq (0.5)^{-1}$

$\Rightarrow 0<x^2-5x+6\leq 2$

$$\begin{cases} x^2-5x+6>0 \\ x^2-5x+6\leq 2 \end{cases} \Rightarrow x\in[1,2)\cup(3,4]$$

Hence, solution set of original inequation : $x\in[1,2)\cup(3,4]$

(b) $\log_{1/3}(\log_4(x^2-5))>0 \Rightarrow 0<\log_4(x^2-5)<1$

$$\begin{cases} 0<\log_4(x^2-5) \Rightarrow x^2-5>1 \\ \log_4(x^2-5)<1 \Rightarrow 0<x^2-5<4 \end{cases} \Rightarrow 1<(x^2-5)<4$$

$\Rightarrow 6<x^2<9 \Rightarrow x\in\left(-3,-\sqrt{6}\right)\cup\left(\sqrt{6},3\right)$

Hence, solution set of original inequation : $x\in\left(-3,-\sqrt{6}\right)\cup\left(\sqrt{6},3\right)$

Ex. 18 If a, b, c ∈ R and equations $ax^2+bx+c=0$ and $x^2+2x+9=0$ have a common root, show that $a:b:c=1:2:9$.

Sol. Given equations are : $x^2+2x+9=0$ .....**(i)**

and $ax^2+bx+c=0$ .....**(ii)**

Clearly roots of equation **(i)** are imaginary since equation **(i)** and **(ii)** have a common root, therefore common root must be imaginary and hence both roots will be common. Therefore equations **(i)** and **(ii)** are identical

$\therefore \frac{a}{1}=\frac{b}{2}=\frac{c}{9}$ $\therefore a:b:c=1:2:9$

Ex. 19 Solve $\frac{x^2+6x-7}{x^2+1}\leq 2$

Sol. $\Rightarrow x^2+6x-7\leq 2x^2+2 \Rightarrow x^2-6x+9\geq 0$

$\Rightarrow (x-3)^2\geq 0 \Rightarrow x\in R$

Ex. 20 A polynomial in x of degree greater than three, leaves remainders 2, 1 and –1 when divided, respectively, by (x – 1), (x + 2) and (x + 1). What will be the remainder when it is divided by (x – 1) (x + 2) (x + 1).

Sol. Let required polynomial be $f(x)=p(x)(x-1)(x+2)(x+1)+a_0x^2+a_1x+a_2$

By remainder theorem, f(1) = 2, f(–2) = 1, f(–1) = –1.

$\Rightarrow a_0+a_1+a_2=2$

$4a_0-2a_1+a_2=1$

$a_0-a_1+a_2=-1$

Solving we get, $a_0=\frac{7}{6}$, $a_1=\frac{3}{2}$, $a_2=\frac{2}{3}$

Remainder when f(x) is divided by (x – 1) (x + 2) (x + 1)

will be $\frac{7}{6}x^2+\frac{3}{2}x+\frac{2}{3}$.

Ex. 21 If $(y^2-5y+3)(x^2+x+1)<2x$ for all $x \in R$, then find the interval in which y lies.

Sol. $(y^2-5y+3)(x^2+x+1)<2x,\ \forall\ x \in R$

$$\Rightarrow \quad y^2-5y+3<\frac{2x}{x^2+x+1}$$

Let $\dfrac{2x}{x^2+x+1}=P$

$$\Rightarrow \quad px^2+(p-2)x+p=0$$

(1) Since x is real, $(p-2)^2-4p^2 \geq 0$

$$\Rightarrow \quad -2 \leq p \leq \frac{2}{3}$$

(2) The minimum value of $2x/(x^2+x+1)$ is -2. So,

$y^2-5y+3<-2 \quad \Rightarrow \quad y^2-5y+5<0$

$$\Rightarrow \quad y \in \left(\frac{5-\sqrt{5}}{2},\frac{5+\sqrt{5}}{2}\right)$$

Ex. 22 If $2x^3+3x^2+5x+6=0$ has roots α, β, γ then find $\alpha+\beta+\gamma$, $\alpha\beta+\beta\gamma+\gamma\alpha$ and $\alpha\beta\gamma$.

Sol. Using relation between roots and coefficients, we get

$$\alpha+\beta+\gamma==-\frac{3}{2}, \qquad \alpha\beta+\beta\gamma+\gamma\alpha=\frac{5}{2}, \qquad \alpha\beta\gamma=-\frac{6}{2}=-3.$$

Ex. 23 Let $x^2-(m-3)x+m=0\ (m \in R)$ be a quadratic equation, then find the values of 'm' for which

(a) both the roots are greater than 2.

(b) both roots are positive.

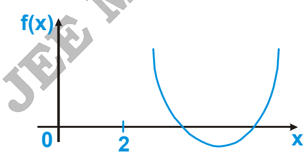
(c) one root is positive and other is negative.

(d) One root is greater than 2 and other smaller than 1

(e) Roots are equal in magnitude and opposite in sign.

(f) both roots lie in the interval (1, 2)

Sol. **(a)**



Condition - I : $D \geq 0 \Rightarrow (m-3)^2-4m \geq 0 \Rightarrow m^2-10m+9 \geq 0$

$\Rightarrow (m-1)(m-9) \geq 0$

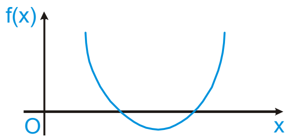
$\Rightarrow m \in (-\infty, 1] \cup [9, \infty)$ .....**(i)**

Condition - II : $f(2)>0 \Rightarrow 4-(m-3)2+m>0 \Rightarrow m<10$ .....**(ii)**

Condition - III : $-\dfrac{b}{2a}>2 \Rightarrow \dfrac{m-3}{2}>2 \Rightarrow m>7$ .....**(iii)**

Intersection of **(i)**, **(ii)** and **(iii)** gives $m \in [9, 10)$

(b)



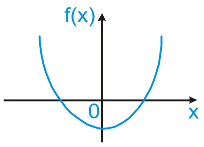
Condition - I $D \geq 0$ $\Rightarrow$ $m \in (-\infty, 1] \cup [9, \infty)$

Condition - II $f(0) > 0$ $\Rightarrow$ $m > 0$

Condition - III $-\frac{b}{2a} > 0$ $\Rightarrow$ $\frac{m-3}{2} > 0$ $\Rightarrow$ $m > 3$

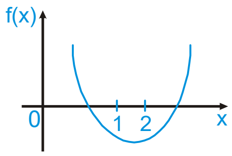
intersection gives $m \in [9, \infty)$ **Ans.**

(c)



Condition - I $f(0) < 0$ $\Rightarrow$ $m < 0$ **Ans.**

(d)



Condition - I $f(1) < 0$ $\Rightarrow$ $4 < 0$ $\Rightarrow$ $m \in \phi$

Condition - II $f(2) < 0$ $\Rightarrow$ $m > 10$

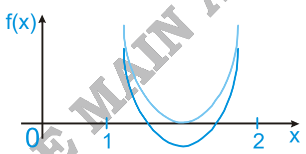
Intersection gives $m \in \phi$ **Ans.**

(e) sum of roots = 0 $\Rightarrow$ $m = 3$

and $f(0) < 0$ $\Rightarrow$ $m < 0$

$\therefore$ $m \in \phi$ **Ans.**

(f)



Condition - I $D \geq 0$ $\Rightarrow$ $m \in (-\infty, 1] \cup [9, \infty)$

Condition - II $f(1) > 0$ $\Rightarrow$ $1 - (m-3) + m > 0$ $\Rightarrow$ $4 > 0$ which is true $\forall\, m \in R$

Condition - III $f(2) > 0$ $\Rightarrow$ $m < 10$

Condition - IV $1 < -\frac{b}{2a} < 2$ $\Rightarrow$ $1 < \frac{m-3}{2} < 2$ $\Rightarrow$ $5 < m < 7$

intersection gives $m \in \phi$ **Ans.**

Ex. 24 Solve the inequation : $\log_{2x+3} x^2 < \log_{2x+3}(2x+3)$

Sol. This inequation is equivalent to the collection of the systems

$$\left[\begin{array}{l}\begin{cases}2x+3>1\\0<x^2<2x+3\end{cases}\\ \text{or}\\ \begin{cases}0<2x+3<1\\x^2>2x+3>0\end{cases}\end{array}\right. \Rightarrow \left[\begin{array}{l}\begin{cases}x>-1\\(x-3)(x+1)<0 \ \ \& \ \ x\neq 0\end{cases}\\ \text{or}\\ \begin{cases}-\dfrac{3}{2}<x<-1\\(x-3)(x+1)>0\end{cases}\end{array}\right. \Rightarrow \left[\begin{array}{l}\begin{cases}x>-1\\-1<x<3\end{cases} \Rightarrow -1<x<3 \ \ \& \ \ x\neq 0\\ \text{or} \qquad\qquad\qquad \text{or}\\ \begin{cases}-\dfrac{3}{2}<x<-1\\x<-1 \text{ or } x>3\end{cases} \Rightarrow -\dfrac{3}{2}<x<-1\end{array}\right.$$

Hence, solution of the original inequation is $x \in \left(-\dfrac{3}{2}, -1\right)\cup(-1, 0)\cup(0, 3)$

Ex. 25 Solve for x : $(1.25)^{1-x} < (0.64)^{2(1+\sqrt{x})}$

Sol. We have $\left(\dfrac{5}{4}\right)^{1-x} < \left(\dfrac{16}{25}\right)^{2(1+\sqrt{x})}$ or $\left(\dfrac{4}{5}\right)^{x-1} < \left(\dfrac{4}{5}\right)^{4(1+\sqrt{x})}$

Since the base $0<\dfrac{4}{5}<1$, the inequality is equivalent to the inequality $x-1>4(1+\sqrt{x})$

$\Rightarrow \quad \dfrac{x-5}{4} > \sqrt{x}$

Now, R.H.S. is positive

$\Rightarrow \quad \dfrac{x-5}{4} > 0 \quad \Rightarrow \quad x>5$ .....(i)

we have $\dfrac{x-5}{4} > \sqrt{x}$ both sides are positive, so squaring both sides

$\Rightarrow \quad \dfrac{(x-5)^2}{16} > x$ or $\dfrac{(x-5)^2}{16} - x > 0$

or $\quad x^2-26x+25>0$ or $(x-25)(x-1)>0$

$\Rightarrow \quad x \in (-\infty, 1)\cup(25, \infty)$ .....(ii)

intersection (i) & (ii) gives $x\in(25,\infty)$

Ex. 26 Find the range of rational expression $y = \dfrac{x^2-x+1}{x^2+x+1}$ if x is real.

