PROPERTIES OF TRIANGLES – II

Class: IX. Mathematics

SOLUTIONS

F+

TEACHING TASK

NOTE: In thich chapter, majority of the problems can be solved, by considering the given triangle (as an equilateral triangle (unless otherwise given). In the equilateral triangle, we have the following asumptions)

① $a=b=c=1$

② $s=\frac{3}{2}$

③ $\Delta=\frac{\sqrt{3}}{4}$

④ $p_1=p_2=p_3=\frac{\sqrt{3}}{2}$

⑤ $r=\frac{\sqrt{3}}{6}$

⑥ $r_1=r_2=r_3=\frac{\sqrt{3}}{2}$

$R=\frac{1}{\sqrt{3}}$

$\cancel{A_1=A_2=A_3=}$

TEACHING TASK

01 $\frac{\Delta^2}{a^2+b^2+c^2}\left\{\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}+\frac{1}{r^2}\right\}$

(Consider this is an equilateral triangle)

$$\frac{\left(\frac{\sqrt{3}}{4}\right)^2}{3(1)^2}\left\{3\left(\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}\right)+\frac{1}{\left(\frac{\sqrt{3}}{6}\right)^2}\right\}$$

$$=\frac{1}{16}\left\{4+\frac{36}{3}\right\}=\frac{16}{16}=1$$

Ans: D

02 $\frac{s-a}{\Delta}=\frac{1}{8},\ \frac{s-b}{\Delta}=\frac{1}{12},\ \frac{s-c}{\Delta}=\frac{1}{24}$

$$\frac{s-a+s-b+s-c}{\Delta}=\frac{1}{8}+\frac{1}{12}+\frac{1}{24}$$

$$\frac{s}{\Delta}=\frac{3+2+1}{24}=\frac{1}{4}$$

$\boxed{r=4}$

$\Delta=\sqrt{rr_1r_2r_3}$

$\Delta=\sqrt{4.8.12.24}=96$

$\therefore s=24$

$\frac{s-b}{\Delta}=\frac{1}{12}\Rightarrow\boxed{b=16}$

Ans: A

03 Let △ABC be an equilateral △le

$$\frac{\sqrt{r\cdot r_1 r_2 r_3}}{2Rr(\sin A+\sin B+\sin C)}$$

$$=\frac{\left(\frac{\sqrt{3}}{4}\right)}{2\times\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{6}\left(3\times\frac{\sqrt{3}}{2}\right)}=\frac{1}{2}$$

Ans: D

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04 $\left(\frac{1}{r}-\frac{1}{r_1}\right)\left(\frac{1}{r}-\frac{1}{r_2}\right)\left(\frac{1}{r}-\frac{1}{r_3}\right)$

$$=\left(\frac{6}{\sqrt{3}}-\frac{2}{\sqrt{3}}\right)^3=\left(\frac{4}{\sqrt{3}}\right)^3=\frac{64}{3\sqrt{3}}$$

opt: A) $\frac{abc}{\Delta^3}=\frac{1.1.1}{\left(\frac{\sqrt{3}}{4}\right)^3}=\left(\frac{4}{\sqrt{3}}\right)^3=\frac{64}{3\sqrt{3}}$

Ans: A

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05 $c=2R\sin C$

$c=2R\sin 90^\circ$

$\boxed{c=2R}$

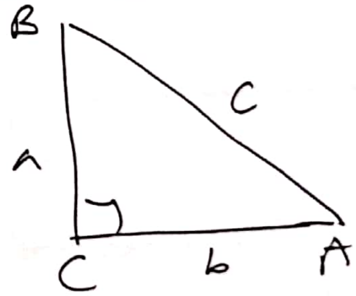
$\tan\frac{C}{2}=\frac{r}{s-c}$

$\tan 45^\circ=\frac{r}{s-c}$

$\Rightarrow s-c=r$

$\Rightarrow 2s-2c=2r$

$\Rightarrow \boxed{a+b-c=2r}$



Now

$2r+2R$

$=a+b-c+c$

$=a+b$

Ans: A, B

06

③

07 $\frac{1}{\sqrt{A_1}}+\frac{1}{\sqrt{A_2}}+\frac{1}{\sqrt{A_3}}$

$=\frac{1}{\sqrt{\pi r_1^2}}+\frac{1}{\sqrt{\pi r_2^2}}+\frac{1}{\sqrt{\pi r_3^2}}$

