6th std advanced

cos-2 · Error analysis

Task

① Given number = 3400

All zero to the right of last non-zero digit are insignificant.

So the number of significant figures in 3200 are 2

② Given number = 0.0160 kg

zero right to the decimal point is insignificant if it followed by a non-zero digit (or) left to the non zero digit after decimal point

So the number of significant figure are 3

③ Given Least count [error] = 0.1 cm = Δl

Least count of vernier calliperse = 0.01 cm = Δr

$l = 4$ cm ; $r = 2$ cm.

The Percentage error in volume $\frac{\Delta V}{V}\% = 2\frac{\Delta r}{r} \times 100 + \frac{\Delta l}{l} \times 100$

$\Rightarrow \frac{\Delta V}{V}\% = 2 \times \frac{(0.01)}{2} \times 100 + \frac{0.1}{4} \times 100$

$\frac{\Delta V}{V}\% = 1 + 2 = 3\%$

④ The maximum error in force is $\frac{\Delta F}{F}\% = 4\%$

error in length $\frac{\Delta l}{l}\% = 2\%$

From the defination pressure = Force / Area

For a square plate area = [length]^2

$$\therefore \quad P = \frac{F}{L^2}$$

$$\Rightarrow \frac{\Delta P}{P}\% = \frac{\Delta F}{F} \times 100 + 2\frac{\Delta l}{l} \times 100$$

$$= 4\% + 2(2)\%$$

$$= 8\%$$

(5) Given % error in the measurement of length $\frac{\Delta l}{l}\% = 1\%$

% error in the measurement of radius $\frac{\Delta r}{r}\% = 2\%$

From defination $R = s\frac{l}{A} = s\frac{l}{\pi r^2}$

$\therefore$ % error in Resistance $\frac{\Delta R}{R} \times 100 = \frac{\Delta l}{l} \times 100 + 2\frac{\Delta r}{r} \times 100$

$$\frac{\Delta R}{R} \times 100 = 1\% + 2(2\%)$$

$$= 1\% + 4\%$$

$$= 5\%$$

(6) Given formula for measurement of length

$$L = k(m \times n)$$

Given % error in m ie) $\frac{\Delta m}{m}\% = 3\%$

% error in n ie) $\frac{\Delta n}{n}\% = 5\%$

$\therefore$ The error in the measurement of L

$$\frac{\Delta L}{L} \times 100 = \frac{\Delta m}{m} \times 100 + \frac{\Delta n}{n} \times 100$$

$$= 3\% + 5\%$$

The % error in length $\frac{\Delta L}{L} \times 100 = 8\%$

(7) Given Diameter = D = 0.25 cm

Least count = Error in measurement of Diameter

$\therefore \Delta D = 0.005$ cm

$\therefore$ percentage error in the measurement of Diameter

$$\frac{\Delta D}{D} \times 100 = \frac{0.005}{0.25} \times 100 = 2\%$$

(8) Given the sides of a rectangle $l \pm \Delta l = 5.5 \pm 0.1$ cm

$b \pm \Delta b = 3.5 \pm 0.2$ cm

$\therefore$ Perimeter $= 2(l+b) = 2(5.5+3.5) = 18$ cm

The maximum possible error in Perimeter $= 2(\Delta l + \Delta b)$

{ If $x = a+b$; $a = a \pm \Delta a$, $b = b \pm \Delta b$] $= 2(0.1+0.2)$

then $\Delta x = \Delta a + \Delta b$ $= 0.6$

$\therefore$ It's Perimeter $= (18 \pm 0.6)$ cm

(9) The initial temperature $= (t_1 + \Delta t_1)^\circ C = (30.4 \pm 0.2)^\circ C$

The final temperature $= (t_2 + \Delta t_2)^\circ C = (50.6 \pm 0.3)^\circ C$.

$\therefore$ The rise in temperature $= t_2 - t_1 = 50.6 - 30.4$

$= 20.2^\circ C$

The maximum possible error in rise in temperature

$\Delta t = \Delta t_1 + \Delta t_2 = 0.2 + 0.3$

$= 0.5^\circ C$.

