Hints and Solutions

Exercise - 1
Single Answer Type Questions

1. Avg. velocity $= \dfrac{20\times 3 + 4\times 20 + 5\times 20}{20+20+20} = 4$ m/s

2. $\hat{v} = \dfrac{(4-1)\hat{i} + (2+2)\hat{j} + (3-3)\hat{k}}{\sqrt{3^2+4^2+0^2}} = \dfrac{3\hat{i}+4\hat{j}}{5}$

$\vec{v} = |\vec{v}|\ \hat{v} = 10\left(\dfrac{3\hat{i}+4\hat{j}}{5}\right) = 6\hat{i}+8\hat{j}$

3. $v_i = 2\hat{i} \Rightarrow v_f = 4\cos 60°\hat{i} + 4\sin 60°\hat{j}$

$= \dfrac{4}{2}\hat{i} + \dfrac{4\sqrt{3}}{2}\hat{j} = 2\hat{i} + 2\sqrt{3}\hat{j}$

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = 2\hat{i} + 2\sqrt{3}\hat{j} - 2\hat{i} = 2\sqrt{3}\hat{j}$

$\langle\vec{a}\rangle = \dfrac{2\sqrt{3}\hat{j}}{2} = \sqrt{3}\hat{j}$ m/s²

4. $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$ here $v_x = \dfrac{dx}{dt} = 2ct$; $v_y = \dfrac{dy}{dt} = 2bt$

Therefore $|\vec{v}| = \sqrt{4t^2(c^2+b^2)} = 2t\sqrt{(c^2+b^2)}$

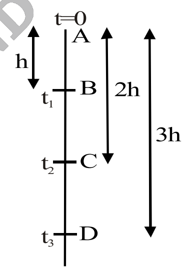
5. For v = 0, x = 1,4 and $a = v\dfrac{dv}{dx}$

so $a|_{x=1} = 0 \times \dfrac{dv}{dx} = 0$; $a|_{x=4} = 0 \times \dfrac{dv}{dx} = 0$

6. $t_1 = \sqrt{\dfrac{2h}{g}}$

$t_2 = \sqrt{\dfrac{2\times 2h}{g}}$

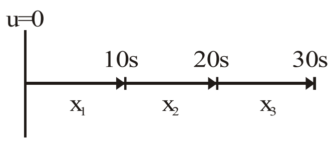
$t_3 = \sqrt{\dfrac{2\times 3h}{g}}$



Required ratio $t_1 : (t_2 - t_1) : (t_3 - t_2)$

$= 1 : \left(\sqrt{2}-1\right) : \left(\sqrt{3}-\sqrt{2}\right)$

7.



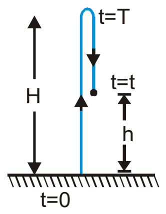
$x_1 = \dfrac{1}{2}a(10)^2 \Rightarrow x_1 + x_2 = \dfrac{1}{2}a(20)^2$

$x_1 + x_2 + x_3 = \dfrac{1}{2}a(30)^2 \Rightarrow x_1 : x_2 : x_3 = 1 : 3 : 5$

8. $\vec{v}_{(1)} = (3 + 4\times 1)\hat{i} + (4 + (-3)\times 1)\hat{j} = 7\hat{i} + \hat{j}$

$|\vec{v}_{(1)}| = \sqrt{49+1} = 5\sqrt{2}$ m/s

9.



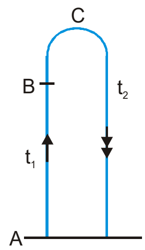
$h = H - \dfrac{1}{2}g(t-T)^2$

10. $t_{AC} = \dfrac{t_1+t_2}{2}$; $t_{BC} = \dfrac{t_2-t_1}{2}$

$\therefore AB = AC - BC$

$= \dfrac{1}{2}g\left(\dfrac{t_1+t_2}{2}\right)^2 - \dfrac{1}{2}g\left(\dfrac{t_2-t_1}{2}\right)^2$

$= \dfrac{1}{2}gt_1t_2$



11. Velocity after 10 sec is equal to

$0 + (10)(10) = 100$ m/s

Distance covered in 10 sec is equal to

$\dfrac{1}{2}(10)(10)^2 = 50$m

Now from $v^2 = u^2 + 2as$.

$\Rightarrow v^2 = (100)^2 - 2(2.5)(2495-400) = 25 \Rightarrow v = 5\text{ms}^{-1}$

12. It happens when in this time interval velocity becomes zero in vertical motion

$\Rightarrow \dfrac{u}{g} = 5 \Rightarrow u = 5 \times 9.8 = 49$ m/s

13. $S_B = S_A + 10.5$

$\dfrac{t^2}{2} = 10t + 10.5$

$t^2 = 20t + 21$

$t^2 - 20t - 21 = 0$

t = 21 sec

14. Displacement $= \dfrac{1}{2}[4+2]\times 4 - \dfrac{1}{2}[4+3]\times 2$

$= 12 - 7 = 5$ m

Distance $= 12 + 7 = 19$ m

15. Two values of velocity (at the same instant) is not possible.

16. When the secant from P to that point becomes the tangent at that point

17. $a = \dfrac{d^2x}{dt^2}$ = change in velocity w.r.t. the time

For OA $\rightarrow$ velocity decreases so a is negative

For AB $\rightarrow$ velocity constant so a is zero.

For BC $\rightarrow$ velocity constant so a is zero.

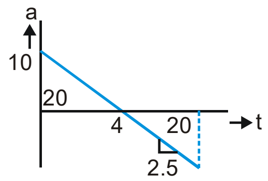
For CD $\rightarrow$ velocity increases so a is positive.

18. Initially velocity increases downwards (negative) and after rebound it becomes positive and then speed is decreasing due to acceleration of gravity ($\downarrow$)

20. Initially the speed decreases and then increases.

21. Upward area of a-t graph gives the change in velocity = 20 m/s for acquiring initial velocity, it again changes by same amount in negative direction.

Slope of curve = – 10/4 = – 2.5



$\therefore$ time = $\sqrt{\dfrac{2 \times 20}{2.5}}$ = 4sec

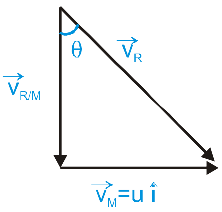
Total time = 4 + 4 =8 sec

22. For shortest time to cross, velocity should be maximum towards north as river velocity does not take any part to cross.

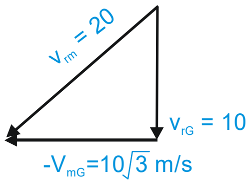
23. $\Rightarrow \vec{v}_{R/M} = \dfrac{u}{\tan\theta}\hat{j}$

$\therefore \vec{v}_R = \vec{v}_{R/M} + \vec{v}_M$

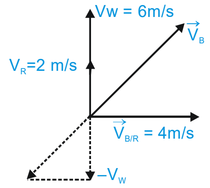
$\Rightarrow \vec{v}_R = u\hat{i} - \dfrac{u}{\tan\theta}\hat{j}$



24. $v_{mG} = \sqrt{(v_{rm})^2 - (v_{rG})^2} = \sqrt{(20)^2 + (10)^2} = 10\sqrt{3}$ m/s



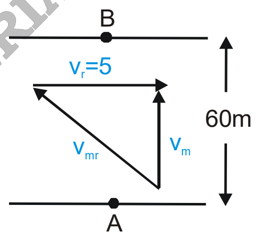
25. Flag blows in the direction of resultant of $\vec{V}_W$ & $-\vec{V}_B$



$\vec{V}_W - \vec{V}_B = 6\hat{j} - (4\hat{i} + 2\hat{j}) = 4(-\hat{i} + \hat{j})$NW

$\Rightarrow$ N-W direction.

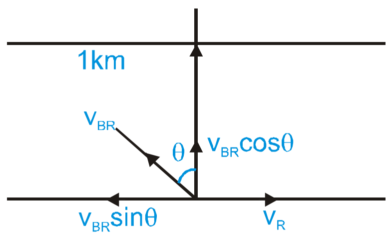
26. The resultant velocity should be in the direction of resultant displacement



So time = $\dfrac{60}{\sqrt{v_m^2 - 5^2}} = 5 \quad \because v_{rm}$ = 13 m/s

27. For shortest time then maximum velocity is in the direction of displacement.

28.



s = ut

$1 = v_{BR}\cos\theta \ t \quad \Rightarrow \quad 1 = 5\cos\theta \ \dfrac{1}{4}$

$\cos\theta = \dfrac{4}{5} \quad \Rightarrow \quad \theta = 37°$

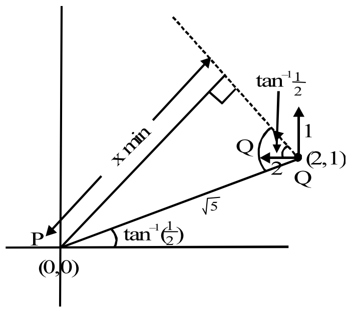
$v_R = v_{BR}\sin 37° = 5 \times \dfrac{3}{5}$ =3 km/hr

29. Time of collision of two boat = 20/2 = 10 sec.

As given in question i.e. the time of flight of stone is also equal to 10 sec. so vertical component of stone initially is 50 m/s and the horizontal component w.r.t. motorboat equals to 2 m/s.

Hence $\vec{v}_{BG} = 3\hat{i} + 50\hat{j}$

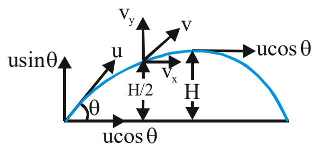
30. $\vec{v}_{QP} = -\hat{i} + 2\hat{j} - \hat{i} - \hat{j} = -2\hat{i} + \hat{j}$



So from sine rule $\dfrac{\sqrt{5}}{\sin 90°} = \dfrac{x_{min}}{\sin\theta} \Rightarrow x_m$

$= \sqrt{5} \times 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} = \sqrt{5} \times 2 \times \dfrac{1}{\sqrt{5}} \times \dfrac{2}{\sqrt{5}} == \dfrac{4}{\sqrt{5}}$

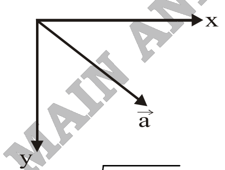
31. $v_y^2 = u^2\sin^2\theta - 2g \times \dfrac{H}{2}$; $v_x^2 = u^2\cos^2\theta$



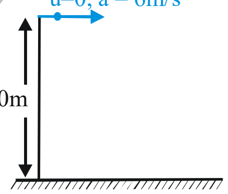
$\therefore$ ucosθ $= \sqrt{\dfrac{6}{7}}\left[\sqrt{v_x^2 + v_y^2}\right]$

$\Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2}$ *or* $\theta = 30°$

32. $\vec{a}_x = a_1\hat{i}$; $\vec{a}_y = -a_2\hat{j}$



33. Time to reach the ground $= \sqrt{\dfrac{2 \times 20}{10}} = 2$ sec



So horizontal displacement $= 0 + \frac{1}{2} \times 6 \times 4 = 12$m

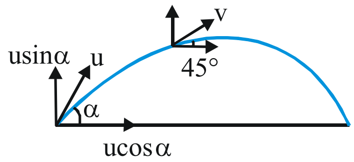
34. $\vec{v} = a\hat{i} + (b - ct)\hat{j}$

Time to reach maximum height (when $\hat{j}$ comp. of velocity becomes zero)

$\therefore b - ct = 0 \Rightarrow t = \dfrac{b}{c}$ $\quad\therefore$ Time of flight $= \dfrac{2b}{c}$

range = horizontal velocity × Time of flight $= a \times \dfrac{2b}{c}$

35. $\vec{v} = u\cos\alpha\,\hat{i} + (u\sin\alpha - gt)\hat{j}$ $\because$ $\vec{v} = \vec{v}_x = \vec{v}_y$



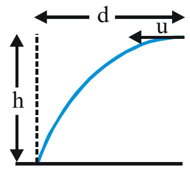
ucosα = usinα - gt $\Rightarrow t = \dfrac{u}{g}(\sin\alpha - \cos\alpha)$

36. $-1500 = \dfrac{-500}{3}\sin 37° \times t - \dfrac{1}{2} \times 10 \times t^2$; t = ?

Distance $= \dfrac{500}{3}\cos 37° \times t$ (Horizontal)

$\Rightarrow x = \dfrac{4000}{3}$m

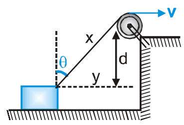
37. Time to reach at ground $= \sqrt{\dfrac{2h}{g}}$



In this time horizontal displacement

$d = u \times \sqrt{\dfrac{2h}{g}} \Rightarrow d^2 = \dfrac{u^2 \times 2h}{g}$

38. $x^2 = y^2 + d^2$



$\Rightarrow 2x\dfrac{dx}{dt} = 2y\dfrac{dy}{dt}$

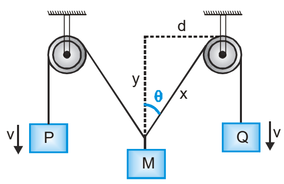
$\Rightarrow \dfrac{dy}{dt} = \left(\dfrac{x}{y}\right)\left(\dfrac{dx}{dt}\right) = \dfrac{v}{\sin\theta}$

OR

Component of velocity along string must same so

$v_M\cos\theta(90-\theta) = v \Rightarrow v_M = \dfrac{v}{\sin\theta}$

39.



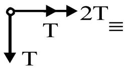
Here $x^2 = y^2 + d^2$.

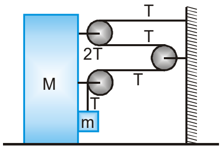
So $2x\frac{dx}{dt} = 2y\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \left(\frac{x}{y}\right)\left(\frac{dx}{dt}\right) = \left(\frac{x}{y}\right)(v) = \frac{v}{\cos\theta}$

OR

Component of velocity along string must be same

so $v_M \cos\theta = v \Rightarrow v_M = \frac{v}{\cos\theta}$

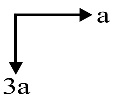
40. Net tension on M  $\equiv \sqrt{(3T)^2 + T^2} = \sqrt{10}T$



Now from acceleration × Tension = constant

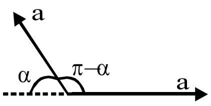
$\Rightarrow a_M(\sqrt{10}T) = a_m(T) \Rightarrow a_m = (\sqrt{10})a_M = \sqrt{10}a$

OR

Net acceleration of m 

$\equiv \sqrt{a^2 + (3a)^2} = \sqrt{10}\ a$

41. Net acceleration of load



$= 2a\cos\left(\frac{\pi - \alpha}{2}\right) = 2a\sin\left(\frac{\alpha}{2}\right)$

42. Given $\omega = \theta^2 + 2\theta$

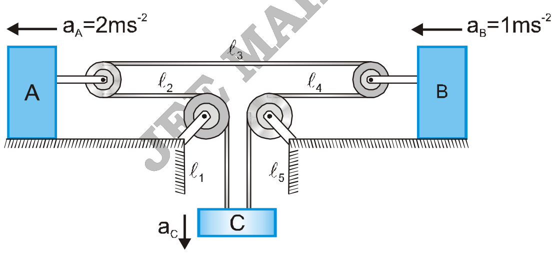
$\frac{d\omega}{d\theta} = 2\theta + 2$

$\alpha = \omega\frac{d\omega}{d\theta} = (\theta^2 + 2\theta)(2\theta + 2)$

at $\theta = 1$

$\alpha = 12$ rad/sec^2

43.



$\ell_1 + \ell_2 + \ell_3 + \ell_4 + \ell_5 =$ constant

$\Rightarrow \ddot{\ell}_1 + \ddot{\ell}_2 + \ddot{\ell}_3 + \ddot{\ell}_4 + \ddot{\ell}_5 = 0$

$\Rightarrow a_C + a_A + (a_A - a_B) + (-a_B) + a_C = 0$

$\Rightarrow 2a_C + 2a_A - 2a_B = 0$

$\Rightarrow a_C = a_B - a_A = 1 - 2 = -1$ ms^{-2}

$\Rightarrow$ Acceleration of C is 1 ms^{-2} upwards

44. $\omega = \frac{14 \times 2\pi}{25}$

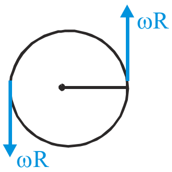
$\therefore$ magnitude of acceleration

$= \omega^2 r = \left(\frac{14 \times 2\pi}{25}\right)^2 \times \frac{80}{100} \simeq 9.9$ m/s^2

45. Centripetal acceleration $= \frac{v^2}{R}$

$\frac{v_1^2}{R_1} = \frac{v_2^2}{R_2} = \frac{v_1}{v_2} = \sqrt{\frac{R_1}{R_2}} = \sqrt{\frac{1}{2}}$

46. ω = constant , $a_T = 0$



$\left[\frac{2\omega^2 rx}{\pi}, \omega = \frac{2\pi}{T}, \frac{T}{2} = \frac{\pi}{\omega}\right]$

$a_{av} = \frac{2\omega R}{\pi / \omega} = \frac{2\omega^2 R}{\pi}$; $a_{inst} = \omega^2 R$

So ratio $= \frac{a_{av}}{a_{inst}} = \frac{2}{\pi}$

47. Given $r = \frac{20}{\pi}$ m

Angular velocity after second revolution

$\omega = \frac{v}{r} = \frac{50\pi}{20} = \frac{5\pi}{2}$

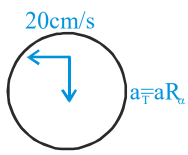
$\omega_{final}^2 = \omega_{initial}^2 + 2\alpha\theta$

$\frac{25}{4}\pi^2 = 2\alpha(4\pi) \Rightarrow \alpha = \frac{25\pi}{32}$

$a_t = \alpha r = \frac{25\pi}{32} \times \frac{20}{\pi} = 15.6$

48. ω and α remain same but v and a_T is proportional to r thus at half the radius,

$v' = \frac{v}{2}$ & $a_T' = \frac{a_T}{2}$



49. $\ell = 6$cm, v = ?, $\omega = \frac{2\pi}{60} = \frac{\pi}{30}$ rad/s.

So $v = \omega\ell = \frac{\pi}{30} \times 6 = \frac{\pi}{5}$ cm/s $= 2\pi$ mm/s

Difference $= \sqrt{2}\frac{\pi}{5}$ cm/s $= 2\sqrt{2}\,\pi$ mm/s

50. Angular velocity ω about centre $= 2\omega$

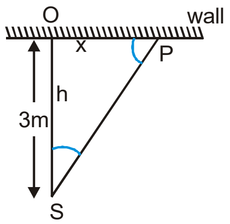
$= 2 \times 0.40 = 0.30$ rad/sec

$v = \omega R$

$= 0.80 \times \frac{1}{2} = 0.40$ m/s

$a = \frac{v^2}{R} = \frac{0.40 \times 0.40 \times 100}{50} = 0.32$ cm/s^2

51. Let x is the distance of point P from O, the, from figure



$\tan\phi = \frac{x}{h}$ or $x = h\tan\phi$

$\Rightarrow \frac{dx}{dt} = h\sec^2\phi \frac{d\theta}{dt}$

$\left[\frac{d\phi}{dt} = \omega\right] \Rightarrow v = h\sec^2\phi\,\omega$

So putting values

$h = 3, \phi = 180 - (90 + 45) = 45°$

we get $v = (3\sqrt{2})^2 \times 0.1 = 0.6$ m/s

Exercise - 2
Segment-I Multiple Answer Type

1. $\alpha = -av^2 = \int_u^v \frac{dv}{v^2} = -a\int_{t=0}^{t} dt$

$\Rightarrow -\left[\frac{1}{v}\right]_u^v = -at \Rightarrow \frac{1}{u} - \frac{1}{v} = -at$

$\Rightarrow v = \frac{u}{1 + aut} \quad \int_0^x dx = \int_{t=0}^{t} \frac{udt}{1 + aut}$

$\Rightarrow x = \frac{u}{au}[\ell n(1 + aut)]_0^t = \frac{1}{a}\ell n(1 + aut)$

2. $u_x = u_0$; $u_y = a\omega\cos\omega t$

$x = u_0 t$; $\int_0^y dy = a\omega\int_{t=0}^{t} \cos\omega t\, dt$

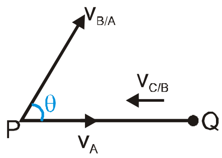
$y = a\frac{w}{w}\sin\omega t = a\sin\left(\frac{\omega x}{u_0}\right)$

3. $t = \alpha x^2 + \beta x \Rightarrow 1 = (2\alpha x + \beta)v \Rightarrow v = \frac{1}{\beta + 2\alpha x}$

$\therefore$ Acceleration $= \frac{2\alpha}{(\beta + 2\alpha x)^2} v = 2\alpha v^3$

4. $|\vec{v}_A| = 10$ m/s

$\vec{v}_C = \vec{v}_{C/B} + \vec{v}_{B/A} + \vec{v}_A$

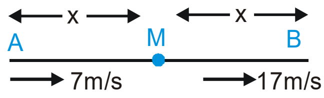


$= 12(-\hat{i}) + 6 \times \frac{15}{24}(\hat{i}) + \left(6 \times \frac{\sqrt{351}}{24}\hat{j}\right) + 10\hat{i}$

$= \left(\frac{15}{4} - 2\right)\hat{i} + \frac{\sqrt{351}}{4}\hat{j}$

$|\vec{v}_C| = \frac{\sqrt{7^2 + (\sqrt{351})^2}}{4} = 3$ m / s

5.