$=\frac{1}{\sqrt{\pi}}\left(\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)=\frac{1}{\sqrt{\pi}}\cdot\frac{1}{r_1}=\frac{1}{\sqrt{\pi r_\phi^2}}=\frac{1}{\sqrt{A}}$

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08 $\frac{\cos A}{P_1}+\frac{\cos B}{P_2}+\frac{\cos D}{P_3}$

$=3\times\frac{\cos 60^\circ}{\left(\frac{\sqrt{3}}{2}\right)}=3\times\frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}=\frac{3}{\sqrt{3}}=\sqrt{3}$

Ans : B

$\therefore\ \frac{1}{R}=\sqrt{3}$

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09 $P_1\cdot P_2\cdot P_3=(P_1)^3=\left(\frac{\sqrt{3}}{2}\right)^3=\frac{3\sqrt{3}}{8}$

opt. D : $\frac{2\Delta^2}{R}=\frac{2\cdot 3}{16\times\frac{1}{\sqrt{3}}}=\frac{3\sqrt{3}}{8}$ Ans. D

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10 $r_1 r_2 r_3 - r(r_1 r_2 + r_2 r_3 + r_3 r_1)$

$=(r_1)^3 - r(3\cdot r_1 r_2)$ (Consider $\triangle ABC$ is an equilateral)

$=\left(\frac{\sqrt{3}}{2}\right)^3-\frac{\sqrt{3}}{6}\left(3\cdot\left(\frac{\sqrt{3}}{2}\right)^2\right)$

Ans : A

$=\frac{3\sqrt{3}}{8}-\frac{3\sqrt{3}}{8}=0$

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11 $\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$ $\therefore\ \boxed{r=4}$

$=\frac{1}{8}+\frac{1}{12}+\frac{1}{24}$ $\Delta^2=r\cdot r_1\cdot r_2\cdot r_3$

$=4\cdot 8\cdot 12\cdot 24$

$=\frac{3+2+1}{24}$ $\therefore\ \boxed{\Delta=96}$

$=\frac{6}{24}=\frac{1}{4}$ $\boxed{s=24}$

$r_1 = \frac{\Delta}{s-a}$ | $r_2 = \frac{\Delta}{s-b}$ | $r_3 = \frac{\Delta}{s-c}$ (4)

$8 = \frac{96}{24-a}$ | $12 = \frac{96}{24-b}$ | $24 = \frac{96}{24-c}$

$\Rightarrow 24-a = 12$ | $24-b = 8$ | $24-c = 4$

$\Rightarrow a = 12$ | $b = 16$ | $c = 20$

Ans: A, B, C, D

12 $(r_2 - r_1)(r_3 - r_1) = 2r_2 r_3$

$\Rightarrow r_2 r_3 - r_1 r_2 - r_1 r_3 + r_1^2 = 2r_2 r_3$

$\Rightarrow r_1^2 = r_1 r_2 + r_2 r_3 + r_3 r_1$

$\Rightarrow \frac{\Delta^2}{(s-a)^2} = \frac{\Delta^2}{(s-a)(s-b)} + \frac{\Delta^2}{(s-b)(s-c)} + \frac{\Delta^2}{(s-c)(s-a)}$

$\Rightarrow \frac{1}{(s-a)^2} = \frac{s-c+s-a+s-b}{(s-a)(s-b)(s-c)}$

$\Rightarrow \frac{1}{(s-a)^{\cancel{2}}} = \frac{s}{\cancel{(s-a)}(s-b)(s-c)}$

$\Rightarrow \frac{s(s-a)}{(s-b)(s-c)} = 1$

$\Rightarrow \cot \left(\frac{A}{2}\right) = 1 \Rightarrow \frac{A}{2} = 45^\circ \Rightarrow A = 90^\circ$

$\therefore \angle B + \angle C = 90^\circ$

Ans: B, C

13 Statement I: $a > b > c$

$-a < -b < -c$

$s-a < s-b < s-c$

$\frac{s-a}{\Delta} < \frac{s-b}{\Delta} < \frac{s-c}{\Delta}$

$\frac{1}{r_1} < \frac{1}{r_2} < \frac{1}{r_3}$ (True)