[If $x = a-b$; where $a = a \pm \Delta a$; $b = b \pm \Delta b$]

$\Delta x = \Delta a + \Delta b$

$\therefore$ The final rise in temperature $= (20.2 \pm 0.5)^\circ C$

(10) Given length $l = 50$ cm ; $\Delta l = 0.25$ cm

The % error in length $\frac{\Delta l}{l}\% = \frac{\Delta l}{l} \times 100 = \frac{0.25}{50} \times 100 = 0.5\%$

(13) Given Distance travelled $= (20 \pm 0.2)\,m$
$(x \pm \Delta x)$

Time $= (4.0 \pm 0.04)\,s$
$(t \pm \Delta t)$

Error in distance $= \Delta x = 0.2\ m$

Error in time $= \Delta t = 0.04\ sec$

$$\text{velocity} = \frac{\text{distance}}{\text{time}} = \frac{20}{4} = 5\ m/s$$

Relative error in velocity $\frac{\Delta v}{v} = \frac{\Delta d}{d} + \frac{\Delta t}{t}$

$$\Rightarrow \frac{\Delta v}{5} = \left[\frac{20}{0.2}\ \frac{0.2}{20} + \frac{0.04}{4}\right]$$

$$= 0.01 + 0.01$$

$$\Rightarrow \frac{\Delta v}{5} = 0.02 \Rightarrow \Delta v = 0.02 \times 5$$
$$= 0.1\ m/s$$

$\therefore$ Velocity with error $= [5 \pm 0.1]\ m/s$

(14) (a) Given masses are $m_1 = 2.3\ kg$ [2 significant figures]
$= 2300\ g$

$m_2 = 41.15\ g$ [4 significant figures]

$m_3 = 30.19\ g$ [4 significant figures]

$\therefore$ Total mass $= m_1 + m_2 + m_3 = 2300 + 41.15 + 30.19$

$= 2371.34\ gm$

$= 2.37134\ g$

Round off it to 2 significant figures because in the given masses least number of significant figures are 2 [For m_1]

$\therefore$ Total mass $= 2.4\ kg$

(b) The value of $\frac{\pi}{53.2} = \frac{3.14}{53.2} = 0.0591$

(c) All zero's to the right of last non zero digit are significant if it is a measurement

So in 2.00×10^3 kg significant figures are 3.

(15) (a) All zero's to the right of last non zero digit are significant if it is a measurement. So in 8000 m → 4

(b) The zero right to the decimal point and left to the first non-zero digit are significant. So in 0.0130 → 3

(c) In $\frac{9.27}{41} = \frac{\text{3 significant figures}}{\text{2 significant figures}} = 0.226$ Round off it to 2 significant fig.

$= 0.23$.

(d) In 117.3 → significant figures are = 4

0.0024 → significant figures are = 2

$\Rightarrow 117.3 \times 0.0024 = 0.28152$ Round off it to 2 significant figures

$= 0.28$.

(16) Error in the measurement in length $= \frac{\Delta l}{l}\% = 1\%$

Error in the Measurement in radius $= \frac{\Delta r}{r}\% = 2\%$

∴ The % error in the measurement of cylinder volume

$$\frac{\Delta V}{V} \times 100 = 2\frac{\Delta r}{r} \times 100 + \frac{\Delta l}{l} \times 100$$

$$= 2 \times 2\% + 1\%$$

$$\Rightarrow 5\%$$

(17) Error in the Measurement of radius $= \frac{\Delta r}{r} \times 100 = 0.4\%$

% error in the measurement of volume $= 3 \times \frac{\Delta r}{r} \times 100$

$$\frac{\Delta V}{V} \times 100 = 3 \times 0.4$$

$$= 1.2\%$$

(18) Given radius of Earth = 6347 km

By rounding off = 6400 km $= 64.00 \times 10^5$ km

it to 2 significant figures

19. Given length $l = 25.2$ cm; breadth $b = 16.8$ cm

error $\Delta l = 0.1$ cm; $\Delta b = 0.1$ cm

Relative error in area of object $\frac{\Delta a}{a} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$