Sol. $y = \dfrac{x^2-x+1}{x^2+x+1} \Rightarrow \quad (y-1)x^2+(y+1)x+y-1=0$ .....(i)

Case-I if $y \neq 1$, then equation (i) is quadratic in x

and $\because$ x is real

$\therefore$ $D \geq 0$

$\Rightarrow$ $(y+1)^2-4(y-1)^2 \geq 0 \quad \Rightarrow \quad (y-3)(3y-1) \leq 0$

$\therefore$ $y \in \left[\dfrac{1}{3}, 3\right] - \{1\}$

Case-II if $y=1$, then equation becomes

$2x=0 \Rightarrow x=0$ which is possible as x is real.

$\therefore$ Range $\left[\dfrac{1}{3}, 3\right]$

Ex. 27 Find the values of a for which the expression $\frac{ax^2+3x-4}{3x-4x^2+a}$ assumes all real values for real values of x.

Sol. Let $y = \frac{ax^2+3x-4}{3x-4x^2+a}$

$x^2(a+4y)+3(1-y)x-(4+ay)=0$

If $x \in R, D \geq 0$

$\Rightarrow \quad 9(1-y)^2+4(a+4y)(4+ay) \geq 0 \quad \Rightarrow \quad (9+16a)y^2+(4a^2+46)y+(9+16a) \geq 0$

for all $y \in R$, $(9+16a)>0$ **&** $D \leq 0$

$\Rightarrow \quad (4a^2+46)^2-4(9+16a)(9+16a) \leq 0 \quad \Rightarrow \quad 4(a^2-8a+7)(a^2+8a+16) \leq 0$

$\Rightarrow \quad a^2-8a+7 \leq 0 \quad \Rightarrow \quad 1 \leq a \leq 7$

$9+16a>0$ **&** $1 \leq a \leq 7$

Taking intersection, $a \in [1, 7]$

Now, checking the boundary values of a

For $a = 1$

$$y = \frac{x^2+3x-4}{3x-4x^2+1} = -\frac{(x-1)(x+4)}{(x-1)(4x+1)}$$

$\because \quad x \neq 1 \Rightarrow y \neq -1$

$\Rightarrow \quad a = 1$ is not possible.

If $a = 7$

$$y = \frac{7x^2+3x-4}{3x-4x^2+7} = \frac{(7x-4)(x+1)}{(7-4x)(x+1)} \qquad \because x \neq -1 \Rightarrow y \neq -1$$

So y will assume all real values for some real values of x.

So $a \in (1,7)$

Ex. 28 If α, β are the roots of $x^2 + px + q = 0$, and γ, δ are the roots of $x^2 + rx + s = 0$, evaluate $(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)$ in terms of p, q, r and s. Deduce the condition that the equations have a common root.

Sol. α, β are the roots of $x^2 + px + q = 0$

$\therefore \quad \alpha + \beta = -p, \alpha\beta = q$ .....(i)

and γ, δ are the roots of $x^2 + rx + s = 0$

$\therefore \quad \gamma + \delta = -r, \gamma\delta = s$ .....(ii)

Now, $(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta)$

$= [\alpha^2 - \alpha(\gamma + \delta) + \gamma\delta][\beta^2 - \beta(\gamma + \delta) + \gamma\delta]$

$= (\alpha^2 + r\alpha + s)(\beta^2 + r\beta + s)$

$= \alpha^2\beta^2 + r\alpha\beta(\alpha + \beta) + r^2\alpha\beta + s(\alpha^2 + \beta^2) + sr(\alpha + \beta) + s^2$

$= \alpha^2\beta^2 + r\alpha\beta(\alpha + \beta) + r^2\alpha\beta + s((\alpha + \beta)^2 - 2\alpha\beta)) + sr(\alpha + \beta) + s^2$

$= q^2 - pqr + r^2q + s(p^2 - 2q) + sr(-p) + s^2$

$= (q - s)^2 - rpq + r^2q + sp^2 - prs$

$= (q - s)^2 - rq(p - r) + sp(p - r)$

$= (q - s)^2 + (p - r)(sp - rq)$

For a common root (Let $\alpha = \gamma$ or $\beta = \delta$) .....(iii)

then $(\alpha - \gamma)(\alpha - \delta)(\beta - \gamma)(\beta - \delta) = 0$ .....(iv)

from (iii) and (iv), we get

$(q - s)^2 + (p - r)(sp - rq) = 0$

$\Rightarrow \quad (q - s)^2 = (p - r)(rq - sp)$, which is the required condition.

Ex. 29 Solve $\left|\dfrac{x^2-3x-1}{x^2+x+1}\right| < 3.$

Sol. $\dfrac{|x^2-3x-1|}{x^2+x+1} < 3.$

$\because$ $(x^2+x+1) > 0, \forall x \in R$

$\therefore$ cross multiplication is valid

$\therefore$ $|x^2-3x-1| < 3(x^2+x+1)$

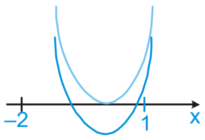
$\Rightarrow$ $(x^2-3x-1)^2 - \{3(x^2+x+1)\}^2 < 0$

$\Rightarrow$ $(4x^2+2)(-2x^2-6x-4) < 0$

$\Rightarrow$ $(2x^2+1)(x+2)(x+1) > 0$ $\Rightarrow$ $x \in (-\infty, -2) \cup (-1, \infty)$

Ex. 30 Find all the values of 'a' for which both the roots of the equation $(a-2)x^2 + 2ax + (a+3) = 0$ lies in the interval $(-2, 1)$.

Sol. **Case - I**



When $a-2>0$ $\Rightarrow$ $a>2$

Condition-I : $f(-2)>0$ $\Rightarrow$ $(a-2)4-4a+a+3>0$

$\Rightarrow$ $a-5>0$ $\Rightarrow$ $a>5$

Condition-II : $f(1)>0$ $\Rightarrow$ $4a+1>0$ $\Rightarrow$ $a > -\dfrac{1}{4}$

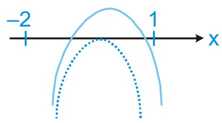
Condition-III : $D \ge 0$ $\Rightarrow$ $4a^2-4(a+3)(a-2) \ge 0$ $\Rightarrow$ $a \le 6$

Condition-IV : $-2 < -\dfrac{b}{2a} < 1$ $\Rightarrow$ $a \in (-\infty, 1) \cup (4, \infty)$

Intersection gives $a \in (5, 6]$. **Ans.**

Case-II when $a-2<0$

$a<2$



Condition - I $f(-2)<0$ $\Rightarrow$ $a<5$

Condition - II $f(1)<0,$ $\Rightarrow$ $a < -\dfrac{1}{4}$

Condition - III $-2 < -\dfrac{b}{2a} < 1$ $\Rightarrow$ $a \in (-\infty, 1) \cup (4, \infty)$

Condition - IV $D \ge 0$ $\Rightarrow$ $a \le 6$

intersection gives $a \in \left(-\infty, -\dfrac{1}{4}\right)$

complete solution is $a \in \left(-\infty, -\dfrac{1}{4}\right) \cup (5, 6]$ **Ans.**

Excercise-1

Questions with only one option correct

1. The roots of the quadratic equation $(a + b - 2c) x^2 - (2a - b - c) x + (a - 2b + c) = 0$ are -
(A) $a + b + c$ & $a - b + c$ (B) $1/2$ & $a - 2b + c$
(C) $a - 2b + c$ & $1/(a + b - 2c)$ (D) none of these

2. If one root of the quadratic equation $px^2 + qx + r = 0$ $(p \neq 0)$ is a surd $\frac{\sqrt{a}}{\sqrt{a}+\sqrt{a-b}}$, where p, q, r; a, b are all rationals then the other root is -
(A) $\frac{\sqrt{b}}{\sqrt{a}-\sqrt{a-b}}$ (B) $a + \frac{\sqrt{a(a-b)}}{b}$ (C) $\frac{a+\sqrt{a(a-b)}}{b}$ (D) $\frac{\sqrt{a}-\sqrt{a-b}}{\sqrt{b}}$

3. $ax^2 + bx + c = 0$ has real and distinct roots α and $\beta(\beta > \alpha)$. Further $a > 0$, $b < 0$ and $c < 0$, then -
(A) $0 < \beta < |\alpha|$ (B) $0 < |\alpha| < \beta$ (C) $\alpha + \beta < 0$ (D) $|\alpha| + |\beta| = \left|\frac{b}{a}\right|$

4. Number of values of 'p' for which the equation $(p^2 - 3p + 2) x^2 - (p^2 - 5p + 4) x + p - p^2 = 0$ possess more than two roots, is:
(A) 0 (B) 1 (C) 2 (D) none

5. The roots of the equation $(b - c) x^2 + (c - a) x + (a - b) = 0$ are
(A) $\frac{c-a}{b-c}, 1$ (B) $\frac{a-b}{b-c}, 1$ (C) $\frac{b-c}{a-b}, 1$ (D) $\frac{c-a}{a-b}, 1$

6. Let $p, q \in \{1, 2, 3, 4\}$. Then number of equation of the form $px^2 + qx + 1 = 0$, having real roots, is
(A) 15 (B) 9 (C) 7 (D) 8

7. If x, y are rational number such that $x + y + (x - 2y)\sqrt{2} = 2x - y + (x - y - 1)\sqrt{6}$, then
(A) x and y connot be determined (B) $x = 2, y = 1$
(C) $x = 5, y = 1$ (D) none of these

8. For the equation $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to:
(A) 1/3 (B) 1 (C) 3 (D) 2/3

9. If $\alpha \neq \beta$, $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the equation whose roots are α/β & β/α, is
(A) $x^2 + 5x - 3 = 0$ (B) $3x^2 + 12x + 3 = 0$ (C) $3x^2 - 19x + 3 = 0$ (D) none of these

10. If one solution of the equation $x^3 - 2x^2 + ax + 10 = 0$ is the additive inverse of another, then which one of the following inequalities is true ?
(A) $-40 < a < -30$ (B) $-30 < a < -20$ (C) $-20 < a < -10$ (D) $-10 < a < 0$

