WS- 12 For 9th class
Rolling motion
T Task

① $K.E_{rot} = 8J$

$m = 1\ kg$; $R = 0.2m$

$k = \frac{1}{\sqrt{2}} R$

$\Rightarrow k^2 = \frac{1}{2} R^2$

$\Rightarrow \frac{k^2}{R^2} = \frac{1}{2}$

$\therefore K.E_{rot} = \frac{1}{2} m v^2 \left[\frac{k^2}{R^2}\right]$

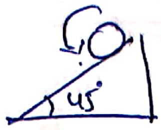
$= \frac{1}{2} \times 1 \times v^2 \left(\frac{1}{2}\right)$

$8 \Rightarrow \frac{1}{4} \times v^2$

$\Rightarrow v^2 = 8 \times 4$

$\Rightarrow v = 4\sqrt{2}\ m/s$

②



$k = \sqrt{\frac{2}{5}} R$

As sphere is rolling down. i.e. it is rotating about it's natural axis

$\therefore a_{cm} = \frac{g \sin\theta}{1 + \frac{k^2}{R^2}}$

$= \frac{g \sin 45°}{1 + \frac{(\sqrt{\frac{2}{5}}R)^2}{R^2}}$

$\Rightarrow \frac{\frac{g}{\sqrt{2}}}{1 + \frac{\frac{2}{5}\cancel{R^2}}{\cancel{R^2}}}$

$a_{cm} = \frac{5g}{7\sqrt{2}}$

③ $K.E_{Total} = \frac{1}{2} m v^2 \left[1 + \frac{k^2}{R^2}\right]$

$K.E_{rot} = \frac{1}{2} m v^2 \frac{k^2}{R^2}$

$k = \frac{R}{\sqrt{2}}$ [For disc rotating about natural axis]

$\therefore \frac{K.E_{rot}}{K.E_{tot}} = \frac{\frac{1}{2}\cancel{mv^2} \frac{k^2}{R^2}}{\frac{1}{2}\cancel{mv^2}\left(1 + \frac{k^2}{R^2}\right)}$

$= \frac{\frac{\left(\frac{R}{\sqrt{2}}\right)^2}{R^2}}{1 + \frac{\left(\frac{R}{\sqrt{2}}\right)^2}{R^2}}$

$\frac{K.E_{rot}}{K.E_{Tot}} = \frac{\frac{\frac{R^2}{2}}{R^2}}{1 + \frac{R^2}{2R^2}} = \frac{1}{3}$

④

For a Ring Rotating about it's natural axis $k = R$.

$V_{com} = \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}} \Rightarrow V_{cm} = \sqrt{\frac{2gh}{1 + \frac{\cancel{R^2}}{\cancel{R^2}}}} = \sqrt{\frac{2gh}{1+1}} = \sqrt{gh}$

⑦ Given $m = 4\ kg$; $V = 2\ m/s$

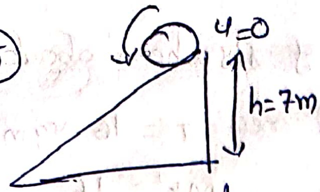
$K.E_{rot} = \frac{1}{2} m v^2 \frac{k^2}{R^2}$

For a ring $k = R$

$= \frac{1}{\cancel{2}} \times 4 \times (2)^{\cancel{2}} \frac{\cancel{R^2}}{\cancel{R^2}}$

$K.E_{rot} = 8J$

(5)



For a solid sphere rotating about it's natural axis

$k = \sqrt{\frac{2}{5}} R.$

$$V_{cm} = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$$

$$V^2 = \frac{2g \times 7}{1+\frac{(\sqrt{\frac{2}{5}}R)^2}{R^2}}$$

$$V^2 = 10g$$

$$K.E_{rot} = \frac{1}{2} m V^2 \frac{k^2}{R^2}$$

$$= \frac{1}{2}(1) 10g \; \frac{(\sqrt{\frac{2}{5}}R)^2}{R^2}$$

$$\Rightarrow 5g \times \frac{2}{5}$$

$$K.E_{rot} \Rightarrow 20 J$$

$$V^2 = \frac{2g \times 7}{1+\frac{\frac{2}{5}R^2}{R^2}}$$

$$= \frac{2g \times 7}{1+\frac{2}{5}}$$

$$\Rightarrow \frac{2g \times 7}{\frac{7}{5}}$$

(6) Given $a_{cm} = \frac{5g}{14}$; $\theta = ?$

For a solid sphere rotating about natural axis $k = \sqrt{\frac{2}{5}} R^2$

$$k^2 = \frac{2}{5} R^2$$

$$a_{cm} = \frac{g \sin\theta}{1+\frac{k^2}{R^2}} \Rightarrow \frac{g \sin\theta}{1+\frac{\frac{2}{5}R^2}{R^2}}$$

$$a_{cm} \; \frac{5g}{14} = \frac{g \sin\theta}{1+\frac{2}{5}}$$

$$\Rightarrow \frac{5}{14_2} = \frac{5 \sin\theta}{7}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