Area $a = l \times b$

$= 25.2 \times 16.8$

$= 423.36$

$\Rightarrow \frac{\Delta a}{a} = \frac{0.1}{25.2} + \frac{0.1}{16.8}$

$\Rightarrow \frac{\Delta a}{a} = [0.00397] + [0.00595]$

$\Rightarrow \frac{\Delta a}{a} = 0.009922$

$\simeq 0.01$

(20) % error in area $\frac{\Delta a}{a} \times 100 = 0.01 \times 100$

$= 1\%$

(18) In 6347, first we look at the last 2 digits in 6347. i-e → 47 which will be rounded off to 50. so now we get 6350. Again when we round off 350 in $350 \rightarrow \geq 5$ so we can add 1 to Previous (3) number it becomes → 4400 which now gives us the final answere

$6400 \text{ km} = 64 \times 10^5 \text{ km}$

L Task

CUQ's

(5) Trailing zero's in a number without decimal are not significant in $340500 \text{ m} \Rightarrow 3405 \times 10^2$

So in 3405 No. of significant figures = 4 m

(6) Given Diameter = $D = 0.5$ cm

Least count = $\Delta D = 0.001$ cm

∴ Percentage error in Diameter $= \frac{\Delta D}{D} \times 100$

$= \frac{0.001}{0.5} \times 100$

$= \frac{1}{5} = 0.2\%$

(7) All trialing zero's are insignificant

so in $2.64 \times 10^{24}\ m^2$ Number of significant figures are = 3.

SAQ's

(11) Given % error in current $\frac{\Delta I}{I} = 4\%$

% error in voltage $\frac{\Delta V}{V} = 2\%$

From Power = V I

∴ % error in the measurement of power

$$\frac{\Delta P}{P} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$

$$= 2\% + 4\%$$

$$= 6\%$$

(12) Given $Q = k L^{-2} M^2 N^3$

% error in $L = \frac{\Delta L}{L} = 1\%$

% error in $M = \frac{\Delta M}{M} = 2\%$

% error in $N = \frac{\Delta N}{N} = 1\%$

∴ % error in Q $\Rightarrow \frac{\Delta Q}{Q} \times 100 = 2 \times \frac{\Delta L}{L} \times 100 + 2\frac{\Delta m}{M} \times 100 + 3\frac{\Delta N}{N} \times 100$

$$= 2(1) + 2(2) + 3(1)$$

$$= 2 + 4 + 3 = 9\%$$

(13) Given length $l = 5$ cm ∴ Least count $= \Delta l = 0.01$ cm $= 0.1$ mm

∴ The percentage error in length $\frac{\Delta l}{l} \times 100 = \frac{0.01}{5} \times 100$

$= 0.2\%$

(14) Given Number 57.986

Here After decimal point 3rd digit (ie 6) is > 5 so we can add 1 to Previous digit (ie to 8) so we get

57.99 → significant figures - 4

(15) Given 44.8 − 21.235

No. of significant figures → 5

3

By doing = 44.800
− 21.235
23.565

So we have to round off minimum no. of signi-ficant figures (3) ∴ 23.6

23.565
+1
⇒ 23.57
+1
⇒ 23.6

(16) Given Numbers are

124.2 → No. of significant fig = 4

52.487 → No. of significant fig = 5

By doing = 124.200
52.487
176.687 → By rounding off to minimum no. of significant figures (4) = 176.7

first we look at last two digits in 176.687 ie 87 → which will be rounded off to 90. so now we get 176.690 Again we rounded off 690 to 700

In 690 → >5 so we can add 1 to 6 we get 700.

∴ Finally The number is 176.700 = 176.7

(17) In 117.3 → No. of significant figures are - 4

0.0024 → No. of significant figures are - 2

By doing 117.3 × 0.0024 = 0.2815 By rounding off to 2 significant figures we get 0.28.

(18) Given length $= 6 \pm 0.1$ cm ; breadth $= 4 \pm 0.2$ cm

$l \pm \Delta l$ $\qquad$ $b \pm \Delta b$

$\therefore$ Area of the plates $= l \times b \Rightarrow 6 \times 4 = 24\ cm^2 = a$