$r_1 > r_2 > r_3$

⑤

14) statement II: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$\Rightarrow \dfrac{3}{\sin 30^\circ} = \dfrac{4}{\sin B}$

$\Rightarrow \sin B = \frac{3}{2}$, which is positive

$\sin B$ is positive in Q_1 and Q_2

$\therefore B$ can take two angles

Hence two △les (True)

Ans: A

(14) statement II: Consider it is an equilateral △le

$$= \frac{(a+b)\cos C + (b+c)\cos A + (c+a)\cos B}{\sin A + \sin B + \sin C}$$

$$= \frac{\cancel{3}(a+b)\cos C}{\cancel{3}\sin A}$$

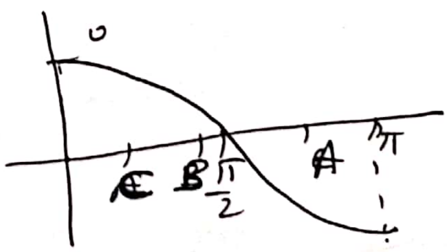
$$= \frac{(1+1)\cos 60^\circ}{\sin 60^\circ} = \frac{2\times\frac{1}{2}}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}}$$

But $R = \frac{1}{\sqrt{3}}$ (statement false)

13. statement II: In △ABC, we have $A+B+C=180^\circ$

We have $\cos x$ is a monotonically decreasing function in the interval 0° to 180°

$\therefore \cos A < \cos B < \cos C$



Given $a > b > c$

$A > B > C$

$\cos A < \cos B < \cos C$ (True

15. Statement I: $r_1 = r_2 = r_3$

$$\Rightarrow \frac{\Delta}{s-a} = \frac{\Delta}{s-b} = \frac{\Delta}{s-c}$$

$$\Rightarrow s-a = s-b = s-c$$

$$\Rightarrow a = b = c$$

G ABC, is equilateral Δle (False)

Statement II: $r r_1 = r_2 r_3$

$$\frac{\Delta}{s} \cdot \frac{\Delta}{s-a} = \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c}$$

$$\Rightarrow \frac{s(s-a)}{(s-b)(s-c)} = 1$$

$$\Rightarrow \cot\left(\frac{A}{2}\right) = 1 \Rightarrow \frac{A}{2} = 45^\circ$$

$$\Rightarrow A = 90^\circ$$

(False)

Ans: B

16 We know AM $\geq$ GM

$$\Rightarrow \frac{\frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}}{3} \geq \sqrt[3]{\frac{1}{P_1} \cdot \frac{1}{P_2} \cdot \frac{1}{P_3}}$$

$$\Rightarrow \frac{1}{6} \geq \sqrt[3]{\frac{1}{P_1} \cdot \frac{1}{P_2} \cdot \frac{1}{P_3}}$$

$$\Rightarrow \frac{1}{P_1 \cdot P_2 P_3} \leq \left(\frac{1}{6}\right)^3$$

$$\Rightarrow \frac{1}{P_1 P_2 P_3} \leq \frac{1}{216}$$

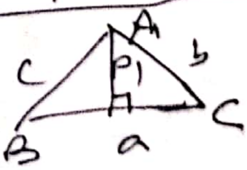
$$\Rightarrow P_1 P_2 P_3 \geq 216$$

least value = 216

Ans: D

17. $$\frac{b^2 P_1}{c} + \frac{c^2 P_2}{a} + \frac{a^2 P_3}{b}$$

$$= 20\left(\frac{b^2}{ac} + \frac{c^2}{ab} + \frac{a^2}{bc}\right)$$



$\Delta = \frac{1}{2} \cdot a \cdot P_1$

$P_1 = \frac{2\Delta}{a}$

We know AM $\geq$ G.M

7

$$\frac{\frac{b^2}{bc}+\frac{c^2}{ab}+\frac{a^2}{bc}}{3} \geq \sqrt[3]{\frac{b^2}{bc}\cdot\frac{c^2}{ab}\cdot\frac{a^2}{bc}}$$

$$\Rightarrow \frac{a^2}{abc}+\frac{b^2}{ca}+\frac{c^2}{ab} \geq 3$$

$$\therefore 20\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right) \geq 60$$

$\therefore$ minimum value $= 60$

opt: D

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18 $P_1, P_2, P_3 \rightarrow AP$

$$\Rightarrow 2P_2 = P_1 + P_3$$

$$\Rightarrow 2\cdot\frac{20}{b} = \frac{20}{a}+\frac{20}{c}$$

$$\Rightarrow \frac{2}{b} = \frac{1}{a}+\frac{1}{c}$$

$$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c} \rightarrow AP$$

$a, b, c \rightarrow$ H.P.