$v_m^2 = (7)^2 + 2 \times a \times x$; $v_m = 13$ m/s

$(17)^2 = (7)^2 + 2 \times a \times 2x$

$\frac{13 - 7}{a} = t_1$; $t_2 = \frac{17 - 13}{a}$; $\frac{t_1}{t_2} = \frac{6}{4} = \frac{3}{2}$

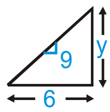
6. Distance covered by :

train I = (Area of Δ)$_{\text{train I}}$ = 200 m

train II = (Area of Δ)$_{\text{train II}}$ = 80 m

So the seperation = 300 – (200 + 80) = 20 m.

7. As given $9 = y/6 \Rightarrow$ $\boxed{y = 54\text{m}}$



Average velocity of particle

$B = \frac{\text{Displacement}}{\text{time}} = \frac{54}{6} = 9$m/s

8. Time of fall of stone $= \sqrt{\frac{2 \times 20}{10}} = 2$ sec

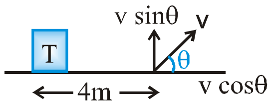
Horizontal displacement of truck in 2 sec

$\Rightarrow S = 2 \times 2 + \frac{1}{2} \times 1 \times 4$.

Length of truck = 6m

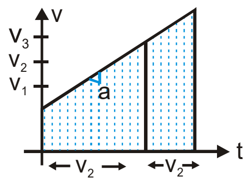
9. $\vec{r} = (t^2 - 4t + 6)\hat{i} + t^2\hat{j}$; $\vec{v} = (2t - 4)\hat{i} + 2t\hat{j}$

$\vec{a} = 2(\hat{i} + \hat{j})$; when $\vec{a} \perp \vec{v}$ then $\vec{a} \cdot \vec{v} = 0$; t = 1s



10. When acceleration is constant the instantaneous velocity is equal to the average velocity in mid of the time interval.

$a = \frac{v_2 - v_1}{\frac{t_1}{2} + \frac{t_2}{2}} = \frac{v_3 - v_2}{\frac{t_2}{2} + \frac{t_3}{2}}$.



11. Time to cross 2m is $\left(\frac{2}{v\sin\theta}\right)$ .....

To avoid an accident

Displacement $= 4 + v\cos\theta \times \frac{2}{v\sin\theta}$

$8 \times \dfrac{2}{v\sin\theta} = 4 + 2\cot\theta$

$v\sin\theta = \dfrac{16\sin\theta}{4\sin\theta + 2\cos\theta}$

$v_{min} = \dfrac{16}{\sqrt{4^2+2^2}} = 1.6\sqrt{5}$ m/s

[$\because$ (a cos θ + b sin θ) has max. value = $\sqrt{a^2+b^2}$]

12. $\vec{v} = 4t\hat{i} + 3t\hat{j}$ ($\because$ x = at² & y = 3/2t²)

$v(1) = 4\hat{i} + 3\hat{j}$; $v(2) = 8\hat{i} + 6\hat{j}$

$\therefore \langle v \rangle = \dfrac{12\hat{i}+9\hat{j}}{2} = (6\hat{i} + 4.5\hat{j})$ m/s

13. $x = 40 + 12t - t^3$.

Speed $\dfrac{dx}{dt} = 0 + 12 - 3t^2 \Rightarrow t = \pm 2$sec

$\therefore$ x(2) = 40 + 12 × 2 – 2³

= 64 – 8 = 56 m. at t = 0, x(0) = 40

Δx = x(2) – x(0) = 16

14. $\langle v_{space} \rangle = \dfrac{\int v ds}{\int ds} = \dfrac{\int \sqrt{2as}ds}{\int ds} = \dfrac{2}{3}v$

$\langle v_{time} \rangle = \dfrac{\int v dt}{\int dt} = \dfrac{\int at dt}{\int dt} = \dfrac{v}{2}$ $\therefore \dfrac{\langle v_s \rangle}{\langle v_t \rangle} = 4:3$

15. v↑ ↓ ↓g m/s² v↓ ↓g m/s² ↑2m/s²
2m/s²

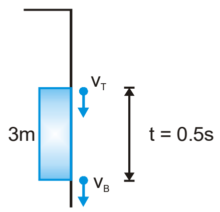
$\therefore$ time of ascent = $\sqrt{\dfrac{2h}{g+2}}$

time of descent = $\sqrt{\dfrac{2h}{g-2}} \Rightarrow \therefore \dfrac{t_a}{t_d} = \sqrt{\dfrac{8}{12}} = \sqrt{\dfrac{2}{3}}$

16. $v_B = v_T + 9.8 \times 0.5 = v_T + 4.9$

$v_B - v_T = 4.9$ m/s and

$v_B^2 - v_T^2 = 2gs = 2 \times 9.8 \times 3 = 58.8$

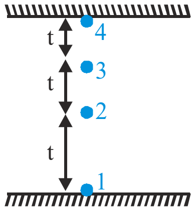


$\Rightarrow (v_B + v_T) \times (v_B - v_T) = 2 \times 9.8 \times 3$

$\Rightarrow v_B + v_T = 12$ m/s

17. Time taken to reach the drop to ground

$9 = 0 + \dfrac{1}{2} \times 10 \times (3t)^2$



$\sqrt{\dfrac{9}{5}} = 3t$

$\boxed{\dfrac{\sqrt{1.8}}{3} = t}$

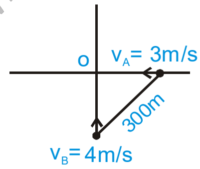
$x_2 = \dfrac{1}{2} \times 10 \times (2t)^2 = 20t^2 = 20 \times \dfrac{1.8}{9} = 4$m

$x_3 = \dfrac{1}{2} \times 10 \times (t)^2 = 5t^2 = 5 \times \dfrac{1.8}{9} = 1$m

18. $300^2 = (3t)^2 + (4t)^2$

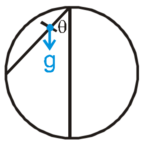
$300 \times 300 = 25t^2$

t = 60



Ratio = $\dfrac{3 \times 2\sqrt{3}}{4 \times 2\sqrt{3}} = 3:4$

19. Time to fall = $\sqrt{\dfrac{2 \times 2R\cos\theta}{g\cos\theta}}$



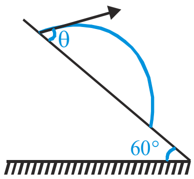
so it does not depend on θ i.e. the chord position.

20. For man on trolley $\dfrac{3}{2}vt = L \Rightarrow t = \dfrac{2L}{3v}$

with respect to ground : $vt + \dfrac{3}{2}vt = L + \dfrac{2L}{3} = \dfrac{5L}{3}$

$\therefore \dfrac{3}{2}vt - vt = L - \dfrac{2L}{3} = \dfrac{L}{3}$ $\therefore \Delta S = \dfrac{5L}{3} - \dfrac{L}{3} = \dfrac{4L}{3}$

21. Time of flight $\boxed{4 = \dfrac{2u\sin\theta}{g\cos 60°}}$ ...(i)



(angle of projection = θ)
Distance travelled by Q on incline in 4 secs is

$= 0 + \dfrac{1}{2} \times \dfrac{\sqrt{3}g}{2} \times 4^2 = 40\sqrt{3}$

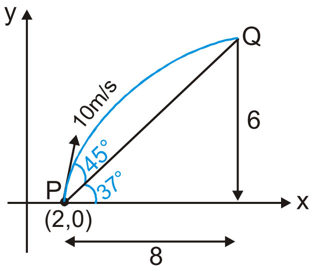
& the range of particle 'P' is $40\sqrt{3}$

$= u\cos\theta \times 4 + \dfrac{1}{2}\dfrac{\sqrt{3}g}{2} \times 4^2 = 40\sqrt{3}$

= u cos θ = 0 ; so θ = 90°

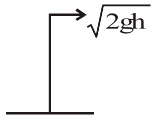
from equation (i) u = 10 m/s

22. $PQ = R = \dfrac{u^2 \sin 90°}{g} = \dfrac{100 \times 1}{10} = 10 \Rightarrow PQ = 10$



$(x,y) = (10, 6, 0)$

23. Time to fall $= \sqrt{\dfrac{2 \times h}{g}}$

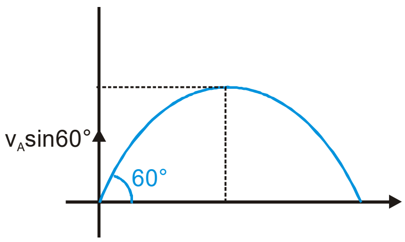


Range $=$ Horizontal velocity $\times$ time

$$x = \sqrt{2gh} \times \sqrt{\frac{2h}{g}} = 2h$$

24. At maximum height vertical component of velocity becomes zero.

$v^2 = u^2 + 2as$



For A : $0 = v_A^2 \sin^2 60° - 2gh$

$2gh = v_A^2 \sin^2 60° = v_A^2\ (3/4)$

$$v_A = \sqrt{\frac{8gh}{3}}$$

For B : $0 = v_B^2 - 2gh$

$$v_B = \sqrt{2gh}\ ;\ \frac{v_A}{v_B} = \frac{2}{\sqrt{3}}$$

25. $x = 10\sqrt{3}t\ ;\ \ y = 10t - t^2;\ \dfrac{dx}{dt} = 10\sqrt{3}$

$v_y = \dfrac{dy}{dt} = 10 - 2t \quad \Rightarrow \quad$ at $t = 5$ sec.

v_y becomes zero at maximum height

$\Rightarrow y = 10 \times 5 - 5^2 = 25m.$

26. $\vec{r} = t^2\hat{i} + (t^3 - 2t)\hat{j}\ ;$

$$\vec{v} = \frac{d\vec{r}}{dt} = 2t\hat{i} + (3t^2 - 2)\hat{j}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = 2\hat{i} + 6t\hat{j}$$

$\vec{a}.\vec{v} = 4t + 18t^3 - 12t = 0$ (For $\perp$)

$\therefore t = \pm 2/3,\ 0.$

For parallel to x-axis $\Rightarrow \dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{3t^2 - 2}{2}$

$\therefore$ at $t = \sqrt{\dfrac{2}{3}}$ sec it becomes zero so (c)

$\vec{a}_{(4,4)} = 2\hat{i} + 6 \times 2\hat{j} = 2\hat{i} + 12\hat{j}$

27. Acceleration = Rate of change of velocity **i.e.** velocity can be changed by changing its direction, speed **or** both.

28. Area of the curve gives distance.

29. $x = t^3 - 3t^2 - 9t + 5.\ x(5) > 0$ and $x(3) > 0$

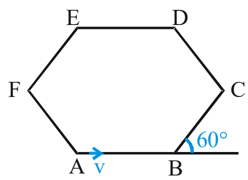
so [A] $v = dx/dt = 3t^2 - 6t - 9$

$\Rightarrow t = -1, 3$ so $t = 3$

Hence particle reverses its direction only once average acc. = change in velocity /time.

In interval ($t = 3$ to $t = 6$), particle does not reverse its velocity and **also** moves in a straight line so distance = displacement.

30. Av. velocity $= \dfrac{\text{Displacment}}{\text{time}}$



31. $x = 2 + 2t + 4t^2,\ y = 4t + 8t^2$

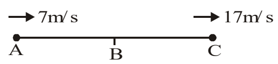
$v_x = \dfrac{dx}{dt} = 2 + 8t,\ v_y = \dfrac{dy}{dt} = 4 + 16t$

$a_x = 8;\ a_y = 16;\ \vec{a} = 8\hat{i} + 16\hat{j}$ = constant

$y = 2(2t + 4t^2);\ y = 2(x - 2)$ ($\because x = 2 + 2t + 4t^2$)

which is the equation of straight line.

32. Motion A to C $\Rightarrow 17^2 = 7^2 + 2as$



Motion A to B $\Rightarrow v_B^2 = 7^2 + 2a\left(\dfrac{s}{2}\right) = \dfrac{17^2 + 7^2}{2}$

(A) $v_B = \sqrt{\dfrac{289 + 49}{2}} = 13$ m/s

(B) $<v_{AB}> = \dfrac{7 + 13}{2} = 10$ m/s

(C) $t_1 = \dfrac{13 - 7}{a},\ t_2 = \dfrac{17 - 13}{a},\ \dfrac{t_1}{t_2} = \dfrac{6}{4} = \dfrac{3}{2}$

(D) $<v_{BC}> = \dfrac{13 + 17}{2} = 15$ m/s

33. $x = u(t-2) + a(t-2)^2$ ...(i)

$\Rightarrow v = \frac{dx}{dt} = u + 2a(t-2)$

Therefore $v(0) = u - 4a$

$a = \frac{d^2x}{dt^2} = 2a.$ **Hence [C]**

$x(2) = 0$ [From (i)]. **Hence [D]**

34. [A] $\because$ Distance $\geq$ Displacement

$\therefore$ Average speed $\geq$ Average velocity

[B] $|\vec{a}| \pm 0 \Rightarrow \Delta\vec{v} \pm 0$

velocity can change by changing its direction

[C] Average velocity depends on displacement in time interval e.g. circular motion $\rightarrow$ after one revolution displacement become zero Hence average velocity but instantaneous velocity never becomes zero during motion.

[D] In a straight line motion ; there must be reversal of the direction of velocity to reach the destination point for making displacement zero and Hence instantaneous velocity has to be zero at least once in a time interval.

35. $\vec{v}(t) = (3 - 1 \times t)\hat{i} + (0 - 0.5t)\hat{j}$ ...(i)

For maximum positive x coordinate when v_x becomes zero

$\therefore 3 - t = 0 \Rightarrow t = 3$ sec

then $\vec{r}(3) = 4.5\hat{i} - 2.25\hat{j}.$

36. $v = \sqrt{x}$; $\int_4^x \frac{dx}{\sqrt{x}} = \int_{t=0}^{t} dt \Rightarrow [2\sqrt{x}]_4^x = t$

$\Rightarrow x = \left(\frac{t+4}{2}\right)^2$ at $t = 2 \Rightarrow x = 9m$

$a = v\frac{dv}{dx} = \sqrt{x} \times \frac{1}{2\sqrt{x}} = \frac{1}{2}$ m/s^2

at $x = 4 \Rightarrow v = 2$m/s & it increases as x increases so it never becomes negative.

37. $\vec{v} = |\vec{v}|\hat{v}$; $[|\vec{v}| \rightarrow$ speed]

Velocity may change by changing either speed **or** direction and by both.

38. For returning, the starting point

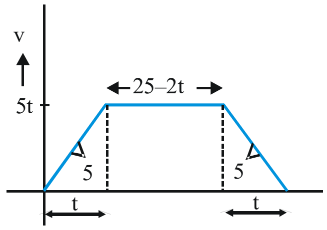
Area of (ΔOAB) = Area of (ΔBCD)

$\frac{1}{2} \times 20 \times 25 = \frac{1}{2} \times t \times 4t \Rightarrow t = 5\sqrt{5} \simeq 11.2$

$\therefore$ Required time = 25 + 11.2 = 36.2

39. Average velocity

$= \frac{\text{Displacement}}{\text{time interval}} = \frac{\text{Area under v} - \text{t curve}}{\text{time}}$



$20 = \frac{\frac{1}{2}[25 + 25 - 2t] \times 5t}{25} \Rightarrow t = 5, 20$

41. As air drag reduces the vertical component of velocity so time to reach maximum height will decrease and it will decrease the downward vertical velocity Hence time to fall on earth increases.

42. As given horizontal velocity = 40m/s

$u\cos\theta \times t = 40$; $t = 1$ sec

At t = 1, height = 50 m

$\therefore 50 = u\sin\theta \times 1 - 1/2 \times g \times 1 \Rightarrow u\sin\theta = 55$

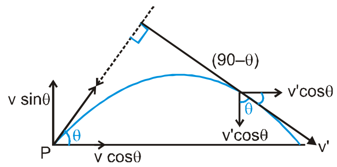
$\therefore$ Initial vertical component = $u\sin\theta = 55$ m/s

As hoop is on same height of the trajectory.

So by symmetry x will be 40 m.

43. $\because$ Horizontal component of velocity remains constant

$\therefore v'\sin\theta = v\cos\theta$ (from figure) $\therefore$ $\boxed{v' = v\cot\theta}$



So from $v_y = u_y + a_y t \rightarrow -v'\cos\theta$

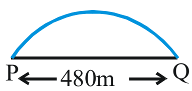
$= \sin\theta - gt - v\frac{\cos^2\theta}{\sin\theta} = v\sin\theta - gt \therefore \boxed{t = \frac{v}{g}\text{cosec}\theta}$

44. Range $= \frac{u^2\sin 2\theta}{g}$

For θ & $(90 - \theta)$ angles, range will be same so for 30° & (90 – 30°)≡60°, projections both strike at the same point. For time of flight, vertical components are responsible

45. Range $= \frac{u^2\sin 2\theta}{g} \Rightarrow 480 = \frac{4900}{980} \times \sin 2\theta$

$(90 - \theta)$ projection angle has same range.



Time of flight :

$T_1 = \dfrac{2u\sin\theta}{g}$; $T_2 = \dfrac{2u\sin(90-\theta)}{g}$

$\dfrac{h_1}{h_2} = \dfrac{u^2\sin^2\theta_1}{u^2\sin^2\theta_2} = \dfrac{\sin^2 30}{\sin^2 60} = \dfrac{1}{3}$

46. After t = 1 sec, the speed increases with
$a = g\sin 37° = 6\ m/s^2$
$\therefore v_Y = g\sin 37° \times 1 = 6$ m/s
$\therefore$ speed $= \sqrt{8^2+6^2} = 10$m/s

47. $y = x^2$, $y_{x=\frac{1}{2}} = \dfrac{1}{4}$; $\dfrac{dy}{dt} = 2x\dfrac{dx}{dt} = 2x\,v_x$

$v_y = 2 \times \dfrac{1}{2} \times 4$ (at $x = \dfrac{1}{2}$, $v_x = 4$)

$v_y = 4$m/s ; $\vec{v}_{x=\frac{1}{2}} = 4\hat{i} + 4\hat{j}$; $|\vec{v}| = 4\sqrt{2}$

Slope of line $4x - 4y - 1 = 0$ is tan 45° = 1
and **also** the slope of velocity is 1.

48. $h_{max} = \dfrac{u^2}{2g} \Rightarrow u = 12 \times 10 \times 5 = 10$ m/s

$t_H = \sqrt{\dfrac{2\times5}{10}} = 1$s so no. of balls in one min.
$= 1 \times 60 = 60$

49. New horizontal range

$= R + \dfrac{1}{2} \times \dfrac{g}{2} \times T^2 = R + \dfrac{g}{4} \times \dfrac{4u^2\sin^2\theta}{g^2}$

$= R + 2H$ ($\because H = \dfrac{u^2\sin^2\theta}{2g}$)

50. Let acceleration of B $\vec{a}_B = a_B\,\hat{i}$
Then acceleration of A w.r.t.
$B = \vec{a}_A - \vec{a}_B = (15 - a_B)\hat{i} + 15\hat{j}$
This acceleration must be along the inclined plane

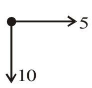
so tan 37° $= \dfrac{15}{15 - a_B} \Rightarrow \dfrac{3}{4} = \dfrac{15}{15 - a_B}$ $a_B = -5$

51. $a = -kv + c$ [k > 0, c > 0]

$\displaystyle\int \frac{dv}{-kv + c} = \int dt \Rightarrow -\frac{1}{k}\ell n(-kv + c) = t$

$\Rightarrow kv = c - e^{-kt} \Rightarrow \vec{a}_B = -5\hat{i}$

52. For B :
Net acceleration



$= \sqrt{5^2 + 10^2} = \sqrt{125} = 5\sqrt{5}\ ms^{-2}$

53. $(4T)\,a_A = (2T)\,(a_B)$

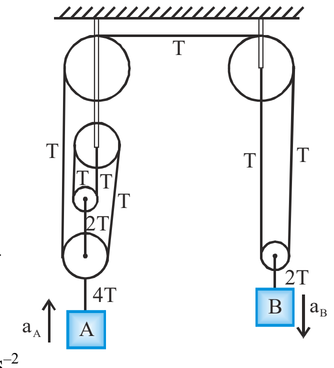
$\Rightarrow a_A = \dfrac{a_B}{2}$

but $a_B = \dfrac{dv_B}{dt}$

$= t + \dfrac{t^2}{2} \Rightarrow a_A = \dfrac{t}{2} + \dfrac{t^2}{4}$

At t = 2s,

$a_A = \dfrac{2}{2} + \dfrac{(2)^2}{4}$ 1+1=2 ms^{-2}

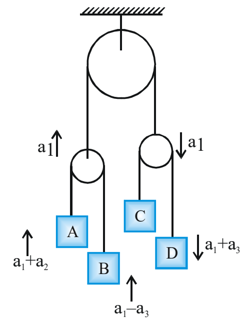


54. Block B will again comes to rest if
$v_A = v_c$ **i.e.** $3t = (12t)t \Rightarrow t = ½$ s

55. $a_1 + a_2 = 1 \Rightarrow a_1 - a_2 = 7$

$\Rightarrow a_3 - a_1 = 2 \Rightarrow a_1 = 4,$

$a_2 = -3,\ a_3 = 6$
Acceleration of D = $a_1 + a_3$
$= 4 + 6 = 10\ ms^{-2}$ downwords



56. $\tan\alpha = \dfrac{v^2}{R} \times \dfrac{1}{dv/dt} = \dfrac{a^2 s}{Rav/2\sqrt{s}} = \dfrac{2s}{R}$

57. Given $\dfrac{dv}{dt} = \dfrac{v^2}{r} \Rightarrow \dfrac{dv}{ds} = \dfrac{v^2}{r}$; $-\displaystyle\int_{v_0}^{v}\frac{1}{v}dv = \int_{o}^{s}\frac{ds}{r}$

$\Rightarrow \ell n\left[\dfrac{v_0}{v}\right] = \dfrac{S}{r} \Rightarrow \dfrac{v_0}{v} = e^{S/r}$

$\Rightarrow v_0 = ve^{S/r} \Rightarrow v = v_0 e^{-S/r}$

Exercise - 2

Segment-II Assertion & Reason Type

1. Whenever a particle having two ⊥ components of velocity then the path of projectile will be parabolic, if particle is projects vertically upwards then the path of projectile will be straight.

