# QUADRATIC EXPRESSION - II

## TEACHING TASK
## JEE MAINS LEVEL

1. Given $-5x^2+2x+3$ here $-5<0$, the above expression will have maximum value

Maximum value $= \dfrac{4ac-b^2}{4a}$

$$= \frac{4(-5)3-2^2}{4(-5)}$$

$$= \frac{16}{5}$$

Ans : A

2. The maximum value of $= \dfrac{1}{4x^2+2x+1}$

$$= \frac{1}{\text{minimum value of } 4x^2+2x+1}$$

$$= \frac{1}{\left(\dfrac{4.4.4-2^2}{4.4}\right)}$$



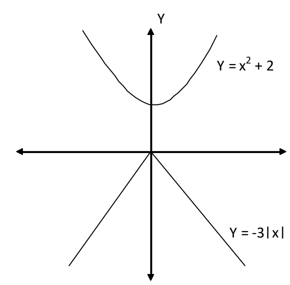
$$= \frac{4}{3}$$

Ans: A

3. Given $x^2+3|x|+2=0$

$\Rightarrow x^2+2= -3|x|$

Clearly the curves $y = x^2+2$ and
$y = -3|x|$ do not touch each other
therefore $x^2+3|x|+2 = 0$ do not have any real solutions.




Ans: B

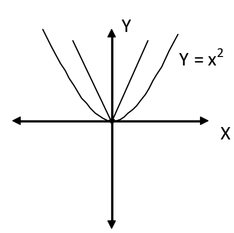
4. Given $x^2-2|x|= 0$ ,

$\Rightarrow x^2 = 2|x|$

clearly the curves

$y = x^2$ and $y = 2|x|$ meets at (0,0)

$\therefore$ $x^2-2|x|=0$ has one solution.




Ans: A

5. Given y = $-x^2+2x+3$ here a = -1<0 and $\Delta = b^2 - 4ac$

$= (2)^2-4(-1)(3) = 16>0$

$\therefore$ The graph of y = $-x^2+2x+3$ is open down word and intersects the x-axis at two distinct points.

Ans :D

6. y = $x^2+2x+3$

The minimum value occurs at $x = \frac{-b}{2a}$

$= \frac{-2}{2.1}$

$= -1$

Ans : B

7. a,b are the roots of $x^2+ax+b=0$

we have a+b = -a and a.b = b

$\Rightarrow$ 2a+b = 0 and a =1

$\Rightarrow$ 2.1+b = 0

$\Rightarrow$ b = -2

Now, $x^2+ax+b$

The least value $= \frac{4AC - B^2}{4A}$

$= \frac{4.1.b - a^2}{4.1}$

$= \frac{4(-2) - 1^2}{4.1}$

$= \frac{-9}{4}$

Ans : B

8. we have $\alpha + \beta = a - 2$

and $\alpha\beta = -(a+1)$

now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (a-2)^2 - 2(-(a+1))$

$= (a-2)^2 + 2(a+1)$

$= a^2 - 2a + 6$

The least value $= \frac{4.1.6 - 4}{4.1}$

$= 5$

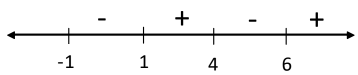
Ans : C

9. $\frac{14x}{x+1}-\frac{9x-30}{x-4}<0$

$$\Rightarrow \frac{14x(x-4)-(x+1)(9x-30)}{(x+1)(x-4)}<0$$

$$\Rightarrow \frac{x^2-7x+6}{(x+1)(x-4)}<0$$

$$\Rightarrow \frac{(x-1)(x-6)}{(x+1)(x-4)}<0$$




Solution set $=(-1,1)\cup(4,6)$

Ans : D

10. f(x) = $x^2+2bx+2c^2$

Minimum value $=\frac{4.1.2c^2-(2b)^2}{4.1}$

$=\frac{8c^2-4b^2}{4}$

$=2c^2-b^2$

$g(x)=-x^2-2cx+2^2$

maximum value $=\frac{4.(-1).b^2-(-2c)^2}{4(-1)}$

$=b^2+c^2$

Given min {f(x)} > max {g(x)}

$\Rightarrow 2c^2-b^2>b^2+c^2$

$\Rightarrow c^2>2b^2$

$\Rightarrow |c|>\left|\sqrt{2}\right|b$

Ans : B

11. Given y = $-x^2+3x+4$
Since a = -1<0, it is open downward parabola.

Ans : A,B

12. $\left(3+2\sqrt{2}\right)^{x^2-4}+\left(3-2\sqrt{2}\right)^{x^2-4}=6$

Here $\left(3+2\sqrt{2}\right)+\left(3-2\sqrt{2}\right)=6$ and $\left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right)=9-8=1$

We have $x^2-4=\pm 1$

$\Rightarrow x^2-4=1$ and $x^2-4=-1$

$\Rightarrow x^2=5$ and $x^2=3$

$\Rightarrow x=\pm\sqrt{5}$ and $x=\pm\sqrt{3}$

Ans : A,B,C,D

13 **Statement : I**

Given y = $-x^2+3x+2$

Max. value $=\dfrac{4ac-b^2}{4a}$

$=\dfrac{4(-1)(2)-3^2}{4(-1)}$

$=\dfrac{17}{4}$

Hence statement – I is true.



