INTEGRATED +

Class: 9th. MATHEMATICS

6. STATISTICS

TEACHING TARK (JEE Mains)

01 $\frac{a+b+c+d+e}{5} = 30$

$\Rightarrow a+b+c+d+e = 150 \rightarrow ①$

$\frac{a+b+c+d}{4} = 28$

$a+b+c+d = 112 \rightarrow ②$

Solving ① & ②

$\Rightarrow 112 + e = 150$

$\Rightarrow e = 38$

Ans. B

02 Combined mean $= \frac{m\bar{x} + n\bar{y}}{m+n}$

$= \frac{6\times12 + 4\times18}{6+4} = \frac{144}{10} = 14.4$

Ans: B

03 $\text{mean} = \frac{8\times20 + 12\times30 + 20\times40 + 10\times50 + 6\times60 + 4\times70}{8+12+20+10+6+4}$

$= 41$

Ans: A

04 from table

$17 + f_1 + 32 + f_2 + 19 = 120$

$\Rightarrow f_1 + f_2 = 52$

optimA: $28 + 24 = 52$

Ans. A

05

Class	Frequency
0–10	5
10–20	15
20–30	20
30–40	$20 \to f_0$
$l \leftarrow$ 40–50	$32 \to f_1$
50–60	$14 \to f_2$
60–70	4

$$\text{mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 40 + \frac{32 - 20}{2.32 - 20 - 14} \times 10$$

$$= 44$$

Ans: D

06 mode

Ans: C

07

$$\frac{x_1 + x_2 + \cdots + x_{10}}{10} = 20$$

$$\Rightarrow x_1 + x_2 + \cdots + x_{10} = 200$$

Now $\Rightarrow x_1 + x_2 + \cdots + x_{10} + 30 = 230.$

$$\text{New mean} = \frac{230}{11} = 20.90$$

Ans. A

08 Total Students = 100

Boys = 70 $\to \bar{B} = 75$

Girls = 30 $\to \bar{G} = ?$

Combined $\bar{C} = 72$

we know, combined mean.

$$\bar{C} = \frac{m\bar{B} + n \cdot \bar{G}}{m + n}$$

$$72 = \frac{70 \times 75 + 30 \cdot \bar{G}}{100}$$

$$\Rightarrow \bar{G} = 65$$

Ans. B

09. Combined mean

$$45 = \frac{100 \times 70 + 100 \times 20 + 100 \times \bar{x}}{300}$$

$$\Rightarrow \bar{x} = 45$$

Ans: A

10 No mode

Ans D

JEE ADVANCED

11. $A.M = \frac{6 \times 4 + 5 \times 2 + 8 \times 5 + 3 \times 1}{4+2+5+1}$

$= 6.42$

Ans: C

12. mean = 6, No. of observations = 10

$\therefore$ mean = $\frac{\text{sum}}{\text{No. of obses}}$

$6 = \frac{\text{sum}}{10} \Rightarrow$ Incorrect sum = 60.

~~we have, correct sum = 70~~

Correct sum = Incorrect sum − Incorrect observation + Correct observation

$\therefore 70 = 60 - 9 +$ Correct observation

$\Rightarrow$ Correct observation = 19

Ans: A

13. Statement I: Conceptual (True)

Statement II: Conceptual (True)

Ans A

$2.10 - (4+6)$

$= 10.5 + 3$

$= 13.5$

Ans: C

19. Mode = 8

Ans: B

14. Statement I: ~~$\sum n = n(n+1)(2n+1)(3n+2)$~~.

Verification technique.

Take one term: $1 \cdot 3 \cdot 5 = 1 \times 3 \times 5 = \underline{15}$

put $n=1$ in $3n^3 + 6n^2 + 7n - 2$

$= 3(1)^3 + 6(1)^2 + 7(1) - 2 = \underline{13}$.

Hence, the given statement is false

Statement II: Conceptual (True)

Ans: D

15 Assertion: We have mean $\leq$ median $\leq$ mode (True)

Reason: Conceptual (True)

Ans: A

16 Assertion: Conceptual (True)

Reason: Conceptual (False)

Ans: C

17 $$\text{mode} = l + \frac{f - f_1}{2f - (f_1 + f_2)} \times C$$

$$= 3499.5 + \frac{37 - 35}{2 \times 37 - (35 + 29)} \times 1500$$

$$= 3799.5$$

Ans: A

18 from the table $l = 10.5, f = 10, f_1 = 4, f_2 = 6$

$C = 5$

$$\text{mode} = 10.5 + \frac{10 - 4}{2 \cdot 10 - (4 + 6)} \times 5$$

$$= 10.5 + 3$$

$$= 13.5$$

Ans: C

19. mode = 8

Ans: B

20 Conceptual (5) Ans: D

21. New mean = 20 + 5 = 25 Ans. 25

22.