Ans: B

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19 Let $\triangle ABC$ be an equilateral triangle

$$\frac{a}{\tan A}+\frac{b}{\tan B}+\frac{c}{\tan C}$$

$$= 3\frac{a}{\tan A} \Rightarrow 3\cdot\frac{1}{\tan 60} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

opt: A: $2r = 2\times\frac{\sqrt{3}}{6} = \frac{1}{\sqrt{3}}$

opt B: $r+2R = \frac{\sqrt{3}}{6}+2\cdot\frac{1}{\sqrt{3}} = \frac{5}{2\sqrt{3}}$

opt C: $2r+R = 2\cdot\frac{\sqrt{3}}{6}+\frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

opt D: $2(r+R) = 2\left(\frac{\sqrt{3}}{6}+\frac{1}{\sqrt{3}}\right) = \sqrt{3}$

opt: D

20 Let ΔABC be an equilateral triangle (8)

$\cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2}$

$= 3\cos^2\frac{A}{2}$

$= 3\cos^2 30^\circ$

$= 3\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{9}{4}$

opt C: $2 + \frac{r}{2R} = 2 + \frac{\left(\frac{\sqrt{3}}{6}\right)}{2\times\frac{1}{\sqrt{3}}} = 2 + \frac{1}{4} = \frac{9}{4}$

Ans: C

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21. Let ΔABC be an equilateral triangle

$\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$

$= 3\sin^2\left(\frac{A}{2}\right)$

$= 3\sin^2 30^\circ$

$= 3\left(\frac{1}{2}\right)^2 = \frac{3}{4}$

opt: D $1 - \frac{r}{2R} = \cancel{1} - \frac{1}{4} = \frac{3}{4}$ opt: D

22

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23 r_1, r_2, r_3 are in HP

$\frac{1}{r_1}, \frac{1}{r_2}, \frac{1}{r_3} \Rightarrow$ A.P

$\Rightarrow \frac{2}{r_2} = \frac{1}{r_1} + \frac{1}{r_3}$

$\Rightarrow \frac{2}{r_2} = \frac{1}{r} - \frac{1}{r_2}$

$\Rightarrow \frac{3}{r_2} = \frac{1}{r}$

$\Rightarrow \frac{r_2}{r} = 3$

Ans: 3

(9)

24 a) let $\triangle ABC$ be an equilateral $\triangle$le

$(r_1 - r)(r_2 - r)(r_3 - r)$

$= (r_1 - r)^3$

$= \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{6}\right)^3 = \frac{\sqrt{3}}{9}$

opt: q : $4Rr^2 = 4 \times \frac{1}{\sqrt{3}} \times \left(\frac{\sqrt{3}}{6}\right)^2 = \frac{\sqrt{3}}{9}$

b) let $\triangle bc$ be an equilateral $\triangle$le

$(r_1 + r_2)(r_2 + r_3)(r_3 + r_1)$

$= (r_1 + r_2)^3$

$= (2r_1)^3$

$= 8 r_1^3$

$= 8 \cdot \left(\frac{\sqrt{3}}{2}\right)^3 = \frac{8 \times 3\sqrt{3}}{8} = 3\sqrt{3}$

opt: $\cancel{p}$ $\Rightarrow 4R = 4 \times$

opt: r $\Rightarrow 4Rs^2$

$= 4 \times \frac{1}{\sqrt{3}} \times \left(\frac{3}{2}\right)^2$

$= 4 \times \frac{1}{\sqrt{3}} \times \frac{9}{4} = 3\sqrt{3}$

c) $r_1 + r_2 + r_3 - r = 4R$ (Conceptual Result)

d) $r + r_3 + r_1 - r_2 = 4R\cos B$ (Conceptual) (Result)

