# 7. TRIGONOMETRIC IDENTITIES (2025|26)

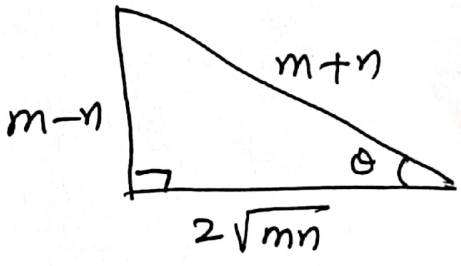
## Class: VIII . MATHEMATICS

## FOUNDATION: SOLUTIONS

### TEACHING TASK
### JEE MAINS LEVEL

01. $\sec\theta = \frac{m+n}{2\sqrt{mn}}$

$\therefore \sin\theta = \frac{m-n}{m+n}$




Ans. C

02 $\frac{(1+\tan^2 A).\cot A}{\operatorname{cosec}^2 A} =$

Shortcut trick put $A = 30^\circ$

$\frac{\left(1+\left(\frac{1}{\sqrt{3}}\right)^2\right).\sqrt{3}}{(2)^2} = \frac{1}{\sqrt{3}}$

$\therefore$ option A. $\tan A = \tan 30^\circ = \frac{1}{\sqrt{3}}$

Ans: B

03 put $A = 45^\circ$

$(\sqrt{2})^2(\sqrt{2})^2 - (1^2+1^2) = 4-2 = 2$ Ans. A

04. put $\theta = 30^\circ$ (2)

$\left(\left(\frac{1}{2}\right)^2 - 1\right)\left(\frac{2}{\sqrt{3}}\right)^2 = -1$ Ans: C

05 $\sec^2\theta - \cos^2\theta$

$= (1 + \tan^2\theta) - (1 - \sin^2\theta)$

$= \tan^2\theta + \sin^2\theta$ Ans: A

06 $(1 + \cot^2\theta)(1 + \tan^2\theta)$

$= \operatorname{cosec}^2\theta \times \sec^2\theta$ Ans. A

07 $(1 - \cos^2 A)\cos^2 B - \cos^2 A(1 - \cos^2 B)$

$= \cos^2 B - \operatorname{cosec}^2 A$ Ans: D

08 Short cut Trick

let $\theta = 30^\circ$

$$\frac{\left(\frac{1}{\sqrt{3}}\right)}{\left(1 + \frac{1}{3}\right)^2} + \frac{\sqrt{3}}{(1+3)^2} = \frac{\sqrt{3}}{4}$$

option: C $\sin 30^\circ \cdot \cos 30^\circ = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$ Ans: C

09. $\sec\theta + \tan\theta = p \rightarrow (1)$

$\sec\theta - \tan\theta = \frac{1}{p} \rightarrow (2)$

$(1) + (2) \Rightarrow \sec\theta = \frac{p^2+1}{2p}$

$(1) - (2) \Rightarrow \tan\theta = \frac{p^2-1}{2p}$

$\frac{\tan\theta}{\sec\theta} = \sin\theta = \frac{p^2-1}{p^2+1}$

Ans: B

10 **Short cut method**

Let $\alpha = 45^\circ$

$$= \frac{(\sqrt{2})^2}{1+(1)^2} + \frac{(1)^2}{1+(1)^2} + \left(\frac{1}{\sqrt{2}}\right)^2 = 2$$

Ans: D

11 **Short cut method**

Let $\theta = 0^\circ$ | Let $\theta = 90^\circ$

$\boxed{b = 1}$ | $\boxed{a = 1}$

Now, $a + 2b = 1 + 2 \cdot 1 = 3$

Ans: C

## JEE ADVANCED LEVEL

01. $\sin^2\alpha \cdot \cos^2\gamma - \cos^2\alpha \cdot \sin^2\gamma$

$= \sin^2\alpha(1 - \sin^2\gamma) - (1 - \sin^2\alpha) \cdot \sin^2\gamma$

$= \sin^2\alpha - \sin^2\gamma$

$= (1 - \cos^2\alpha) - (1 - \cos^2\gamma)$

$= \cos^2\gamma - \cos^2\alpha$

Ans: A, C

02 $\alpha + \beta = 90^\circ$

$\alpha - \beta = 30^\circ$

$\alpha = 60^\circ, \beta = 30^\circ$

Ans: A, C

03 $a\sin^2\theta + b\cos^2\theta = c$

$\div \cos^2\theta$

$a\tan^2\theta + b = c\sec^2\theta$

$a\tan^2\theta + b = c + c\tan^2\theta$

$\therefore \tan^2\theta = \frac{c-b}{a-c}$

Ans: B

06 Statement I: Conceptual (True)

Statement II:

$$LHS = \frac{1}{\operatorname{cosec}A - \cot A} - \frac{1}{\sin A}$$

$$= \operatorname{cosec}A + \cot A - \operatorname{cosec}A = \cot A$$

$$RHS = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec}A + \cot A}$$

$$= \operatorname{cosec}A - (\operatorname{cosec}A - \cot A)$$

$$= \operatorname{cosec}A - \operatorname{cosec}A + \cot A$$

$$= \cot A \quad (\text{True})$$

Ans: A

07. Assertion: $\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + (2)^2 = 5$ (True)

Reason: $\cot^2\theta + 1 = \operatorname{cosec}^2\theta \quad \forall \theta \in R - \{n\pi, n \in Z\}$ (False)

Ans: C

08 Assertion: $\sin^2\theta + \cos^2\theta = 1 \quad \forall \theta \in R$ (False)

Reason: Conceptual (False)

Ans. B

09 $\left.\begin{aligned} \sin A + \cos A &= \sqrt{2} \\ \tan A + \cot A &= 2 \end{aligned}\right\} \quad A = 45^\circ$

$$\sin A \cdot \cos A = \sin 45^\circ \cdot \cos 45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

Ans: —

10 $\sec A \operatorname{cosec} A = \sec 45^\circ \cdot \operatorname{cosec} 45^\circ$

$= \sqrt{2} \cdot \sqrt{2} = 2$

Ans.: A

11 $\sin^2 A - \cos^2 A = \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 = 0$

Ans. A

12 $\sin\theta + \cos\theta = \frac{b}{a}$ ; $\sin\theta\cos\theta = \frac{c}{a}$

$(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)$

$= (\sin\theta + \cos\theta)\sqrt{(\sin\theta + \cos\theta)^2 - 4\sin\theta\cos\theta}$

$= \left(\frac{b}{a}\right)\sqrt{\left(\frac{b}{a}\right)^2 - \frac{4c}{a}}$

$= \left(\frac{b}{a}\right)\sqrt{\frac{b^2 - 4ac}{a^2}} = \frac{b\sqrt{b^2 - 4ac}}{a^2}$

Ans. D

13 $\sin^4\theta + \cos^4\theta$

$= (\sin^2\theta)^2 + (\cos^2\theta)^2$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - 2\left(\frac{c}{a}\right)^2 = \frac{a^2 - 2c^2}{a^2}$

Ans. A

14 $\sec^2\theta = 1 + \tan^2\theta = 1 + \frac{25}{144} = \frac{169}{144}$

$\sec^2\theta = \frac{169}{144}$ ⟶ Numerator → 169

Ans: 169

15 1

Ans. 1

16 a) $2\sec^2\theta + 2\tan^2\theta - 7$

$= 2\sec^2\theta + 2(\sec^2\theta - 1) - 7$

$= 4\sec^2\theta - 9$

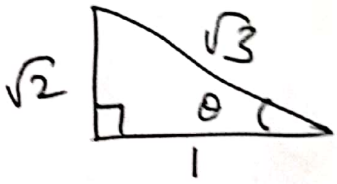
$= 4\left(\frac{3}{2}\right)^2 - 9 = 0$ (S)

b) $\underbrace{\frac{\tan 55^\circ}{\cot 35^\circ}}_{1} + \underbrace{\cot 1^\circ . \cot 2^\circ . \ldots \boxed{\cot 90^\circ} \to 0}_{0}$

$= 1 + 0 = 1$ (Q)

c) $x(1)\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}} \Rightarrow x = 1$ (Q)

d) $\sec^2\theta = 3 \Rightarrow \sec\theta = \frac{\sqrt{3}}{1}$




Now $\dfrac{\left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 - \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2}{(\sqrt{2})^2 + \left(\frac{\sqrt{3}}{\sqrt{2}}\right)^2} = \frac{1}{7}$ (P)

Ans: S, Q, Q, P

17 a) $\sin(n\pi + \theta) = (-1)^n . \sin\theta$ (t)

b) $\cos(n\pi + \theta) = (-1)^n . \cos\theta$ (r)

c) $\tan\left[(2n+1)\frac{\pi}{2} - \theta\right) = \cot\theta$ (P)

d) $\cos\left[(2n+1)\frac{\pi}{2} + \theta\right] = (-1)^{n+1} \sin\theta$ [S]