2. For max. range $\left(\dfrac{u^2\sin2\theta}{g}\right)$, the projection angle(θ) should be 45°.

So initial velocity ai + bj $\Rightarrow$ tan 45° $= \dfrac{b}{a} \Rightarrow a = b$

3. Acceleration depends on change in velocity not on velocity.

4. To meet, co-ordinates must be same. So in frame of one particle, second particle should approach it.

5. If displacement is zero in given time interval then its average velocity **also** will be zero. e.g. particle projects vertically upwards.

6. Because initial vertical velocity component is zero in both cases.

7. Yes, river velocity does not any help to cross the river in minimum time.

8. In air, the relative acceleration is zero. The relative velocity becomes constant which increases distance linearly which time.

9. Inclined plane, in downwards journey. The component of gravity is along inclined supports in displacement but not in the other case.

10. If the acceleration acts opposite to the velocity then the particle is slowing down.

11. Maximum height depends on the vertical component of velocity which is equal for both.

12. Speed is the magnitude of velocity which can't be negative.

13. Free fall implies that the particle moves only in presence of gravity.

Exercise - 3

Segment-I Matrix Matching Type

1. Slope of v.t. curve gives acceleration (instantaneous) at that point $\vec{a} = \frac{d\vec{v}}{dt}$

2. [A] $X = 3t^2 + 2 \Rightarrow V = \frac{dx}{dt} = 6t \Rightarrow a = \frac{d^2x}{dt^2}$

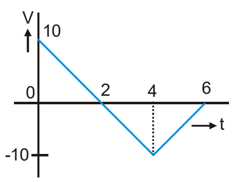
   [B] $V = 8t \Rightarrow a = \frac{dv}{dt} = 8$

   [D] For changing the direction $6t - 3t^2 = 0$

   $\Rightarrow t = 0, 2$ sec

3. At $t = 0$, $v(0) = 10$ m/s; $t = 0$; $v(6) = 0$

   Change $v(6) - v(0)$; $\Delta v = 0 - 10 = -10$ m/s



Average acceleration $= \frac{\text{charge in velocity}}{\text{time}}$

$= \frac{-10}{6} = \frac{-5}{3}$ m/s^2

Average velocity $= \frac{\text{Displacement}}{\text{time interval}}$

Total displacement = Area of Δ's (with +ve **or** –ve)

$= \frac{1}{2} \times 2 \times 10 - \frac{1}{2} \times 4 \times 10 = -10$ m (units)

$\therefore$ Average velocity $= \frac{-10}{6} = -\frac{-5}{3}$ m/s

a(3) = slope of line which exist at $t = 0$ $I_0 t = 4$

$a = \tan\theta = \frac{-10}{2} = -5$

4. Velocity & height of the balloon after 2 sec:

   $v = 0 + 10 \times 2 = 20$ m/s $\uparrow$

   $h = 1/2 \times 10 \times 4 = 20$ m

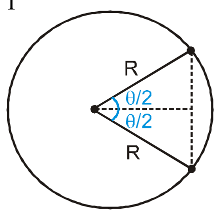
   Initial velocity of drop particle is equals to the velocity of balloon = 20 m $\Rightarrow$ $\because$ $u_s = 20$ m/s $\boxed{a_s = g \downarrow}$

   After further 2s $\boxed{v_s = 0}$

   $\therefore$ height $= \frac{u_s + v_s}{2} \times 2 = 20$m from initial position of balloon

   $\therefore$ Height from ground = 20 + 2v = 40m

5. $R\theta = vt$; $\theta = \frac{4 \times 1}{1} = 4$ radian



$\therefore$ Displacement = 2R sin θ/2 = 2 sin 2

Distance = vt = 4m

Average velocity $= \frac{\text{Displacement}}{\text{time}} = 2 \sin 2$

Average acceleration =

$\frac{\text{Change in velocity}}{\text{time}} = \frac{2 \times 4 \sin 2}{1} = 8 \sin 2$

Exercise - 3

Segment-II Comprehension Type

Comprehension #1

1. Positive slopes have positive acceleration, negative slopes have negative acceleration.

2. Accelerated motion having positive area on v-t graph has concave shape.

3. Maximum displacement = total area of graph
$= 20 + 40 + 60 + 80 - 40 = 160$ m

4. Average speed

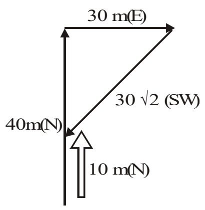
$$= \frac{\text{Distance}}{\text{time}} = \frac{20+40+60+80+40}{70} = \frac{24}{7}\,\text{m/s}$$

5. Time interval of retardation = 30 to 70.

Comprehension#2

1. $\frac{\text{Distance}}{\text{Displacement}} = \frac{\pi d/2}{d} = \frac{\pi}{2}$

2.



3. $x_1 = 1, y_1 = 4; x_2 = 2, y_2 = 16$

$\therefore$ Displacement $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{1^2 + 12^2} = \sqrt{145} \cong 12\text{m}$

Comprehension # 3

1. $y = \sqrt{3}x - 2x^2$

Trajectory equation is $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$\tan\theta = \sqrt{3} \Rightarrow \boxed{\theta = 60°}$ & $\frac{g}{2u^2\cos^2\theta} = 2$

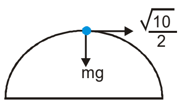
$$\Rightarrow u = \frac{5}{2\times\frac{1}{4}} = \sqrt{10}$$

2. Max. height $H = \frac{u^2\sin^2\theta}{2g} = \frac{10\times\left(\frac{\sqrt{3}}{2}\right)^2}{2\times 10} = \frac{3}{8}$ m

3. Range of A $= \frac{u^2\sin 2\theta}{g} = \frac{10\times\sin 120°}{10} = \frac{\sqrt{3}}{2}$

4. Time of flight $= \frac{2u\sin\theta}{g} = \frac{2\times\sqrt{10}\times\frac{\sqrt{3}}{2}}{10} = \frac{\sqrt{3}}{10}$

5. At the top most point $v = u\cos\theta = \sqrt{10}\cos 60° = \frac{\sqrt{10}}{2}$



$$\therefore mg = \frac{mv^2}{R}\ ;\ R = \frac{\left(\frac{\sqrt{10}}{2}\right)^2}{10} = \frac{10}{40}\quad \boxed{R = \frac{1}{4}\text{m}}$$

Comprehension # 4

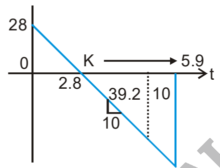
1. If the projection angle is increased, maximum height will increase.

2. Projection angle is 45° & $V_y = 21$ m/s, projection speed is $V_0 \sin 45° = 21 \Rightarrow V_0 = 21\times\sqrt{2} = 30$m/s

3. By the $v_y - t$ graph the acceleration is

$$\frac{-21}{2.1} = -10 = -g$$

4.



5. Initial kinetic energy = $1/2\,mV_0^2$
If mass doubles, then we can sec from $(v_y - t)$ curve then velocity becomes half of previous.

$$\therefore \frac{1}{2}\times 2m\times\left(\frac{v_0}{2}\right)^2 = \frac{1/2\,mv_0^2}{2}$$ **Hence [B]**

6. Position of the cable at the max. height point.

$$H = \frac{(V_0\sin 45)^2}{2g} = \frac{V_0^2}{4g}$$

Comprehension #5

1. $R = Cv_0^n$
Putting data from table: $8 = C\times 10^n$

$\Rightarrow 31.8 = C\times 20^n \Rightarrow \frac{31.8}{8} = 3.9 \cong 4 = 2^n \Rightarrow n=2$

2. C depends on the angle of projection.

3. $R = C\times v_0^n \Rightarrow 8 = C\times 10^n$ and

$R = C\times 5^n \Rightarrow R = \frac{8}{2^2} = 2\text{m}$

Comprehension#6

1. In vertical direction $h = (u\sin\theta)t - \frac{1}{2}gt^2$

$$\Rightarrow t^2 - \left(\frac{2u\sin\theta}{g}\right)t + \frac{2h}{g} = 0$$

$$\Rightarrow t_1 + t_2 = \frac{2u\sin\theta}{g} \quad .....(i)$$

In horizontal direction $x = (u\cos\theta)t - \frac{1}{2}at^2$

$\Rightarrow t^2 - \left(\frac{2u\cos\theta}{a}\right)t + \frac{2x}{a} = 0$

$\Rightarrow t_3 + t_4 = \frac{2u\cos\theta}{a}$ ....**(ii)**

From **(i)** and **(ii)** $\theta = \tan^{-1}\left[\frac{g(t_1+t_2)}{a(t_3+t_4)}\right]$

2. At maximum height $v_y = 0$

$\Rightarrow H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{g}{8}(t_1+t_2)^2$

3. At maximum range vertical displacement = 0

$\Rightarrow t = \frac{2u\sin\theta}{g}$. So range R

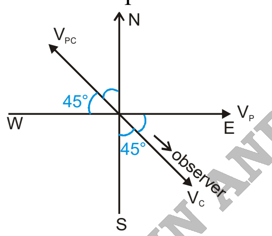
$= (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right) - \frac{1}{2}a\left(\frac{2u\sin\theta}{g}\right)^2$

$= \frac{2u^2\sin\theta\cos\theta}{g}\left(\frac{g}{a} - \tan\theta\right)3$

Comprehenison # 7

1. In ground frame [A] it is simply a projectile motion. But in [B] frame horizontal component of the displacement is zero **i.e.** in this frame only vertical comp. appear which is responsible for the maximum height.

2. As observer observes that particle moves north-wards.



3. Frame [D], which is attached with particles itself so the minimum distance is equal to zero.

4. $\uparrow a_b = 20$ m/s² ; $\downarrow a_D = 10$ m/s² $\Rightarrow a_{bD} = 30$ m/s² $\uparrow$

$\therefore$ Force acting on a body = 10 × 20 = 200N

Exercise - 4

Subjective Type Questions

1. By observing the graph, position of A (Q) is greater than position of B (P) **i.e.** B lives farther than A and **also** the slope of x-t curve for A & B gives their velocities $v_B > v_A$.

2. By observation, for equal interval of time the magnitude of slope of line in x-t curve is greatest in interval 3.

3. $a = a_0\left(1 - \frac{t}{T}\right)$ where a_0 & T are constants

$\int_0^v dv = a_0\int_{t=0}^{t}\left(1 - \frac{t}{T}\right)dt \Rightarrow v = a_0\left[t - \frac{t^2}{2T}\right]$

$\Rightarrow \int dx = a_0\int_{t=0}^{t}\left[t - \frac{t^2}{2T}\right]dt$

For $a = 0 \Rightarrow 1 - \frac{t}{T} = 0$ $\boxed{t = T}$ $= a_0\left[\frac{t^2}{2} - \frac{t^3}{6T}\right]$

$\therefore \langle v \rangle = \frac{\int_0^T v\,dt}{\int_0^T dt} = \frac{a_0\left[\frac{T^2}{2} - \frac{T^3}{6T}\right]}{T} = \frac{a_0 T}{3}$

4. After 3 sec distance covered = 1/2 × 2 × 9 = 9m velocity of lift = 2 × 3 = 6 m/s↓ $\therefore u_p$ = 6m/s↓,

a = g ↓ height = (100–9) = 91 m

$\therefore$ Time to reach the ground

$= 91 = 6t + \frac{1}{2} \times g \times t^2$ t = 3.7 sec

Total time taken by object to reach the ground = 3 + 3.7 = 6.7 sec.

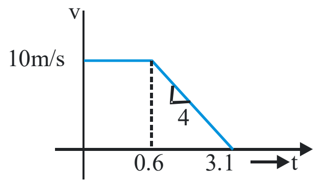
Time to reach on the ground by lift

$= \frac{1}{2} \times 2 \times t^2 = 100 \Rightarrow t = 10$ sec.

So interval = 10 – 6.7 = 3.3 sec

5. $S_n = u + \frac{a}{2}(2n-1)$ by putting the value of n=7 and 9, find the value of u & a, u=7 m/s & a =2 m/s².

6. $\Delta t = t - 0.6 = \frac{0-10}{-4} = 2.5$



Stopping distance = 0.6 × 10 + $\frac{1}{2}$ × 2.5 × 10

6 + 12.5 m = 18.5 m

7. Deceleration of train,

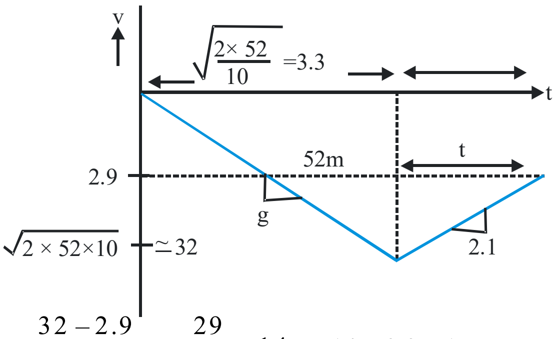
$a = \left|\frac{v^2 - u^2}{2s}\right| = \frac{20 \times 20}{2 \times 2} = 100$ km/hr²

Time to reach platform $= \frac{20}{100} = \frac{1}{5}$hr

$\therefore$ Total distance travelled by the bird

$= vt = 60 \times \frac{1}{5} = 12km$

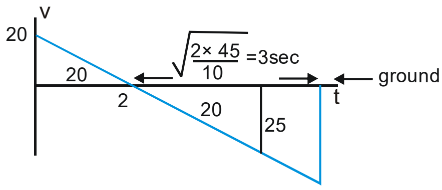
8. **(i)**



$2.1 = \frac{32-2.9}{t}\,;\, t = \frac{29}{2.1} \simeq 14 \;\; \therefore\; 14 + 3.3 \simeq 17$

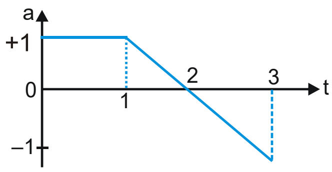
(ii) Height $= 52 + \frac{1}{2}\,[32 + 2.9] \times 14 = 293.8$

9. **(i)** Height = upward area under v–t curve = 20m



(ii) Total time of flight = 2 + 3 = 5sec

10.



u = 0

(i) Area under a – t curve the change in velocity

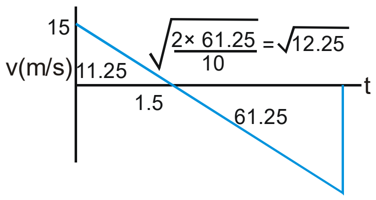
$\Delta u = 1 \times 1 + \frac{1}{2} \times 1 \times 1;\; u_2 - u_0 = 1.5$ m/s

$\boxed{u_2 = 1.5\, m/s}$ $(\because u_0 = 0)$

upto 3 sec : $\Delta u = 1.5 - \frac{1}{2} \times 1 \times 1 = 1m/s$

$u_3 - u_0 = 1m/s \Rightarrow u_3 = 1m/s \quad (\because u_0 = 0)$

11. Total time = 1.5 + 3.5 = 5s



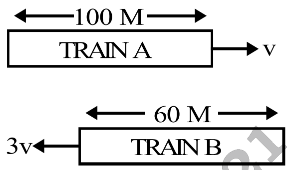
12. From given situation :

(i) $a_{avg} = \frac{60-20}{1.00-0.75} = \frac{4000}{25} = 160$ km/hr^2

(ii) Area $= \frac{1}{2} \times [20 + 60] \times 0.25$

$= 40 \times \frac{25}{100} = 10km$

13. Let $v_{AB} = v - (-3v) = 4v$

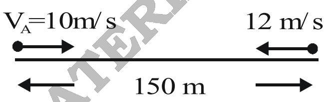


time $= \frac{160}{4v} = 4$ sec $\boxed{v = 10 m/s}$

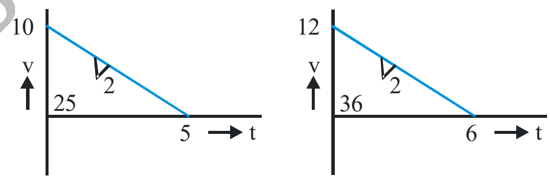
velocity of train $v_A = 10$ m/s

$v_B = 3 \times v = 30$ m/s

14.



Distance travelled to stop



Car A Car B

Total distance = 25 + 36 = 61 m covers by both car

$\therefore$ Remaining distance = 150 – 61 = 89 m

15. **For A :** $30t_1 = S/2 = 60\,(2 - t_1) \Rightarrow t_1 = 4/3$ hr

(Here S is the total distance and t_1 is time up to which A's speed is 30 km/hr)

For B : $\frac{1}{2} \times a \times 2^2 = \left(30 \times \frac{4}{3}\right) \times 2 = S \Rightarrow a = 40$ km/hr^2

(i) **(a)** $v_B = 40t = 30 \Rightarrow t = 0.75$ hr

(b) $v_B = 40t = 60 \Rightarrow t = 1.5$ hr

(ii) There is no overtaking.

16. Direction of flag = Resultant direction of the wind velocity and the opposite of boat velocity

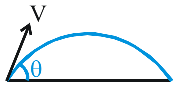
$\Rightarrow \vec{v}_W - \vec{v}_B = \frac{72}{\sqrt{2}}(\hat{i} + \hat{j}) - 51\hat{j}$

$= 36\sqrt{2}\hat{i} + (36\sqrt{2} - 51)\hat{j} = 36\sqrt{2}\hat{i}\;\;(EAST)$

17. Relative velocity of A w.r.t to B,

$$V_{AB} \text{ time} = \frac{a}{v - v\cos\theta} = \frac{a}{v(1-\cos\theta)} \qquad \boxed{\theta = \frac{2\pi}{n}}$$

18. $\vec{v}(0) = v\cos\theta\hat{i} + v\sin\theta\hat{j}$



$\vec{v}(t) = v\cos\theta\hat{i} + (v\sin\theta - gt)\hat{j}$

$|\vec{v}(t)| = \sqrt{v^2\cos^2\theta + (v\sin\theta - gt)^2}$

$$<\vec{v}(t)> = \frac{\vec{v}(t) + \vec{v}(0)}{2} = v\cos\theta\ \hat{i} + \frac{(2v\sin\theta - gt)}{2}\hat{j}$$

According to question $\sqrt{(v\cos\theta)^2 + (v\sin\theta - gt)^2}$

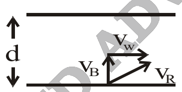
$$= \sqrt{(v\cos\theta)^2 + \left(\frac{2v\sin\theta - gt}{2}\right)^2}$$

$$v^2\cos^2\theta + (v\sin\theta - gt)^2 = v^2\cos^2\theta + \left(\frac{2v\sin\theta - gt}{2}\right)^2$$

$$v\sin\theta - gt = -v\sin\theta + \frac{gt}{2} \Rightarrow \frac{3gt}{2} = 2v\sin\theta$$

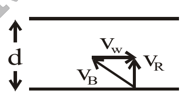
$$\boxed{t = \frac{4}{3}\left(\frac{v\sin\theta}{g}\right)}$$

19. $t = \dfrac{d}{v_B} = 600s$, drift$= v_w \times \dfrac{d}{v_B}$



$120 = v_w \times 600s$; $v_w = \dfrac{1}{5}\dfrac{m}{sec}$

$$t = \frac{d}{\sqrt{v_B^2 - v_W^2}} = 750$$

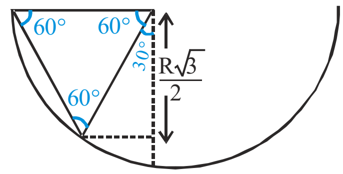


$$\sqrt{1 - \left(\frac{v_W}{v_B}\right)^2} = \frac{4}{5} \Rightarrow \left(\frac{v_W}{v_B}\right)^2 = \frac{9}{25}$$

$$\boxed{\frac{v_W}{v_B} = \frac{3}{5}} \Rightarrow \boxed{v_B = \frac{1/5}{3/5} = \frac{1}{3}m/\sec}$$

$$\frac{d}{v_B} = 600 = \boxed{d = 600 \times \frac{1}{3} = 200m}$$

20. $\therefore$ Vertical displacement of particle $= \dfrac{R\sqrt{3}}{2}$



$$\text{Time for this} = \sqrt{\frac{2 \times R\frac{\sqrt{3}}{2}}{g}} = \sqrt{\frac{\sqrt{3}R}{g}}$$

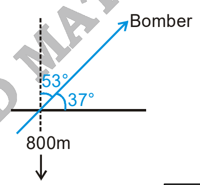
$$\vec{v}(t) = u\hat{i} + gt\hat{j} = u\hat{i} + g \times \sqrt{\frac{\sqrt{3}R}{g}}\hat{j} = u\hat{i} + \sqrt{\sqrt{3}Rg}\,\hat{j}$$

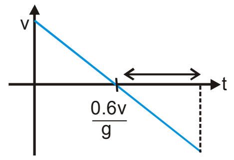
21. $u\sin\theta \times 1 - \frac{1}{2}g(1)^2 = u\sin\theta \times 3 - \frac{1}{2} \times g \times (3)^2$

$2u\sin\theta = 40 \Rightarrow u\sin\theta = 20m/s$

$$\text{Max. height} = \frac{u^2\sin^2\theta}{2g} = \frac{20 \times 20}{20} = 20m$$

22.





$$20 = \frac{0.6v}{g} + \sqrt{\frac{2}{g} \times \left[\frac{(0.6v)^2}{2g} + 800\right]} \quad ....(i)$$

(i) By solving equation **(i)**, we get v = 100 m/s.