11. The quadratic equation $x^2 - 1088x + 295680 = 0$ has two positive integral roots whose greatest common divisor is 16. The least common multiple of the two roots is
(A) 18240 (B) 18480 (C) 18960 (D) 19240

12. The number of positive integral solutions of the inequation $\frac{x^2(3x-4)^3\ (x-2)^4}{(x-5)^5\ (2x-7)^6} \leq 0$ is -
(A) 2 (B) 0 (C) 3 (D) 4

13. If the roots of the quadratic equation $x^2 + 6x + b = 0$ are real and distinct and they differ by atmost 4 then the least

value of b is -

(A) 5 (B) 6 (C) 7 (D) 8

14. For every $x \in R$, the polynomial $x^8 - x^5 + x^2 - x + 1$ is :

(A) positive (B) never positive

(C) positive as well as negative (D) negative

15. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real & less than 3 then -

(A) $a < 2$ (B) $2 \le a \le 3$ (C) $3 < a \le 4$ (D) $a > 4$

16. The number of integral values of m, for which the roots of $x^2 - 2mx + m^2 - 1 = 0$ will lie between -2 and 4 is -

(A) 2 (B) 0 (C) 3 (D) 1

17. Number of real solutions of the equation $x^4 + 8x^2 + 16 = 4x^2 - 12x + 9$ is equal to -

(A) 1 (B) 2 (C) 3 (D) 4

18. If α, β are the roots of quadratic equation $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$, then $(\alpha - \gamma) . (\alpha - \delta)$ is equal to :

(A) $q + r$ (B) $q - r$ (C) $-(q + r)$ (D) $-(p + q + r)$

19. If $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ has equal root, then a, b, c are in

(A) A.P. (B) G.P. (C) H.P. (D) none of these

20. If the roots of the quadratic equation $ax^2 + bx + c = 0$ are imaginary then for all values of a, b, c and $x \in R$, the expression $a^2x^2 + abx + ac$ is -

(A) positive (B) non-negative

(C) negative (D) may be positive, zero or negative

21. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the value of $x^3 - 6x^2 + 6x$ is

(A) 3 (B) 2 (C) 1 (D) -2

22. If a, b, c are integers and $b^2 = 4(ac + 5d^2)$, $d \in N$, then roots of the equation $ax^2 + bx + c = 0$ are

(A) Irrational (B) Rational & different

(C) Complex conjugate (D) Rational & equal

23. Let a, b and c be real numbers such that $4a + 2b + c = 0$ and $ab > 0$. Then the equation $ax^2 + bx + c = 0$ has

(A) real roots (B) imaginary roots (C) exactly one root (D) none of these

24. If $\log_{1/3} \dfrac{3x - 1}{x + 2}$ is less than unity then x must lie in the interval -

(A) $(-\infty, -2) \cup (5/8, \infty)$ (B) $(-2, 5/8)$

(C) $(-\infty, -2) \cup (1/3, 5/8)$ (D) $(-2, 1/3)$

25. If $\alpha, \beta, \gamma, \delta$ are roots of $x^4 - 100x^3 + 2x^2 + 4x + 10 = 0$, then $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}$ is equal to -

(A) $\frac{2}{5}$ **(B)** $\frac{1}{10}$ **(C)** 4 **(D)** $-\frac{2}{5}$

26. If the roots of the equation, $x^3 + Px^2 + Qx - 19 = 0$ are each one more than the roots of the equation, $x^3 - Ax^2 + Bx - C = 0$, where A, B, C, P & Q are constants then the value of $A + B + C =$

(A) 18 **(B)** 19 **(C)** 20 **(D)** none

27. The expression $\frac{x^2+2x+1}{x^2+2x+7}$ lies in the interval ; $(x \in R)$ -

(A) $[0, -1]$ **(B)** $(-\infty, 0] \cup [1, \infty)$ **(C)** $[0, 1)$ **(D)** none of these

28. If exactly one root of the quadratic equation $x^2 - (a+1)x + 2a = 0$ lies in the interval $(0, 3)$ then the set of values 'a' is given by

(A) $(-\infty, 0) \cup (6, \infty)$ **(B)** $(-\infty, 0] \cup (6, \infty)$ **(C)** $(-\infty, 0] \cup [6, \infty)$ **(D)** $(0, 6)$

29. Consider the equation $x^2 + 2x - n = 0$, where $n \in N$ and $n \in [5, 100]$. Total number of different values of 'n' so that the given equation has integral roots, is

(A) 4 **(B)** 6 **(C)** 8 **(D)** 3

30. If the A.M. of the roots of a quadratic equation is $\frac{8}{5}$ and A.M. of their reciprocals is $\frac{8}{7}$, then the quadratic equation is -

(A) $5x^2 - 8x + 7 = 0$ **(B)** $5x^2 - 16x + 7 = 0$ **(C)** $7x^2 - 16x + 5 = 0$ **(D)** $7x^2 + 16x + 5 = 0$

31. If $\sin\alpha$ & $\cos\alpha$ are the roots of the equation $ax^2 + bx + c = 0$ then -

(A) $a^2 - b^2 + 2ac = 0$ **(B)** $a^2 + b^2 + 2ac = 0$ **(C)** $a^2 - b^2 - 2ac = 0$ **(D)** $a^2 + b^2 - 2ac = 0$

32. If the roots of $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ are equal then a, b, c are in

(A) A.P. **(B)** G.P. **(C)** H.P. **(D)** none of these

33. If a, b, p, q are non−zero real numbers, then two equations $2a^2x^2 - 2abx + b^2 = 0$ and $p^2x^2 + 2pqx + q^2 = 0$ have:

(A) no common root **(B)** one common root if $2a^2 + b^2 = p^2 + q^2$

(C) two common roots if $3pq = 2ab$ **(D)** two common roots if $3qb = 2ap$

34. For the equation, $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to:

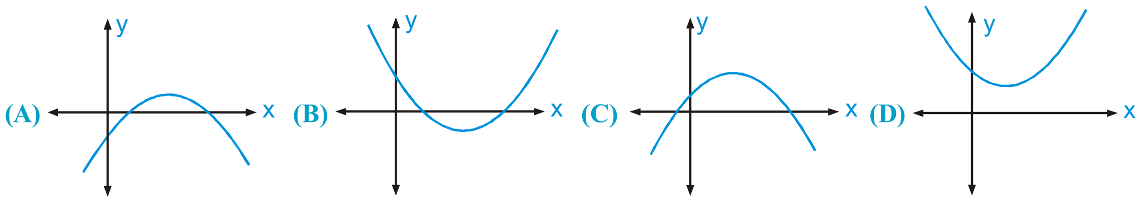
(A) 1/3 **(B)** 1 **(C)** 3 **(D)** 2/3

35. Graph of the function $f(x) = Ax^2 - BX + C$, where

$A = (\sec\theta - \cos\theta)(\csc\theta - \sin\theta)(\tan\theta + \cot\theta)$,

$B = (\sin\theta + \csc\theta)^2 + (\cos\theta + \sec\theta)^2 - (\tan^2\theta + \cot^2\theta)$ &

$C = 12$, is represented by



36. The equation whose roots are the squares of the roots of the equation $ax^2 + bx + c = 0$ is -

(A) $a^2x^2 + b^2x + c^2 = 0$

(B) $a^2x^2 - (b^2 - 4ac)x + c^2 = 0$

(C) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

(D) $a^2x^2 + (b^2 - ac)x + c^2 = 0$

37. If α, β are the roots of the equation $x^2 - 3x + 1 = 0$, then the equation with roots $\frac{1}{\alpha - 2}, \frac{1}{\beta - 2}$ will be

(A) $x^2 - x - 1 = 0$ (B) $x^2 + x - 1 = 0$ (C) $x^2 + x + 2 = 0$ (D) none of these

38. If α, β be the roots of $4x^2 - 16x + \lambda = 0$, where $\lambda \in R$, such that $1 < \alpha < 2$ and $2 < \beta < 3$, then the number of integral solutions of λ is

(A) 5 (B) 6 (C) 2 (D) 3

39. If $x^2 - 11x + a$ and $x^2 - 14x + 2a$ have a common factor then 'a' is equal to

(A) 24 (B) 1 (C) 2 (D) 12

40. If $b > a$, then the equation, $(x - a)(x - b) - 1 = 0$, has :

(A) both roots in [a, b]

(B) both roots in $(-\infty, a)$

(C) both roots in $[b, \infty)$

(D) one root in $(-\infty, a)$ & other in $(b, +\infty)$

41. Let 'a' be a real number. Number of real roots of the equation $(x^2 + ax + 1)(3x^2 + ax - 3) = 0$, is

(A) at least two (B) atmost two (C) exactly two (D) all four.

42. The value of 'a' for which the sum of the squares of the roots of $2x^2 - 2(a - 2)x - a - 1 = 0$ is least is -

(A) 1 (B) 3/2 (C) 2 (D) –1

43. If x be the real number such that $x^3 + 4x = 8$, then the value of the expression $x^7 + 64x^2$ is

(A) 124 (B) 125 (C) 128 (D) 132

44. Exhaustive set of value of x satisfying $\log_{|x|}(x^2 + x + 1) \geq 0$ is -

(A) $(-1, 0)$

(B) $(-\infty, 1) \cup (1, \infty)$

(C) $(-\infty, \infty) - \{-1, 0, 1\}$

(D) $(-\infty, -1) \cup (-1, 0) \cup (1, \infty)$

45. Let α, β, γ be the roots of $(x - a)(x - b)(x - c) = d$, $d \neq 0$, then the roots of the equation $(x - \alpha)(x - \beta)(x - \gamma) + d = 0$ are :

(A) $a + 1, b + 1, c + 1$ (B) a, b, c (C) $a - 1, b - 1, c - 1$ (D) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$

Excercise-2(Segment-I) Questions with multi options correct

1. If α is a root of the equation $2x(2x + 1) = 1$, then the other root is -

(A) $3\alpha^3 - 4\alpha$ **(B)** $-2\alpha(\alpha + 1)$ **(C)** $4\alpha^3 - 3\alpha$ **(D)** none of these

2. If $1/2 \le \log_{0.1} x \le 2$, then -

(A) the maximum value of x is **(B)** x lies between and

(C) x does not lie between and **(D)** the minimum value of x is

3. $x^2 + x + 1$ is a factor of $a x^3 + b x^2 + c x + d = 0$, then the real root of above equation is $(a, b, c, d \in R)$