$a \to q,\ b \to r,\ c \to p,\ d = t$

25 a) $r_1 = 3,\ r_2 = 10,\ r_3 = 15$

$\frac{1}{r} = \frac{1}{3} + \frac{1}{10} + \frac{1}{15} \Rightarrow \boxed{r = 2}$

$\Delta^2 = r \cdot r_1 \cdot r_2 \cdot r_3 \Rightarrow \boxed{\Delta = 30}$

$r = \frac{\Delta}{s} \Rightarrow \boxed{s = 15}$

$r_1 = \frac{\Delta}{s-a}$

$\Rightarrow 3 = \frac{30}{15-a} \Rightarrow \boxed{a=5}$ similarly $\boxed{b=12}$ & $\boxed{c=13}$

$\because \Delta = \frac{abc}{4R}$

$\Rightarrow R = \frac{abc}{4\Delta} = \frac{13}{2}$

b) $\frac{\sqrt{r r_1 r_2 r_3}}{2Rr(\sin A + \sin B + \sin C)}$ (Consider ΔABC is an equilateral Δ)

$= \frac{\Delta}{2Rr(3\sin A)}$

$= \frac{\frac{\sqrt{3}}{4}}{2 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{6} \times 3 \times \frac{\sqrt{3}}{2}} = \frac{1}{2}$

c) r_1, r_2, r_3 HP $\Rightarrow \frac{r_2}{r} = 3$

d) we have $\sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$

$= \frac{3}{2} - \frac{1}{2}(\cos A + \cos B + \cos C)$

Also, $\cos A + \cos B + \cos C = 1 + \frac{r}{R}$

$= 1 + \frac{3}{7}$ since $7r = 3R$, $\frac{r}{R} = \frac{3}{7}$

$= \frac{10}{7}$

$\therefore \sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2} = \frac{3}{2} - \frac{1}{2}\left(\frac{10}{7}\right)$

$= \frac{3}{2} - \frac{5}{7}$

$= \frac{21-10}{14} = \frac{11}{14}$

Ans: s, p, t, q

LEARNERS TASK

CUQ'S

01. $a = 13,\ b = 14,\ c = 15$

$$s = \frac{13 + 14 + 15}{2} = \frac{42}{2} = 21$$

$r = \frac{\Delta}{s}$ Now $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= 84.$

$$r = \frac{\Delta}{s} = \frac{84}{21} = 4$$

Ans: A

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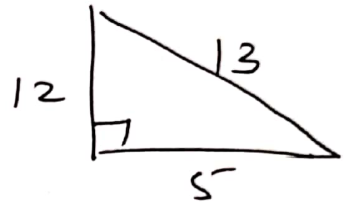
02 $a = 5,\ b = 12,\ c = 13$



$$\Delta = \frac{1}{2} \times 12 \times 5 = 30.$$

$$S = \frac{5 + 12 + 13}{2} = \frac{30}{2} = 15$$

$$\therefore r_1 = \frac{\Delta}{s-a} = \frac{30}{15-5} = \frac{30}{10} = 3$$

Ans: B

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03 $r_1 = \frac{\Delta}{s-c}$

$$= \frac{48}{16-4} = \frac{48}{12} = 4$$

Ans: C

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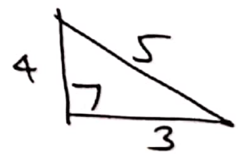
04 $a = 4,\ b = 5,\ c = 3$



$$\Delta = \frac{1}{2} \times 4 \times 3 = 6$$

$$S = \frac{4 + 5 + 3}{2} = \frac{12}{2} = 6$$

$$\therefore r = \frac{\Delta}{S} = \frac{6}{6} = 1$$

Ans: A

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05 $r_2 = \frac{\Delta}{s-b}$

$$= \frac{24}{15-7} = \frac{24}{8} = 3$$

Ans: D

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(12)

06 $\frac{1}{r} = \frac{1}{8} + \frac{1}{12} + \frac{1}{24}$ $\quad \therefore \boxed{r=4}$

$= \frac{3+2+1}{24} = \frac{6}{24} = \frac{1}{4}$

$\Delta^2 = 8\cdot 12 \cdot 24 \cdot 4 \quad \therefore \boxed{\Delta = 96}$

$r = \frac{\Delta}{s} \Rightarrow s = \frac{\Delta}{r} = \frac{96}{4} = 24$ $\quad \boxed{s=24}$