(ii) Maximum height :

$$= 800 + \frac{(0.6v)^2}{2g} = 800 + \frac{(0.6 \times 100)^2}{20} = 980m$$

(iii) horizontal distance

= Horizontal velocity × time of flight

= 100 cos 37° × 20 = 1600m

(iv) horizontal component

$v_H = u_H = 100\cos 37° = 80$ m/s

$v_v = u_v - 10 \times 20 = 100\sec 37° - 200$

= 140 m/s

$\therefore v_{strike} = 80\hat{i} - 140\hat{j}$, $|\vec{v}| = \sqrt{80^2 + 140^2}$

23. $780 = u\sin\theta \times 6 + \frac{1}{2} \times g \times 36$

$780 - 180 = u\sin\theta \times 6$

$u\sin\theta = \frac{600}{6} = 100$ m/sec

i.e. food package dropped before 10 secs

$1000 = u\times 10 \Rightarrow u = 100$ m/s

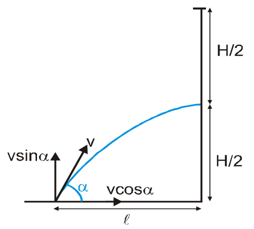
$\therefore h = \frac{g\times(16)^2}{2} = 1280$ m.

24. $\frac{d}{10\sqrt{2}\cos45^\circ+10} = \frac{10}{10\sqrt{2}\sin45^\circ}$

$d = 20\times 1 = 20$ m.

25. $\frac{H}{2}$ distance covered by free falling body

$\frac{H}{2} = \frac{1}{2}gt^2$; $t = \sqrt{\frac{H}{g}}$



In same time, projectile **also** travel vertical distance

$\frac{H}{2}$, then $\frac{H}{2} = v\sin\alpha\sqrt{\frac{H}{2}} - \frac{1}{2}g\frac{H}{g}$

$v\sin\alpha = \sqrt{gH}$ ...(i)

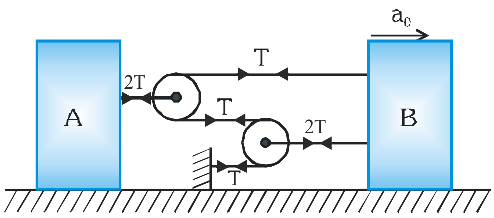
also $\ell = v\cos\alpha\sqrt{\frac{H}{g}}$; $v\cos\alpha = \ell\sqrt{\frac{g}{H}}$ ...(ii)

From equation (i) and (ii)

$\tan\alpha = \frac{H}{\ell}v^2\sin^2\alpha + v^2\cos^2\alpha = gH + \ell^2\frac{g}{H}$

$v = \sqrt{gH\left(1+\frac{\ell^2}{H^2}\right)}t$

26. Here $a_B(3T) = (a_A)(2T)$ $a_A = \frac{3}{2}a_B$



$a_{AB} = a_A - a_B = \frac{3}{2}a_0 - a_0 = \frac{a_0}{2}$

27. $v = 2t^2$; $a_T = \frac{dv}{dt} = 4t \Rightarrow a_T(1) = 4$

$a_N = \frac{v^2}{R} = \frac{(2\times1^2)^2}{1} = 4$

$a = \sqrt{a_T^2 + a_N^2} = \sqrt{(4)^2+4^2} = \sqrt{32}$

$\boxed{a = 4\sqrt{2}}$

28. $\vec{a}_t = 6\hat{i} = \vec{\alpha}\times\vec{R} = \vec{\alpha}\times 2\hat{j} \Rightarrow \vec{\alpha} = -3\hat{k}$ rad/s²

$\vec{a}_r = -8\hat{j} = \vec{\omega}\times\vec{v} = \vec{\omega}\times(\omega R)\hat{i} \Rightarrow \vec{\omega} = -2\hat{k}$ rad/s

29. $a_t = ar$; $\alpha r = \omega^2 r$; $\alpha = \alpha^2 t^2 \Rightarrow \alpha = \frac{1}{t^2}$

30. $(v_A+v_B)t = 2\pi R$, $(0.7+1.5)t = 2\times\frac{22}{7}\times 5$

$t = \frac{2\times22\times5}{7\times2.2}\times 10 = \frac{100}{7}$ sec $= 14.3$ sec

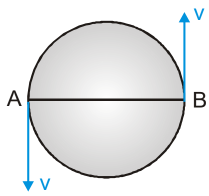
Acceleration of B $= \frac{v_B^2}{R} = \frac{1.5^2}{5} = 0.45$ m/s²

31. According to

$\theta = \frac{1}{2}\times\frac{72v^2}{25\pi R}\times t^2 = \pi R \Rightarrow t = \frac{5\pi R}{6v}$

Using $R\theta = vt + \left(\frac{1}{2}\right)\frac{72v^2}{25\pi R}\times\frac{25\pi^2R^2}{36v^2}$

$a_T = \frac{72v^2}{25\pi R}$



$R\theta = \frac{v5\pi R}{6v} + \pi R = \frac{11}{6}\pi$

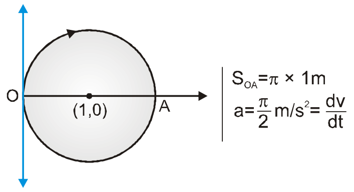
Angular velocity : $\omega = \omega_v + \alpha t$

$= \frac{v}{R} + \frac{72v^2}{25\pi R^2}\times\frac{5\pi R}{6v} = \frac{v}{R} + \frac{12v}{5R} = \frac{17v}{5R}$

Angular acceleration $\alpha = \omega^2 R = \frac{289v^2}{25R}$

32. (a) $\pi = 0 + \frac{1}{2}\times\frac{\pi}{2}t^2 \Rightarrow t = 2$sec

(b) $v = 0 + \frac{\pi}{2}\times 2 = \pi$ m/s



33. $r = 2.5$ m, $a_{net} = 25$ m/s^2

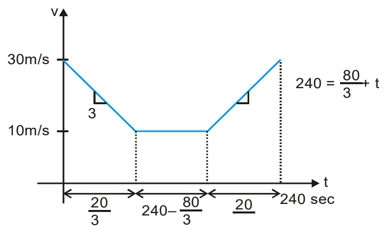
(a) Radial acceleration $= 25\cos\theta = 25 \times \frac{\sqrt{3}}{2}$ m/s^2

(b) $25\frac{\sqrt{3}}{2} = \frac{v^2}{25} \Rightarrow v = \left(125\frac{\sqrt{3}}{4}\right)^{1/2}$ m/s

(c) Tangential acceleration $= 25\sin\theta = 25 \times \frac{1}{2}$ m/s^2

34. (i) Area $= \frac{1}{2}[10+30] \times \frac{20}{3\lambda} + 10 \times \left(240 - \frac{80}{3\lambda}\right)$

$+ \frac{1}{2}[10+30] \times \frac{20}{\lambda} = 4000$

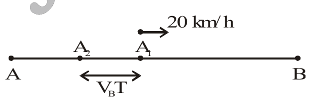


$$\frac{400}{3\lambda} + 2400 - \frac{800}{3\lambda} + \frac{400}{\lambda} = 4000$$

$$\frac{400 - 800 + 1200}{3\lambda} = 1600 \Rightarrow 3\lambda = \frac{800}{1600} = \frac{1}{2};\ \lambda = \frac{1}{6}$$

(ii) Dist. travelled $= 10\left(240 - \frac{80}{3 \times 1/6}\right) = 800$ m

35. $\frac{V_B T}{V_B - 20} = 18,\ \frac{V_B T}{V_B + 20} = 6;\ \frac{V_B + 20}{V_B - 20} = 3$



$\Rightarrow V_B + 20 = 3V_B - 60$ $\boxed{V_B = 40 \text{km/h}}$

$\therefore T = \frac{6(V_B + 20)}{V_B} = \frac{6 \times 60}{40} = 9$ min

36. –25 m/s After 5 sec

height of balloon = 25 × 5 = 125 m

(i) Minimum speed

$125 = \frac{(u-25)^2}{2g} \Rightarrow (u-25)^2 = 2500;$

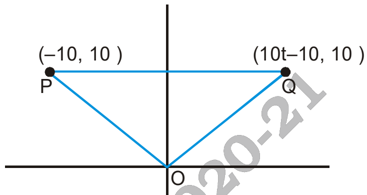
$u - 25 = 50;\ u = 75$ m/s

(ii) $u = 2 \times 75 = 150$ m/s

$125 = (150 - 25)t - 5t^2$

$125 = 125t - 5t^2 \Rightarrow t^2 - 25t + 25 = 0$

37. It's velocity is $10\hat{i}$



$\therefore$ displacement after time 't' $= 10\hat{i} \times t$

Velocity of second ship $= u \times \frac{(\hat{i} + 2\hat{j})}{\sqrt{5}}$

$$\tan\theta = \frac{2u/5}{\left(10 - \frac{u}{\sqrt{5}}\right)} = \frac{2 \times 10\sqrt{5}}{10\sqrt{5} - 10\sqrt{5}}$$

(i) $t = \frac{10}{20} = \frac{1}{2}$ sec, minimum distance = 10 km

38. Let t = time of accelerated motion of the helicopter.
Distance travelled by helicopter
= Distance travelled by sound

$\Rightarrow \frac{1}{2} \times 3 \times t^2 = 320(30 - t) \Rightarrow t = \frac{80}{3}$ sec

Final velocity of helicopter

$$v = u + at = 0 + 3 \times \frac{80}{3} = 80 \text{ m/s}$$

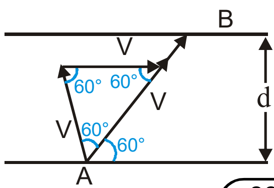
39. $v_{12} = v_1 - v_2 = v_1 - (-v_2) = v_1 + v_2$

$$\boxed{\ell_{max} = \frac{(v_1 + v_2)^2}{2(a_1 + a_2)}}$$

$a_{12} = -a_1 - a_2 = -(a_1 + a_2)$

40. From figure (a) 120°

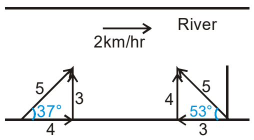
time to cross $= \frac{2d}{\sqrt{3}V}$



Minimum time $t = \frac{d}{v}$

$\therefore$ Ratio $= 2\sqrt{3}$

41. $V_A = (4+2)\,\hat{i} + 3\hat{j},\ V_B = (-3+2)\,\hat{i} + 4\hat{j}$



Time to cross the river $t_A = \frac{100}{3}$; $t_B = \frac{100}{4}$

Drift $= \frac{100}{3} \times 6 = 200$ m ; Drift $= -1 \times \frac{100}{4}$

Remaining distance = 300 – 200 ; 25 m

$(t_{total})_A = \frac{100}{3} + \frac{100}{8}$; $t_B = \frac{100}{4} + \frac{100}{6}$

$t_A = \frac{800+300}{24} = \frac{1100}{24}$; $t_B = \frac{600+400}{24} = \frac{1000}{24}$

$\boxed{t_A = 165 \text{ sec}}$ $\boxed{t_B = 150 \text{ sec}}$

42. Range (OA) $= \frac{u^2 \sin 2\theta}{g} = \frac{1600 \times \sqrt{3}}{10 \times 2} = 80\sqrt{3}$

$h = 80\sqrt{3} \times \tan 60° = \frac{10 \times 80 \times 80 \times 3}{2 \times v^2 \cos 60°}$

Time to strike $\Rightarrow$ vcos 60°× t = $80\sqrt{3}$

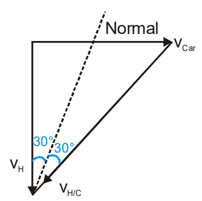
$\Rightarrow t = \frac{80\sqrt{3} \times 2}{v \times 1} = \frac{10\sqrt{3}}{v}$

$h = 9\sqrt{3} \times \frac{160\sqrt{3}}{v} \frac{480 \times 9}{v} = \frac{240^2 - 38400}{v^2}$

$v^2 - 1600 - 18v = 0 \Rightarrow v = \frac{18 \pm \sqrt{324 + 6400}}{2}$

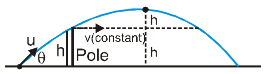
$\Rightarrow \boxed{v = 50 \text{ m/s}}$

43. $\tan 60° = \frac{v_{Car}}{v_H} \Rightarrow \sqrt{3} = \frac{v_C}{10} \Rightarrow v_C = 10\sqrt{3}$ m/s



44. For stone : $2h = \frac{(u\sin\theta)^2}{2g}$ & $h = (u \sin \theta)t - \frac{1}{2}gt^2$

$\Rightarrow t = \frac{\sqrt{40h} \pm \sqrt{20h}}{10} \Rightarrow \Delta t = \sqrt{0.8h} = \frac{2}{10}\sqrt{20h}$

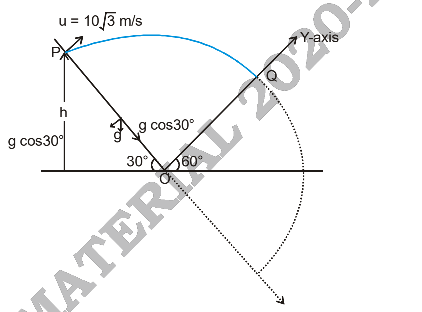


Horizontal displacement : $vt_2 = u \cos \theta\, \Delta t$

$\Rightarrow \frac{v(\sqrt{2}+1)\sqrt{20h}}{10} = u\cos\theta \times \frac{2\sqrt{20h}}{10}$

$\Rightarrow \frac{v}{u\cos\theta} = \frac{2}{\sqrt{2}+1}$

45.



(i) $v(t) = (u - g \cos 30° t)\,\hat{i} - g \sin\theta\, t\,\hat{j}$

From given situation

$u - g \cos 30°\ t = 0$ $t = 2$ sec

(ii) Velocity $u_x = 0$, $a_x = g \cos 30° = \frac{g}{2}$

$\therefore v_x = 0 + \frac{g}{2} \times 2 = 10$m/s

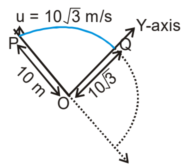
(iii) Distance $PO = 10\sqrt{3}\cos 90° \times t + \frac{1}{2} \times g\sin 30° \times (2)^2$

PO = 10 m $\therefore$ h = 10 sin 30° = 5 m

(iv) Maximum height $= h + \frac{u(\sin 60°)^2}{2g}$

$= 5 + \frac{\left(10\sqrt{3} \times \frac{\sqrt{3}}{2}\right)^2}{20} = 16.25$ m

(v) Distance PQ



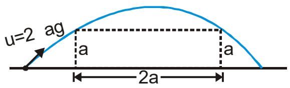
$OQ = \frac{(10\sqrt{3})^2}{2g\cos 30°}$

$OQ = 10\sqrt{3}$

$\therefore PQ = \sqrt{(PO)^2 + (O)^2}$

$= \sqrt{10^2 + (10\sqrt{3})^2} = 20$ m

46. $s = ut + \frac{1}{2}at^2 \Rightarrow a = (u\sin\theta)t - \frac{1}{2}gt^2$



$\Rightarrow t = \frac{u\sin\theta \pm \sqrt{u^2\sin^2\theta - 2ag}}{g}$

$\Delta t = \frac{2\sqrt{u^2\sin^2\theta - 2ag}}{g}$

For horizontal motion : $2a = u\cos\theta \times \Delta t$

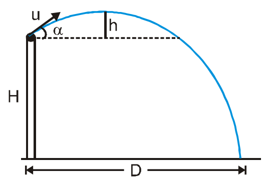
$\Rightarrow 2a = \frac{u\cos\theta \times 2\sqrt{u^2\sin^2\theta - 2ag}}{g} \Rightarrow \theta = 60°$

$\therefore \Delta t = \frac{2a}{u\cos\theta} = \frac{2a}{2\sqrt{ag} \times \frac{1}{2}} = 2\sqrt{\frac{a}{g}}$

47. $u\cos\alpha t = D$ ....**(i)** $u\sin\alpha\, t - \frac{1}{2}gt^2 = -H$ ....**(ii)**

$\Rightarrow t = \frac{2u\sin\alpha \pm \sqrt{u^2\sin^2\alpha + 2gH}}{g} = \frac{D}{u\cos\alpha}$

$h = \frac{(u\sin\alpha)^2}{2g} = \frac{D^2\tan^2\alpha}{4(H + D\tan\alpha)}$



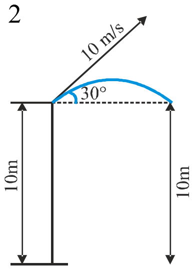
$\therefore H_{max} = h + H = H + \frac{D^2\tan^2\alpha}{4(H + D\tan\alpha)}$

Exercise - 5
Segment-I Previous Year AIEEE

1. Kinetic energy of a projectile at the highest point = $E\cos^2(\theta)$ where E is the kinetic energy of projection, θ is the angle of projection.

$E_{\text{highest point}} = E(\cos45°)^2 = E\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{E}{2}$

2. $R = \frac{u^2\sin2\theta}{g}$ Þ $R = \frac{(10)^2\sin60°}{10}$



$R = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} = 8.66$ m

3. Both horizontal direction speed is same

$v_0\cos\theta = \frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

4. When a body is projected at an angles θ and 90–θ; the ranges for both angles are equal and the corresponding time of flights for the two ranges are t_1 and t_2.

$R = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{1}{2}g\left(\frac{2u\sin\theta}{g}\right)\left(\frac{2u\sin(90° - \theta)}{g}\right)$

$= \frac{1}{2}gt_1t_2 \Rightarrow R \propto t_1t_2$

5. $K_{\text{highest point}} = [K_{\text{Point of projection}}]\cos^2\theta$

$K_H = K(\cos60°) \Rightarrow K_H = \frac{K}{4}$

6. $\vec{v} = K(y\hat{i} + x\hat{j})$; $v_x = Ky$; $\frac{dx}{dt} = Ky$

similarly $\frac{dy}{dt} = Kx$

Hence $\frac{dy}{dx} = \frac{x}{y} \Rightarrow y\,dy = x\,dx$,

by integrating $y^2 = x^2 + c$.

7. $R_{max} = \frac{u^2}{g}$; Area = $\pi r^2 = \frac{\pi u^2 R_{max}^2}{g^2}$

8. $H_{max} = \frac{u^2}{2g} = 10$ m and $R_{max} = \frac{u^2}{g} = 20$ m

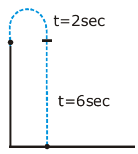
9. $u = \sqrt{5}$ and $\tan\theta = 2$

so by $y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta)$

$\Rightarrow y = 2x - \frac{10x^2}{2 \times 5}(1+4) \Rightarrow y = 2x - 5x^2$

10. 1

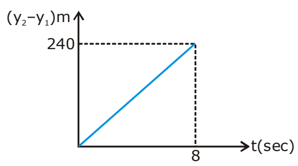
11. Ist stone

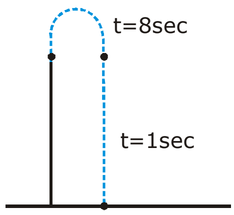


$0 \le t \le 8$ sec

$v_r = 40 - 10 = 30$ m/s $\Rightarrow a_r = 0$

$\Rightarrow s_r = v_r \times t = 30 \times 8 = 240$ m





$8 \text{ sec} < t \le 12 \text{ sec}$

v_r increases in magnitude and relative acceleration is g downwards

Exercise - 5

Segment-II Previous Year JEE Ques.

Single Choice

1. $v_{av} = \dfrac{\text{total displacement}}{\text{total time}} = \dfrac{2}{1} = 2\text{m/s}$

2. $v^2 = 2gh$ [it is parabola]
and direction of speed (velocity) changes.