(A) $-d/a$ **(B)** d/a **(C)** $(b - a)/a$ **(D)** $(a - b)/a$

4. For $x \in [1, 5]$, $y = x^2 - 5x + 3$ has -

(A) least value $= -1.5$ **(B)** greatest value $= 3$ **(C)** least value $= -3.25$ **(D)** greatest value $= 3.5$

5. If $b^2 \ge 4ac$ for the equation $ax^4 + bx^2 + c = 0$, then all roots of the equation will be real if -

(A) $b > 0, a < 0, c > 0$ **(B)** $b < 0, a > 0, c > 0$ **(C)** $b > 0, a > 0, c > 0$ **(D)** $b > 0, a < 0, c < 0$

6. If $\cos \alpha$ is a root of the equation $25x^2 + 5x - 12 = 0$, $-1 < x < 0$, then the value of $\sin 2\alpha$ is -

(A) $12/25$ **(B)** $-12/25$ **(C)** $-24/25$ **(D)** $24/25$

7. If the quadratic equations, $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root then the equation containing their other roots is/are :

(A) $x^2 + a(b + c)x - a^2bc = 0$ **(B)** $x^2 - a(b + c)x + a^2bc = 0$

(C) $a(b + c)x^2 - (b + c)x + abc = 0$ **(D)** $a(b + c)x^2 + (b + c)x - abc = 0$

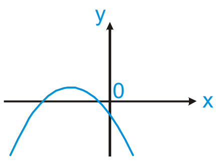
8. Let α, β be the roots of $x^2 - ax + b = 0$, where a & $b \in R$. If $\alpha + 3\beta = 0$, then -

(A) $3a^2 + 4b = 0$ **(B)** $3b^2 + 4a = 0$ **(C)** $b < 0$ **(D)** $a < 0$

9. Integral real values of x satisfying $\log_{1/2}(x^2 - 6x + 12) \ge -2$ is -

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5

10. The graph of the quadratic polynomial $y = ax^2 + bx + c$ is as shown in the figure. Then :



(A) $b^2 - 4ac > 0$ **(B)** $b < 0$ **(C)** $a > 0$ **(D)** $c < 0$

11. For the quadratic polynomial $f(x) = 4x^2 - 8kx + k$, the statements which hold good are

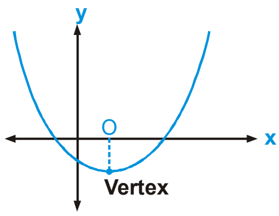
(A) there is only one integral k for which $f(x)$ is non negative $\forall\ x \in R$

(B) for $k < 0$ the number zero lies between the zeros of the polynomial.

(C) $f(x) = 0$ has two distinct solutions in $(0, 1)$ for $k \in (1/4, 4/7)$

(D) Minimum value of y $\forall\ k \in R$ is $k(1 + 12k)$

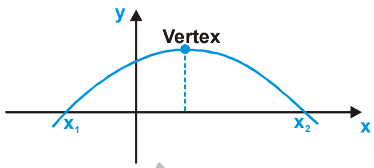
12. Graph of $y = ax^2 + bx + c = 0$ is given adjacently. What conclusions can be drawn from this graph -



(A) $a > 0$ **(B)** $b < 0$

(C) $c < 0$ **(D)** $b^2 - 4ac > 0$

13. The adjoining figure shows the graph of $y = ax^2 + bx + c$. Then -



(A) $a > 0$ **(B)** $b > 0$

(C) $c > 0$ **(D)** $b^2 < 4ac$

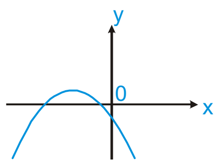
14. Let $f(x) = x^2 + ax + b$. If the maximum and the minimum values of $f(x)$ are 3 and 2 respectively for $0 \le x \le 2$, then the possible ordered pair(s) of (a, b) is/are

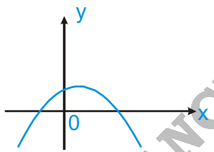
(A) (–2, 3) **(B)** (– 3/2, 2) **(C)** (– 5/2, 3) **(D)** (– 5/2, 2)

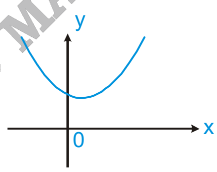
15. If p & q are distinct reals, then $2\{(x-p)(x-q) + (p-x)(p-q) + (q-x)(q-p)\} = (p-q)^2 + (x-p)^2 + (x-q)^2$ is satisfied by -

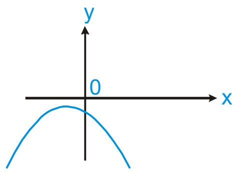
(A) no value of x **(B)** exactly one value of x **(C)** exactly two values of x **(D)** infinite values of x

16. For which of the following graphs of the quadratic expression $y = ax^2 + bx + c$, the product a b c is negative ?

(A) 

(B) 

(C) 

(D) 

17. For every $x \in R$, the polynomial $x^8 - x^5 + x^2 - x + 1$ is -

(A) positive **(B)** never positive

(C) positive as well as negative **(D)** negative

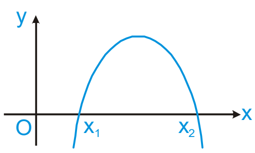
18. If $\log_{\left(\frac{x^2-12x+30}{10}\right)}\left(\log_2 \frac{2x}{5}\right) > 0$ then x belongs to interval -

(A) $\left(\frac{5}{2}, 6+\sqrt{6}\right)$ **(B)** $\left(\frac{5}{2}, 6-\sqrt{6}\right)$ **(C)** $(6, 6+\sqrt{6})$ **(D)** $(10, \infty)$

19. If one of the root of the equation $4x^2 - 15x + 4p = 0$ is the square of the other then the value of p is

(A) 125/64 **(B)** –27/8 **(C)** –125/8 **(D)** 27/8

20. The adjoining figure shows the graph of $y = ax^2 + bx + c$. Then



(A) $a < 0$ **(B)** $b^2 < 4ac$ **(C)** $c > 0$ **(D)** a and b are of opposite sign

21. The correct statement is / are -

(A) If x_1 & x_2 are roots of the equation $2x^2 - 6x - b = 0$ $(b > 0)$, then $\frac{x_1}{x_2} + \frac{x_2}{x_1} < -2$
(B) Equation $ax^2 + bx + c = 0$ has real roots if $a < 0$, $c > 0$ and $b \in R$
(C) If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + bx + c$, where $ac \neq 0$ and $a, b, c \in R$, then $P(x).Q(x)$ has at least two real roots.
(D) If both the roots of the equation $(3a + 1)x^2 - (2a + 3b)x + 3 = 0$ are infinite then $a = 0$ & $b \in R$

22. Equation $2x^2 - 2(2a + 1)x + a(a + 1) = 0$ has one root less than 'a' and other root greater than 'a', if
(A) $0 < a < 1$ (B) $-1 < a < 0$ (C) $a > 0$ (D) $a < -1$

23. If $x^2 + Px + 1$ is a factor of the expression $ax^3 + bx + c$ then -
(A) $a^2 + c^2 = -ab$ (B) $a^2 - c^2 = -ab$ (C) $a^2 - c^2 = ab$ (D) none of these

24. For the equation $|x|^2 + |x| - 6 = 0$, the correct statement(s) is (are) :
(A) sum of roots is 0 (B) product of roots is -4
(C) there are 4 roots (D) there are only 2 roots

25. If a, b are non-zero real numbers and α, β the roots of $x^2 + ax + b = 0$, then
(A) α^2, β^2 are the roots of $x^2 - (2b - a^2)x + a^2 = 0$
(B) $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of $bx^2 + ax + 1 = 0$
(C) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $bx^2 + (2b - a^2)x + b = 0$
(D) $(\alpha - 1), (\beta - 1)$ are the roots of the equation $x^2 + x(a + 2) + 1 + a + b = 0$

Excercise-2(Segment-II) Assertion and Reason Type

These questions contains, Statement-I (assertion) and Statement-II (reason).
(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I
(C) Statement-I is true, Statement-II is false
(D) Statement-I is false, Statement-II is true

1. **Statement-I :** If equation $ax^2 + bx + c = 0$; $(a, b, c \in R)$ and $2x^2 + 3x + 4 = 0$ have a common root, then $a : b : c = 2 : 3 : 4$.
Statement-II : If $p + iq$ is one root of a quadratic equation with real coefficients then $p - iq$ will be the other root ; $p, q \in R$, $i = \sqrt{-1}$

2. Let α, β be the roots of $f(x) = 3x^2 - 4x + 5 = 0$.
Statement-I : The equation whose roots are $2\alpha, 2\beta$ is given by $3x^2 + 8x - 20 = 0$.
Statement-II : To obtain, from the equation $f(x) = 0$, having roots α and β, the equation having roots $2\alpha, 2\beta$ one needs to change x to $\frac{x}{2}$ in $f(x) = 0$.

3. Consider a cubic function $f(x) = ax^3 + bx + c$ where $a, b, c \in R$.
Statement-I: $f(x)$ can not have 3 non negative real roots.
Statement-II : Sum of roots is equal to zero.

4. **Statement-I :** Let $(a_1, a_2, a_3, a_4, a_5)$ denote a re-arrangement of $(1, -4, 6, 7, -10)$. Then the equation

$a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5 = 0$ has at least two real roots.

Statement-II : If $ax^2 + bx + c = 0$ and $a + b + c = 0$, (i.e. in a polynomial the sum of coefficients is zero) then $x = 1$ is root of $ax^2 + bx + c = 0$.

5. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and α, are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$

Statement-I : $(p^2 - q)(b^2 - ac) \geq 0$

Statement-II : $b \neq pa$ or $c \neq qa$

6. **Statement-I:** If $a > b > c$ and $a^3 + b^3 + c^3 = 3abc$ then the quadratic equation $ax^2 + bx + c = 0$ has roots of opposite sign.

Statement-II : If roots of a quadratic equation $ax^2 + bx + c = 0$ are of opposite sign then product of roots < 0 and $|$ sum of roots $| \geq 0$

7. **Statement-I :** If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$.

Statement-II : If a, b, c are odd integer then the roots of the equation $4abc\,x^2 + (b^2 - 4ac)x - b = 0$ are real and distinct.