$r_1 = \frac{\Delta}{s-a}$

$\Rightarrow 8 = \frac{96}{24-a}$

$\Rightarrow 24 - a = 12$

$\boxed{a=12}$

Ans : B

07 $r \cdot r_1 \cdot r_2 \cdot r_3 = \Delta^2$ $\quad r = \frac{\Delta}{s}$

$r_1 \cdot r_2 \cdot r_3 = \frac{\Delta^2}{r} = \frac{\Delta\cdot\Delta}{r} =$

$= \frac{(rs)^2}{r} = rs^2$

Ans: C

08 $\frac{1}{r_1} : \frac{1}{r_2} : \frac{1}{r_3}$ $\quad s = \frac{30+24+18}{2}$

$= 36$

$= \frac{s-a}{\Delta} : \frac{s-b}{\Delta} : \frac{s-c}{\Delta}$

$= s-a : s-b : s-c$

$= 36-30 : 36-24 : 36-18$

$= 6 : 12 : 18$

$= 1 : 2 : 3$

Ans: A

09 $r = \frac{\Delta}{s} = \frac{8}{1.5} = \frac{80}{15} = \frac{16}{3}$ Ans. D

10. $\Delta^2 = 2\times 4\times 6\times 12 = 24$ Ans. B

JEE MAINS LEVEL QUESTIONS (B)

01. Conceptual. $r = (s-b)\tan\frac{B}{2}$ — Ans. B

02. $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$ — Ans. B

03. $(r_1 - r)(r_2 - r)(r_3 - r) = 4Rr^2$ — Ans: B

04. $\sqrt{\frac{r r_1}{r_2 r_3}} = \sqrt{\frac{\frac{\Delta}{s}\cdot\frac{\Delta}{s-a}}{\frac{\Delta}{s-b}\cdot\frac{\Delta}{s-c}}} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \tan\left(\frac{A}{2}\right)$

Ans: B

05. $r_1 - r = 4R\sin^2 A/2$

$7r - r = 6r = 4R\sin^2 A/2$

$6 \times R/6 = 4R\sin^2 A/2$

$\sin^2\frac{A}{2} = \frac{1}{4}$

$\frac{A}{2} = 30^\circ$

$\therefore A = 60^\circ$

$A = \frac{\pi}{3}$

06. $r\left(\cot\frac{B}{2} + \cot\frac{C}{2}\right)$

$= r\left(\sqrt{\frac{s(s-b)}{(s-a)(s-c)}} + \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}\right)$

$= r\left(\frac{\sqrt{s}\cdot(s-b) + \sqrt{s}(s-c)}{\sqrt{(s-a)(s-b)(s-c)}}\right) \times \frac{\sqrt{s}}{\sqrt{s}}$

$= \frac{r}{\Delta}\left(s(2s-b-c)\right) = \frac{r}{\cancel{\Delta}}\cancel{s}\cdot a = a$

Ans: A

07. Consider $\triangle ABC$ is an equilateral $\triangle$le

$r^2 \cdot \cot\frac{A}{2} \cdot \cot\frac{B}{2} \cdot \cot\frac{C}{2}$

$= r^2 \cdot \cot^3\left(\frac{A}{2}\right)$

$= \left(\frac{\sqrt{3}}{6}\right)^2 \cdot \cot^3 30$

$= \frac{3}{36} \times 3\sqrt{3} = \frac{\sqrt{3}}{4} = \Delta$

Ans: C

08. $r_2 + r_3 = 4R\cos^2\left(\frac{A}{2}\right)$

$= 4R\cdot\cos^2 45^\circ$

$= 4R\cdot\frac{1}{2}$

$= 2R$

$\therefore \cos^{-1}\left(\frac{R}{2R}\right) = \cos^{-1}\left(\frac{1}{2}\right)$

$= 60^\circ$

Ans: B

(14)