3. $a = -\dfrac{10}{11}t + 10$ at maximum speed a = 0

$\dfrac{10}{11}t + 10 \Rightarrow t = 11$ sec

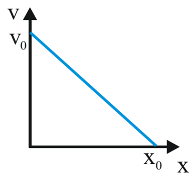
Area under the curve $= \dfrac{1}{2} \times 11 \times 10 = 55$

4. $S_n = u + \dfrac{a}{2}(2n-1) = \dfrac{a}{2}(2n-1)$

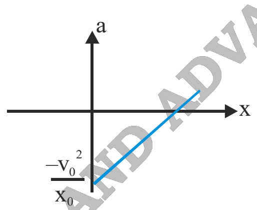
$S_{(n+1)} = x + \dfrac{a}{2}(2n+1) = \dfrac{a}{2}(2n+1) \Rightarrow \dfrac{S_n}{S_{n+1}} = \dfrac{(2n-1)}{(2n+1)}$

5. $v = -\left(\dfrac{v_0}{x_0}\right)x + v_0$

$a = \left[-\dfrac{v_0}{x_0}n + v_0\right]\left[-\dfrac{v_0}{x_0}\right]$



$a = \left(-\dfrac{v_0}{x_0}\right)^2 x - \dfrac{v_0^2}{x_0}$



MCQ's

1. x = a cospt ; y = bsinpt; $\vec{r} = a\cos(pt)\hat{i} + b\sin(pt)\hat{j}$

$\because \sin^2 pt + \cos^2 pt = 1 \Rightarrow \therefore \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (ellipse)

$\vec{v} = -ap\sin(pt)\hat{i} + bp\cos(pt)\hat{j}$; $v_t = \dfrac{\pi}{2p} = -ap\hat{i}$

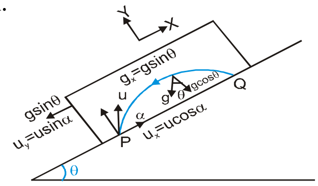
$\vec{a} = -ap^2(pt)\hat{i} - bp^2\sin(pt)\hat{j}$; $a_t = \dfrac{\pi}{2p} = -bp^2\hat{j}$

$\vec{a} \cdot \vec{v} = 0$

$\vec{a} = -p^2\left[a\cos pt\hat{i} + b\sin pt\hat{j}\right] = -p^2\vec{r}$

Subjective

1. (i) u is the relative velocity of the particle with respect to the box.



u_x is the relative velocity of particle with respect to the box in x-direction. u_y is the relative veloicty with respect to the box in y-direction. Since there is no velocity of the box in the y-direction, therefore this is the vertical velocity of the particle with respect to ground **also**.

Y-direction motion

(Taking relative terms w.r.t. box)

$u_y = +u\sin\alpha$; $a_y = -g\cos\theta$

$s = ut + \dfrac{1}{2}at^2 \Rightarrow 0 = (u\sin\alpha)t - \dfrac{1}{2}g\cos\theta \times t^2$

$\Rightarrow t = 0$ or $t = \dfrac{2u\sin\alpha}{g\cos\theta}$

X-direction motion (taking relative terms w.r.t box)

$u_x = +u\cos\alpha$ & $s = ut + \dfrac{1}{2}at^2$

$a_x = 0 \Rightarrow s_x = u\cos\alpha \times \dfrac{2u\sin\alpha}{g\cos\theta} = \dfrac{u^2\sin 2\alpha}{g\cos\theta}$

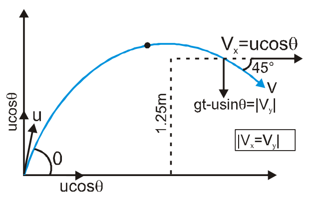
(ii) For the observer (on ground) to see the horizontal displacement to be zero, the distance travelled by the box in time $\left(\dfrac{2u\sin\alpha}{g\cos\theta}\right)$ should be equal to the range of the particle. Let the speed of the box at the time of projection of particle be u. Then for the motion of box with respect to ground.

$u_x = -v$, $s = vt + \frac{1}{2}at^2$, $a_x = -g\sin\theta$

$s_x = \dfrac{-u^2\sin 2\alpha}{g\cos\theta} = -v\left(\dfrac{2u\sin\alpha}{g\cos\theta}\right) - \dfrac{1}{2}g\sin\theta\left(\dfrac{2u\sin\alpha}{g\cos\theta}\right)^2$

On solving we get $v = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$

2. Let 't' be the time after which the stone hits the object and θ be the angle which the velocity vector $\vec{u}$ makes with horizontal. According to question, we have following three conditions.



(i) Vertical displacement of stone is 1.25 m.

$\because 1.25 = (u\sin\theta)t - \frac{1}{2}gt^2$ where g=10 m/s^2

$\Rightarrow (u\sin\theta)t = 1.25 + 5t^2$ ...**(i)**

(ii) Horizontal displacement of stone

= 3 + displacement of object A.

Therefore $(u\cos\theta)t = 3 + \frac{1}{2}at^2$

where a= 1.5 m/s^2 $\Rightarrow (u\cos\theta)t = 3 + 0.75t^2$ ...**(ii)**

(iii) Horizontal component of velocity (of stone)

= vertical component (because velocity vector is inclined) at 45° with horizontal).

Therefore $(u\cos\theta) = gt - (u\sin\theta)$ ...**(iii)**

The right hand side is written gt–usinθ because the stone is in its downward motion.

Therefore, $gt > u\sin\theta$.

In upward motion $u\sin\theta > gt$.

Multiplying equation **(iii)** with t we can write,

$(u\cos\theta)t + (u\sin\theta)t = 10t^2$ ...**(iv)**

Now **(iv)-(ii)-(i)** gives $4.25t^2 - 4.25 = 0$ **or** t = 1 s

Substituting t = 1s in **(i)** and **(ii)** we get

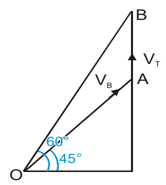
$u\sin\theta = 6.25$ m/s

$\Rightarrow u_y = 6.25$ m/s and $u\cos\theta = 3.75$ m/s

$\Rightarrow u_x = 3.75$ m/s therefore $\vec{u} = u_x\hat{i} + u_y\hat{j}$

$\Rightarrow \vec{u} = (3.75\hat{i} + 6.25\hat{j})$ m/s

3. **(a)** From the diagram $\vec{V}_{BT}$ makes an angle of 45° with the x-axis.



(b) Using sine rule

$$\frac{V_B}{\sin 135°} = \frac{V_T}{\sin 15°} \Rightarrow V_B = 2 \text{ m/s}$$

Integer Type questions

1. With respect to train :

Time of flight : $T = \frac{2v_y}{g} = \frac{2 \times 5\sqrt{3}}{10} = \sqrt{3}$

By using $s = ut + \frac{1}{2}at^2$

we have $1.15 = 5T - \frac{1}{2}aT^2 \Rightarrow a = 5$ m/s^2

2. 5

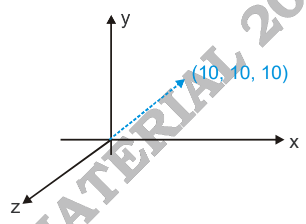
3. 2 **or** 8

4. 4

Comprehension : **1.** A **2.** B

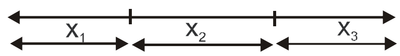
Rectilinear Motion Practice Test

1. Displacement vector is $10\hat{i} + 10\hat{j} + 10\hat{k}$



$\Rightarrow$ Magnitude = $\sqrt{10^2 + 10^2 + 10^2} = 10\sqrt{3}$ Ans.

2.



Starting from rest $x_1 = \frac{1}{2}a(10)^2$ ....(1)

$x_1 + x_2 = \frac{1}{2}a(20)^2$ ....(2)

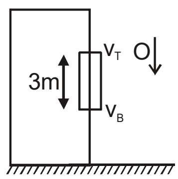
$x_1 + x_2 + x_3 = \frac{1}{2}a(30)^2$ ....(3)

From (2) – (1) $\Rightarrow x_2 = \frac{1}{2}a(300)$

From (3) – (2) $\Rightarrow x_3 = \frac{1}{2}a(500)$

$\Rightarrow x_1 : x_2 : x_3 :: 1 : 3 : 5$ **Ans.**

3. $s = \frac{(u+v)}{2}t$



$3 = \frac{(v_T + v_B)}{2} \times 0.5$

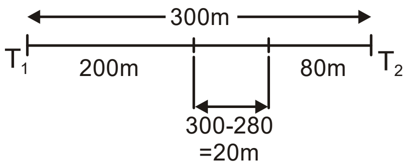
$v_T + v_B = 12$ m/s

Also, $v_B = v_T + (9.8)(0.5)$ .... (2)

$v_B - v_T = 4.9$ m/s

4. Initial distance between trains is 300 m.
Displacement of 1st train is calculated by area under V-t. curve of train 1 = $\frac{1}{2} \times 10 \times 40 = 200$ m.

Displacement of train 2 = $\frac{1}{2} \times 8 \times (-20) = -80$ m.



Which means it moves towards left.

$\therefore$ Distance between the two is 20 m.

5. At $t = \frac{T}{4}$ and $t = \frac{3T}{4}$, the stone is at same height,
Hence average velocity in this time interval is zero.
Change in velocity in same time interval is same for a particle moving with constant acceleration.
Let H be maximum height attained by stone, then distance travelled from t = 0 to $t = \frac{T}{4}$ is $\frac{3}{4}$H and

from $t = \frac{T}{4}$ to $t = \frac{3T}{4}$ distance travelled is $\frac{H}{2}$.

From $t = \frac{T}{2}$ to t = T sec distance travelled is H

and from $t = \frac{T}{2}$ to $t = \frac{3T}{4}$ distance travelled is $\frac{H}{4}$.

6. The retardation is given by $\frac{dv}{dt} = -av^2$

integrating between proper limits

$$\Rightarrow -\int_u^v \frac{dv}{v^2} = \int_0^t a\,dt \text{ or } \frac{1}{v} = at + \frac{1}{u}$$

$$\Rightarrow \frac{dt}{dx} = at + \frac{1}{u} \Rightarrow dx = \frac{u\,dt}{1+aut}$$

integrating between proper limits

$$\Rightarrow \int_0^s dx = \int_0^t \frac{u\,dt}{1+aut} \Rightarrow S = \frac{1}{a}\ell n(1+aut)$$

7. Let a be the retardation produced by resistive force, t_a and t_d be the time of ascent and descent respectively.

If the particle rises upto a height h

then $h = \frac{1}{2}(g+a)t_a^2$ and $h = \frac{1}{2}(g-a)t_d^2$

$$\therefore \frac{t_a}{t_d} = \sqrt{\frac{g-a}{g+a}} = \sqrt{\frac{10-2}{10+2}} = \sqrt{\frac{2}{3}} \quad \textbf{Ans. } \sqrt{\frac{2}{3}}$$

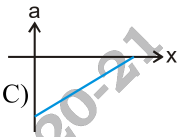
8. The linear relationship between V and x is
V = – mx + C where m and C are positive constants.

$\therefore$ Acceleration

$a = V\frac{dV}{dx} = -m(-mx + C)$

$\Rightarrow \therefore a = m^2x - mC$

Hence the graph relating a to x is :



9. $x_A = x_B$

$10.5 + 10t = \frac{1}{2}at^2$ $a = \tan 45° = 1$

$t^2 - 20t - 21 = 0 \quad \Rightarrow t^2 - 21t + t - 21 = 0$

$t(t-21) + 1(t-21) = 0 \Rightarrow \quad t = 21, -1$

rejecting negative value t = 21 sec.

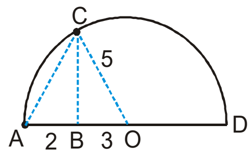
10. From triangle BCO $\Rightarrow$ BC = 4

From triangle BCA $\Rightarrow$

$AC = \sqrt{2^2 + 4^2} = 2\sqrt{5}$

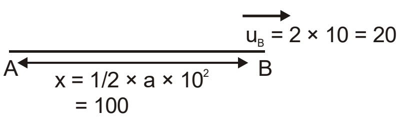
$AC = u_1 t$, $BC = u_2 t$

$$\therefore \frac{u_1}{u_2} = \frac{AC}{BC} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{\sqrt{4}}$$



11. After 10 sec $\Rightarrow$ Now $x_A = (40\,t)$

$x_B = 100 + (ut) + \frac{1}{2}(2)t^2 = 100 + 20t + t^2$



A will be ahead of B when

$x_B < x_A \quad \Rightarrow 100 + 20t + t^2 < 40t$

$\Rightarrow t^2 - 20t + 100 < 0 \quad \Rightarrow t^2 - 10t - 10t + 100 < 0$

$t(t-10) - 10(t-10) < 0 \Rightarrow (t-10)^2 < 0$

which is not possible

12. From given graphs : a_x is +ve & a_y is –ve, as v_x is increasing in +ve direction and v_y in –ve direction. (Checked from slope)

13. Distance travelled from time 't–1' sec to 't' sec is

$S = u + \frac{a}{2}(2t-1)$ ....(1)

from given condition S = t .... (2)

(1) & (2) $\Rightarrow t = u + \frac{a}{2}(2t-1) \Rightarrow u = \frac{a}{2} + t(1-a)$.

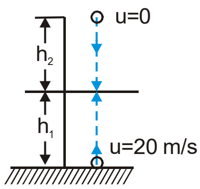
Since u and a are arbitrary constants, and they must be constant for every time.

$\Rightarrow$ coefficient of t must be equal to zero.

$\Rightarrow 1 - a = 0 \Rightarrow a = 1$ for a = 1, $u = \frac{1}{2}$ unit

Initial speed is $\frac{1}{2}$ unit. Ans.

14. Height of the building



$H = h_1 + h_2$

$= \frac{1}{2}gt^2 + ut - \frac{1}{2}gt^2$

= ut = 60 m.

15. $\vec{r} = (t^2 - 4t + 6)\hat{i} + t^2\hat{j}$; $\vec{v} = \frac{d\vec{r}}{dt} = (2t-4)\hat{i} + 2t\hat{j}$,

$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 2\hat{j}$ if $\vec{a}$ and $\vec{v}$ are perpendicular

$\vec{a}.\vec{v} = 0 \Rightarrow (2\hat{i} + 2\hat{j}).((2t-4)\hat{i} + 2t\hat{j}) = 0$

$8t - 8 = 0 \Rightarrow t = 1$ sec. **Ans.**

16. At t = 0

$\frac{dx}{dt} = 0$ for particles 1,2 and 3 and $\left|\frac{d^2x}{dt^2}\right| > 0$ for t > 0

and $\frac{dx}{dt} = -3.4$ m/s for particle 4 and $\frac{d^2x}{dt^2}$ is negative for t > 0

Therefore for t > 0 ; $\left|\frac{dx}{dt}\right|$ is increasing in all.

17. |Displacement| ≤ Distance.

So, average speed of a particle in a given time $\left(\text{i.e. } \frac{\Delta(\text{distance})}{\Delta t}\right)$ is never less than magnitude of average velocity $\left(\text{i.e } \left|\frac{\Delta}{\Delta t}(\text{displacement})\right|\right)$

It is possible to have a situation in which $\left|\frac{d\vec{v}}{dt}\right| \neq 0$ (**i.e.,** |acceleration| ≠ 0) but $\frac{d|\vec{v}|}{dt} = 0$ (**i.e.,** $\frac{d}{dt}$ (speed) = 0). A particle moving in a circle with constant speed follow the upper statement.

A partcile revolving in a circle has zero average velocity every time it reaches the starting point.

18. **(A)** Magnitude of velocity is changing **Hence** acceleration is present.

(C) Velocity is changing, it can happen by change in direction, as in the case of uniform circular motion.

Hence acceleration is present.

19. $\upsilon = \sqrt{x} \Rightarrow \frac{dx}{dt} = \sqrt{x}$

$\frac{dx}{x^{1/2}} = dt \Rightarrow 2\sqrt{x} = t + C$

but given at t = 0 ; x = 4 $\Rightarrow$ c = 4

$x = \frac{(t+4)^2}{4} \Rightarrow x = \frac{(6)^2}{4} = \frac{36}{4} = 9$ m

[Putting t = 2 sec.]

$a = v\frac{dv}{dx} = \sqrt{x} \times \frac{1}{2\sqrt{x}} = \frac{1}{2}$ m/s^2

20. Slope of displacement-time curve gives velocity.

(A) During OA slope is +ve but decreasing **Hence** velocity is positive and acceleration is negative.

(C) During BC slope is – ve and going to zero **Hence** velocity is – ve but acceleration is +ve.

(D) During DE slope is +ve and increasing **Hence** vel. is +ve and increasing $\therefore$ +ve acceleration

21.

time		distance left
t = 0	→	x_0
t = T	→	$x_0/2$
t = 2T	→	$x_0/2^2$

$t = nT \rightarrow \frac{x_0}{(2)^n} = \frac{x_0}{(2)^{t/T}} = x_0(2)^{-t/T} = x_0(2)^{-t}$

($\because$ T = 1s)

$\therefore$ distance travelled in time $t = x = x_0 - x_0(2)^{-t} = x_0(1 - 2^{-t})$

$v = \frac{dx}{dt} = x_0 2^{-t} \times \ell n(2) = \frac{x_0 \ell n2}{2^t}$

($\therefore$ slope of x-t curve is positive and decreasing with time)

$a = \frac{dv}{dt} = -x_0 2^{-t} \times (\ell n2)^2 \Rightarrow |a| = x_0 2^{-t} \times (\ell n2)^2$

22. (i) $V\dfrac{dv}{dx} = -\beta V \Rightarrow dv = -\beta dx$

$\Rightarrow \int_{v_0}^{0} dv = -\beta \int_0^x dx \Rightarrow -v_0 = -\beta x$

$x = \dfrac{v_0}{\beta}$ [when V = 0, accelaration = 0,

so x is total direction

(ii) $a = -\beta V \Rightarrow \dfrac{dv}{dt} = -\beta V$ $\int_{v_0}^{v} \dfrac{dv}{v} = -\beta \int_0^t dt \Rightarrow \ell n\left(\dfrac{V}{V_0}\right) = -\beta t \Rightarrow V = V_0 e^{-\beta t}$

$V = \dfrac{V_0}{e^{\beta t}}$ at $t \to \infty$ $V = 0$.

$\therefore$ A & B are correct answer

23. Average velocity $= \dfrac{\text{displacement}}{\text{Time}}$, and

average speed $= \dfrac{\text{distance}}{\text{time}} \Rightarrow |\text{Displacement}| \le \text{Distance}$.

24. $a = \dfrac{dv}{dt}$

If a = 0

v may **or** may not be zero.

25. A particle is projected vertically upwards. In duration of time from projection till it reaches back to point of projection, average velocity is zero. **Hence** statement I is false.

26. The expression for velocity and time can be expressed as v = (t – 2) (t – 4)

The speed is **therefore** zero at t = 2. **Hence** speed is minimum at t = 2.

But $\dfrac{dv}{dt} = 2t - 6$ is zero at t = 3 seconds.

Hence statement I is true, **also** we know statement II is true but II is not a correct explanation of I.

27. (A)

28. (B)

29. $a = \sin\pi t$

$\therefore \int dv = \int 2\sin\pi t\, dt$ or $v = -\dfrac{2}{\pi}\cos\pi t + C$

at t = 0, v = 0 $\therefore C = \dfrac{2}{\pi}$ or $v = \dfrac{2}{\pi}(1 - \cos\pi t)$

Note : Velocity is always non-negative as $\cos\theta \le 1$ **Hence** particle always moves along positive x-direction

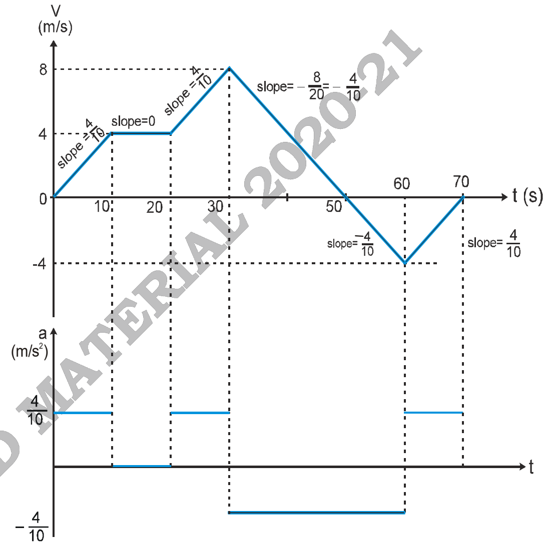
$\therefore$ Distance from time t = 0 to t is

$$S = \int_0^t \frac{2}{\pi}(1-\cos\pi t)dt = \frac{2}{\pi}\left[t - \frac{1}{\pi}\sin\pi t\right]_0^t = \frac{2}{\pi}t - \frac{2}{\pi^2}\sin\pi t$$

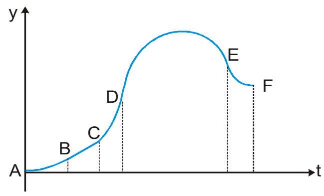
also displacement from time t = 0 to $t = \dfrac{2t}{\pi} - \dfrac{2}{\pi^2}\sin\pi t$

Distance from time t = 0 to t = 1 s $= \dfrac{2}{\pi}$ meters

30. a = slope of v–t curve so



31.



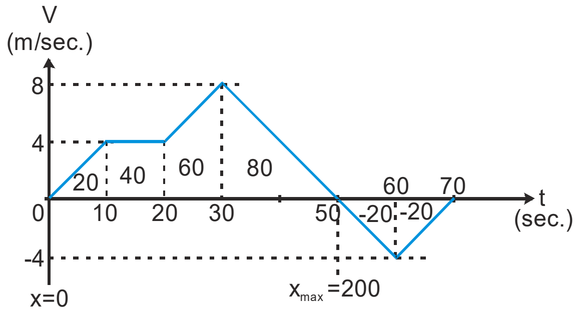
AB = Slope increasing, BC = Slope constant,

CD = Slope increasing

DE = Slope decreasing, EF = Slope increasing

F = Slope is 0

32.



Positive increase in area of v-t curve shows positive increase in displacement. So displacement is increasing till t = 50 s.

$\therefore$ max displacement = positive v-t area.

33. In case A and B acceleration is constant but speed first decreases and then increases.

In case C and D, the slope does not change sign **Hence** direction of acceleration is constant. Speed and magnitude of acceleration decreases with time.

34. **(A)** $v = 6t + 2$ m/s $v(t = 1) = 8$ m/s

$a = 6$ m/s^2

$v > 0$

(B) $v(t = 1) = 8$ m/s

$a = 8$ m/s^2

$v > 0$

(C) a is variable and +ve

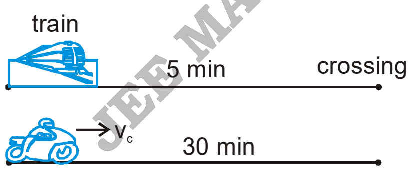
$v = \int a\,dt = 8t^2$, $v(t = 1s) = 8$ m/s

(D) $v(t = 1s) = 3$ m/s

$a = 6 - 6t$, variable.

