VARIATIONS OF GRAPHS

Class: IX, Mathematics

IIT ADVANCED

SOLUTIONS

TEACHING TASK

01. $\sin x \cdot \cos\left(\frac{\pi}{4} - x\right)$

$= \sin x\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right)$

$= \frac{1}{\sqrt{2}}\left(\sin x \cos x + \sin^2 x\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{\sin 2x}{2} + \frac{1 - \cos 2x}{2}\right)$

$= \frac{1}{2\sqrt{2}}\left(\sin 2x - \cos 2x + 1\right)$

period of $\sin 2x = \frac{2\pi}{2} = \pi$

period of $\cos 2x = \frac{2\pi}{2} = \pi$

hence period of given function $= \pi$

Ans. A

02. $y = \sin^2\theta + \cos^4\theta$

$y = \sin^2\theta + (1 - \sin^2\theta)^2$

$y = \sin^4 - \sin^2\theta + 1$

$y = \sin^2\theta(\sin^2\theta - 1) + 1$

$y = -\sin^2\theta\cos^2\theta + 1$

$y = -\frac{1}{4}(2\sin\theta\cos\theta)^2 + 1$

$y = -\frac{1}{4}\sin^2 2\theta + 1$

$0 \le \sin^2 2\theta \le 1$

$0 \ge -\frac{1}{4}\sin^2\theta \ge -\frac{1}{4}$

$1 \ge y \ge -\frac{1}{4} + 1$

$\frac{3}{4} \le y \le 1$

Ans. D.

03 $f(\theta) = \dfrac{\tan\theta - \frac{1}{3}\tan^3\theta}{\frac{1}{3} - \tan^2\theta}$

$= \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

$= \tan 3\theta$

$\therefore$ period $= \dfrac{\pi}{3}$.

Ans: A

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04 period of $\cos nx \Rightarrow \dfrac{2\pi}{n}$

period of $\sin\left(\frac{x}{n}\right) \Rightarrow \dfrac{2\pi}{\left(\frac{1}{n}\right)} = 2\pi n$.

L.c.m $\left\{ \dfrac{2\pi}{n}, 2\pi n \right\} = \dfrac{2n\pi}{1} = 2n\pi$

~~Ans~~:

Given $2n\pi = 4\pi$

$\Rightarrow n = 2$

Ans: C

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05 The least value of $\dfrac{1}{5\cos x + 12\sin x + 15}$

max. value of $5\cos x + 12\sin x + 15$

$= 15 + \sqrt{25 + 144}$

$= 15 + 13$

$= 28$.

Ans: B

$\therefore$ Ans: $\dfrac{1}{28}$

06 $\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2$

$= (\cos^2\theta + \sin^2\theta) - 3\cdot(2\sin\theta\cos\theta) + 2\sin^2\theta + 2$

$= 3 - 3.\sin 2\theta + 2\sin^2\theta$

$= 4 - 3\sin 2\theta + 2\sin^2\theta - 1$

$= 4 - 3\sin 2\theta + \cos 2\theta$

minimum value $= c - \sqrt{a^2+b^2}$

$= 4 - \sqrt{9+1}$

$= 4 - \sqrt{10}$

Ans: B

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07. $f(x) = \cos^2 x + \sec^2 x$

$= (\cos x - \sec x)^2 + 2.\cos x.\sec x$

$= (\cos x - \sec x)^2 + 2.$

We know $(\cos x - \sec x)^2 \geq 0.$

$f(x) = 0+2 = 2 \rightarrow$ minimum value

hence Range $= [2, \infty)$

Ans: C

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08 opt: A $f(x) = \sin 3\pi x$

period $= \frac{2\pi}{3\pi} = \frac{2}{3}.$

Ans. A

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09. retn $\sin^2 x + \cos^2 x = 1$ $\forall x \in R$

$\sin^2\frac{3x}{2} + \cos^2\frac{3x}{2} = 1$ $\forall x \in R$

hence. $\frac{\text{minimum value is 1}}{\text{maximum value is 1}} = 1$

Ans: A

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10. A) $f(x) = \sin x + |\sin x|$

$f(x) = \sin(2\pi + x) + |\sin(2\pi + x)|$

$f(x) = \sin x + |\sin x|$

hence $f(x)$ is periodic period with period 2π

b) $g(x) = \sin x + \cos x$

$g(\cancel{2\pi + x})\ \sin(2\pi + x) + \cos(2\pi + x)$

$= \sin x + \cos x$

hence $g(x)$ is periodic with period 2π

c) $h(x) = \tan x$ is periodic

d) $p(x) = \sin^4 x + \cos^4 x$

$p(\pi + x) = \sin^4(\pi + x) + \cos^4(\pi + x)$

$= \sin^4 x + \cos^4 x$

hence, it is periodic with period π

(04)