09. $r_1 + r = r_2 + r_3$

$$\frac{\Delta}{s-a} + \frac{\Delta}{s} = \frac{\Delta}{s-b} + \frac{\Delta}{s-c}$$

$$\Rightarrow \frac{s+s-a}{s(s-a)} = \frac{2s-b-c}{(s-b)(s-c)}$$

$$\Rightarrow \frac{b+c}{a} = \frac{s(s-a)}{(s-b)(s-c)}$$

$$\Rightarrow \frac{b+c}{a} = \cot^2\left(\frac{A}{2}\right) = \cot^2\left(\frac{60^\circ}{2}\right) = 3$$

$$\Rightarrow b+c = 3a$$

$$\Rightarrow a+b+c = 4a$$

$$\Rightarrow 2s = 4a$$

$$\Rightarrow \frac{s}{a} = 2$$

Ans: B

10 $r_1 = 2r_2 = 3r_3$ ~~= k~~

$$\frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c}$$

$$\Rightarrow \frac{1}{s-a} = \frac{2}{s-b} = \frac{3}{s-c} = k$$

$$s-a = \frac{1}{k}, \quad s-b = \frac{2}{k}, \quad s-c = \frac{3}{k}$$

$$3s-(a+b+c) = \frac{6}{k}$$

$$\boxed{s = \frac{6}{k}}$$

$$s-a = \frac{1}{k}$$

$$a = s - \frac{1}{k} = \frac{6}{k} - \frac{1}{k}$$

$$a = \frac{5}{k}$$

$$b = \frac{6}{k} - \frac{2}{k} = \frac{4}{k}$$

$$c = \frac{6}{k} - \frac{3}{k} = \frac{3}{k}$$

Now $\frac{a}{b} = \frac{\left(\frac{5}{k}\right)}{\left(\frac{4}{k}\right)} = \frac{5}{4}$

$$\frac{b}{c} = \frac{\left(\frac{4}{k}\right)}{\left(\frac{3}{k}\right)} = \frac{4}{3}$$

$$\frac{c}{a} = \frac{\left(\frac{3}{k}\right)}{\left(\frac{5}{k}\right)} = \frac{3}{5}$$

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = \frac{5}{4} + \frac{4}{3} + \frac{3}{5}$$

$$= \frac{191}{60}$$

Ans: D

11. Conceptual — Ans: A, B

12. Conceptual — Ans: A, C

13. Statement I: let $a = 2,\ b = 3$

$$\Delta = \frac{1}{2} ab \sin C$$
$$= \frac{1}{2} \cdot 2 \cdot 3 \cdot \sin C$$
$$= 3 \sin C$$

We know $-1 \le \sin C \le 1$

$-3 \le 3 \sin C \le 3$

Area can not exceed $= 3$

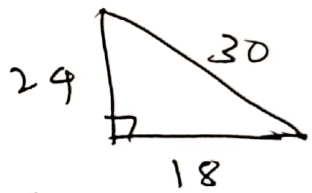
~~Ans 3~~ (True)

Statement II: Conceptual true — Ans: A

14. Statement I: $a = 18,\ b = 24,\ c = 30$

Now $s = 36$

$$\Delta = \frac{1}{2} \times 18 \times 24 = 216$$



$$r_1 = \frac{\Delta}{s-a} = 12,\quad r_2 = 18,\quad r_3 = 36$$

Now $l = r_2 - r_1 = 18 - 12 = 6$ | $m = r_3 - r_2 = 36 - 18 = 18$ | $n = r_3 - r_1 = 36 - 12 = 14$

$\therefore$ Ascending order l, m, n (False)

Statement II: $a = 13,\ b = 14,\ c = 15$

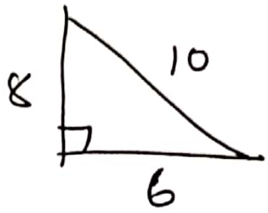
$$\text{Now, } s = \frac{13+14+15}{2} = 21$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = 84$$

$$\therefore r_1 = \frac{\Delta}{s-a} = \frac{84}{21-13} = \frac{84}{8} = \frac{21}{2} \text{ (true)}$$