**Statement – II :** The maximum value of y = $ax^2+bx+c$, where a<0 is $\dfrac{4ac-b^2}{4a}$

Hence, statement –II is the best explanation of statement –I.

Ans : A

14. **Statement – I**

Given $\left(\log_5^x\right)^2+\log_5^x<2$

Let $\log_5^x=t$

$\Rightarrow t^2+t-2<0$

$\Rightarrow (t+2)(t-1)<0$

$\Rightarrow\ -2<t<1$

$\Rightarrow\ -2<\log_5^x<1$

$\Rightarrow\ 5^{-2}<x<5^1$

$\Rightarrow\ \dfrac{1}{25}<x<5$

Hence, statement – I is TRUE.

**Statement – II:**
Clearly, the statement – II is TRUE.
Also, the statement – II is the best explanation of statement - I

Ans: A

15. Let $y = \dfrac{x^2 - 6x + 5}{x^2 + 2x + 1}$

$\Rightarrow y x^2 + 2yx + y - x^2 + 6x - 5 = 0$

$\Rightarrow (y-1)x^2 + (2y+6)x + y - 5 = 0$

since, $x \in R,\ \Delta \geq 0$

$(2y+6)^2 - 4(y-1)(y-5) \geq 0$

$\Rightarrow 3y + 1 \geq 0$

$\Rightarrow y \geq -\dfrac{1}{3}$

$\therefore$ The laeast value $= \dfrac{-1}{3}$

Ans: D

16. $\dfrac{A}{B} = \dfrac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \dfrac{\left(x - \frac{1}{x}\right)^2 + 2}{x - \frac{1}{x}}$

$= \left(x - \dfrac{1}{x}\right) + \dfrac{2}{\left(x - \frac{1}{x}\right)}$

$= P + \dfrac{2}{P} \geq 2\sqrt{2}$ where $p = x - \dfrac{1}{x}$

since $A.M \geq G.M$

$\therefore$ The least value $2\sqrt{2}$

Ans: D

17. Given $2^{x^2-5x+8} + 3^{x^2-5x+8} = 13$

$\Rightarrow 2^{x^2-5x+8} + 3^{x^2-5x+8} = 2^2 + 3^2$

We have $x^2 - 5x + 8 = 2$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow (x-2)(x-3) = 0$

$\Rightarrow x = 2 \text{ or } 3$

Ans : A

18. Given $\left(2\sqrt{2}\right)^{x^2-4x+6}+\left(3\sqrt{2}\right)^{x^2-4x+6}=26$

$$\Rightarrow\left(2\sqrt{2}\right)^{x^2-4x+6}+\left(3\sqrt{2}\right)^{x^2-4x+6}=\left(2\sqrt{2}\right)^2+\left(3\sqrt{3}\right)^2$$

$$\Rightarrow x^2-4x+6=2$$

$$\Rightarrow x^2-4x+4=0$$

$$\Rightarrow x=2$$

Ans : D

19. $(x-19)(x-97)-p=(x-\alpha)(x-\beta)$

$$\Rightarrow(x-\alpha)(x-\beta)+p=(x-19)(x-97)$$

$\therefore$ The roots are 19 and 97

Ans : 19

20. Given $x=\dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}\text{x}\dfrac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

$$=\frac{7+3+2\sqrt{21}}{7-3}$$

$$=\frac{10+2\sqrt{21}}{4}$$

$$=\frac{5+\sqrt{21}}{2}$$

Similarly $y=\dfrac{5-\sqrt{21}}{2}$

We have x+y = 5 and xy = 1

Now, $x^4+y^4+(x+y)^4$

$$=\left(x^2+y^2\right)^2-2(xy)^2+(x+y)^4$$

$$=\left[(x+y)^2-2xy\right]^2-2(xy)^2+(x+y)^4$$

$$=(25-2)^2-2+5^4=1152$$

Ans : 1152

21.a) $y = x^2+3x+4$

here a = 1 > 0

It is a open upward parabola.