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11 $u^2 = a^2 + b^2 + 2\sqrt{(a^2\cos^2\theta + b^2\sin^2\theta)(a^2\sin^2\theta + b^2\cos^2\theta)}$

$= a^2 + b^2 + 2\sqrt{a^4\cos^2\theta\sin^2\theta + b^4\sin^2\theta\cos^2\theta + a^2b^2(\sin^4\theta + \cos^4\theta)}$

$= a^2 + b^2 + 2\sqrt{(a^4 + b^4)\sin^2\theta\cos^2\theta + a^2b^2(1 - 2\sin^2\theta + \cos^2\theta)}$

$= a^2 + b^2 + 2\sqrt{\sin^2\theta\cos^2\theta(a^2 - b^2) + a^2b^2}$

$= a^2 + b^2 + 2\sqrt{a^2b^2 + (a^2 - b^2)\cdot\sin^2 2\theta/4}$

$u^2_{max} = a^2 + b^2 + 2\sqrt{a^2b^2(a^2 - b^2)/4} = 2(a^2 + b^2)$

$u^2_{min} = a^2 + b^2 + 2\sqrt{a^2b^2} = (a+b)^2$

Difference $= 2(a^2 + b^2) - (a+b)^2 = (a-b)^2$

Ans. B

13. **Statement I:**

period of $\sin 8x = \frac{2\pi}{8} = \frac{\pi}{4}$

period of $\cos 2x = \frac{2\pi}{2} = \pi$

period = L.C.M. $\left\{ \frac{\pi}{4}, \pi \right\} = \pi$. (True)

Ans: A

Statement II: Conceptual (True)

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14 $3\cos a + 4\sin a + 5$

Max. value $= 5 + \sqrt{3^2+4^2} = 5 + 5 = 10$

Ans: C

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15 $5\sin^2\frac{\theta}{2} + 4\cos^2\frac{\theta}{2}$

$= 5\sin^2\frac{\theta}{2} + 4\left(1 - \sin^2\frac{\theta}{2}\right)$

$= 5\sin^2\frac{\theta}{2} + 4 - 4\sin^2\frac{\theta}{2}$

$= \sin^2\frac{\theta}{2} + 4.$

$0 \le \sin^2\frac{\theta}{2} \le 1$

$4 \le 4 + \sin^2\frac{\theta}{2} \le 5$

minimum value = 4.

Ans: D

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16 $5|\sec x| - 7|\csc x|$

$a \neq b \Rightarrow$ hence period $= \pi$

Ans: A

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17 $29|\tan x| + \sqrt{841}\,|\cot x|$

$29 \neq \sqrt{841}$

hence period $\rightarrow \pi$

Ans: A

06

18 $\sqrt{13}\left(|\sin x|+|\cos x|\right)$

here $\sqrt{13}=\sqrt{13}$ i.e. $a=b$

Ans: D

period $=\frac{\pi}{2}$

19 $\sin^2\theta+5\sin\theta\cos\theta+5\cos^2\theta$

$=(\sin^2\theta+\cos^2\theta)+\frac{5}{2}(2\sin\theta\cos\theta)+4\cos^2\theta$

$=1+\frac{5}{2}(\sin 2\theta)+2(2\cos^2\theta-1)+2$

$=3+\frac{5}{2}(\sin 2\theta)+2\cos 2\theta$

min. value $=3-\sqrt{\left(\frac{5}{2}\right)^2+2^2}$

$=3-\sqrt{\frac{41}{4}}=3-\frac{\sqrt{41}}{2}=\frac{6-\sqrt{41}}{2}$

$\therefore$ max value of $f(x)=\frac{1}{\left(\frac{6-\sqrt{41}}{2}\right)}=\frac{2}{6-\sqrt{41}}$.

20

20 a) $\dfrac{7+6\tan\theta-\tan^2\theta}{1+\tan^2\theta}$

$= \dfrac{7+6\tan\theta-\tan^2\theta}{\sec^2\theta}$

$= 7\cos^2\theta + 6.\sin\theta\cos\theta - \sin^2\theta$

$= 6\cos^2\theta + \cos 2\theta + 3.\sin 2\theta$

$= 3(2\cos^2\theta - 1) + \cos 2\theta + 3\sin 2\theta + 3$

$= 3.\cos 2\theta + \cos 2\theta + 3\sin 2\theta + 3$

$= 3 + 3\sin 2\theta + 4\cos\theta$

max $= 3 + \sqrt{3^2+4^2} = 3+5 = 8 \to m$

min $= 3 - \sqrt{3^2+4^2} = 3-5 = -2 \to M$

$\therefore m + M = 8 - 2 = 6$

b) $5\cos\theta + 3\cos\left(\theta+\frac{\pi}{3}\right) + 3$

$= 5\cos\theta + 3\left(\cos\theta\cdot\frac{1}{2} - \sin\theta\cdot\frac{\sqrt{3}}{2}\right) + 3$