Ans: D

15 $a=6,\ b=10,\ c=8$



$$s=\frac{6+10+8}{2}=\frac{24}{2}=12$$

$$\Delta=\frac{1}{2}\times 6\times 8=\cancel{32}\ 24$$

$$r=\frac{\Delta}{s}=\frac{24}{12}=2$$

Ans: B

16 $$r_1=\frac{\Delta}{s-a}=\frac{24}{12-6}=\frac{24}{6}=4$$

Ans: A

17 $$r_3=\frac{\Delta}{s-c}$$

$$=\frac{24}{12-8}=\frac{24}{4}=6$$

Ans: C

18 Conceptual

Ans: A

19. ~~Conceptual~~

~~Ans: B~~

20 Consider ΔABC, is an equilateral Δle

$$r_1\cot\frac{A}{2}+r_2\cot\frac{B}{2}+r_3\cot\frac{C}{2}$$

$$=3.\ r_1\cot\frac{A}{2}$$

$$=3.\frac{\sqrt{3}}{2}.\cot 30^\circ=\frac{3\sqrt{3}}{2}\times\sqrt{3}=\frac{9}{2}=3\times\frac{3}{2}=3s$$

Ans: A

21. Zero, since in an equilateral Δle, orthocentre and circumcentre coincides

Ans: 0

22 $$\frac{a}{r_1}=\frac{1}{\frac{\sqrt{3}}{2}}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}.$$

23 a) $\Delta=400\sqrt{3}$, $s=20(2+\sqrt{3})$

$$r=\frac{\Delta}{s}=\frac{400\sqrt{3}}{20(2+\sqrt{3})}=\frac{20\sqrt{3}}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=20(2\sqrt{3}-3)$$

b) $\Delta = 400\sqrt{3}$, $\cancel{s = 20(2+\sqrt{3})}$, $s - a = 20\sqrt{3}$

$$r_1 = \frac{\Delta}{s-a} = \frac{400\sqrt{3}}{20\sqrt{3}} = 20$$

c) $a + b - c = 40(2-\sqrt{3})$

$$2s - 2c = 40(2-\sqrt{3})$$

$$\Rightarrow s - c = 20(2-\sqrt{3})$$

$$r_3 = \frac{\Delta}{s-c} = \frac{400\sqrt{3}}{20(2-\sqrt{3})} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$= 20\sqrt{3}(2+\sqrt{3})$$

d) $$\Delta = \frac{abc}{4R}$$

$$\Rightarrow 400\sqrt{3} = \frac{64000\sqrt{3}}{4R} \Rightarrow R = 40$$

$\therefore$ Ans: a, s, r, p

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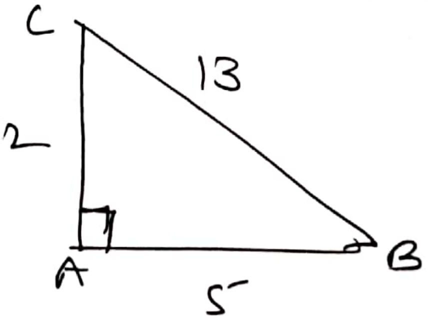
24 a) $A = 90^\circ$



$$\therefore \sin\left(\frac{A}{2}\right) = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

b) $2b = a + c$

$\Rightarrow 3b = 2s$

$$\tan\left(\frac{A}{2}\right) \cdot \tan\frac{C}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)} \times \frac{(s-a)(s-b)}{s(s-c)}}$$

$$= \frac{s-b}{s}$$

$$= \frac{2s-2b}{2s} = \frac{3b-2b}{3b} = \frac{b}{3b} = \frac{1}{3}$$

c) $3a = b + c$

$\Rightarrow 4a = 2s$

$\Rightarrow 2a = s$

$$\cot\frac{B}{2} \cdot \cot\frac{C}{2} = \sqrt{\frac{s(s-b)}{(s-a)(s-c)} \cdot \frac{s(s-c)}{(s-a)(s-b)}}$$

$$= \frac{s}{s-a}$$

$$= \frac{2a}{2a-a} = \frac{2a}{a} = 2$$

d) $$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$= \frac{1}{8} + \frac{1}{12} + \frac{1}{24}$$

$$= \frac{3+2+1}{24} = \frac{6}{24} = \frac{1}{4}$$

$\therefore \boxed{r = 4}$

$\Delta^2 = r r_1 r_2 r_3 \Rightarrow$ $\boxed{\therefore \Delta = 96}$

$$r = \frac{\Delta}{s}$$

$$4 = \frac{96}{s}$$

$$\Rightarrow \boxed{s = 24}$$

$$r_1 = \frac{\Delta}{s-a} \Rightarrow 8 = \frac{96}{24-a}$$

$$\Rightarrow \boxed{a = 12}$$

Ans: r, p, t, q

⇒ THE END ⇐