Class Interval	mid value x_i	frequency f_i	$f_i x_i$
0–10	5	8	40
10–20	15	4	60
20–30	25	6	150
30–40	35	3	105
40–50	45	4	180
		25	535

$\text{mean} = \frac{535}{25}$

$= 21.4$

Ans: 21

23. i) $\text{mean} = \frac{(x+5)+(x+6)+(x+10)+(x+11)+(x+14)+(x+20)}{6}$

$= \frac{6x+66}{6} = x+11$ (q)

ii) $\bar{x} = \frac{20\times 8 + 30\times 10}{20+30} = 9.2$ (t)

iii) Let the No. of Rays be x, No. of sixth be y

Given $15 = \frac{x\times 14 + y\times 17}{x+y} \Rightarrow \frac{x}{y} = \frac{2}{1}$

$\Rightarrow x : y = 2 : 1$ (p)

iv) $\frac{(x+2)+(2x+3)+(3x+4)+(4x+5)}{4} = x+2$

$\Rightarrow x = -1$ (r)

Ans: q, t, p, r

24 i) $\text{mean} = \dfrac{\left(\frac{a+b}{2}\right)+\left(\frac{a-b}{2}\right)}{2} = \dfrac{a}{2}$ (q)

ii) Sum of observations $= 12 \times 15 = 180$

New sum $= 180 - 20 - 25 = 135$

New mean $= \dfrac{135}{10} = 13.5$ (r)

iii) Sum of first n odd natural numbers $= n^2$

$$\text{mean} = \frac{n^2}{n} = n$$

Given $n = \dfrac{n^2}{81} \Rightarrow n = 81$ (p)

iv) No. of students $= 50$.

let No. of Boys $= x$

$\therefore$ No. of girls $= 50 - x$

Given $$76 = \frac{x \times 70 + (50-x) \times 80}{50}$$

$$\Rightarrow y = 20 \text{ (t)}$$

Ans: q, r, p, t

LEARNERS TASK (CUQ's)

01.	10	Ans: A
02	Conceptual	Ans: A
03	Mode is the one of the observation value, so it must lie within the data range	Ans: B
04	Mode	Ans: C
05	Mean	Ans: A
06	mean $= 5x + 1$	Ans: C

07 Given $\frac{a+b+c}{3} = b$ $\Rightarrow a+c = 2b$

$\Rightarrow a+b+c = 3b$ Ans: C

08 $\frac{2+4+6+8+x+y}{6} = 5$

$\Rightarrow x+y = 10$ Ans: A

09 We know, mean of first n natural number $= \frac{n+1}{2}$

$\Rightarrow \frac{n+1}{2} = \frac{5n}{9}$ $\quad n = 9$

$\Rightarrow 9n+9 = 10n$ Ans: C

10 Does not exists Ans: D

JEE MAINS

01. We know $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$

$\therefore$ mean $= \frac{\left[\frac{n(n+1)(2n+1)}{6}\right]}{n} = \frac{(n+1)(2n+1)}{6}$ Ans: B

02 $\frac{20+30}{2} = 25$ Ans: B

03 total sum $= 5 \times 18 = 90$ Ans: A

04 Wrong sum $= 20 \times 15 = 300$

Correct sum $= 300 - 3 - 6 + 8 + 4$

$= 303$

Correct mean $= \frac{303}{20} = 15.15$ Ans: B

05 Mean increases Ans: A

06 Original Observations

$15-3, 15+0, 15-2, 15+4, 15+6, 15+1, 15+1$

$= 12, 15, 13, 19, 21, 16, 16$

$\text{mean} = \frac{12+15+13+19+21+16+16}{7} = 16$ Ans: C

07 5 Ans: D

08 10 students got 6 marks each.

i.e ~~Hence,~~ 6 is repeated 10 times

Hence, mode = 6 Ans: A

09 From table

$17 + f_1 + 32 + f_2 + 19 = 120$

$\Rightarrow f_1 + f_2 = 52$

28, 24 satisfies above eqn Ans: A

10. Not defined Ans: C

JEE ADVANCED

11. $\frac{x_1 + x_2 + \cdots + x_n}{n} = M$

$\Rightarrow x_1 + x_2 + \cdots + x_n = Mn$

$\Rightarrow (x_1 + x_2 + x_3 + x_4) + a = Mn$

$\Rightarrow x_1 + x_2 + x_3 + x_4 = Mn - a$

$\therefore \text{Mean} = \frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{Mn - a}{4} = \frac{Mn}{4} - \frac{a}{4}$

Ans: C, D

(4)

12 A) mean $= \frac{6+8+10+12+14}{5} = 10$ ✓

B) Conceptual ✓

C) New mean = 20 ✗

Ans A, B

13 Statement I: mode = 8 (True)

Statement II: conceptual (True)

Ans.. A

14 Statement I: mean $= \frac{5+10+15}{3} = \frac{30}{3} = 10$ (True)

Statement II: conceptual (True)

Ans.. A

15 Assertion: Conceptual (False)

Reason: conceptual (True)