b) $y = -x^2+x+2$

here a = -1 < 0

$\therefore$ It is a opendown word parabola

c) $y = x^2+5x+6$

min. value $= \frac{4.1.6-5^2}{4.1}$

$= \frac{-1}{4}$

d) $y = -x^2-5x+6$

max. value $= \frac{4(-1)6-(-5)^2}{4(-1)}$

$= \frac{49}{4}$

Ans : a-t, b-s, c-r, d-q

22.a) $x^2-5x+6 < 0$

$\Rightarrow (x-2)(x-3) < 0$

$\Rightarrow 2 < x < 3$

b) $x^2 + x - 12 > 0$

$\Rightarrow (x+4)(x-3) > 0$

$\Rightarrow x < -4 \text{ or } x > 3$

c) $x^2+9x+18 < 0$

$\Rightarrow (x+3)(x+6) < 0$

$\Rightarrow -6 < x < -3$

d) $x^2-x-6 > 0$

$\Rightarrow (x-3)(x+2) > 0$

$\Rightarrow x < -2, \text{ or } x > 3$

Ans : a-p, b-r, c-t, d-q



## LEARNER'S TASK

**CUQ'S :**

1. The graph lies entirely above the x-axis.

Ans : B

2. The graph touches the x-axis and lies above it

Ans : B

3. The graph lies below the x-axis

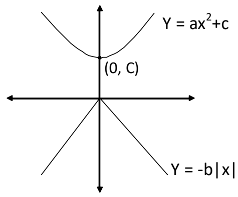
Ans : A

4. The graph cuts the x-axis in two real points.

Ans : A

5. Given $ax^2+b|x|+c = 0$

$\Rightarrow ax^2 + c = -b|x|$




Both the graphs do not truch each other. There fore number of real roots = 0.

Ans : C

6. $(x-\alpha)(x-\beta)<0$

$\Rightarrow \alpha<x<\beta$

Ans : B

7. Given $x^2-6x+5$

Least value $= \dfrac{4.1.5-(-6)^2}{4.1}$

= -4

Ans : A

8. Quadratic inepuation has infinitely many solutions.

Ans : C

9. Maximum value of $f(x) = \dfrac{1}{\text{minimum value of } f(x)}$

Ans : D

10. Extreem point of $y = px^2 + qr + r$ is $\left(\dfrac{-q}{2p}, \dfrac{4pr-q^2}{4p}\right)$

Ans : A

**JEE MAINS LEVEL QUESTIONS**

1. y = (x-2)(x-3)

= $x^2$-5x+6

The graph is a parabola

Ans :B

2. y = $x^2$-2x+3

Minimum value occurs at $x = \dfrac{-b}{2a}$

$$= \frac{-(-2)}{2.1}$$

$$= 1$$

Ans : C

3. y = $-x^2+2x+3$

here a = -1 < 0, y has maximum value

Maximum value $= \frac{4ac-b^2}{4a}$

$$= \frac{4.(-1).3-2^2}{4.(-1)}$$

= 4

Ans : C

4. y = $x^2-4x+3$

since a = 1 > 0 and

$$\Delta = (-4)^2 - 4.1.3$$

= 4 > 0

The graph is open upward and intersects the x-axis at two distinct points

Ans : B

5. y = $-x^2+3x+4$

since a = -1 < 0 and

$$\Delta = (3)^2 - 4(-1).4$$

= 25 > 0

The graph is open downward and inter sects the x-axis at two distinct points

Ans : A

6. y = $16x^2+8(a+5)x-7a-5$

We have $\Delta = b^2 - 4ac < 0$

$$\Rightarrow [8(a+5)]^2 - 4.16.(-7a-5) < 0$$

$$\Rightarrow a^2 + 17a + 30 < 0$$

$$\Rightarrow (a+15)(a+2) < 0$$

$$\Rightarrow -15 < a < -2$$

Ans : D

7. Given $|x|^2 - 3|x| + 2 = 0$

$$\Rightarrow (|x|-2)(|x|-1) = 0$$

$\Rightarrow |x|-2=0 \quad or \quad |x|-1=0$

$\Rightarrow \ |x| \ =2 \quad or \quad |x|=1$

$\Rightarrow \ x \ = \ \pm 2 \quad or \quad x=\pm 1$

Ans : C

8. Given $-x^2+x+6 > 0$

$\Rightarrow \ x^2-x-6<0$

$\Rightarrow \ (x-3)(x+2)<0$

$\Rightarrow \ -2<x<3$

Ans : D

9. Given $2^{x^2-3} = 2^1$

$\Rightarrow \ x^2-3=1$

$\Rightarrow \ x^2 \ = 4$

$\Rightarrow \ x = \ \pm 2$

Ans : A

10. Given $(x-2)^2-2|x-2|-15=0$

$\Rightarrow \ |x-2|^2-2|x-2|-15=0$

Let |x-2| = t

$\Rightarrow \ t^2-2t-15=0$

$\Rightarrow \ (t-5)(t+3)=0$

$\Rightarrow \ t=5 \quad or \quad t=-3$

Now, |x-2| = 5

$|x-2|=5$

$\Rightarrow \ x-2=\pm 5$

$\Rightarrow \ x=7 \ or \ -3$

$\therefore$ sum of the

solutions $=7-3$

$= \ 4$

$|x-2|=-3$

This is not possible

since modulus of

areal number is never

negative

Ans : A

**Multi Answer Correct Type Questions :**

11. The graph of y = $ax^2+bx+c$ where a > 0 and $b^2-4ac < 0$ is a open upward parrabola and does not touches the x-axis