(8) For a solid cylinder

$$k^2 = \frac{1}{2} R^2.$$

$$\frac{k^2}{R^2} = \frac{1}{2}$$

$$V = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$$

$$\Rightarrow 14 = \sqrt{\frac{2 \times 9.8 \times h}{1+\frac{1}{2}}}$$

$$\Rightarrow 14^2 = \frac{2 \times 9.8 \times h}{\frac{3}{2}}$$

$$\Rightarrow 14^2 = \frac{4 \times 9.8}{3} h$$

By simplifying

$h = 15 m$

(9) For a Ring

$$k^2 = R^2$$

$$\frac{k^2}{R^2} = 1$$

$$t = \sqrt{\frac{2h}{g \sin^2\theta}}$$

For rolling down

$$t_d = \sqrt{\frac{2h(1+\frac{k^2}{R^2})}{g \sin^2\theta}}$$

$$= \sqrt{\frac{2h}{g \sin^2\theta}} \sqrt{1+\frac{k^2}{R^2}}$$

$$t_d = t \sqrt{1+\frac{R^2}{R^2}}$$

$$= t\sqrt{1+1}$$

$$\boxed{t_d = \sqrt{2}\, t}$$

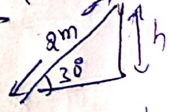
(15) For a solid sphere in rolling $k^2 = \frac{2}{5} R^2$

$$\frac{k^2}{R^2} = \frac{2}{5}$$

$$a_{cm} = \frac{g \sin\theta}{1+\frac{k^2}{R^2}} = \frac{g \sin 30}{1+\frac{2}{5}}$$

$$a_{cm} = \frac{g(\frac{1}{2})}{\frac{7}{5}} \Rightarrow \frac{5g}{14}$$

(16)



$$V_{cm} = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$$

$\sin 30 = \frac{h}{l}$

$h = l \sin 30$

$= 2 \times \frac{1}{2}$

$= 1 m$

$$V_{cm} = \sqrt{\frac{2g(1)}{1+\frac{2}{5}}}$$

$$V_{cm} = \sqrt{\frac{2g}{\frac{7}{5}}}$$

$$\Rightarrow \sqrt{\frac{10g}{7}} = \frac{g}{\sqrt{7}}$$

$[g = 10 m/s^2]$

For solid sphere $k^2 = \frac{2}{5} R^2$

$$= \frac{k^2}{R^2} = \frac{2}{5}$$

(17) For hollow sphere

$k^2 = \frac{2}{3} R^2$

$\Rightarrow \frac{k^2}{R^2} = \frac{2}{3}$

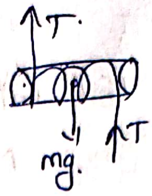
$K \cdot E_{rot} = \frac{1}{2} m v^2 \frac{k^2}{R^2}$

$K \cdot E_{tran} = \frac{1}{2} m v^2$

$\therefore \frac{K \cdot E_{rot}}{K \cdot E_{tran}} = \frac{\frac{1}{2} \cancel{m v^2} \frac{k^2}{R^2}}{\frac{1}{2} \cancel{m v^2}}$

$\frac{K \cdot E_{rot}}{K \cdot E_{tran}} = \frac{k^2}{R^2} = \frac{2}{3}$

(18)



$F_{net} = mg - T$

$\Rightarrow ma = mg - T$

$\Rightarrow ma = mg - \frac{ma}{2}$

$\Rightarrow mg = ma + \frac{ma}{2}$

$\Rightarrow \cancel{m} g = 3 \frac{\cancel{m} a}{2} \Rightarrow \frac{3a}{2} = g$

$\Rightarrow a = \frac{2g}{3}$

Torque acting on cylinder

$\tau = F \times R$

$I \alpha = T R$

$\Rightarrow \frac{M R^{\cancel{2}}}{2} \frac{a}{\cancel{R}} = T R$

$\Rightarrow T = \frac{ma}{2}$

LTask

(1) Ball is in the form of sphere so In rolling motion for a sphere

$k^2 = \frac{2}{5} R^2$ & Given $\theta = 30^\circ$

$\Rightarrow \frac{k^2}{R^2} = \frac{2}{5}$

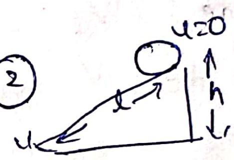
$a = \frac{g \sin\theta}{1 + \frac{k^2}{R^2}}$

