9th 8th advanced ws-1

8th foundation ①

I Task

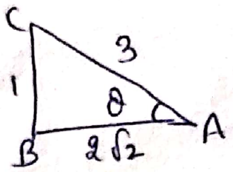
① Given $\sin\theta = \frac{1}{2}$

i.e. we know that $\sin 30^\circ = \frac{1}{2}$

$\therefore \theta = 30^\circ$

$\Rightarrow \operatorname{cosec}\theta = \operatorname{cosec} 30^\circ = 2$

② Given $\sin\theta = \frac{1}{3}$

From 

Acc to Pythagoraus theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow 3^2 = AB^2 + 1$

$\Rightarrow 9 = AB^2 + 1 \Rightarrow AB^2 = 9 - 1 = 8$

$\Rightarrow AB = \sqrt{8}$ or $2\sqrt{2}$

$\therefore \tan\theta = \frac{\text{opp side}}{\text{adj side}}$

$= \frac{BC}{AB} \Rightarrow \tan\theta = \frac{1}{2\sqrt{2}}$

④ Given $\sec\theta = \frac{2}{\sqrt{3}}$

$\sin 30^\circ = \frac{1}{2}$; $\cos 30^\circ = \frac{\sqrt{3}}{2}$

we know that $\sec 30^\circ = \frac{2}{\sqrt{3}}$; $\tan 30^\circ = \frac{1}{\sqrt{3}}$; $\cot 30^\circ = \sqrt{3}$

$\Rightarrow \theta = 30^\circ$

⑥ Given $5\tan^2 A - 5\sec^2 A + 1 = ?$

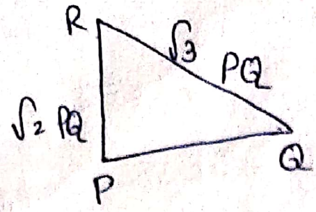
we know that $\sec^2 A - \tan^2 A = 1$

$\therefore$ From the given equation $5\tan^2 A - 5\sec^2 A + 1$

$\Rightarrow 5(\tan^2 A - \sec^2 A) + 1$

$\Rightarrow 5(-1) + 1 = -5 + 1 = -4$.

⑤



From Pythagoras theorem

$QR^2 = PQ^2 + PR^2$

$\Rightarrow (\sqrt{3}\,PQ)^2 = PQ^2 + PR^2$

$\Rightarrow 3PQ^2 = PQ^2 + PR^2$

$\Rightarrow 3PQ^2 - PQ^2 = PR^2$

$\Rightarrow PR^2 = 2PQ^2$

$\Rightarrow PR = \sqrt{2}\,PQ$

$\therefore \sin Q = \frac{\text{opp side}}{\text{hyp}}$

$= \frac{PR}{RQ}$

$\sin\theta = \frac{\sqrt{2}\,PQ}{\sqrt{3}\,PQ} = \sqrt{\frac{2}{3}}$

$\operatorname{cosec} Q = \frac{1}{\sin Q}$

$\Rightarrow \operatorname{cosec} Q = \frac{1}{\sqrt{\frac{2}{3}}}$

$= \sqrt{\frac{3}{2}}$

$\cos Q = \frac{\text{adj side}}{\text{hyp}} = \frac{PQ}{RQ}$

$\Rightarrow \cos Q = \frac{PQ}{\sqrt{3}\,PQ} = \frac{1}{\sqrt{3}}$

and

$\cot Q = \frac{\text{adj side}}{\text{opp side}} = \frac{PQ}{PR}$

$\cot Q = \frac{PQ}{\sqrt{2}\,PQ} = \frac{1}{\sqrt{2}}$.

⑦

Given $\cos(\alpha+\beta) = 0$,

we know $\cos 90^\circ = 0$

$\therefore \alpha + \beta = 90^\circ$

$\Rightarrow \alpha = 90^\circ - \beta$

$\therefore \sin(\alpha - \beta) = \sin(90 - \beta - \beta)$

$= \sin(90 - 2\beta)$

$\Rightarrow \cos 2\beta$

⑧

Given $\cos\theta - \sin\theta = 0$

$\Rightarrow \cos\theta = \sin\theta$

$\Rightarrow \frac{\cos\theta}{\sin\theta} = 1$

$\Rightarrow \cot\theta = 1$

$\Rightarrow \theta = 45^\circ$

Now we have to find

$(\sin^4\theta + \cos^4\theta) = ?$

$\Rightarrow (\sin 45^\circ)^4 + (\cos 45^\circ)^4$

$\Rightarrow [(\frac{1}{\sqrt{2}})^4 + (\frac{1}{\sqrt{2}})^4]$

$\Rightarrow \frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$

⑨ Given $\sin x + \sin^2 x = 1$.

From here $\sin x = 1 - \sin^2 x \rightarrow ①$

we know $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$.

$\therefore$ From ① $\sin x = \cos^2 x$

: Now find the value of

$\cos^2 x + \cos^4 x$

$\Rightarrow \cos^2 x + (\cos^2 x)^2$

$\Rightarrow \sin x + (\sin x)^2$

$= \sin x + \sin^2 x$

$= 1$.

⑩ we have to find out the value of

$\cot\theta - \tan\theta$

$\Rightarrow \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} \Rightarrow \frac{\cos^2\theta - \sin^2\theta}{\cos\theta \sin\theta}$ $\left[\begin{array}{l}\cos^2\theta + \sin^2\theta = 1 \\ \sin^2\theta = 1 - \cos^2\theta\end{array}\right]$

$\Rightarrow \frac{\cos^2\theta - (1 - \cos^2\theta)}{\cos\theta \sin\theta} = \frac{\cos^2\theta - 1 + \cos^2\theta}{\sin\theta \cos\theta}$

$\Rightarrow \frac{2\cos^2\theta - 1}{\sin\theta \cos\theta}$.

⑪ (A) From $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$.

(B) $1 + \frac{\sin^2\theta}{\cos^2\theta} = \sec^2\theta$

L.H.S

$\Rightarrow 1 + \left(\frac{\sin\theta}{\cos\theta}\right)^2$

$\Rightarrow 1 + \tan^2\theta$

$= \sec^2\theta =$ R.H.S

(C) we know that

$\sin^2\theta + \cos^2\theta = 1 \rightarrow$ L.H.S

R.H.S $\rightarrow \sec^2\theta \times \cos^2\theta$

$\Rightarrow \frac{1}{\cancel{\cos^2\theta}} \times \cancel{\cos^2\theta}$

$= 1$ $\therefore$ L.H.S = R.H.S

(D) R.H.S $\Rightarrow \frac{\sec^2\theta}{\cos^2\theta} \Rightarrow \sec^2\theta \times \frac{1}{\cos^2\theta}$

$= \sec^2\theta \times \sec^2\theta = \sec^4\theta$

L.H.S $\neq$ R.H.S (or) $\frac{1}{\cos^4\theta}$