Ans: t, r, P, S

# LEARNERS TASK (CUQ'S)

**01.** $\sin\theta = +\sqrt{1-\cos^2\theta}$ — Ans: B

**02** $\dfrac{1}{\operatorname{cosec}\theta - \cot\theta}$ — Ans. A

**03** $-1$ — Ans. D

**04** $\cot\theta = \pm\sqrt{\operatorname{cosec}^2\theta - 1}$ — Ans: D

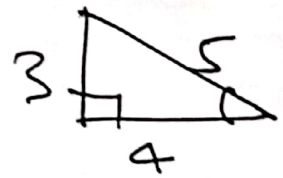
**05** $\dfrac{\sec\theta - 1}{\tan\theta} \times \dfrac{\sec\theta + 1}{\sec\theta + 1} = \dfrac{\sec^2\theta - 1}{\tan\theta(\sec\theta + 1)} = \dfrac{\tan\theta}{\sec\theta + 1}$ — Ans. B

**06** $\dfrac{\operatorname{cosec}\theta + 1}{\cot\theta} \times \dfrac{\operatorname{cosec}\theta - 1}{\operatorname{cosec}\theta - 1}$

$= \dfrac{\operatorname{cosec}^2\theta - 1}{\cot\theta(\operatorname{cosec}\theta - 1)} = \dfrac{\cot\theta}{\operatorname{cosec}\theta - 1}$ — Ans. B

**07** $4\sin\theta = 3\cos\theta$

$\Rightarrow \tan\theta = \dfrac{3}{4}$




$= \sin^2\theta - \cos^2\theta$

$= \left(\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$

$= \dfrac{-7}{25}$ — Ans. D

**08** $\tan^2 45^\circ + \dfrac{1}{\tan^2 45^\circ} = 1 + 1 = 2$ — Ans. B

**09.** $\cos\theta = \sec\theta \Rightarrow \cos^2\theta = 1$ ; $\sin\theta = \operatorname{cosec}\theta \Rightarrow \sin^2\theta = 1$

$\therefore \sin^2\theta - \cos^2\theta = 0$ — Ans: C

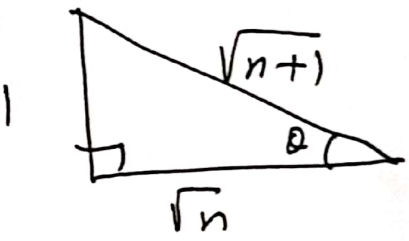
108 $(1-\sin^2\theta)\sec^2\theta = \cos^2\theta \cdot \sec^2\theta = 1$ Ans. C

## JEE MAINS LEVEL (9)

01. $\frac{1}{\sec^2\theta} + \frac{1}{\operatorname{cosec}^2\theta} = \cos^2\theta + \sin^2\theta = 1$ Ans. B

02 $\operatorname{cosec}\theta = \frac{\sqrt{n+1}}{1}$

$\therefore \cos\theta = \sqrt{\frac{n}{n+1}}$




Ans. D

03 $\left(1 - \frac{\sin^2 x}{1+\cos x}\right) + \left(\frac{1+\cos x}{\sin x} - \frac{\sin x}{1-\cos x}\right)$

$= \left(1 - \frac{1-\cos^2 x}{1+\cos x}\right) + \left(\frac{1-\cos^2 x - \sin^2 x}{\sin x(1-\cos x)}\right)$

$= \left[1 - (1-\cos x)\right] + 0$

$= \cos x$

Ans: B

04 Short trick method

put $\theta = 45^\circ$

$\frac{1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}} = 1+\sqrt{2}$

Opt: D: $\frac{\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$

Ans. D

05 We know

$\sin^4\theta + \cos^4\theta = (\sin^2\theta)^2 + (\cos^2\theta)^2$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - 2\sin^2\theta\cos^2\theta$

∴ given expression

$1 - 2\sin^2\theta\cos^2\theta = l + 3 + K\sin^2\theta\cdot\cos^2\theta$

Comparing

$l + 3 = 1$

$\Rightarrow l = -2$

$K = -2$

$\therefore l = K$

$\Rightarrow l - K = 0$

Ans. B

06 $\operatorname{cosec}\theta + \cot\theta + \operatorname{cosec}\theta - \cot\theta$

$= 2\operatorname{cosec}\theta$

Ans: B

07 $\sec^4 A - \sec^2 A$

$= \sec^2 A(\sec^2 A - 1)$

$= (1 + \tan^2 A)(\tan^2 A) = \tan^2 A + \tan^4 A$

Ans: C

08 Short trick method

put $A = 45^\circ$

$\left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2 = \frac{9}{2} + \frac{9}{2} = 9$

option: B: $(1 + \sqrt{2}\cdot\sqrt{2})^2 = 9$

Ans: B

09 $(\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C) = k^2$ (II)