$a_{cm} = \frac{g \sin 30}{1 + \frac{2}{5}}$

$= \frac{5g \times \frac{1}{2}}{\frac{7}{5}} = \frac{25}{7}$

$a_{cm} = 3.55 \ m/s^2$

(2)



For disc $k^2 = \frac{R^2}{2}$

$\frac{k^2}{R^2} = 2$

$R = 0.25m$; $M = 2kg$

$K \cdot E_{rot} = 4 J$

$\Rightarrow \frac{1}{2} m v^2 \frac{k^2}{R^2} = 4$

$\Rightarrow \frac{1}{\cancel{2}} \times \cancel{2} \times v^2 \times \frac{1}{\cancel{2}} = 4$

$\Rightarrow \frac{\cancel{2} v^2}{2} = 4$

$\Rightarrow v^2 = 2 \times 4$

$\Rightarrow v = 2\sqrt{2} \ m/s$

(3) For a spherical ~~sol~~ ball $k^2 = \frac{2}{5} R^2$

$\frac{k^2}{R^2} = \frac{2}{5}$

$\frac{K_{rot}}{K_{Total}} = \frac{k^2}{R^2 + k^2} = \frac{\frac{2}{5} R^2}{R^2 + \frac{2}{5} R^2} = \frac{\frac{2}{5} \cancel{R^2}}{\frac{7}{5} \cancel{R^2}}$

$= \frac{2}{7}$

(4)

$l = 6m$

For hollow sphere

$k^2 = \frac{2}{3} R^2 \Rightarrow \frac{k^2}{R^2} = \frac{2}{3}$

$V_{cm} = \sqrt{\frac{2gl \sin\theta}{1 + \frac{k^2}{R^2}}}$

$V_{cm} = \sqrt{\frac{2 g \times 6 \times \sin 30}{1 + \frac{2}{3}}} = \sqrt{\frac{2 g \times 6 \times \frac{1}{2}}{\frac{5}{3}}} = \sqrt{\frac{6g}{\frac{5}{3}}} = \sqrt{36} = 6 \ m/s$

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⑤ For Ring

$\Rightarrow k^2 = R^2$.

Given $K.E_{rot} = K.E_{tran}$

$\Rightarrow \frac{1}{2} M\cancel{v_{cm}^2}\frac{k^2}{R^2} = \frac{1}{2}\cancel{Mv_{cm}^2}$

$\Rightarrow \frac{k^2}{R^2} = 1$

$\Rightarrow k^2 = R^2$

$\Rightarrow \boxed{k = R}$ ring

⑥ Given:

$M_{Ring} = M_{Disc}$.

For Ring: $k^2 = R^2$, $\frac{k^2}{R^2} = 1$

For Disc: $k^2 = \frac{R^2}{2}$ $\Rightarrow \frac{k^2}{R^2} = \frac{1}{2}$.

$\therefore K.E_{roll} = \frac{1}{2}Mv^2\left(1+\frac{k^2}{R^2}\right)$; $K.E_{Disc} = \frac{1}{2}mv^2\left(1+\frac{k^2}{R^2}\right)$

$\Rightarrow K.E_{ring} = \frac{1}{2}Mv^2[1+1]$; $K.E_{Disc} = 4J\left[1+\frac{1}{2}\right]$

$\Rightarrow 4\cancel{8} = \frac{1}{2}mv^2 \times \cancel{2}$; $= \cancel{4}^2\left[\frac{3}{\cancel{2}}\right]$

$\Rightarrow \frac{1}{2}mv^2 = 4J$. ; $K.E_{Disc} = 6J$

⑦ For hollow sphere

$k^2 = \frac{2}{3}R^2$

$\Rightarrow \frac{k^2}{\cancel{p}}$

$\frac{K.E_{tran}}{K.E_{Tot}} = \frac{R^2}{R^2+k^2}$

$= \frac{R^2}{R^2+\frac{2}{3}R^2} = \frac{\cancel{R^2}}{\frac{5}{3}\cancel{R^2}}$

$= \frac{3}{\cancel{5}} \times \cancel{100}^{20} = 60\%$.

⑧ Given $h = \frac{3v^2}{4g}$

$v = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$

$\Rightarrow v^2 = \frac{2gh}{1+\frac{k^2}{R^2}}$

$\Rightarrow 1+\frac{k^2}{R^2} = \frac{2gh}{v^2}$

$\Rightarrow 1+\frac{k^2}{R^2} = \frac{\cancel{2g}}{\cancel{v^2}} \frac{3\cancel{v^2}}{_2 4\cancel{g}}$

$1+\frac{k^2}{R^2} = \frac{3}{2}$.