8. **Statement-I :** The nearest point from x - axis on the curve $f(x) = x^2 - 6x + 11$ is (3, 2)

Statement-II : If $a > 0$ and $D < 0$, then $ax^2 + bx + c > 0\ \forall\ x \in R$.

9. **Statement-I :** If f(x) is a quadratic expression such that $f(1) + f(2) = 0$. If -1 is a root of $f(x) = 0$ then the other root is $\frac{8}{5}$

Statement-II : If $f(x) = ax^2 + bx + c$ then sum of roots $= -\frac{b}{a}$ and product of roots $= \frac{c}{a}$

10. Consider two quadratic functions $f(x) = ax^2 + ax + (a + b)$ and $g(x) = ax^2 + 3ax + 3a + b$, where a and b are non-zero real numbers having same sign.

Statement-I : Graphs of both $y = f(x)$ and $y = g(x)$ either completely lie above x-axis or lie completely below x-axis $x \in R$

Statement-II : If discriminant of $f(x)$, $D < 0$, then $y = f(x)$ is of same sign $x \in R$ and $f(x + 1)$ will also be of same sign as that of $f(x) x \in R$.

11. **Statement-I :** If $a + b + c > 0$ and $a < 0 < b < c$, then the roots of the equation $a(x - b)(x - c) + b(x - c)(x - a) + c(x - a)(x - b) = 0$ are of both negative.

Statement-II : If both roots are negative, then sum of roots < 0 and product of roots > 0

Excercise-3(Segment-I) Matrix Matching Type Questions

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **one or more** statement(s) in **Column-II**.

1. Consider the equation $x^2 + 2(a-1)x + a + 5 = 0$, where 'a' is a parameter. Match of the real values of 'a' so that the given equation has

	Column-I		Column-II
(A)	imaginary roots	(p)	$\left(-\infty, -\frac{8}{7}\right)$
(B)	one root smaller than 3 and other root greater than 3	(q)	$(-1, 4)$
(C)	exactly one root in the interval (1, 3) & 1 and 3 are not the root of the equation	(r)	$\left(-\frac{4}{3}, -\frac{8}{7}\right)$
(D)	one root smaller than 1 and other root greater than 3	(s)	$\left(-\infty, -\frac{4}{3}\right)$

2. It is given that α, β $(\beta \ge \alpha)$ are the roots of the equation $f(x) = ax^2 + bx + c$. Also $af(t) > 0$.
Match the condition given in column-I with their corresponding conclusions given in column-II.

	Column-I		Column-II
(A)	$a > 0$ and $b^2 > 4ac$	(p)	$t \neq \alpha$
(B)	$a > 0$ and $b^2 = 4ac$	(q)	no solution
(C)	$a < 0$ and $b^2 > 4ac$	(r)	$\alpha < t < \beta$
(D)	$a < 0$ and $b^2 = 4ac$	(s)	$t < \alpha$ or $t > \beta$

3.

	Column – I		Column – II
(A)	If the roots of $x^2 - bx + c = 0$ are two consecutive integers, then value of $b^2 - 4c$	(p)	1
(B)	If $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ $(a \neq 0)$ have a common root, then value of $a + b$	(q)	7
(C)	If a, b are roots of $x^2 - x + 3 = 0$ then value of $a^4 + b^4$	(r)	17
(D)	If a, b, c are the roots of $x^3 - 7x^2 + 16x - 12 = 0$ then value of $a^2 + b^2 + c^2$	(s)	-1

4. Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$

	Column I		Column II
(A)	If $-1 < x < 1$, then $f(x)$ satisfies	(p)	$0 < f(x) < 1$
(B)	If $1 < x < 2$, the $f(x)$ satisfies	(q)	$f(x) < 0$
(C)	If $3 < x < 5$, then $f(x)$ satisfies	(r)	$f(x) > 0$
(D)	If $x > 5$, then $f(x)$ satisfies	(s)	$f(x) < 1$

5. **Column – I** | **Column – II**

Column – I		Column – II	
(A)	If α, $\alpha + 4$ are two roots of $x^2 - 8x + k = 0$, then possible value of k is	**(p)**	2
(B)	Number of real roots of equation $x^2 - 5\|x\| + 6 = 0$ are 'n', then value of $\frac{n}{2}$ is	**(q)**	3
(C)	If $3 - i$ is a root of $x^2 + ax + b = 0$ $(a, b \in R)$, then b is	**(r)**	12
(D)	If both roots of $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then 'k' may be equal to	**(s)**	10

6.

Column-I		Column-II	
(A)	Given $x, y \in R$, $x^2 + y^2 > 0$. If the maximum and minimum value of the expression $E = \frac{x^2 + y^2}{x^2 + xy + 4y^2}$ are M and m, and A denotes the average value of M and m, then the value of (2007)A, equals	**(p)**	1210
		(q)	1338
(B)	Given the cubic equation $x^3 - 2kx^2 - 4kx + k^2 = 0$. If one root of the equation is less than 1, other root is in the interval (1, 4) and the 3^{rd} root is greater than 4, then the value of k lies in the interval $\left(a + \sqrt{b}, b(a + \sqrt{b})\right)$ where $a, b \in N$. The value of $(a + b)^3 + (ab + 2)^2$ equals	**(r)**	2007
(C)	If roots of the equation are a, b and those of $x^2 - 10ax - 11b = 0$ are c, d, then the value of $a + b + c + d$, is (where a, b, c and d are distinct numbers)	**(s)**	2009

7. Let $f(x) = x^2 - 2px + p^2 - 1$, then

Column-I		Column-II.	
(A)	both the roots of $f(x) = 0$ are less than 4, if $p \in$	**(p)**	$(-1, \infty)$
(B)	both the roots of $f(x) = 0$ are greater than -2 if $p \in$	**(q)**	$(-\infty, 3)$
(C)	exactly one root of $f(x) = 0$ lie in $(-2, 4)$, if $p \in$	**(r)**	$(0, 2)$
(D)	1 lies between the roots of $f(x) = 0$, if $p \in$	**(s)**	$(-3, -1) \cup (3, 5)$

8.

Column-I		Column-II	
(A)	The minimum value of $\dfrac{\left(x + \frac{1}{x}\right)^6 - \left(x^6 + \frac{1}{x^6}\right) - 2}{\left(x + \frac{1}{x}\right)^3 + x^3 + \frac{1}{x^3}}$ for $x > 0$, is	**(p)**	2
(B)	The integral values of the parameters c for which the inequality $1 + \log_2\left(2x^2 + 2x + \frac{7}{2}\right) \geq \log_2(cx^2 + c)$ has at least one solution, is	**(q)**	4
		(r)	6
(C)	Let $P(x) = x^2 + bx + c$, where b and c are integer. If P (x) is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, then the value of P(1) equals	**(s)**	8

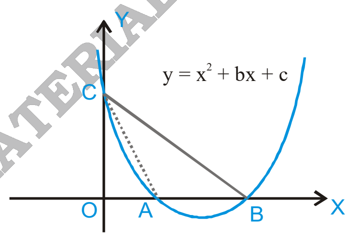
9.

	Column-I		Column-II
(A)	α, β are the roots of the equation $K(x^2 - x) + x + 5 = 0$. If K_1 & K_2 are the two values of K for which the roots α, β are connected by the relation $(\alpha/\beta) + (\beta/\alpha) = 4/5$. The value of $(K_1/K_2) + (K_2/K_1)$ equals.	(p)	146
		(q)	254
(B)	If the range of the function $f(x) = \dfrac{x^2 + ax + b}{x^2 + 2x + 3}$ is $[-5, 4]$, $a, b \in N$, then the value of $(a^2 + b^2)$, is	(r)	277
(C)	Suppose a cubic polynomial $f(x) = x^3 + px^2 + qx + 72$ is divisible by both $x^2 + ax + b$ and $x^2 + bx + a$ (where a, b, p, q are constants and $a \neq b$). The sum of the squares of the roots of the cubic polynomial, is	(s)	298

Excercise-3(Segment-II) Comprehension Type Questions

Comprehension # 1

In the given figure ΔOBC is an isosceles right triangle in which AC is a median, then answer the following questions :



1. Roots of $y = 0$ are

(A) {2, 1} (B) {4, 2} (C) {1, 1/2} (D) {8, 4}

2. The equation whose roots are $(\alpha + \beta)$ & $(\alpha - \beta)$, where α, β $(\alpha > \beta)$ are roots obtained in previous question, is

(A) $x^2 - 4x + 3 = 0$ (B) $x^2 - 8x + 12 = 0$ (C) $4x^2 - 8x + 3 = 0$ (D) $x^2 - 16x + 48 = 0$

3. Minimum value of the quadratic expression corresponding to the quadratic equation obtained in Q. No. 2 occurs at x =

(A) 8 (B) 1 (C) 4 (D) 2

Comprehension # 2

If α, β, γ be the roots of the equation $ax^3 + bx^2 + cx + d = 0$. To obtain the equation whose roots are $f(\alpha), f(\beta), f(\gamma)$, where f is a function, we put $y = f(\alpha)$ and simplify it to obtain $\alpha = g(y)$ (some function of y).

Now, α is a root of the equation $ax^3 + bx^2 + cx + d = 0$, then we obtain the desired equation which is $a\{g(y)\}^3 + b\{g(y)\}^2 + c\{g(y)\} + d = 0$

For example, if α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0$. To find equation whose roots are

$\dfrac{1}{\alpha}, \dfrac{1}{\beta}, \dfrac{1}{\gamma}$ we put $y = \dfrac{1}{\alpha}$ $\Rightarrow$ $\alpha = \dfrac{1}{y}$

As α is a root of $ax^3 + bx^2 + cx + d = 0$

we get $\dfrac{a}{y^3} + \dfrac{b}{y^2} + \dfrac{c}{y} + d = 0$ $\Rightarrow$ $dy^3 + cy^2 + by + a = 0$

This is desired equation.