$\therefore k^2 = 1 \Rightarrow k = \pm 1$ Ans: C

10 $(\sec A - \tan A)\ \_\sec A = -\tan A$ Ans. C

## JEE ADVANCED LEVEL

01. $\sec\theta = \frac{1}{\cos\theta} = \sqrt{1+\tan^2\theta}$

$= \frac{1}{\sqrt{1-\sin^2\theta}}$ Ans: B, C, D

02 ~~$\cos^2\theta - \sin^2\theta$~~

$\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1-\sin\theta)$

$= \frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta} = \cos\theta$ or $\frac{1}{\sec\theta}$

Ans: A, C

03 Statement I: Let $\theta = 0^\circ$

$a = \cos 0^\circ = 1, \quad b = \sin 0^\circ = 0$

$(a-b)^4 = 1 + 4a^2b^2 - 4ab$

$\Rightarrow 1 = 1$ (True)

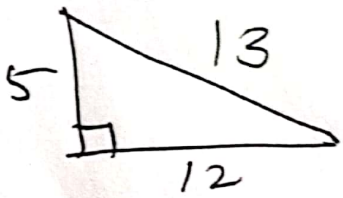
$a^6 + b^6 = 1 - 3a^2b^2$

$\Rightarrow 1 = 1$ (True)

Statement: Let $\theta = 0^\circ \Rightarrow 3+4+6 = 13$ (True)

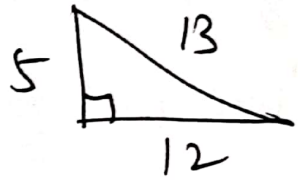
Ans: A

04. Statement I: $\tan\theta = \frac{5}{12}$




$\therefore \sec\theta = \frac{13}{12}$ (True)

Statement:




(True) Ans. A

---

05 (Repeated) Assertion: $\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + (2)^2 = 5$ (True)

Reasin: $\cot^2\theta + 1 = \operatorname{cosec}^2\theta \quad \forall \theta \in R - \{n\pi, n \in I\}$ (False)

Ans. C

---

06 Assertion: Conceptual (True)

Reason: conceptual (True) Ans. A

---

07. $\angle A + \angle C = 180°, \quad \angle B + \angle D = 180°$

$\sin A + \sin B = \sin(180° - C) + \sin(180° - D)$

$= \sin C + \sin D$ Ans: A

---

08 $\tan\left(\frac{A+B}{4}\right) = \tan\left(\frac{360° - (C+D)}{4}\right)$

$= \tan\left[90° - \frac{C+D}{4}\right]$

$= \cot\left(\frac{C+D}{4}\right)$ Ans: D

---

09 ~~Cancelled~~

09. $\cos\left(\frac{A+B}{2}\right) + \cos\left(\frac{C+D}{2}\right)$

$= \cos\left[\frac{360^\circ - (C+D)}{2}\right] + \cos\left(\frac{C+D}{2}\right)$

$= \cos\left(180^\circ - \frac{C+D}{2}\right) + \cos\left(\frac{C+D}{2}\right)$

$= -\cos\left(\frac{C+D}{2}\right) + \cos\left(\frac{C+D}{2}\right) = 0$ Ans: A

10. NOTE: Please change the questions as

$\sin(A+B) = 1$ and $\cos(A-B) = 0$

$\Rightarrow A+B = 90^\circ$ and $A-B = 0$

$\Rightarrow A = B = 45^\circ$

$\sin 2A + \cos 2A$

$= \sin 90^\circ + \cos 90^\circ = 1+0 = 1$ Ans: A

11. $\sin^2 45^\circ + \cos^2 45^\circ = \frac{1}{2} + \frac{1}{2} = 1$ Ans: C

12 $\tan^2 90^\circ + \tan^2 45^\circ$ = Not defined Ans: D

13 $(\tan 1^\circ \cdot \tan 89^\circ)(\tan 2^\circ \cdot \tan 88^\circ) \ldots \ldots \tan 45^\circ$

$= 1$ Ans: 1

14 $\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \ldots \boxed{\cos 90^\circ} \ldots \cos 180^\circ = 0$