Ans: D

16. Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans A

17. mean $= \frac{6+8+10+12+14}{5} = \frac{50}{5} = 10$

Ans. A

18 New mean = 10 + 2 = 12

Ans: C

19. $l = 3499.5$, $f_1 = 37$, $f_0 = 35$, $f_2 = 21$, $h = 1500$.

$$\text{mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 3499.5 + \frac{37-35}{2\times37-35-21} \times 1500$$

$$= 3499.5 + 166.67$$

$$= 3666.17$$

Ans: —

20 Form table (iv)

$l = 10.5, \quad f_0 = 4, \quad f_1 = 10, \quad f_2 = 6, \quad h = 5$

$$\text{mode} = 10.5 + \frac{10-4}{2\times10-4-6} \times 5$$

$$= 10.5 + \frac{6}{10-2} \times 5 \qquad = 10.5 + 3 = 13.5$$

Ans. A

21. $$100 = \frac{30\times120 + 20\times\bar{x}}{50}$$

$$\Rightarrow \bar{x} = 70$$

Ans: 70

22 $\bar{x} = 25$

Ans: 25

23 i) $$\frac{148+146+144+\cdots \text{ n term}}{n} = 125$$

$$\Rightarrow 148 + 146 + 144 + \cdots \text{ n term} = 125n$$

$$\Rightarrow \frac{n}{2}\left[2\times148 + (n-1)\times(-2)\right] = 125n$$

$$= n(148 - n + 1) = 1 \qquad \Rightarrow n = 24 \ (r)$$

ii) The number i repeated i times

i.e. 1 repeats 1 time $= 1$

2 repeats 2 times $= 2+2 = 2^2$

3 repeats 3 times $= 3+3+3 = 3^2$

i repeats n times $= n^2$

$$\text{mean} = \frac{1^2+2^2+3^2+\cdots+n^2}{1+2+3+\cdots+n} = \frac{\left[\frac{n(n+1)(2n+1)}{6}\right]}{\left(\frac{n(n+1)}{2}\right)}$$

$$= \frac{2n+1}{3} \ (P)$$

iii)

$$\text{mean} = \frac{1(1^2+1) + 2(2^2+2) + 3(3^2+3) + \cdots + n(n^2+n)}{(1^2+1) + (2^2+2) + \cdots + (n^2+n)}$$

$$= \frac{(1^3+2^3+\cdots+n^3) + (1^2+2^2+3^2+\cdots+n^2)}{(1^2+2^2+\cdots+n^2) + (1+2+3+\cdots+n)}$$

$$= \frac{\frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}$$

$$= \frac{\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{2n+1}{3}\right]}{\frac{n(n+1)}{2}\left[\frac{2n+1}{3} + 1\right]}$$

$$= \frac{3n(n+1)+2(2n+1)}{6} \times \frac{3}{(2n+1+3)}$$

$$= \frac{3n^2+7n+2}{2} \times \frac{1}{2n+4}$$

$$= \frac{(3n+1)(n+2)}{2 \cdot 2(n+2)} = \frac{3n+1}{4} \quad (S)$$

Short cut method

Take two terms and find their mean

$$\text{mean} = \frac{1(1^2+1)+2(2^2+2)}{(1^2+1)+(2^2+2)} = \frac{7}{4} \checkmark$$

put $n=2$ in $\frac{3n+1}{4} = \frac{7}{4}$ ✓

(iv) Data consists of six 6's.

∴ 6 repeats 6 times.

Hence, mode = 6 (Q)

Ans: R, P, S, Q

24 i) Sum of the observation = $10 \times 16.3 = 163$

Correct sum = $163 + 23 - 32 = 154$

∴ Correct mean = $\frac{154}{10} = 15.4$ (P)

(ii) $\frac{x_1 + x_2 + \cdots + x_n}{n} = \bar{x}$

New observations $(x_1+2), (x_2+2), \cdots (x_n+2)$

$$\text{New mean} = \frac{(x_1+2)+(x_2+2)+\cdots+(x_n+2)}{n}$$

$$= \left(\frac{x_1+x_2+\cdots+x_n}{n}\right) + \left(\frac{2+2+\cdots+2}{n}\right)$$

$$= \bar{x} + \frac{2n}{n} = \bar{x} + 2$$

iii)

iii) Given $\frac{x_1 + x_2 + \cdots + x_{10}}{10} = 20$

$\Rightarrow x_1 + x_2 + \cdots + x_{10} = 200$

New mean $= \frac{(x_1+4)+(x_2+8)+(x_3+12)+\cdots+(x_{10}+40)}{10}$

$= \frac{(x_1 + x_2 + \cdots x_{10}) + (4+8+12+\cdots+40)}{10}$

$= \frac{x_1 + x_2 + \cdots x_{10}}{10} + \frac{4(1+2+3+\cdots+10)}{10}$

$= 20 + \frac{4}{10} \times \frac{10 \times 11}{2}$

$= 20 + 22 = 42$ (r)

iv) modes are → 5, 7, 6

∴ No. of modes = 3 (s)

Ans: P, ~~Q~~, S

⇒ THE END ⇐