$v < 0$ for $t > 2$ s.

35.



$t_{cycle} = \dfrac{10\text{km}}{20\text{kmh}^{-1}} = \dfrac{1}{2}$ h = 30 min

$\Delta t = 5\text{ min} = \dfrac{5}{60}$ hr

Train running as per shedule

So $V_{train} = \dfrac{10}{(5/60)} = \dfrac{10 \times 60}{5} = 120\text{ kmh}^{-1}$

36. Acceleration of the particle $a = 2t - 1$.

The particle retards when acceleration is opposite to velocity.

$\Rightarrow a.v < 0 \Rightarrow (2t - 1)(t^2 - t) < 0 \Rightarrow t(2t - 1)(t - 1) < 0$

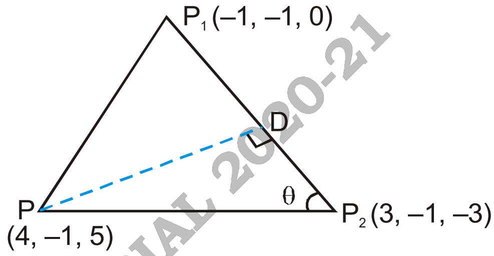
now t is always positive $\therefore (2t - 1)(t - 1) < 0$

$\Rightarrow$ either $2t - 1 < 0$ & $t - 1 > 0 \Rightarrow t < \dfrac{1}{2}$ & $t > 1$

This is simultaneously not possible.

or $2t - 1 > 0$ & $t - 1 < 0 \Rightarrow \dfrac{1}{2}$ **$< t < 1$ Ans.**

37. Let the position of bird 'P' and the two positions P_1 and P_2 are as shown in figure



Let the bird flies and reaches point D where it is collinear with P_1 and P_2.

$\left|\overrightarrow{P_2P_1}\right| = \sqrt{(-1-3)^2 + (-1+1)^2 + (0+3)^2}$

$= \sqrt{4^2 + 3^2} = 5$

$\left|\overrightarrow{P_2P}\right| = \sqrt{(4-3)^2 + (-1+1)^2 + (5+3)^2}$

$= \sqrt{65}$

Here $\angle P_1P_2P = \theta$

$\therefore \cos\theta = \dfrac{(\overrightarrow{P_2P}).(\overrightarrow{P_2P_1})}{\left|\overrightarrow{P_2P}\right|\left|\overrightarrow{P_2P_1}\right|} = \dfrac{(\hat{i} + 8\hat{k}).(-4\hat{i} + 3\hat{k})}{\sqrt{65}\,.\,5}$

$= \dfrac{4}{\sqrt{65}}$

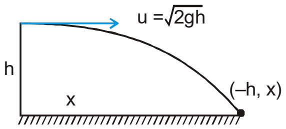
$\cos\theta = \dfrac{4}{\sqrt{65}}$, $\sin\theta = \dfrac{7}{\sqrt{65}}$

$\therefore$ From ΔPDP_2, $PD = PP_2 \sin\theta = \sqrt{65} \times \dfrac{7}{\sqrt{65}} = 7$m

$\therefore$ Time taken by bird $= \dfrac{7}{2}$ sec = 3.5 sec

Projectile Motion
Practice Test

1. Using equation of trajectory :



$$-h = x\tan(0°) - \frac{gx^2}{2(2gh)(\cos^2 0°)}$$

$\Rightarrow x = 2h$ **Ans.**

Method II

time of flight $T = \sqrt{\frac{2h}{g}}$

horizontal distance covered during time of flight is

$$x = u_x t = \sqrt{\frac{2h}{g}} \times \sqrt{2hg} = 2h$$

2. Ranges for complementary angles are same

$\therefore$ Required angle $= \frac{\pi}{2} - \frac{5\pi}{36} = \frac{13\pi}{36}$ Ans.

3. Use $\alpha = \beta = 45°$ in the formula for Range down the incline plane.

4. Time of flight $T = \frac{2u_y}{g}$

$$T = \frac{2 \times 20\sin 37°}{10} = 4 \times \frac{3}{5} = \frac{12}{5} \text{ sec.}$$

Range $R = u_x \times T = \frac{12}{5} \times (20\cos 37° + 10)$

$$R = \frac{12}{5} \times \left(20 \times \frac{4}{5} + 10\right) = 26 \times \frac{12}{5} = 62.4 \text{ m}$$

5. Use the given data in the formulae for projection up the inclined plane.

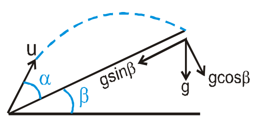
Let the inclination of the inclined plane $= \beta$

$u\cos\alpha = 10$ .... (1)

Time of flight $\frac{2u\sin\alpha}{g\cos\beta} = 2$ .... (2)

maximum height

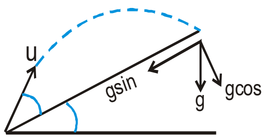
$\frac{u^2\sin^2\alpha}{g\cos\beta} = 5$ .... (3)



$u\sin\alpha = 5 \;\; \therefore u = 5\sqrt{5}$

$u\cos\alpha = 10$ .... (1)

$\frac{2u\sin\alpha}{g\cos\beta} = 2$ .... (2)



$\frac{u^2\sin^2\alpha}{g\cos\beta} = 5$ .... (3)

$u\sin\alpha = 5 \;\; \therefore u = 5\sqrt{5}$

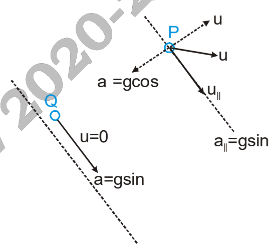
6. Path will not be straight line but parabolic **Hence** neither stone will hit any person. Condition of collision will depend upon direction as well as magnitude of velocities of projection which are not given.

7. It can be observed from figure that P and Q shall collide if the initial component of velocity of P on inclined plane i.e along incline. $u_{\parallel} = 0$ that is particle is projected perpendicular to incline.

$\therefore$ Time of flight

$$T = \frac{2u_{\perp}}{g\cos\theta}$$

$$= \frac{2u}{g\cos\theta}$$



$\therefore u = \frac{gT\cos\theta}{2} = 10$ m/s.

8. $\tan\theta = \frac{9-1}{4-0} = 2$ Where θ is the angle of projection

Displacement in y-direction $s_y = u_y t + \frac{1}{2}a_y t^2$

now, $-1 = u\sin\theta\,(1) - \frac{1}{2}g(1)^2$

$u\sin\theta = 4$ and from triangle

$\sin\theta = \frac{2}{\sqrt{5}} \Rightarrow u = 2\sqrt{5}$ m/s

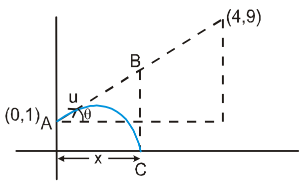


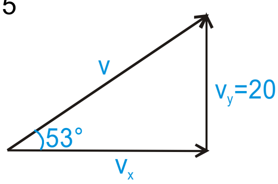
fig. (A)

Displacement in x-direction $s_x = u_x t + \frac{1}{2}a_x t^2$

now, $x = u\cos\theta\,(1) = (2\sqrt{5}) \times \frac{1}{\sqrt{5}} = 2$m

9. Two second before maximum height $v_y = g \times 2 = 20$ m/s

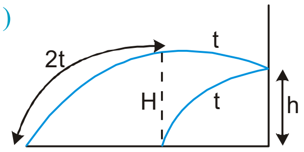
$\tan 53° = \frac{20}{v_x}$ $v_x = 15$ m/s



velocity at maximum height $v = v_x = 15$ m/s

10. For B to C

$H = \frac{1}{2} g (2t)^2 = 2gt^2$ ....(1)



$h' = \frac{1}{2} g t^2$ ....(2)

$h = H - h' \Rightarrow h = H - \frac{1}{2} gt^2$ ....(3)

By (1) & (2)

$h = H - \frac{H}{4} = \frac{3H}{4}$

11. velocity component $u_x = 400/3\ \hat{i}$, $u_y = 100\ \hat{j}$

Applying equation is y direction

$-1500 = -100t - \frac{1}{2} \times 10t^2 \Rightarrow \frac{t^2}{2} + 10t - 150 = 0$

$t = \frac{-20 \pm 40}{2}$

So t = 10 sec **i.e.** horizontal distance

$u_x \times t = \frac{500}{3} \times \frac{4}{5} \times 10 = \frac{4000}{3}$ m.

12. For minimum number of jumps, range must be maximum.

maximum range $= \frac{u^2}{g} = \frac{(\sqrt{10})^2}{10} = 1$ meter.

Total distance to be covered = 10 meter

So minimum number of jumps = 10

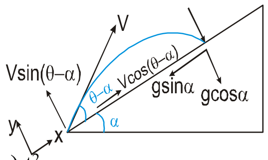
13. $y = bx^2$

$\frac{dy}{dt} = 2bx.\frac{dx}{dt} \Rightarrow \frac{d^2y}{dt^2} = 2b\left(\frac{dx}{dt}\right)^2 + 2bx\frac{d^2x}{dt^2}$

$a = 2bv^2 + 0 \Rightarrow v = \sqrt{\frac{a}{2b}}$

14. Applying equation of motion perpendicular to the incline for y = 0.

$y = u_y t + \frac{1}{2} a_y t^2$



$0 = V \sin(\theta)t + \frac{1}{2}(-g\cos\alpha)t^2$

$\Rightarrow t = 0\ \&\ \frac{2V\sin(\theta-\alpha)}{g\cos\alpha}$

At the moment of striking the plane, as velocity is perpendicular to the inclined plane **Hence** component of velocity along incline must be zero.

$V_x = u_x + a_x t$

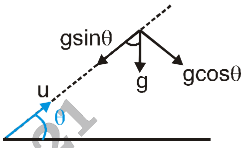
$0 = v\cos(\theta-\alpha) + (-g\sin\alpha).\frac{2V\sin(\theta-\alpha)}{g\cos\alpha}$

$v\cos(\theta-\alpha) = \tan\alpha.\ 2V\sin(\theta-\alpha)$

$\cot(\theta-\alpha) = 2\tan\alpha$ **Ans. (D)**

15. $0 = u - g\sin\theta .t$

(a) $t = \frac{2u\sin\theta}{g} = \frac{2.(10)\sin 30^\circ}{10} = 1$ sec.



(b) $t = \frac{2.10\sqrt{3}.\sqrt{3}}{10.2} = 3$ sec.

(c) $t = \frac{u}{g\sin\theta} = \frac{10\sqrt{3}}{10(\sqrt{3}/2)} = 2$ sec.

t is less than time of flight

(d) $t = \frac{u}{g\sin\theta} = \frac{10}{10.\frac{1}{2}} = 2$ sec.

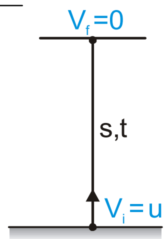
But it's time of flight is 1sec

16. **(A)** Total displacement is zero **Hence** its average velocity is zero.

(B) Displacement is zero.

(C) Total distance travelled is 2s and total time taken is 2t.

$\text{<speed>} = \frac{\text{total distance travelled}}{\text{total time taken}}$



$0^2 = u^2 - 2gs$

$\therefore s = \frac{u^2}{2g} \Rightarrow 2s = u^2/g$

also $0 = u - gt \Rightarrow t = u/g$

$\therefore 2t = \frac{2u}{g}$ $\text{<speed>} = \frac{u^2}{g} \Big/ \frac{2u}{g} = \frac{u}{2}$

17. On the curve

$y = x^2$ at $x = 1/2 \Rightarrow y = \frac{1}{4}$

Hence the coordinate $\left(\frac{1}{2}, \frac{1}{4}\right)$

Differentiating : $y = x^2 \Rightarrow v_y = 2xv_x$

$v_y = 2\left(\frac{1}{2}\right)(4) = 4$ m/s

Which satisfies the line

$4x - 4y - 1 = 0$ (tangent to the curve)

& magnitude of velocity :

$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = 4\sqrt{2}$ m/s

As the line 4x – 4y = 1 does not pass through the origin, **therefore** (D) is not correct.

18. Let u_x and u_y be horizontal and vertical components of velocity respectively at t = 0. Then,
$v_y = u_y - gt$
Hence, v_y – t graph is straight line.
$x = v_x t$
Hence, x - t graph is straight line passing through origin.
The relation between y and t is $y = u_y t - \frac{1}{2} gt^2$
Hence y-t graph is parabolic.
v_x = constant
Hence, v_x-t graph is a straight line.

19. $R_1 = \dfrac{2u^2 \sin\alpha \cos(\alpha+\theta)}{g\cos^2\theta}$ and $h_1 = \dfrac{u^2\sin^2\alpha}{2g\cos\beta}$

$R_2 = \dfrac{2u^2 \sin\alpha \cos(\alpha-\theta)}{g\cos^2\theta}$ and $h_2 = \dfrac{u^2\sin^2\alpha}{2g\cos\beta}$

Hence $h_1 = h_2$
$R_2 - R_1 = g \sin\theta\, T_2^2$

$R_2 - R_1 = g \sin\theta\, T_1^2$

20. Total time taken by the ball to reach at bottom = $\sqrt{\dfrac{2H}{g}} = \sqrt{\dfrac{2x80}{10}} = 4$ sec.

Let time taken in one collision is t
Then t x 10 = 7
t = 0.7 sec.

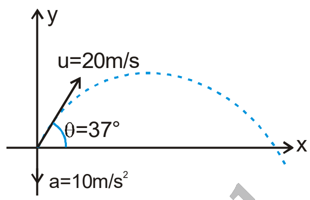
No. of collisions = $\dfrac{40}{7} = 5\dfrac{5}{7}$ (5th collisions from wall B)
Horizontal distance travelled in between 2 successive collisions = 7 m
∴ Horizontal distance travelled in 5/7 part of collisions
$= \dfrac{5}{7} x 7 = 5$ m
Distance from A is 2 m. **Ans.**

21. Both the stones cannot meet (collide) because their horizontal component of velocities are different. Hence statement I is false.

22. If particle moves with constant acceleration $\vec{a}$, then change in velocity in every one second is numerically equal to $\vec{a}$ by definition. Hence statement-2 is true and correct explanation of statement-1.

23. Velocity of a particle is independent of its position vector rather it depends on change in position vector while position vector depends on choice of origin. Hence statement-1 is false.

24. The question can be reframed as shown in figure.
The path of particle is parabolic.



$\therefore \vec{a} \perp \vec{v}$ at maximum height, that is at half time of flight

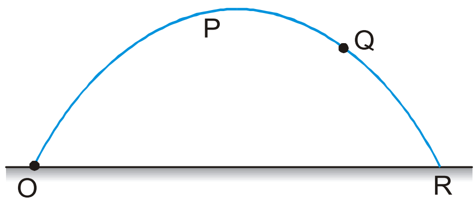
Hence $t_0 = \dfrac{u\sin\theta}{a} = \dfrac{20 \times 3/5}{10} = 1.2$ sec.

25. Speed is least at maximum height, that is at instant t_o=1.2 sec.

26. acceleration and displacement are mutually perpendicular at instant $2t_0$ = 2.4 sec.

27. $H_A = H_C > H_B$
Obviously A just reaches its maximum height and C has crossed its maximum height which is equal to A as u and θ are same. But B is unable to reach its max. height.

28. Time of flight of A is 4 seconds which is same as the time of flight if wall was not there.
Time taken by B to reach the inclined roof is 1 sec.



$T_{OR} = 4 \qquad T_{QR} = 1$
$\therefore T_{OQ} = T_{OR} - T_{QR}$ = 3 seconds.

29. From above $T = \dfrac{2u\sin\theta}{g} = 4$ s
∴ $u\sin\theta$ = 20 m/s ⇒ vertical component is 20 m/s for maximum height
$v^2 = u^2 + 2as \Rightarrow 0^2 = 20^2 - 2 \times 10 \times s$ s = 20 m.

30. **(A)** $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{100\sqrt{3}}{2(10)} = 5\sqrt{3}\text{m}$

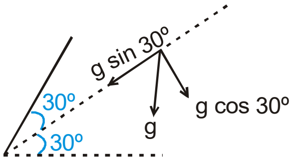
(B) $11.25 = -10\sin 60^\circ t + \dfrac{1}{2}(10)t^2$

$\Rightarrow 5t^2 - 5\sqrt{3}\,t - 11.25 = 0$

$t = \dfrac{5\sqrt{3} \pm \sqrt{25(3) + 4(5)(11.25)}}{10}$

$= \dfrac{5\sqrt{3} \pm \sqrt{3}(10)}{10}$

$= \dfrac{15}{10}\sqrt{3} = \dfrac{3}{2}\sqrt{3}$



$R = (10\cos 60)\left(\dfrac{3}{2}\sqrt{3}\right) = 7.5\sqrt{3}$ m

(C) $t = \dfrac{2u\sin 30^\circ}{g\cos 30^\circ} = \dfrac{2(10)\left(\frac{1}{2}\right)}{10\left(\frac{\sqrt{3}}{2}\right)} = \dfrac{2}{\sqrt{3}}$ sec.

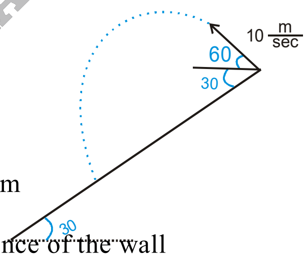
$R = 10\cos 30^\circ\, t - \dfrac{1}{2}g\sin 30^\circ\, t^2$

$= \dfrac{10\sqrt{3}}{2}\left(\dfrac{2}{\sqrt{3}}\right) - \dfrac{1}{2}(10)\left(\dfrac{1}{2}\right)\dfrac{4}{3} = 10 - \dfrac{10}{3} = \dfrac{20}{3}$ m

(D) $T = \dfrac{2(10)}{g\cos 30} = \dfrac{2(10)}{10\left(\frac{\sqrt{3}}{2}\right)} = \dfrac{4}{\sqrt{3}}$ sec.

$R = \dfrac{1}{2}g\sin 30^\circ t^2$

$= \dfrac{1}{2}(10)\left(\dfrac{1}{2}\right)\dfrac{16}{3} = \dfrac{40}{3}$ m



31. Range of the ball in absence of the wall

$= \dfrac{u^2 \sin 2\theta}{g} = \dfrac{20^2 \sin 150^\circ}{10}$ m $= 20$ m

When d < 20 m, ball will hit the wall. when d = 25 m, ball will fall 5m short of the wall.
When d < 20 m, the ball will hit the ground , at a distance, x = 20 m – d in front of the wall.

32. From graph (1) : $v_y = 0$ at $t = \dfrac{1}{2}$ sec.

i.e., time taken to reach maximum height H is

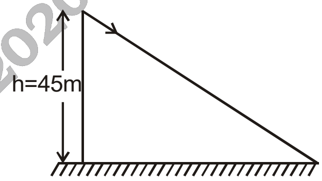
$t = \dfrac{u_y}{g} = \dfrac{1}{2} \Rightarrow u_y = 5$ m/s **...Ans.(i)**

from graph (2) : $v_y = 0$ at $x = 2$m
i.e., when the particle is at maximum height, its displacement along horizontal x = 2 m

$x = u_x \times t \Rightarrow 2 = u_x \times \dfrac{1}{2}$

$\Rightarrow u_x = 4$ m/s **....Ans (ii)**

33. **(a)** Taking motion in vertical direction
$u = 0$, $g = 10$ m/s^2, $h = 45$ m
$h = ut + ½ gt^2$
$\Rightarrow h = 0 + ½ gt^2$

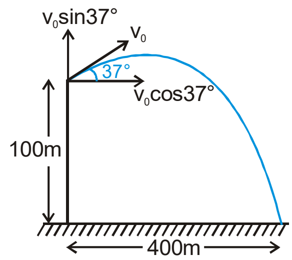


$\Rightarrow t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 45}{10}}$

$\Rightarrow t = 3$ sec.

34. Let t be the time after which projectile reaches the ground. Taking motion in horizontal direction
$400 = (v_0 \cos 37°)\, t$
$400 = v_0 (4/5)\, t$
$\Rightarrow v_0 t = 500$ .... (1)
Taking motion in vertical direction
$h = ut + ½ gt^2$
$\Rightarrow 100 = (-v_0 \sin 37°)\, t + ½ (10) t^2$

$\Rightarrow 100 = -\dfrac{3}{5}(v_0 t) + 5t^2$



Putting $v_0 t = 500$
from equation (1) ;

$\Rightarrow 100 = -\dfrac{3}{5}(500) + 5t^2$

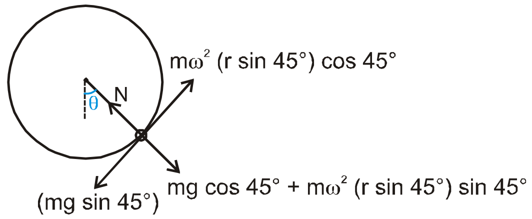
$\Rightarrow 5t^2 = 400 \Rightarrow t = \dfrac{20}{\sqrt{5}}$sec

From eqn (1) ; $v_0 = 500 \times \dfrac{\sqrt{5}}{20}$

$v_0 = 25\sqrt{5}$ m/s

Circular Motion
Practice Test

1. The maximum angular speed of the hoop corresponds to the situation when the bead is just about to slide upwards.
The free body diagram of the bead is



For the bead not to slide upwards.