$\downarrow$

$0$

Ans: 0

15 put $\alpha = 0^\circ$

$3(1+0) - 2(0+1) = 1$

Ans: 1

16 $\cos A + \cos^2 A = 1$

$\Rightarrow \cos A = 1 - \cos^2 A$

$\Rightarrow \cos A = \sin^2 A$

$\Rightarrow \cos^2 A = \sin^4 A$

$\sin^2 A + \sin^4 A$

$= \sin^2 A + \cos^2 A$

$= 1$

Ans: 1

17 a) $(\sin^2 A)^2 - (\cos^2 A)^2$

$= (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A)$

$= 1 - \cos^2 A - \cos^2 A$

$= 1 - 2\cos^2 A$ (r)

b) $1 - \frac{(1-\sin^2 A)}{1+\sin A} = 1 - \frac{(1+\sin A)(1-\sin A)}{(1+\sin A)}$

$= 1 - 1 + \sin A = \sin A$ (q)

c) $\tan^2 A + \tan^4 A$

$= \tan^2 A (1 + \tan^2 A)$

$= (\sec^2 A - 1)(\sec^2 A) = \sec^4 A - \sec^2 A$ (p)

d) $\left(\frac{\cos A}{\sin A}\right) \cdot \sin A = \cos A$ (t)

Ans: r, q, p, t

18 a) $\sqrt{\frac{1-\sin\alpha}{1+\sin\alpha} \times \frac{1-\sin\alpha}{1-\sin\alpha}} = \frac{1-\sin\alpha}{\cos\alpha}$

$= \sec\alpha - \tan\alpha$ (s)

b) $\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha} \times \frac{1+\sin\alpha}{1+\sin\alpha}} = \frac{1+\sin\alpha}{\cos\alpha}$

$= \sec\alpha + \tan\alpha$ (p)

c) $\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha} \times \frac{1+\cos\alpha}{1+\cos\alpha}} = \frac{1+\cos\alpha}{\sin\alpha}$

$= \operatorname{cosec}\alpha + \cot\alpha$ (t)

d) $\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha} \times \frac{1-\cos\alpha}{1-\cos\alpha}} = \frac{1-\cos\alpha}{\sin\alpha}$

$= \operatorname{cosec}\alpha - \cot\alpha$ (q)

Ans: s, p, t, q

## ADDITIONAL PRACTICE QUESTIONS

01. $\cos^2\theta + \frac{1}{\operatorname{cosec}^2\theta} = \cos^2\theta + \sin^2\theta = 1$ Ans.- D

02 $\sec\alpha(1-\sin\theta)(\sec\theta+\tan\theta)$

$= (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$

$= \sec^2\theta - \tan^2\theta$

$= 1$

Ans: D

03 $\sin^2\theta + \frac{1}{\sec^2\theta} = \sin^2\theta + \cos^2\theta = 1$ Ans. A

(1b)

04 $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$

$= \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$

$= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$

$= 2\sec\theta$

Ans: B

05 $\tan\theta \times \cot\theta = 1$ Ans: C

06 put $\theta = 45^\circ$

$(1+1-\sqrt{2})(1+1+\sqrt{2})$

$= (2-\sqrt{2})(2+\sqrt{2}) = 4-2 = 2$ Ans: C

07 put $\alpha = 45^\circ$

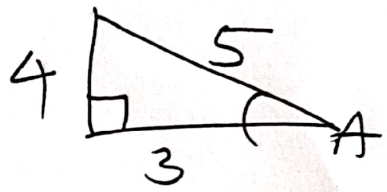
$= (1)^2\left[\frac{\sqrt{2}-1}{1+\frac{1}{\sqrt{2}}}\right] + (\sqrt{2})^2\left[\frac{\frac{1}{\sqrt{2}}-1}{1+\sqrt{2}}\right]$

$= \sqrt{2}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) + 2\left(\frac{1-\sqrt{2}}{\sqrt{2}(1+\sqrt{2})}\right)$

$= \sqrt{2}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) - \sqrt{2}\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = 0$ Ans: A

08 $3\tan A + 4 = 0$

$\Rightarrow \tan A = \frac{-4}{3}$




$A \in Q_2$

$\therefore \alpha = \sec A = -\frac{5}{3}$

$\beta = \operatorname{cosec} A = \frac{5}{4}$

Quadratic eq. $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - \left(-\frac{5}{3} + \frac{5}{4}\right)x + \left(-\frac{5}{3}\right)\left(\frac{5}{4}\right) = 0$

$\Rightarrow 12x^2 + 5x - 25 = 0$

Ans: A

09 put $A = 0^\circ$

$= (1)^4 (1-0) - 2(0)$

$= 1$

Ans: B

$\Rightarrow$ THE END $\in$