$\Rightarrow \frac{k^2}{R^2} = \frac{3}{2} - 1$

$\frac{k^2}{R^2} = \frac{1}{2}$

$\Rightarrow k^2 = \frac{R^2}{2}$

$\Rightarrow$ Disc

⑨ For a disc $k^2 = \frac{1}{2}R^2$

$r = 2m$, $M = 100kg$; $v_{cm} = 20\ cm/s$
$= 20\times10^{-2}\ m/s$

$K.E_{roll} = \frac{1}{2}Mv^2\left[1+\frac{k^2}{R^2}\right]$

$= \frac{1}{2}\times100\times(20\times10^{-2})^2\left[1+\frac{1}{2}\right]$

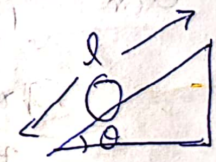
$= \frac{1}{2}\times100\times400\times10^{-4}\times\frac{3}{2}$

$K.E_{roll} = \frac{3}{4}\times4 = 3J$.

⑩ $m = 2kg$; $r = 50cm$
$= 50\times10^{-2}m$

$\theta = 30°$

$v_{cm} = 4\ m/s$



For a cylinder $k^2 = \frac{R^2}{2}$ $\Rightarrow \frac{k^2}{R^2} = \frac{1}{2}$

$v = \sqrt{\frac{2gl\sin\theta}{1+\frac{k^2}{R^2}}}$

$\Rightarrow 4 = \sqrt{\frac{2gl\sin30}{1+\frac{1}{2}}}$

$\Rightarrow 4^2 = \frac{\cancel{2}gl\times\frac{1}{\cancel{2}}}{\frac{3}{2}}$

$\Rightarrow 16 = \frac{gl}{\frac{3}{2}} = \frac{2gl}{3}$

$\Rightarrow l = \frac{3\times\cancel{16}^8}{\cancel{2}\times g} = \frac{3\times8}{g}$

$= \frac{24}{10} = 2.4m$.

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(16) (i) For solid sphere $k^2 = \frac{2}{5}R^2$

$\Rightarrow \frac{k^2}{R^2} = \frac{2}{5}$.

$K.E_{rot} = 40J$

$\therefore \cancel{K.E_{rot}}\ \frac{1}{2}mv^2 \frac{k^2}{R^2} = 40J$

$\Rightarrow \frac{1}{2}mv^2 \frac{2}{5} = 40$

$\Rightarrow \frac{1}{2}mv^2 = 40 \times \frac{5}{2}$

$\Rightarrow \cancel{K.E}\ \frac{1}{2}mv^2 = 100J$.

(ii) $K.E_{tot} = \frac{1}{2}mv^2\left[1+\frac{k^2}{R^2}\right]$

$= 100\left[1+\frac{2}{5}\right] = 100 \times \frac{7}{5}$

$K.E_{tot} = 140J$

(iii) $\frac{K.E_{rot}}{K.E_{tran}} = \frac{\frac{1}{2}\cancel{mv^2}\frac{k^2}{R^2}}{\frac{1}{2}\cancel{mv^2}} = \frac{k^2}{R^2} = \frac{2}{5}$

(17) For solid disc

$k^2 = \frac{1}{2}R^2$

$\frac{k_D^2}{R_D^2} = \frac{1}{2}$

For Ring

$k_R^2 = R_R^2$

$\frac{k_R^2}{R_R^2} = 1$.

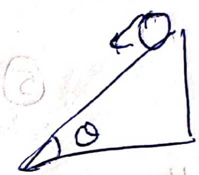
Given $M_R = M_D$; $R_D = R_R$; $V_D = V_R$

$\therefore \frac{K.E_{Ring}}{K.E_{Disc}} = \frac{\frac{1}{2}\cancel{M_R V_R^2}\left[1+\frac{k^2}{R^2}\right]_R}{\frac{1}{2}\cancel{M_D V_D^2}\left[1+\frac{k^2}{R^2}\right]_D}$

$= \frac{1+1}{1+\frac{1}{2}} = \frac{2}{\frac{3}{2}}$

$\frac{K.E_R}{K.E_D} = \frac{4}{3}$.

(18)



$v' = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$

$v' = ?$

$v = \sqrt{2gh}$

For Ring $k^2 = R^2 \Rightarrow \frac{k^2}{R^2} = 1$

$\Rightarrow v' = \sqrt{\frac{2gh}{1+1}} = \sqrt{\frac{2gh}{2}}$

$v' = \frac{v}{\sqrt{2}}$.