Relative error in area is $\frac{\Delta a}{a} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$

$$\Rightarrow \frac{\Delta a}{24} = \left[\frac{0.1}{6} + \frac{0.2}{4}\right]$$

$$\Rightarrow \Delta a = 24\left[\frac{0.4 + 1.2}{24}\right] = 1.6\ cm^2$$

$\therefore$ Area of the plates $(a \pm \Delta a) = (24 \pm 1.6)\ cm^2$

(19) $l_A = 5$cm ; $l_B = 7$cm

$\Delta l_A = 0.1$ cm $\qquad$ $\Delta l_B = 0.2$ cm

Error in their total length $= \Delta l_A + \Delta l_B$

$= 0.1 + 0.2 = 0.3$cm

Even in difference also error is $= \Delta l_A + \Delta l_B$

$= 0.3$cm

[If $x = a + b$ or $x = a - b$ Error in measurement of x is $\Delta x = \Delta a + \Delta b$]

(20) Given $x = \frac{A^2 B^3}{C\sqrt{D}} = \frac{A^2 B^3}{C D^{1/2}}$

Given $\frac{\Delta A}{A} \times 100 = 1\%$; $\frac{\Delta B}{B} = 2\%$; $\frac{\Delta C}{C} = 3\%$; $\frac{\Delta D}{D} = 4\%$

$\therefore$ The Possible % Error in x is

$$\frac{\Delta x}{x} \times 100 = 2\ \frac{\Delta A}{A} \times 100 + 3\ \frac{\Delta B}{B} \times 100 + \frac{\Delta C}{C} \times 100 + \frac{1}{2}\ \frac{\Delta D}{D} \times 100$$

$$\Delta x = 2(1) + 3(2) + 3 + \frac{1}{2}(4)$$

$$= 2 + 6 + 3 + 2$$

$$\frac{\Delta x}{x} = 13\%$$

(23) Given $L_1 = (2.02 \pm 0.01)\,m$; $L_2 = (1.02 \pm 0.01)\,m$.
$L_1 \pm \Delta L_1$ $\quad$ $l_2 \pm \Delta l_2$

Error in $l_1 = \Delta l_1 = 0.01\,m$; Error in $l_2 = \Delta l_2 = 0.01\,m$

Error in $l_1 + 2L_2 = \Delta l_1 + 2\Delta l_2 = 0.01 + 2(0.01)$
$= \underline{0.03\,m}$

(24)

(a) All non-zero digits are significant and all zeros to the right of last non-zero digit after decimal point are significant

so in $6.320 \rightarrow$ Significant figures $\rightarrow 4$

(b) All zeros to the right of decimal point and left of non zero digit are insignificant [eg Number < 1]

so in $0.0006032 \rightarrow$ significant figures $\rightarrow 4$

(c) Zeros between two non-zero digit are significant

so in $48000.50 \Rightarrow 7$ significant figures.

(25) Given Error in the measurement of sphere

$\frac{\Delta r}{r} = 2\%$ $\quad$ we know $V = \frac{4\pi}{3} r^3$

$\therefore$ Error in the measurement of volume =

$\frac{\Delta V}{V} \times 100 = 3\frac{\Delta r}{r} \times 100$

$= 3(2)$

$= 6$

$\therefore \frac{\Delta V}{V} = 6\%$

(26) Given $\frac{\Delta m}{m} = 2\%$; $\frac{\Delta V}{V} = 3\%$.

From $K.E = \frac{1}{2}mv^2$ we can write

$\frac{\Delta K.E}{K.E} \times 100 = \left(\frac{\Delta m}{m} + 2\frac{\Delta V}{V}\right) \times 100$

$= [2\% + 2(3\%)] \times 100$

$\therefore \frac{\Delta K.E}{K.E} = 8\%$

(27) If any number has more than one digit after decimal point are significant. Power of 10 does not affect no. of significant figures.

So in 4.8000×10^4 → 5 significant figures

(28) In 124.2 → significant figures → 4.

52.487 → " " → 5

By doing $124.200 - 52.487 = 71.713$ By rounding to 4 significant figures we get 71.71 [In last two digits (i.e) 13 No number is greater than or equal to 5, so we can not add 1 to either 1 (or) 7.

(29) From problem 23 we got error in $L_1 + 2L_2$ as 0.03.

By doing $L_1 + 2L_2 = 2.02 + 2(1.02)$

$= 2.02 + 2.04$

$\Rightarrow 4.06$

$\therefore L_1 + 2L_2$ with accuracy $= (4.06 \pm 0.03)$ cm