$m\omega^2 (r \sin 45°) \cos 45° - mg \sin 45° < \mu N$ .... **(1)**

where $N = mg \cos 45° + m\omega^2 (r \sin 45°) \sin 45°$ .... **(2)**

From 1 and 2 we get.

$\omega = \sqrt{30\sqrt{2}}$ rad / s.

2. Let v be the speed of particle at B, just when it is about to loose contact.

From application of Newton's second law to the particle normal to the spherical surface.

$\frac{mv^2}{r} = mg \sin \beta$ .... **(1)**

Applying conservation of energy as the block moves from A to B..

$\frac{1}{2} mv^2 = mg (r \cos \alpha - r \sin \beta)$ .... **(2)**

Solving 1 and 2 we get $\Rightarrow 3 \sin \beta = 2 \cos \alpha$

3. As the mass is at the verge of slipping

$\therefore$ $mg \sin 37 - \mu\, mg \cos 37 = m\omega^2 r$

$6 - 8\mu = 4.5$

$\therefore$ $\mu = \frac{3}{16}$

4. As when they collide $vt + \frac{1}{2}\left(\frac{72v^2}{25\pi R}\right)t^2 - \pi R = vt$

$\therefore$ $t = \frac{5\pi R}{6v}$

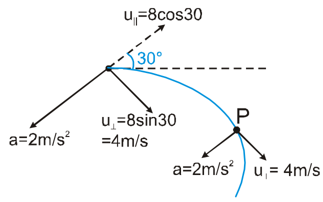
Now angle covered by A = $\pi + \frac{vt}{R}$

Put t $\therefore$ angle covered by A = $\frac{11\pi}{6}$

5. The acceleration vector shall change the component of velocity $u_{\parallel}$ along the acceleration vector.

$r = \frac{v^2}{a_n}$

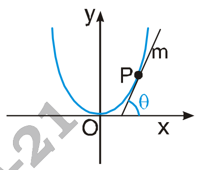
Radius of curvature r_{min} means v is minimum and a_n is maximum. This is at point P when component of velocity parallel to acceleration vector becomes zero, that is $u_{\parallel} = 0$.



$\therefore R = \frac{u_{\perp}^2}{a} = \frac{4^2}{2} = 8$ meters.

6. $x^2 = 4ay$

Differentiating w.r.t. y, we get



$\frac{dy}{dx} = \frac{x}{2a}$

$\therefore$ At (2a, a), $\frac{dy}{dx} = 1 \Rightarrow$ Hence $\theta = 45°$

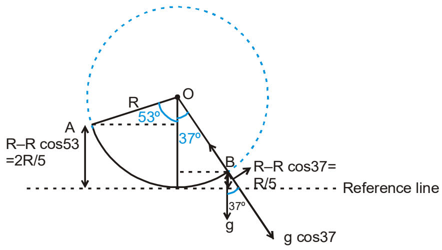
the component of weight along tangential direction is mg sin θ.

Hence tangential acceleration is $g \sin \theta = \frac{g}{\sqrt{2}}$

7. The nature of the motion can be determined only if we know velocity and acceleration as function of time. Here acceleration at an instant is given and not known at other times so D is the correct option

8. By energy conservation between A & B

$\Rightarrow Mg \frac{2R}{5} + 0 = \frac{MgR}{5} + \frac{1}{2} Mv^2 \Rightarrow v = \sqrt{\frac{2gR}{5}}$

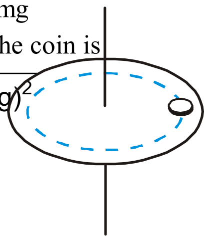


Now, radius of curvature $r = \frac{v_{\perp}^2}{a_r} = \frac{2gR/5}{g\cos 37} = \frac{R}{2}$

9. The friction force on coin just before coin is to slip will be : $f = \mu_s mg$

Normal reaction on the coin ; N = mg

The resultant reaction by disk to the coin is



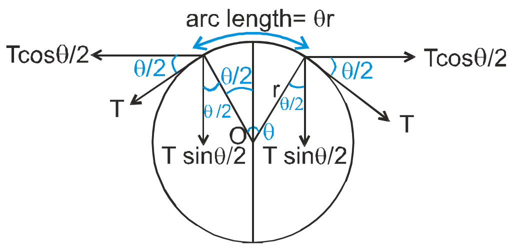
$= \sqrt{N^2 + f^2} = \sqrt{(mg)^2 + (\mu_s mg)^2}$

$= mg\sqrt{1 + \mu_s^2}$

$= 40 \times 10^{-3} \times 10 \times \sqrt{1+\frac{9}{16}}$ **= 0.5 N**

10. As $2T \sin\frac{\theta}{2} = dm\,\omega^2 r$ (for small angle $\sin\frac{\theta}{2} \to \frac{\theta}{2}$)

but $dm = \frac{m}{\ell}\,\theta r$



As $\ell = 2\pi r$ $\quad\therefore T = m\omega^2 r/2\pi$

Put $m = 2\pi$ kg $\quad \omega = 10\,\pi$ radian/s

and $r = 0.25$ m $\quad\therefore T = 250$ N

11. when he applies brakes

$s_1 = \frac{v^2}{2a}$

if μ is the friction coefficient then $a = \mu g$

$\therefore\ s_1 = \frac{v^2}{2\mu g}$

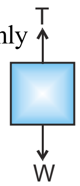
when he takes turn $\frac{mv^2}{r} = \mu mg$

$r = \frac{v^2}{\mu g}$

then we can see $r > s_1$ Hence driver can hit the wall when he takes turn due to insufficient radius of curvature.

12. As tengential acceleration $a = dv/dt = \omega dr/dt$

but $\omega = 4\pi$ and $dr/dt = 1.5$ (reel is turned uniformly at the rate of 2 r.p.s.)



$\therefore\ a = 6\pi$, Now by the F.B.D. of the mass.

$T - W = \frac{W}{g}a$

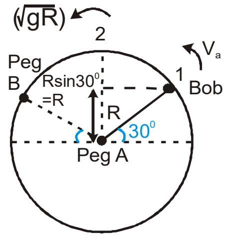
$\therefore\ T = W(1 + a/g)$ put $a = 6\pi$

$\therefore\ T = 1.019$ W

13. For anti-clockwise motion, speed at the highest point should be $\sqrt{gR}$.

Conserving energy at (1) & (2) :

$\frac{1}{2}mv_a^2 = mg\frac{R}{2} + \frac{1}{2}m(gR)$

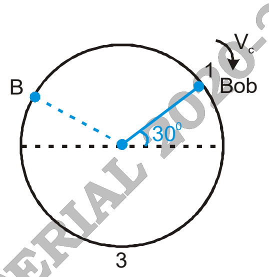


$\Rightarrow v_a^2 = gR + gR = 2gR$

$\Rightarrow v_a = \sqrt{2gR}$

For clock-wise motion, the bob must have atleast that much speed initially, so that the string must not become loose any where until it reaches the peg B.

At the initial position :



$T + mg\cos 60^0 = \frac{mv_c^2}{R}$;

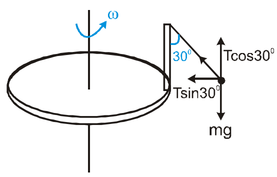
v_C being the initial speed in clockwise direction.

For $v_{C\,min}$: Put T = 0 ;

$\Rightarrow v_C = \sqrt{\frac{gR}{2}} \Rightarrow v_C/v_a = \frac{\sqrt{\frac{gR}{2}}}{\sqrt{2gR}} = \frac{1}{2} \Rightarrow v_C : v_a = 1 : 2$ **Ans.**

14. The bob of the pendulum moves in a circle of radius $(R + R\sin 30^0) = \frac{3R}{2}$

Force equations :



$T\sin 30^0 = m\left(\frac{3R}{2}\right)\omega^2$

$T\cos 30^0 = mg$

$\Rightarrow \tan 30^0 = \frac{3}{2}\frac{\omega^2 R}{g} = \frac{1}{\sqrt{3}} \Rightarrow \omega = \sqrt{\frac{2g}{3\sqrt{3}R}}$ **Ans.**

15. $v_{min} = \sqrt{5gR} = \sqrt{5 \times 10 \times 2} = 10$ m/s

16. $T\cos\theta + N = mg$ ...(1)

and $T\sin\theta = m\,\omega^2 r$ ...(2)

but $T = Kx$

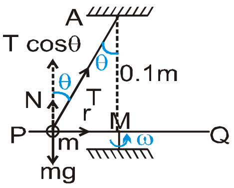
$T = 1.47 \times 10^2 (0.1 \sec\theta - 0.1)$

$(K = 1.47 \times 10^2\ N/m)$

Also $r = 0.1 \tan\theta$

put T, r, m & ω in equation **(2)**

we have $\cos\theta = 3/5$ and $T = 9.8$ N

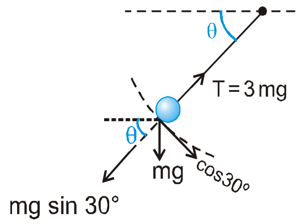


17. $T - mg\sin\theta = \dfrac{mv^2}{R}$

$\Rightarrow 3\,mg - mg\sin 30^\circ$

$= \dfrac{m.(u_0^2 + 2g\ell\sin 30^\circ)}{\ell}$

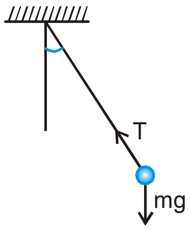
$\therefore\ u_0 = \sqrt{3g/2}$



18. When the acceleration of bob is horizontal, net vertical force on the bob will be zero.

$T\cos\theta - mg = 0$

The tangential force at that instant is



$= mg\sin\theta = mg\sqrt{1 - \cos^2\theta} = \dfrac{mg}{T}\sqrt{T^2 - (mg)^2}$

19. From length constraint on AB

$a\cos 45^\circ = b\cos 45^\circ$

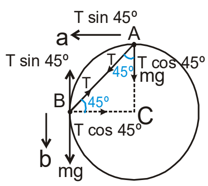
$a = b$

$T\sin 45^\circ = m(a)$ $\quad mg - T\sin 45^\circ = mb$

$mg - ma = ma$

$2ma = mg \qquad a = \dfrac{g}{2}$

$\dfrac{T}{\sqrt{2}} = \dfrac{mg}{2} \qquad T = \dfrac{mg}{\sqrt{2}}$



20. (C)

$V = \sqrt{gR\tan\theta} \Rightarrow (20)^2 = 10 \times 100 \times \tan\theta$

$\Rightarrow \tan\theta = \dfrac{4}{10} = \dfrac{2}{5} \Rightarrow \theta = \tan^{-1}(2/5)$

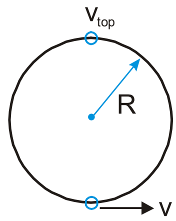
21. In the frame of ring (inertial w.r.t. earth), the initial velocity of the bead is v at the lowest position.

The condition for bead to complete the vertical circle is, its speed at top position $v_{top} \geq 0$

From conservation of energy

$\dfrac{1}{2}\, m\, v_{top}^2 + mg(2R) = \dfrac{1}{2} mv^2$ or

$v = \sqrt{4gR}$

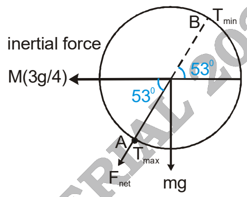


22. $|\Delta V| = \sqrt{v^2 + v^2 - 2v^2\cos 60^\circ} = v$

$a_{av} = \dfrac{|\Delta\vec{v}|}{\Delta t} = \dfrac{v}{t} = \dfrac{3v^2}{\pi R} \Rightarrow a_i = \dfrac{v^2}{R}$;

$\dfrac{a_i}{a_{av}} = \dfrac{v^2\pi R}{R \times 3v^2} = \dfrac{\pi}{3}$

23.

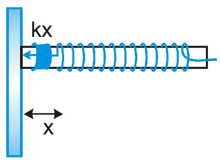


F_{net} is shown in the figure. So, tension will be max. at point A and will be min. at point B.

24. For the ring to move in a circle at constant speed the net force on it should be zero. Here spring force will provide the necessary centripetal force.

$\therefore\ kx = mx\omega^2 \Rightarrow$

$\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{300}{3}} = 10$ rad/sec.

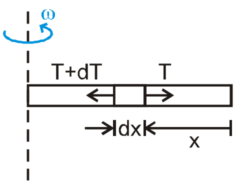


25. $dT = dm(\ell - x)\omega^2 \Rightarrow dT = \dfrac{m}{\ell}.dx(\ell - x)\omega^2$

$\Rightarrow \displaystyle\int_0^T dT = \int_0^{\ell/2} \dfrac{m\omega^2}{\ell}(\ell - x)\,dx$

$= \dfrac{m\omega^2}{\ell}\left[\ell x - \dfrac{x^2}{2}\right]_0^{\ell/2}$

$= \dfrac{m\omega^2}{\ell}\left[\dfrac{\ell^2}{2} - \dfrac{\ell^2}{8}\right]$ $\quad\therefore$ Tension at mid point is :



$T = \dfrac{3}{8}\, m\ell\,\omega^2 \Rightarrow$ stress $= \dfrac{3m\ell\omega^2}{8A}$

$\Rightarrow$ strain $= \dfrac{3m\ell\omega^2}{8AY}$

26. At A : $N_A - mg = \dfrac{mV^2}{R_A}$

$$N_A = mg + \frac{mV^2}{R_A}$$

and At B : $N_B = mg - \frac{mV^2}{R_B}$

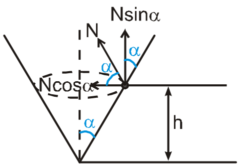
and At C : $N_C = mg + \frac{mV^2}{R_C}$

As by energy conservation ;

$R_A < R_C$

$\therefore$ N_A is greatest among all.

27.



As $N \sin \alpha = mg$ $\Rightarrow$ $N \cos \alpha = m\omega^2 r$

$\tan \alpha = \frac{g}{\omega^2 r}$ $\therefore T^2 \propto \tan \alpha$

$\therefore$ when α increases T **also** increases

Also $T^2 \propto r \tan \alpha$

but $r = h \tan \alpha$

$\therefore$ $T^2 \propto h \tan^2 \alpha$

for constant α

$T^2 \propto h$

Thus when h increases T **also** increases

28. Let N be the normal reaction (Reading of the weighing machine)

at A $\Rightarrow$ $N_A - mg = \frac{mv^2}{r}$

Put v $\therefore N_A - mg = mg \Rightarrow N_A = 2mg = 2W$

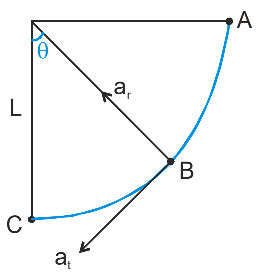
Also, at E, $N_E + mg = \frac{mv^2}{r} = mg$

$\therefore N_E = 0$ **Hence** $N_A > N_E$ by 2W

Now at G, $N_G = mg = W = N_C$

Also $\frac{N_E}{N_A} = 0$ and $\frac{N_A}{N_C} = 2$

29. Between A and B



$mgL \cos \theta = \frac{1}{2} mv_B^2$

$\therefore v_B^2 = 2gL \cos\theta$

Now $a_r = \frac{v_B^2}{L} = 2g \cos\theta$

and $a_t = g \sin\theta$

$\therefore a = \sqrt{a_t^2 + a_r^2} = g\sqrt{1 + 3\cos^2\theta}$

Now, at B $T_B - mg \cos\theta = \frac{mv_B^2}{L}$

Put V_B $\Rightarrow$ $T_B = 3\, mg \cos\theta$

When total acceleration vector directed horizontally

$\tan(90 - \theta) = \frac{a_t}{a_r} = \frac{g\sin\theta}{2g\cos\theta} = \frac{1}{2}\tan\theta$

On solving $\theta = \cos^{-1} 1/\sqrt{3}$

30. For case : $\omega_1 = \frac{5\pi}{6}$ rad/sec.

$\omega_{A/T} = \frac{5\pi}{6}$ rad/sec.

$\omega_{B/G} = \frac{v}{R} = \frac{3.14}{3} = \frac{\pi}{3}$ rad/sec.

$\omega_{T/G} = -\frac{\pi}{6}$ rad/sec (in opposite direction)

$\omega_{A/G} = \omega_{A/T} + \omega_{T/G} = \frac{5\pi}{6} + \left(-\frac{\pi}{6}\right) = \frac{4\pi}{6} = \frac{2\pi}{3}$ rad/s.

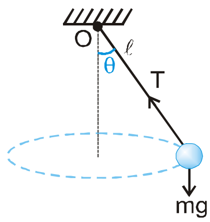
$\omega_{A/B} = \omega_A - \omega_B = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$ rad/sec.

and $\theta_{A/B} = 30° = \frac{\pi}{6}$ rad/sec.

Using ; $\theta_{rel} = \omega_{i\,(rel)} t + \frac{1}{2}\alpha_{rel} t^2$

$\frac{\pi}{6} = \frac{\pi}{3} t + 0$ $\Rightarrow t = 0.5$ sec. **Ans.**

31. For conical pendulum of length ℓ, mass m moving along horizontal circle as shown



$T \cos\theta = mg$ .... (1)

$T \sin\theta = m\omega^2 \ell \sin\theta$ .... (2)

From equation 1 and equation 2,

$\ell \cos\theta = \frac{g}{\omega^2}$

$\ell \cos\theta$ is the vertical distance of sphere below O point of suspension. Hence if ω of both pendulums are same, they shall move in same horizontal plane.

Hence statement-2 is correct explanation of statement-1.

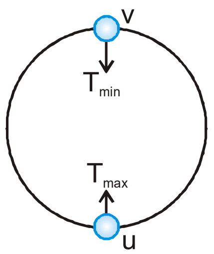
32. The normal reaction is not least at topmost point, Hence statement 1 is false.

33. Let the minimum and maximum tensions be T_{max} and T_{min} and the minimum and maximum speed be u and v.

$\therefore\ T_{max} = \dfrac{mu^2}{R} + mg$

$T_{min} = \dfrac{mv^2}{R} - mg$

$\therefore\ \Delta T = m\left(\dfrac{u^2}{R} - \dfrac{v^2}{R}\right) + 2\,mg.$



From conservation of energy

$\dfrac{u^2}{R} - \dfrac{v^2}{R} = 4g \quad \Rightarrow$ is independent of u.

and $\Delta T = 6\,mg$.

$\therefore$ Statement-2 is correct explanation of statement-1.

34. $v_B = \sqrt{2gL\sin\theta}$ and $v_C = \sqrt{2gL}$

If $\quad v_C = 2v_B$

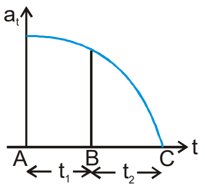
Then $\quad 2gL = 4\ (2gL\sin\theta)$

or $\quad \sin\theta = \dfrac{1}{4}$ **or** $\quad \theta = \sin^{-1}\dfrac{1}{4}$

35. Tangential acceleration is $a_t = g\cos\theta$,

which decreases with time.

Hence the plot of a_t versus time may be as shown in graph.



Area under graph in time interval $t_1 = v_B - 0 = v_B$

Area under graph in time interval $t_2 = v_C - v_B = v_B$

Hence area under graph in time t_1 and t_2 is same.

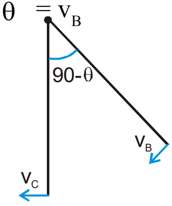
$\therefore\quad t_1 < t_2$

36. $|\vec{v}_B - \vec{v}_C| = \sqrt{v_B^2 + v_C^2 - 2v_Bv_C\sin\theta} = v_B$

$\Rightarrow\ v_B^2 + v_C^2 - 2v_Bv_C\sin\theta = v_B^{\,2}$

$v_C = 2v_B\sin\theta$

$\Rightarrow\ \sqrt{2g\ell} = 2\sqrt{2g\ell\sin\theta}\,\sin\theta$



$\therefore\ \sin^3\theta = \dfrac{1}{4} \Rightarrow \sin\theta = \left(\dfrac{1}{4}\right)^{1/3} \Rightarrow \theta = \sin^{-1}\left(\dfrac{1}{4}\right)^{1/3}$

37. Putting h = 0 and the values we have T = 164 N

38. Putting h = 2R we get T = 144 – 5gR = 44 N.

39. At $\theta = 60°$, $h = R - R\cos 60° = \dfrac{R}{2}$

Putting $h = \dfrac{R}{2}$ in $v^2 = u^2 - 2gh$

We get the result.

40. **(A)** $\vec{F}$ = constant and $\vec{u}\times\vec{F} = 0$

Therefore initial velocity is either in direction of constant force or opposite to it. Hence the particle will move in straight line and speed may increase or decrease. When F and u are antiparallel then particle will come to rest for an instant and will return back

(B) $\vec{u}\cdot\vec{F} = 0$ and $\vec{F}$ = constant

initial velocity is perpendicular to constant force, Hence the path will be parabolic with speed of particle increasing.

(C) $\vec{v}\cdot\vec{F} = 0$ means instantaneous velocity is alway perpendicular to force. Hence the speed will remain constant. And also $|\vec{F}|$ = constant. Since the particle moves in one plane, the resulting motion has to be circular.

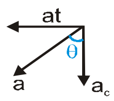
(D) $\vec{u} = 2\hat{i} - 3\hat{j}$ and $\vec{a} = 6\hat{i} - 9\hat{j}$. Hence initial velocity is in same direction of constant acceleration, therefore particle moves in straight line with increasing speed.