On the Basis of Above Information, Answer the Following Questions

1. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the roots of the equation $a(2x+1)^2 + b(2x+1)(x-1) + c(x-1)^2 = 0$ are-

(A) $\frac{2\alpha+1}{\alpha-1}, \frac{2\beta+1}{\beta-1}$ **(B)** $\frac{2\alpha-1}{\alpha+1}, \frac{2\beta-1}{\beta+1}$ **(C)** $\frac{\alpha+1}{\alpha-2}, \frac{\beta+1}{\beta-2}$ **(D)** $\frac{2\alpha+3}{\alpha-1}, \frac{2\beta+3}{\beta-1}$

2. If α, β are the roots of the equation $2x^2 + 4x - 5 = 0$, the equation whose roots are the reciprocals of $2\alpha - 3$ and $2\beta - 3$ is -

(A) $x^2 + 10x - 11 = 0$ **(B)** $11x^2 + 10x + 1 = 0$ **(C)** $x^2 + 10x + 11 = 0$ **(D)** $11x^2 - 10x + 1 = 0$

3. If α, β are the roots of the equation $px^2 - qx + r = 0$, then the equation whose roots are $\alpha^2 + \frac{r}{p}$ and $\beta^2 + \frac{r}{p}$ is-

(A) $p^3x^2 + pq^2x + r = 0$ **(B)** $px^2 - qx + r = 0$ **(C)** $p^3x^2 - pq^2x + q^2r = 0$ **(D)** $px^2 + qx - r = 0$

4. If α, β, γ are the roots of the equation $x^3 - x - 1 = 0$, then the value of $\Pi\left(\frac{1+\alpha}{1-\alpha}\right)$ is equal to -

(A) –7 **(B)** –5 **(C)** –3 **(D)** –1

Excercise-4

Subjective Type Questions

1. If α, β are the roots of the equation $x^2 - 2x + 3 = 0$ obtain the equation whose roots are $\alpha^3 - 3\alpha^2 + 5\alpha - 2, \beta^3 - \beta^2 + \beta + 5$.

2. If one root of the equation $ax^2 + bx + c = 0$ be the square of the other, prove that $b^3 + a^2c + ac^2 = 3abc$.

3. Find the product of the real roots of the equation, $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$

4. Let a, b be arbitrary real numbers. Find the smallest natural number 'b' for which the equation $x^2 + 2(a + b)x + (a - b + 8) = 0$ has unequal real roots for all $a \in R$.

5. Solve the inequality : $\log_3 \frac{|x^2 - 4x| + 3}{x^2 + |x - 5|} \ge 0$

6. Find the range of values of a, such that $f(x) = \frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32}$ is always negative.

7. Suppose $a, b, c \in I$ such that greatest common divisor of $x^2 + ax + b$ and $x^2 + bx + c$ is $(x + 1)$ and the least common multiple of $x^2 + ax + b$ and $x^2 + bx + c$ is $(x^3 - 4x^2 + x + 6)$. Find the value of $(a + b + c)$.

8. For what value of 'a', the equation $(a^2 - a - 2)x^2 + (a^2 - 4)x + (a^2 - 3a + 2) = 0$, will have more than two solutions Does there exist a real value of 'x' for which the above equation will be an identity in 'a' ?

9. The equation $x^2 - ax + b = 0$ & $x^3 - px^2 + qx = 0$, where $b \ne 0, q \ne 0$, have one common root & the second equation has two equal roots. Prove that $2(q + b) = ap$.

10. The equation $x^n + px^2 + qx + r = 0$, where $n \ge 5$ & $r \ne 0$ has roots $\alpha_1, \alpha_2, \alpha_3 ....\alpha_n$.

Denoting $\sum_{i=1}^{n} \alpha_i^k$ by S_k.

(a) Calculate S_2 & deduce that the roots cannot all be real.

(b) Prove that $S_n + pS_2 + qS_1 + nr = 0$ & hence find the value of S_n.

11. The length of a rectangle is 2 metre more than its width. If the length is increased by 6 metre and the width is decreased by 2 metre, the area becomes 119 square metre. Find the dimensions of the original rectangle.

12. Find the product of uncommon real roots of the two polynomials $P(x) = x^4 + 2x^3 - 8x^2 - 6x + 15$ and $Q(x) = x^3 + 4x^2 - x - 10$.

13. Find all values of a for which both roots of the equation $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ are greater than 3.

14. If α, β are the roots of $ax^2 + bx + c = 0$ $(a \ne 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of,

$Ax^2 + Bx + C = 0$ $(A \ne 0)$ for some constant δ, then prove that $\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$.

15. If the roots of the equation $\frac{1}{(x+p)} + \frac{1}{(x+q)} = \frac{1}{r}$ are equal in magnitude but opposite in sign, show that

$p + q = 2r$ & that the product of the roots is equal to $(-1/2)(p^2 + q^2)$.

16. Find all values of a for which the inequality $(a + 4)x^2 - 2ax + 2a - 6 < 0$ is satisfied for all $x \in R$.

17. Two roots of a biquadratic $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$ have their product equal to (–32). Find the value of k.

18. Let the quadratic equation $x^2 + 3x - k = 0$ has roots a, b and $x^2 + 3x - 10 = 0$ has roots c, d such that modulus of difference of the roots of the first equation is equal to twice the modulus of the difference of the roots of the second equation. If the value of 'k' can be expressed as rational number in the lowest form as m/n then find the value of (m + n).

19. If the quadratic equation $ax^2 + bx + c = 0$ has real roots, of opposite sign in the interval (–2, 2) then prove that

$$1+\frac{c}{4a}-\left|\frac{b}{2a}\right|>0 .$$

20. If $a < b < c < d$ then prove that the roots of the equation ; $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are real & distinct.

21. If $x^2 + px + q = 0$ and $x^2 + qx + p = 0$, $(p \neq q)$ have a common root, show that $1 + p + q = 0$; show that their other roots are the roots of the equation $x^2 + x + pq = 0$.

22. If one root of the quadratic equation $ax^2 + bx + c = 0$ is equal to the n^{th} power of the other, then show that $(ac^n)^{1/(n+1)} + (a^n c)^{1/(n+1)} + b = 0$.

23. When $y^2 + my + 2$ is divided by $(y - 1)$ then the quotient is f (y) and the remainder is R_1. When $y^2 + my + 2$ is divided by $(y + 1)$ then quotient is g (y) and the remainder is R_2. If $R_1 = R_2$ then find the value of *m*.

24. If α, β and γ are roots of $2x^3 + x^2 - 7 = 0$, then find the value of $\sum\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$.

25. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then find the value of

$$\left(\alpha-\frac{1}{\beta\gamma}\right)\left(\beta-\frac{1}{\gamma\alpha}\right)\left(\gamma-\frac{1}{\alpha\beta}\right).$$

Excercise-5(Segment-I) Previous Year Questions (AIEEE)

1. If the roots of the equation $x^2 - 5x + 16 = 0$ are α, β and the roots of the equation $x^2 + px + q = 0$ are $(\alpha^2 + \beta^2)$ and $\frac{\alpha\beta}{2}$, then- **[AIEEE-2002]**

(1) p = 1 and q = 56 (2) p = 1 and q = –56
(3) p = –1 and q = 56 (4) p = –1 and q = –56

2. If α and β be the roots of the equation $(x - a)(x - b) = c$ and $c \neq 0$, then roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are **[AIEEE-2002]**

(1) a and c (2) b and c (3) a and b (4) a + b and b + c

3. If $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$ then the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (where $\alpha \neq \beta$) is- **[AIEEE-2002]**

(1) 19/3 (2) 25/3 (3) –19/3 (4) none of these

4. The value of a for which one roots of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is **[AIEEE-2003]**

(1) – 2/3 (2) 1/3 (3) – 1/3 (4) 2/3

5, If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the square of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in **[AIEEE-2003]**

(1) geometric progression (2) harmonic progression
(3) arithmetic-geometric progression (4) arithmetic progression

6. The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$, is- **[AIEEE-2003]**

(1) 4 (2) 1 (3) 3 (4) 2

7. The real number x when added to its inverse gives the minimum value of the sum at x equal to- **[AIEEE-2003]**

(1) 1 (2) –1 (3) –2 (4) 2

8. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation- **[AIEEE-2004]**

(1) $x^2 + 18x - 16 = 0$ (2) $x^2 - 18x + 16 = 0$ (3) $x^2 + 18x + 16 = 0$ (4) $x^2 - 18x - 16 = 0$

9. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its roots are **[AIEEE-2004]**

(1) 0, – 1 (2) – 1, 1 (3) 0, 1 (4) – 1, 2

10. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is- **[AIEEE-2004]**

(1) 3 (2) 12 (3) 49/4 (4) 4

11. If value of a for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is- **[AIEEE-2005]**

(1) 2 (2) 3 (3) 0 (4) 1

12. If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals- **[AIEEE-2005]**

(1) 1 (2) 2 (3) 3 (4) –2

13. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval- **[AIEEE-2005]**

(1) [4, 5] (2) $(-\infty, 4)$ (3) $(6, \infty)$ (4) (5, 6)

14. If the equation $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \ldots + a_1 = 0$ has a positive root, which is- **[AIEEE-2005]**
(1) equal to α (2) greater than or equal to α
(3) smaller than α (4) greater than α

15. All the values of m for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than -2 but less than 4, lie in the interval- **[AIEEE-2006]**
(1) $-1 < m < 3$ (2) $1 < m < 4$ (3) $-2 < m < 0$ (4) $m > 3$

16. If the roots of the quadratic equation $x^2 + px + q = 0$ are $\tan 30°$ and $\tan 15°$, respectively then the value of $2 + q - p$ is- **[AIEEE-2006]**
(1) 0 (2) 1 (3) 2 (4) 3

17. If x is real, then maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is- **[AIEEE-2006]**
(1) 1 (2) $\frac{17}{7}$ (3) $\frac{1}{4}$ (4) 41

18. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is **[AIEEE-2007]**
(1) $(-3, \infty)$ (2) $(3, \infty)$ (3) $(-\infty, -3)$ (4) $(-3, -2) \cup (2, 3)$

19. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[AIEEE-2008]**
(1) 1 (2) 4 (3) 3 (4) 2

20. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is :- **[AIEEE-2009]**
(1) Greater than $-4ab$ (2) Less than $-4ab$ (3) Greater than $4ab$ (4) Less than $4ab$

21. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[AIEEE-2010]**
(1) -2 (2) -1 (3) 1 (4) 2

22. Let for $a \neq a_1 \neq 0$, $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is: **[AIEEE-2011]**
(1) 18 (2) 3 (3) 9 (4) 6

23. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of x to get roots (3, 2). The correct roots of equation are: **[AIEEE-2011]**
(1) $-4, -3$ (2) 6, 1 (3) 4, 3 (4) $-6, -1$

24. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has : **[AIEEE-2012]**
(1) exactly four real roots. (2) infinite number of real roots.
(3) no real roots. (4) exactly one real root.

25. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ **[JEE Main 2014]**
(where [x] denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval :
(1) $(-1, 0) \cup (0, 1)$ (2) $(1, 2)$ (3) $(-2, -1)$ (4) $(-\infty, -2) \cup (2, \infty)$

26. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to :
(1) 3 (2) -3 (3) 6 (4) -6 **[JEE Main 2015]**

27. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2+4x-60} = 1$ is **[JEE Main 2016]**
(1) -4 (2) 6 (3) 5 (4) 3

Excercise-5(Segment-II) Previous Year Questions (JEE Main/Advanced)

1. Let a, b, c be real numbers with $a \neq 0$ and let α, β be the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms of α, β. **[JEE 2001]**

2. The set of all real numbers x for which $x^2 - |x + 2| + x > 0$, is

(A) $(-\infty, -2) \cup (2, \infty)$ **(B)** $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

(C) $(-\infty, -1) \cup (1, \infty)$ **(D)** $(\sqrt{2}, \infty)$ **[JEE 2002]**

3. If $x^2 + (a - b)x + (1 - a - b) = 0$ where $a, b \in R$ then find the values of 'a' for which equation has unequal real roots for all values of 'b'. **[JEE 2003]**

4. (a) If one root of the equation $x^2 + px + q = 0$ is the square of the other, then **[JEE 2004]**

(A) $p^3 + q^2 - q(3p + 1) = 0$ **(B)** $p^3 + q^2 + q(1 + 3p) = 0$

(C) $p^3 + q^2 + q(3p - 1) = 0$ **(D)** $p^3 + q^2 + q(1 - 3p) = 0$

(b) If $x^2 + 2ax + 10 - 3a > 0$ for all $x \in R$, then

(A) $-5 < a < 2$ **(B)** $a < -5$ **(C)** $a > 5$ **(D)** $2 < a < 5$

5. Find the range of values of t for which $2\sin t = \dfrac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$, $t \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$. **[JEE 2005]**

6. (a) Let a, b, c be the sides of a triangle. No two of them are equal and $\lambda \in R$. If the roots of the equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then

(A) $\lambda < \dfrac{4}{3}$ **(B)** $\lambda > \dfrac{5}{3}$ **(C)** $\lambda \in \left(\dfrac{1}{3}, \dfrac{5}{3}\right)$ **(D)** $\lambda \in \left(\dfrac{4}{3}, \dfrac{5}{3}\right)$

[JEE 2006]

(b) If roots of the equation $x^2 - 10cx - 11d = 0$ are a, b and those of $x^2 - 10ax - 11b = 0$ are c, d, then find the value of a + b + c + d. (a, b, c and d are distinct numbers)

7. (a) Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\alpha/2, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of 'r' is **[JEE 2006]**

(A) $\dfrac{2}{9}(p-q)(2q-p)$ **(B)** $\dfrac{2}{9}(q-p)(2p-q)$ **(C)** $\dfrac{2}{9}(q-2p)(2q-p)$ **(D)** $\dfrac{2}{9}(2p-q)(2q-p)$

Match the Column

(b) Let $f(x) = \dfrac{x^2 - 6x + 5}{x^2 - 5x + 6}$

Match the expressions / statements in **Column I** with expressions / statements in **Column II**.

	Column I		Column II
(A)	If $-1 < x < 1$, then $f(x)$ satisfies	**(p)**	$0 < f(x) < 1$
(B)	If $1 < x < 2$, the $f(x)$ satisfies	**(q)**	$f(x) < 0$
(C)	If $3 < x < 5$, then $f(x)$ satisfies	**(r)**	$f(x) > 0$
(D)	If $x > 5$, then $f(x)$ satisfies	**(s)**	$f(x) < 1$

[JEE 2007]

Assertion & Reason

8. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, 1/\beta$ are the roots of the equation $ax^2 + 2bx + c = 0$, where $\beta^2 \notin \{-1, 0, 1\}$

Statement-I : $(p^2 - q)(b^2 - ac) \geq 0$

and

Statement-II : $b \neq pa$ or $c \neq qa$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement 1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement 1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True **[JEE 2008]**

9. The smallest value of k, for which both the roots of the equation, $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is **[JEE 2009]**

10. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is **[JEE 2010]**

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$
(B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$
(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$
(D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

11. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is **[JEE 2011]**

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

12. A value of b for which the equations

$x^2 + bx - 1 = 0$

$x^2 + x + b = 0,$

have one root in common is - **[JEE 2011]**

(A) $-\sqrt{2}$ **(B)** $-i\sqrt{3}$ **(C)** $i\sqrt{5}$ **(D)** $\sqrt{2}$

13. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has **[JEE Ad. 2014]**

(A) only purely imaginary roots **(B)** all real roots
(C) two real and two purely imaginary roots **(D)** neither real nor purely imaginary roots

14. Let $a \in R$ and let $f : R \to R$ be given by $f(x) = x^5 - 5x + a$ Then, **[JEE Ad. 2014]**

(A) f(x) has three real roots of $a > 4$ **(B)** f(x) has only one real root if $a > 4$
(C) f(x) has three real roots if $a < -4$ **(D)** f(x) has three real roots if $-4 < a < 4$

15. Let S be the set of all non-zero real numbers a such that the quadratic equation $ax^2 - x + a = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. **[JEE Ad. 2015]**
Which of the following intervals is(are) a subset(s) of S ?

(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ **(B)** $\left(-\frac{1}{\sqrt{5}}, 0\right)$ **(C)** $\left(0, \frac{1}{\sqrt{5}}\right)$ **(D)** $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

Practice Test

SECTION - I : STRAIGHT OBJECTIVE TYPE

1. If $(1 + k) \tan^2 x - 4 \tan x - 1 + k = 0$ has real roots $\tan x_1$ and $\tan x_2$, where $\tan x_1 \neq \tan x_2$, then

(A) $k^2 < 5, k \neq -1$ (B) $k^2 \leq 5$ (C) $k^2 \leq 5, k \neq -1$ (D) none of these

2. Consider two quadratic expressions $f(x) = ax^2 + bx + c$ and $g(x) = ax^2 + px + q$, $(a, b, c, p, q \in R, b \neq p)$ such that their discriminants are equal. If $f(x) = g(x)$ has a root $x = \alpha$, then

(A) α will be A.M. of the roots of $f(x) = 0$ and $g(x) = 0$ (B) α will be A.M. of the roots of $f(x) = 0$

(C) α will be A.M. of the roots of $f(x) = 0$ or $g(x) = 0$ (D) α will be A.M. of the roots of $g(x) = 0$

3. α, β are roots of the equation $\lambda(x^2 - x) + x + 5 = 0$. If λ_1 and λ_2 are the two values of λ for which the roots α, β are connected by the relation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 4$, then the value of $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1}$ is

(A) 150 (B) 254 (C) 180 (D) 1022

4. If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is

(A) greater than 4ab (B) less than 4ab (C) greater than –4ab (D) less than –4ab

5. $f(x) = x^2 + bx + c$, where $b,c \in R$, if $f(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$ then the least value of $f(x)$ is

(A) 2 (B) 3 (C) 2.5 (D) 4

6. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then the equation $ax^2 - bx(x - 1) + c(x - 1)^2 = 0$ has roots

(A) $\frac{\alpha}{1-\alpha}, \frac{\beta}{1-\beta}$ (B) $\alpha - 1, \beta - 1$ (C) $\frac{\alpha}{\alpha+1}, \frac{\beta}{\beta+1}$ (D) $\frac{1-\alpha}{\alpha}, \frac{1-\beta}{\beta}$

7. If $p, q, r, s \in R$, then equation $(x^2 + px + 3q)(-x^2 + rx + q)(-x^2 + sx - 2q) = 0$ has

(A) 6 real roots (B) at least two real roots

(C) 2 real and 4 imaginary roots (D) 4 real and 2 imaginary roots

8. If x_1 and x_2 are the arithmetic and harmonic mean of the roots of the equations $ax^2 + bx + c = 0$, then quadratic equation whose roots are x_1 and x_2 is

(A) $abx^2 + (b^2 + ac)x + bc = 0$ (B) $2ab\,x^2 + (b^2 + 4ac)x + 2bc = 0$

(C) $2abx^2 + (b^2 + ac)x + bc = 0$ (D) none of these

9. If $x_1 > x_2 > x_3$ and x_1, x_2, x_3 are roots of $\frac{x-a}{b} + \frac{x-b}{a} = \frac{b}{x-a} + \frac{a}{x-b}$; $(a, b > 0)$ and $x_1 - x_2 - x_3 = c$, then a, c, b are in.

(A) A.P. (B) G.P. (C) H.P. (D) None

10. S_1 : The roots of the equation $x^2 + px + q = 0$ are $\tan 22°$ and $\tan 23°$ then $p - q = -1$

S_2 : If α, β be the roots of $x^2 + x + 1 = 0$. Then the equation whose roots are α^{229} and α^{1004} is $x^2 + x + 1$

S_3 : If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation $a^2 = b^2 + 2ac$

S_4 : Range of $\frac{1}{1+x^2}$ is (0, 1]

(A) FTTT (B) TFTF (C) TTFT (D) TTTF

SECTION - II : MULTIPLE CORRECT ANSWER TYPE

11. If $a, b \in R$ and $ax^2 + bx + 6 = 0, a \neq 0$ does not have two distinct real roots, then
(A) Minimum possible value of $3a + b$ is -2
(B) Minimum possible value of $3a + b$ is 2
(C) Minimum possible value of $6a + b$ is -1
(D) Minimum possible value of $6a + b$ is 1

12. Let Δ^2 be the discriminant and α, β be the roots of the equation $ax^2 + bx + c = 0$. Then, $2a\alpha + \Delta$ and $2a\,\beta - \Delta$ can be the roots of the equation
(A) $x^2 + 2b\,x + b^2 = 0$
(B) $x^2 - 2bx + b^2 = 0$
(C) $x^2 + 2bx - 3b^2 + 16\,ac = 0$
(D) $x^2 - 2bx - 3b^2 + 16ac = 0$

13. If the quadratic equation $(ab - bc)\,x^2 + (bc - ca)\,x + ca - ab = 0$, $a, b, c \in R$, has both the roots equal, then
(A) both roots are equal to 0
(B) both roots are equal to 1
(C) a, c, b are in harmonic progresssion
(D) $ab^2c^2, b^2a^2c, a^2c^2b$ are in arithmetic progression

14. $\dfrac{\pi^e}{x-e} + \dfrac{e^\pi}{x-\pi} + \dfrac{\pi^\pi + e^e}{x-\pi-e} = 0$ has
(A) one real root in (e, π) and other in $(\pi - e, e)$
(B) one real root in (e, π) and other in $(\pi, \pi + e)$
(C) two real roots in $(\pi - e, \pi + e)$
(D) No real roots

15. If the roots of the equation $x^3 + bx^2 + cx - 1 = 0$ form an increasing G.P., then
(A) $b + c = 0$
(B) $b \in (-\infty, -3)$
(C) one of the roots is 1
(D) one root is smaller than 1 and one root is more than 1.