Ans : A,B,C

12. Given $ax^2+b|x|+c = 0$

$\Rightarrow ax^2+c=-b|x|$

The graph of $y = ax^2+c$ and $y = -b|x|$ do not touch each other hence, $ax^2+b|x|+c = 0$ has no real roots i.e it has imaginary roots

Ans : A,B

13. **Statement Type :**

**Statement - I:** Everny quadratic equation has atmost 2 solutions.

Hence, statement I is TRUE.

**Statement-II :** The graph of the poly nomial $y = ax^2+bx+c$, never touches the x-axis, when $b^2-4ac < 0$.

Hence, statement-II is TRUE.

But, Statement-II is NOT the correct explanation of statement-I

Ans : B

14. **Statement - I:**

Given $(\cos p-1)x^2+(\cos p)x+\sin p = 0$

Given the above equation has real roots.

We have $\Delta = b^2-4ac \geq 0$

$\Rightarrow \cos^2 p-4(\cos p-1)(\sin p)\geq 0$

$\Rightarrow \cos^2 p-4\cos p\sin p+4\sin p\geq 0$

$\Rightarrow (\cos p-2\sin p)^2-4\sin^2 p+4\sin p\geq 0$

$\Rightarrow (\cos p-2\sin p)^2+4(1-\sin p)\sin p\geq 0$

$\Rightarrow \sin p \geq 0$

$\Rightarrow p\in(0,\pi)$

Hence, statement-I is TRUE

**Statement-II :**

The solution set of $ax^2+bx+c > 0$ is $x<\alpha$ or $x>\beta$

Hence, statement-II is FALSE.

Ans : C

**Comprehension-I**

15. $y = -x^2+3x+4$

The maximum value $= \dfrac{4ac-b^2}{4a}$

$= \dfrac{4(-1)4-3^2}{4(-1)}$

$= \dfrac{25}{4}$

Ans : B

16. $y=-x^2+x+3$

We have

$$x=\frac{-b}{2a} = \frac{-1}{2(-1)} = \frac{1}{2}$$

$$y=\frac{4ac-b^2}{4a} = \frac{4(-1).3-1^2}{4.(-1)} = \frac{13}{4}$$

Hence, turning point $=\left(\frac{1}{2},\frac{13}{4}\right)$

Ans : B

**Comprehension-II**

17. Given $-x^2+5x+24>0$

$\Rightarrow x^2-5x-24<0$

$\Rightarrow (x-8)(x+3)<0$

$\Rightarrow -3<x<8$

Ans : C



18. Given $x\le -2 \quad or \quad x\ge 2$

$\Rightarrow (x+2)(x-2)\ge 0$

$\Rightarrow x^2-4\ge 0$

Ans : B

**Integer Answer Type questions :**

19. Given $x^2-4x+4=0$

$$\Rightarrow (x-2)^2=0$$

$$\Rightarrow x=2$$

Ans : 2

20. The required quadratic equation is (x+2)(x-3) = 0

$\Rightarrow x^2-x-6=0$

$\therefore$ The minimum value $=\alpha=\frac{4ac-b^2}{4a}$

$$=\frac{4.1(-6)-(-1)^2}{4.1}$$

$$= \frac{-25}{4}$$

$$\therefore 4\alpha + 25 = 0$$

21. **Matrix Matching :**

a) $a > 0, \Delta < 0$

The curve y = $ax^2+bx+c$ never touches the x-axis

b) $a < 0, \Delta > 0$

Intersects the x-axis at two distinct points.

c) $a > 0, \Delta = 0$

The curve y = $ax^2+bx+c$ touches the x-axis

d) $a > 0, \Delta > 0$

y = $ax^2+bx+c$ is a open upward parabola.

Ans : a-t, b-s, c-r, d-q

22.

a) $-3 < x < 4$

$\Rightarrow (x+3)(x-4) < 0$

$\Rightarrow x^2-x-12 < 0$

b) $x < -3$ or $x > 4$

$\Rightarrow (x+3)(x-4) > 0$

$\Rightarrow x^2-x-12 > 0$

c) $-4 < x < -3$

$\Rightarrow (x+3)(x+4) < 0$

$\Rightarrow x^2+7x+12 < 0$

d) $x < -4$ or $x > -3$

$\Rightarrow (x+3)(x+4) > 0$

$\Rightarrow x^2+7x+12 > 0$

Ans : a-p, b-q, c-r, d-s