41. $v = 2t^2$

Tangential acceleration $a_t = 4t$

Centripetal acceleration $a_c = \dfrac{v^2}{R} = \dfrac{2t^4}{R}$

Angular speed $\omega = \dfrac{v}{R} = \dfrac{4t}{R}$,



$\tan\theta = \dfrac{a_t}{a_c} = \dfrac{4tR}{4t^4} = \dfrac{R}{t^3}$

42. From graph (a) $\quad\Rightarrow\quad \omega = k\theta$ where k is positive constant

angular acceleration $= \omega\dfrac{d\omega}{d\theta} = k\theta \times k = k^2\theta$

$\therefore$ angular acceleration is non uniform and directly proportional to θ. $\quad\therefore$ (a) q, s

From graph (b) $\quad\Rightarrow\quad \omega^2 = k\theta$.

Differentiating both sides with respect to θ.

$2\omega\dfrac{d\omega}{d\theta} = k \qquad$ **or** $\quad \omega\dfrac{d\omega}{d\theta} = \dfrac{k}{2}$

k is slope of curve Hence angular acceleration is uniform. $\therefore$ (B) p, t

From graph (c) $\quad\Rightarrow\quad \omega = kt$

angular acceleration $= \dfrac{d\omega}{dt} = k$

k is slope of curve Hence angular acceleration is uniform $\Rightarrow$ (C) p, t

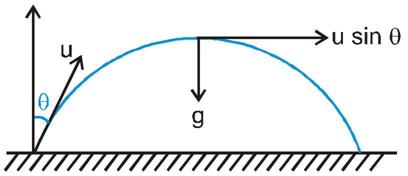
From graph (d) $\Rightarrow$ $\omega = kt^2$

angular acceleration $= \frac{d\omega}{dt} = 2kt$

k is slope of curve Hence angular acceleration is non uniform and directly proportional to t. Slope of the curve is constant (can be seen in given graph) but

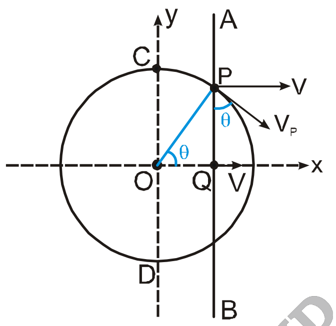
$\alpha = \frac{d\omega}{dt} = 2kt$ increasing with time. $\therefore$ (D) q,r

43.



$$R = \frac{(v_\perp)^2}{a_\perp} = \frac{u^2 \sin^2\theta}{g} = 20\text{ m.}$$

44. (a) (5)



As a rod AB moves, the point 'P' will always lie on the circle.

$\therefore$ Its velocity will be along the circle as shown by 'v_P' in the figure. If the point P has to lie on the rod 'AB' **also** then it should have component in 'x' direction as 'v'.

$\therefore$ $v_P \sin\theta = v$ $\Rightarrow$ $v_P = v \text{ cosec }\theta$

here $\cos\theta = \frac{x}{R} = \frac{1}{R} \cdot \frac{3R}{5} = \frac{3}{5}$

$\therefore \sin\theta = \frac{4}{5}$ $\therefore \text{cosec }\theta = \frac{5}{4}$

$\therefore v_P = \frac{5}{4}v$ **...Ans. x = 5**

(b) $\omega = \frac{V_P}{R} = \frac{5V}{4R}$

Alternative Solution :

(a) Let 'P' have coordinate (x, y)

$x = R\cos\theta,\ y = R\sin\theta.$

$$v_x = \frac{dx}{dt} = -R\sin\theta\frac{d\theta}{dt} = v \Rightarrow \frac{d\theta}{dt} = \frac{-v}{R\sin\theta}$$

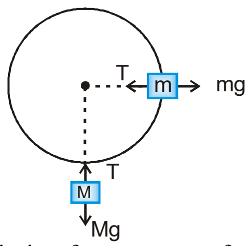
$$\text{and } v_Y = R\cos\theta\frac{d\theta}{dt} = R\cos\theta\left(-\frac{v}{R\sin\theta}\right) = -v\cot\theta$$

$$\therefore\ v_P = \sqrt{v_x^2 + v_y^2} = \sqrt{v^2 + v^2\cot^2\theta} = v\text{ cosec }\theta \quad \textbf{...Ans.}$$

45. As the car travels at a fixed speed 1 m/s, Hence tangential acceleration will be zero. Therefore, there will be no component of friction along tangent.

Case I : If $Mg > \frac{mv^2}{r}$; Hence friction force on car of mass m will be outwards from the centre.

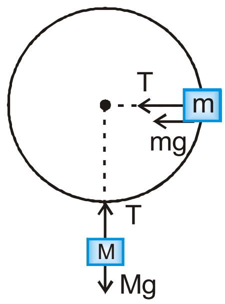
$$T - \mu mg = \frac{mv^2}{r_{max}}$$



$$Mg - \mu mg = \frac{m}{r_{max}} \quad \text{..... (1)}$$

Case II : If $Mg < \frac{mv^2}{r}$; Hence friction force on car of mass m will be towards centre.

$$T + \mu mg = \frac{mv^2}{r_{min}}$$



$$Mg + \mu mg = \frac{m}{r_{min}} \quad \text{.... (2)}$$

From equations **(1)** and **(2)**

$$\Rightarrow \frac{r_{max}}{r_{min}} = \frac{M + \mu m}{M - \mu m}$$

46. By Newton's law at B

$$T - mg\cos\theta = \frac{mv^2}{\ell}$$

By energy conservation b/w A and B

$$mg\ell(1 - \cos\theta) + \frac{1}{2}mv^2 = \frac{1}{2}m(5\ell g)$$

$$mv^2 = m\,5\ell g - 2mg\ell(1 - \cos\theta)$$

Relative Motion
Practice Test

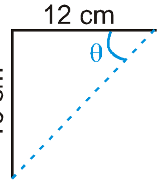
1. Relative to the person in the train, acceleration of the stone is 'g' downward, a (acceleration of train) backwards.

According to him $: x = \frac{1}{2}at^2$, $Y = \frac{1}{2}gt^2$

$\Rightarrow \frac{X}{Y} = \frac{a}{g} \Rightarrow Y = \frac{g}{a}x \Rightarrow$ straight line.

2. $V_{R/G(x)} = 0$, $V_{R/G(y)} = 10$ m/s

Let, velocity of man = v



$\tan\theta = \frac{16}{12} = \frac{4}{3}$

then, $v_{R/man} = v$ (opposite to man)

For the required condition :

$\tan\theta = \frac{V_{R/M(y)}}{V_{R/M(x)}} = \frac{10}{v} = \frac{4}{3}$

$\Rightarrow V = \frac{10 \times 3}{4} = 7.5$ **Ans.**

3. $v = at = 2t$

Velocity of car at t = 3 $v_1 = 6$ m/s

at t = 4 $v_2 = 8$ m/s

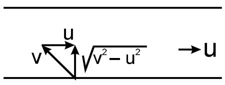
Coin 1 will fall with horizontal velocity 6 m/s & second coin will fall with horizontal velocity 8 m/s. Both will travel 6 m & 8 m horizontally before they fall from the point of release.

Car moves $\frac{(6+8)}{2} \times 1 = 7$ m. In fourth second, position of first coin $x_1 = 6$ $x_2 = 7 + 8 = 15$

$\Rightarrow x_2 - x_1 = 15 - 6 = 9$m

4. Let velocity of man in still water be v and that of water with respect to ground be u.

Velocity of man perpendicular to river flow with respect to ground = $\sqrt{v^2 - u^2}$



Velocity of man downstream = v + u

As given, $\sqrt{v^2 - u^2}\ t = (v + u)\ T$

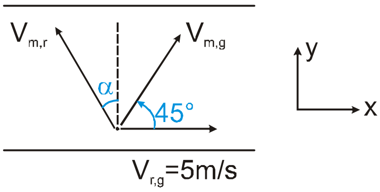
$\Rightarrow (v^2 - u^2)\ t^2 = (v + u)^2\ T^2$

$\Rightarrow (v - u)\ t^2 = (v + u)\ T^2$

$\therefore \frac{v}{u} = \frac{t^2 + T^2}{t^2 - T^2}$

5. $\vec{V}_{m,g} = \vec{V}_{m,r} + \vec{V}_{r,g}$

As resulting velocity $\vec{V}_{m,g}$ is at 45° with river flow



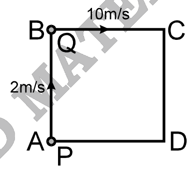
i.e. $V_{r,g} - V_{m,r}\sin\alpha = V_{m,r}\cos\alpha$ ....(1)

and $\frac{60m}{V_{mr}\cos\alpha} = 6$ sec. ....(2)

Solving (1) & (2)

$V_{m,r} = 5\sqrt{5}$ m/s

6.



a = 8 m

They meet when Q moves 8 × 3 m with respect to P

$\Rightarrow$ relative distance = relative speed × time.

$8 \times 3 = (10 - 2)\ t \Rightarrow t = 3$ sec **Ans. 3 sec**

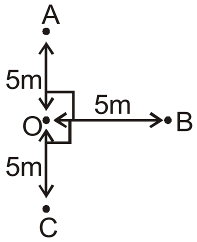
7. Relative velocity of stone = 5 m/s

relative acceleration of stone = 10 + 5 = 15 m/s²

$\therefore v = u + at = 5 + 15 \times 2 = 35$ m/s

$\therefore$ relative velocity after t = 2 second is 35 m/s

8. Let the stones be projected at t=0 sec with a speed u from point O. Then an observer ,at rest at t= 0 and having constant acceleration equal to acceleration due to gravity, shall observe the three stones move with constant velocity as shown.

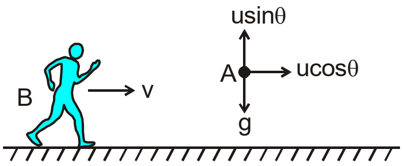


In the given time each ball shall travel a distance 5 metre as seen by this observer. Hence the required distance between A and B will be

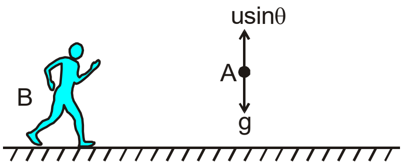
$= \sqrt{5^2 + 5^2} = 5\sqrt{2}$ metre

9. The horizontal and vertical components of initial velocity of projectile are as shown in figure. Since the observer moving with uniform velocity v sees the projectile moving in straight line

Hence $v = u\cos\theta$



velocity of A and B given
from frame of ground



velocity of A given
from frame of B

The time of flight as measured by observer B is T

Hence horizontal range of projectile on ground is

$R = (u\cos\theta)T = vT$

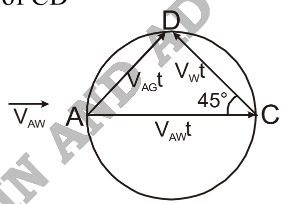
10. Without wind A reaches to C and with wind it reaches to D in same time so wind must deflect from C to D so wind blow in the direction of CD

$\vec{V}_{AG} = \vec{V}_{AW} + \vec{V}_{WG}$

$\Rightarrow \vec{V}_{AG}\,t = \vec{V}_{AW}\,t + \vec{V}_{WG}\,t$

$AC = \vec{V}_{AW}t$

$CD = \vec{V}_{WG}\,t$



11. With respect to lift initial speed = v_0

acc = – 2g

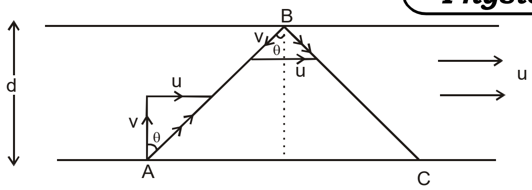
displacement = 0

$\therefore\ S = ut + \frac{1}{2}at^2$

$0 = v_0T' - \frac{1}{2} \times 2g \times T'^2$

$\therefore\ T' = \frac{v_0}{g} = \frac{1}{2} \times \frac{2v_0}{g} = \frac{1}{2}\,T$

12.



V = velocity of man w.r.t. river

u = velocity of river

$A \xrightarrow{t} B = \frac{d}{v} \Rightarrow 10 = \frac{d}{v} \Rightarrow d = 10\,V$ .... **(1)**

$B \xrightarrow{t} C = \frac{d}{v\cos\theta} \Rightarrow 15 = \frac{d}{v\cos\theta}$

$\Rightarrow d = 15\,v\cos\theta$ .... **(2)**

(1) & **(2)** $\Rightarrow \cos\theta = 2/3 \Rightarrow \sec\theta = 3/2$

$\because\ \tan\theta = \frac{u}{v}\ \therefore\ \sqrt{\sec^2\theta - 1} = \frac{u}{v}$

$\Rightarrow \frac{u}{v} = \sqrt{9/4 - 1} = \frac{\sqrt{5}}{2} \Rightarrow \frac{v}{u} = \frac{2}{\sqrt{5}}$

13. No. of taxi = $\frac{240}{10}$ = 24 but when 24th start motion it reach the destination so it will meet 23 only.

14. $v_{rel} = 2v\sin\frac{\theta}{2}$; $\langle v \rangle = \frac{\int_0^{2\pi} 2v\sin\frac{\theta}{2}d\theta}{\int_0^{2\pi} d\theta} = \frac{4v}{\pi}$

15. Let velocity of the aeroplane be $\vec{v}_P = u\cos30^0\hat{i} + u\sin30^0\hat{j}$ and velocity of the wind be v, then

$u\frac{\sqrt{3}}{2}t\hat{i} + \left(\frac{u}{2}t - 5t^2\right)\hat{j} + vt\hat{k} = 400\sqrt{3}\hat{i} + 80\hat{j} + 200\hat{k}$

$\Rightarrow u\frac{\sqrt{3}}{2}t = 400\sqrt{3}$, $\frac{u}{2}t - 5t^2 = 80$, $vt = 200$

$\Rightarrow$ $ut = 800$ and $\frac{u}{2}t - 5t^2 = 80$

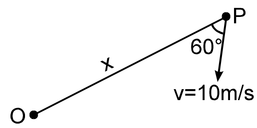
$\Rightarrow$ $400 - 5t^2 = 80$

$\Rightarrow$ $t^2 = 64$

$\Rightarrow$ $t = 8$ sec.

16. Velocity of approach of P and O is

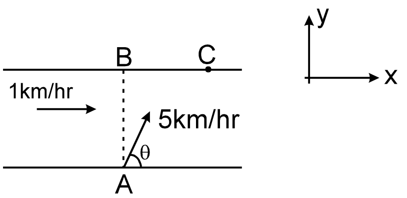
$-\frac{dx}{dt} = v\cos60° = 5$ m/s



It can be seen that velocity of approach is always constant.

$\therefore$ P reaches O after $= \dfrac{100}{5} = 20$ sec.

17.



AB = BC = 400 m = 0.4 km

$v_x = 5\cos\theta + 1$

$v_y = 5\sin\theta$

time taken $(t) = \dfrac{AB}{v_y} = \dfrac{BC}{v_x}$

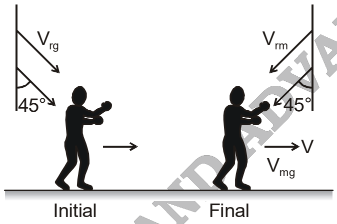
$\Rightarrow v_y = v_x \quad \Rightarrow 5\sin\theta = 5\cos\theta + 1$

$\Rightarrow \theta = 53°$

and $t = \dfrac{0.4}{5\sin(53°)} = 0.1$ hr = 6 min.

18. He can only reach the opposite point if he can cancel up the velocity of river by his component of velocity.

19.



$\vec{V}_{rg} = \vec{V}_{rm} + \vec{V}_{mg}$

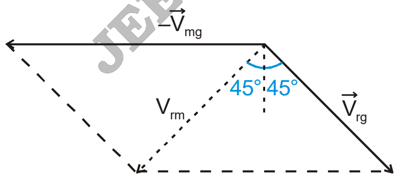
$\vec{V}_{rm} = \vec{V}_{rg} - \vec{V}_{mg}$

$V_{rm}\cos 45° = V_{rg}\cos 45°$

$V_{rm} = 2\sqrt{2}$ m/s $= V_{rg}$

$V_{rm}\cos 45° = V_{mg} - V_{rg}\cos 45°$

$$V_{mg} = 2\sqrt{2}\frac{1}{\sqrt{2}} + 2\sqrt{2}\frac{1}{\sqrt{2}} = 4 \text{ m/s}$$



using $v^2 = u^2 + 2as$ for the motion of man, s = 16 m.

20. While both the stones are in flight, $a_1 = g$ and $a_2 = g$

So $a_{rel} = 0 \Rightarrow V_{rel}$ = constant

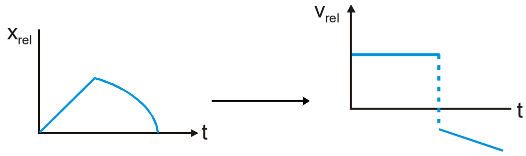
$\Rightarrow x_{rel}$ = (const) t

$\Rightarrow$ Curve of x_{rel} v/s t will be straight line.

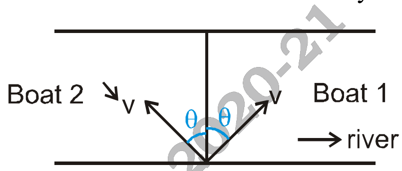
After the first particle drops on ground, the seperation (x_{rel}) will decrease parabolically (due to gravitational acceleration), and finally becomes zero.

and V_{rel} = slope of x_{rel} v/s t

So



21. If component of velocities of boat relative to river is same normal to river flow (as shown in figure) both boats reach other bank simultaneously.



22. Acceleration of each of the projectile $= \vec{g}$. Relative acceleration $\vec{a}_r = \vec{g} - \vec{g} = 0$.

23. Statement-1 is True, Statement-2 is True;

Statement-2 is a correct explanation for Statement-1

In air their relative acceleration is zero. **Hence** they can,t approch the vertical distance between.

24. $t_1 = \dfrac{d}{v_{sw}} = \dfrac{d}{v}$; $t_2 = \dfrac{d}{\sqrt{v^2 - u^2}}$

$$\therefore \frac{t_1}{t_2} = \frac{d/v}{d/\sqrt{v^2-u^2}} = \left(\frac{\sqrt{v^2-u^2}}{v}\right) = \sqrt{1 - \frac{u^2}{v^2}}.$$

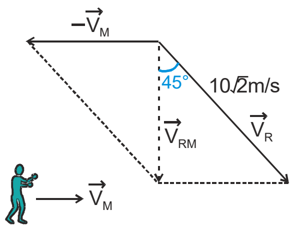
25. $t_1' = \dfrac{d}{v}$; $t_2' = \dfrac{d}{v}$ $\quad \therefore \quad \dfrac{t_1'}{t_2'} = 1.$

26. $T_1 = \dfrac{d}{\sqrt{v^2 - u^2}}$ and $T_2 = \dfrac{d}{(v-u)}$

$$\text{so, } \frac{T_2}{T_1} = \frac{\sqrt{v^2-u^2}}{v-u} = \sqrt{\frac{v+u}{v-u}} = \sqrt{\frac{1+u/v}{1-u/v}}$$

27 to 29

In the first case :



From the figure it is clear that

$\vec{V}_{RM}$ is 10 m/s downwards and

$\vec{V}_M$ is 10 m/s towards right.

In the second case :

Velocity of rain as observed by man becomes $\sqrt{3}$ times in magnitude.

$\therefore$ New velocity of rain

$\vec{V}_{R'} = \vec{V}_{R'M} + \vec{V}_M$

$\therefore$ The angle rain makes with vertical is

$\tan\theta = \dfrac{10}{10\sqrt{3}}$ **or** $\theta = 30°$

$\therefore$ Change in angle of rain = 45 – 30 = 15°.

30. The initial velocity of A relative to B is

$\vec{u}_{AB} = \vec{u}_A - \vec{u}_B = (8\hat{i} - 8\hat{j})$ m/s $\therefore u_{AB} = 8\sqrt{2}$ m/s

Acceleration of A relative to B is -

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = (-2\hat{i} + 2$) m/s^2 $\therefore a_{AB} =$ m/s^2

since B observes initial velocity and constant acceleration of A in opposite directions, Hence B observes A moving along a straight line.

From frame of B

Hence time when $v_{AB} = 0$ is t = = 4 sec.

The distance between A & B when $v_{AB} = 0$ is

S = = m

The time when both are at same position is -

T = = 8 sec.

Magnitude of relative velocity when they are at same position is u_{AB} = m/s.

31.

tan ϕ = = tan ϕ = =

d_{min} =

=

= =

t = =

t =

,

,

= –96 + 96 – 66 t – 176 t + 121 t²

0 = –242 t + 121 t² = 0, t = 0 and t =

32. Let velocity of bodies be v_1 and v_2.

in first case

$u_1 = v_1 + v_2$ .... **(i)**

in second case

$u_2 = v_1 - v_2$ .... **(i)**

$\therefore v_1 =$ and $v_2 =$

Here $u_1 =$ m/s and $u_2 =$

After solving we have

$v_1 =$ m/s and $v_2 =$ m/s.

33. Let $V_w = u$ & $v_{sw} = v$

Time taken by swimmer to go from M to O and O to B = time taken by float to reach B from M

⇒ + = ⇒ + =

⇒ = ⇒ (2v + 2) u = 2 (v + u)

⇒ 2vu + 2u = 2v + 2u ⇒ u = 1 km/h Ans.