SECTION - III : ASSERTION AND REASON TYPE

16. **Statement-I :** The equation $(x - p)(x - r) + \lambda (x - q)(x - s) = 0$, $p < q < r < s$, has non real roots if $\lambda > 0$
Statement-II : The equation $ax^2 + bx + c = 0$, $a, b, c \in R$, has non real roots if $b^2 - 4ac < 0$.
(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False
(D) Statement-I is False, Statement-II is True

17. **Statement-I :** If roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c = 1$.
Statement-II : If a, b, c are odd integer then the roots of the equation $4\,abc\,x^2 + (b^2 - 4ac)\,x - b = 0$ are real and distinct.
(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I
(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I
(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

18. **Statement-I :** If one roots is $\sqrt{5} - \sqrt{2}$ then the equation of lowest degree with rational coefficient is $x^4 - 14x^2 + 9 = 0$.

Statement-II : For a polynomial equation with rational co-efficient irrational roots occurs in pairs

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

19. **Statement-I :** The quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ have one root $x = 1$

Statement-II : If sum of the co-efficients in a quadratic equation vanishes then its one root is $x = 1$

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

20. **Statement-I :** The number of values of 'a' for which $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$ is an identity in x, is 2.

Statement-II : If $a = b = c = 0$, then equation $ax^2 + bx + c = 0$ is an identity in x.

(A) Statement-I is True, Statement-II is True; Statement-II is a correct explanation for Statement-I

(B) Statement-I is True, Statement-II is True; Statement-II is NOT a correct explanation for Statement-I

(C) Statement-I is True, Statement-II is False

(D) Statement-I is False, Statement-II is True

SECTION - IV : MATRIX - MATCH TYPE

21. **Match the Column**

	Column – I		Column – II
(A)	Number of real solution of $\|x + 1\| = e^x$ is	**(p)**	2
(B)	The number of non-negative real roots of $2^x - x - 1 = 0$ equal to	**(q)**	3
(C)	If p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$, then minimum value of $p^2 + q^2$ is equal to	**(r)**	6
(D)	If α and β are the roots of $2x^2 + 7x + c = 0$	**(s)**	4
	and $\|\alpha^2 - \beta^2\| = \frac{7}{4}$, then c is equal to	**(t)**	5

22. Match the Column

	Column - I		Column - II
(A)	The equation $x^3 - 6x^2 + 9x + \lambda = 0$ have exactly one root in (1, 3) then $[\lambda + 1]$ is (where [.] denotes the greatest integer function)	(p)	-3
(B)	If $-3 < \dfrac{x^2 - \lambda x - 2}{x^2 + x + 1} < 2$ for all $x \in R$, then $[\lambda]$ is can be where [.] denotes the greatest integer function	(q)	-2
(C)	If $x^2 + \lambda x + 1 = 0$ and $(b - c)x^2 + (c - a)x + (a - b) = 0$ have both the roots common, then $[\lambda - 1]$ is (where [.] denotes the greatest integer function)	(r)	-1
(D)	If N be the number of solutions of the equation $\lvert x^2 - x - 6 \rvert = x + 2$, then the value of $-N$ is.	(s)	3
		(t)	0

SECTION - V : COMPREHENSION TYPE

23. Read the following comprehension carefully and answer the questions.

Let $f(x) = x^2 + b_1x + c_1$, $g(x) = x^2 + b_2x + c_2$. Real roots of $f(x) = 0$ be α, β and real roots of $g(x) = 0$ be $\alpha + \delta$, $\beta + \delta$. Least value of f(x) be $-\dfrac{1}{4}$. Least value of g(x) occurs at $x = \dfrac{7}{2}$

1. The Least value of g(x) is

(A) -1 (B) $-\dfrac{1}{2}$ (C) $-\dfrac{1}{4}$ (D) $-\dfrac{1}{3}$

2. The value of b_2 is

(A) 6 (B) -7 (C) 8 (D) 0

3. The roots of $g(x) = 0$ are

(A) 3, 4 (B) -3, 4 (C) 3, -4 (D) -3, -4

24. Read the following comprehension carefully and answer the questions.

If roots of the equation $x^4 - 12x^3 + bx^2 + cx + 81 = 0$ are positive, then

1. Value of b is

(A) -54 (B) 54 (C) 27 (D) -27

2. Value of c is

(A) 108 (B) -108 (C) 54 (D) -54

3. Root of equation $2bx + c = 0$ is

(A) $-\dfrac{1}{2}$ (B) $\dfrac{1}{2}$ (C) 1 (D) -1

25. Read the following comprehension carefully and answer the questions.

In the given figure vertices of ΔABC lie on $y = f(x) = ax^2 + bx + c$. The ΔABC is right angled isosceles triangle whose hypotaneous $AC = 4\sqrt{2}$ units, then

1. $y = f(x)$ is given by

(A) $y = \frac{x^2}{2\sqrt{2}} - 2\sqrt{2}$ **(B)** $y = \frac{x^2}{2} - 2$ **(C)** $y = x^2 - 8$ **(D)** $y = x^2 - 2\sqrt{2}$

2. Minimum value of $y = f(x)$ is

(A) $2\sqrt{2}$ **(B)** $-2\sqrt{2}$ **(C)** 2 **(D)** –2

3. Number of integral value of k for which $\frac{k}{2}$ lies between the roots of $f(x) = 0$, is

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12

SECTION - VI : INTEGER TYPE

26. Find all 'm' for which $f(x) \equiv x^2 - (m-3)x + m > 0$ for all values of 'x' in [1, 2].

27. If α, β are roots of the equation $x^2 - 34x + 1 = 0$, evaluate $\sqrt[4]{\alpha} - \sqrt[4]{\beta}$, where $\sqrt[4]{.}$ denotes the principal value.

28. Find the absolute value of the difference of the real roots of the equation

$x^2 - 2^{2010}x + |x - 2^{2009}| + 2(2^{4017} - 1) = 0$

29. Find the values of 'a' for which the equation

$(x^2 + x + 2)^2 - (a-3)(x^2 + x + 2)(x^2 + x + 1) + (a-4)(x^2 + x + 1)^2 = 0$ has at least one real root.

30. Find the number of real roots of $\left(x + \frac{1}{x}\right)^3 + \left(x + \frac{1}{x}\right) = 0$

Answer Key

EXERCISE - 1

1. D 2. C 3. B 4. B 5. B 6. C 7. B 8. C 9. C 10. D 11. B 12. C 13. A
14. A 15. A 16. C 17. A 18. C 19. C 20. A 21. B 22. A 23. A 24. A 25. D 26. A
27. C 28. B 29. C 30. B 31. A 32. B 33. A 34. C. 35. B 36. C 37. A 38. D 39. A
40. D 41. A 42. B 43. C 44. D 45. B

EXERCISE - 2 : Segment-I

1. BC 2. ABD 3. AD 4. BC 5. BD 6. CD 7. BD 8. AC 9. ABC
10. ABD 11. ABC 12. ABCD 13. BC 14. A 15. D 16. ABCD 17. A 18. BD
19. CD 20. AD 21. ABC 22. ACD 23. C 24. ABD 25. BCD

Segment-II

1. A 2. D 3. D 4. A 5. B 6. A 7. B 8. B 9. A 10. A 11. D

EXERCISE - 3 : Segment-I

1. $A \to q$, $B \to prs$, $C \to r$, $D \to s$, 2. $A \to ps$, $B \to ps$, $C \to ps$, $D \to ps$ 3. $A \to p$, $B \to s$, $C \to q$, $D \to r$
4. $A \to prs$, $B \to qs$, $C \to qs$, $D \to prs$ 5. $A \to r$, $B \to p$, $C \to s$, $D \to pq$, 6. $A \to q$, $B \to r$, $C \to p$
7. $A \to q$, $B \to p$, $C \to s$, $D \to r$, 8. $A \to r$, $B \to pqrs$, $C \to q$, 9. $A \to q$, $B \to q$, $C \to p$,

Segment-II

Comprehension # 1 : 1. A 2. A 3. D **Comprehension # 2 :** 1. C 2. B 3. C 4. D

EXERCISE - 4 : Subjective type questions

EXERCISE - 5 : Segment-I

1. 4 2. 3 3. 1 4. 4 5. 2 6. 1 7. 1
8. 2 9. 1 10. 3 11. 4 12. 1 13. 2 14. 3
15. 1 16. 4 17. 4 18. 4 19. 4 20. 1 21. 3
22. 1 23. 2 24. 3 25. 1 26. 1 27. 4

Segment-II

1. $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ or $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$ 2. B 3. $a > 1$ 4. (a) $\to$ D; (b) $\to$ A

5. $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$ 6. (a) A; (b) 1210 7. (a) D; (b) **A** $\to$ p, r, s; **B** $\to$ q, s; **C** $\to$ q, s; **D** $\to$ p, r, s

8. B 9. 2 10. B 11. C 12. B 13. D 14. B, D 15. **D**

MOCK TEST

1. A	**2.** A	**3.** D	**4.** C	**5.** D	**6.** C	**7.** B	**8.** B	**9.** C
10. C	**11.** AC	**12.** AC	**13.** BCD	**14.** BC	**15.** ABCD	**16.** D	**17.** B	**18.** A
19. A	**20.** D	**21.** $A\to q, B\to p, C\to t, D\to r$			**22.** $A\to pqrt, B\to rt, C\to p, D\to p$			
23. **1** C **2** B **3** A		**24** **1** B **2** C		**25** **1** A **2** B **3** C		**26** $-\infty, 10$	**27** ± 2	**28.** 2
29. $5 < a \le \frac{19}{3}$		**30.** 0						