$= 5\cos\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$= 3 - \frac{3\sqrt{3}}{2}\sin\theta + \frac{13}{2}\cos\theta$

max $= 3 + \sqrt{\left(\frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{13}{2}\right)^2}$

$= 3 + 7 = 10 \to m$

min $= 3 - 7 = -4 \to M$

$m + M = 10 - 4 = 6$

c)

(08)

c) $1+\sin\left(\frac{\pi}{4}+\theta\right)+2\cos\left(\frac{\pi}{4}-\theta\right)$

$=1+\left(\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right)+2\left(\frac{1}{\sqrt{2}}\cos\theta-\frac{1}{\sqrt{2}}\sin\theta\right)$

$=1+\left(\frac{1}{\sqrt{2}}+\sqrt{2}\right)\cos\theta+\left(\frac{1}{\sqrt{2}}-\sqrt{2}\right)\sin\theta$

max. value $=1+\sqrt{\left(\frac{1}{\sqrt{2}}+\sqrt{2}\right)^2+\left(\frac{1}{\sqrt{2}}-\sqrt{2}\right)^2}$

$=1+\sqrt{2.\left[\left(\frac{1}{\sqrt{2}}\right)^2+(\sqrt{2})^2\right]}$

$=1+\sqrt{5}\rightarrow \lambda$ $\quad\therefore \lambda=$

min v.alue $=1-\sqrt{5}\rightarrow M$

$\lambda+M=2.$

d) $\sin^2(120^\circ+\theta)+\sin^2(120^\circ-\theta)$

$=\left[\sin(120^\circ+\theta)+\sin(120^\circ-\theta)\right]^2-2\sin(120^\circ+\theta).\sin(120^\circ-\theta)$

$=\left[2\sin120^\circ.\cos\theta\right]^2-2\left[\sin^2 120^\circ-\sin^2\theta\right]$

$=\left[2.\frac{\sqrt{3}}{2}.\cos\theta\right]^2-2\left[\frac{3}{4}-\sin^2\theta\right]$

$=3\cos^2\theta-\frac{3}{2}+2\sin^2\theta$

$=2+\cos^2\theta-\frac{3}{2}$

$=\frac{1}{2}+\cos^2\theta$

$\therefore\ 0\leq\cos^2\theta\leq 1$

$\frac{1}{2}\leq\frac{1}{2}+\cos^2\theta\leq 1+\frac{1}{2}$

$\frac{1}{2}\leq\frac{1}{2}+\cos^2\theta\leq\frac{3}{2}$

max $\rightarrow\frac{3}{2}\rightarrow\lambda$

min $\rightarrow\frac{1}{2}\rightarrow M$

$\lambda+M=\frac{3}{2}+\frac{1}{2}=2$

Ans: γ,γ,P,P

21) a) $2\sin^2 x + \cos^2 x$

$= \sin^2 x + 1$

$0 \le \sin^2 x \le 1$

$1 \le 1 + \sin^2 x \le 2$

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21) a) $2\sin^2 x + \cos^2 x$

$f(\pi + x) = 2\sin^2(\pi + x) + \cos^2(\pi + x)$

$= 2\sin^2 x + \cos^2 x \rightarrow$ period π

b) $f(\theta) = \sin(\pi \sin\theta)$

$f(2\pi + \theta) = \sin(\pi \sin(2\pi + \theta))$

$= \sin(\pi \sin\theta) \longrightarrow$ period 2π

c) $\sin x . \sin(120^\circ + x) . \sin(120^\circ - x)$

$= \sin x . (\sin^2 120^\circ - \sin^2 x)$

$= \sin x \left(\frac{3}{4} - \sin^2 x\right)$

$= \frac{3\sin x - 4\sin^3 x}{4} = \frac{1}{4}\sin 3x$

period $= \frac{2\pi}{3}$

d) $f(x) = |\sin x| . |\cos x|$

$f(\frac{\pi}{2} + x) = \left|\sin\left(\frac{\pi}{2} + x\right)\right| . \left|\cos\left(\frac{\pi}{2} + x\right)\right|$

$= |\cos x| |\sin x| = f(x)$

period $= \frac{\pi}{2}$

Ans : s, p, q, r

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LEARNERS TASK

CUQ's

01. period of $\sin(ax+b) = \frac{2\pi}{|a|}$ Ans: C

02 $\sin 2x \rightarrow$ period $= \frac{2\pi}{2} = \pi$ Ans: C

03 $\tan kx$, $\cot kx \rightarrow$ period $= \frac{\pi}{|k|}$ Ans: C

04 $-\cot x \rightarrow$ period $= \pi$ Ans: D

05 Amplitude $\rightarrow \frac{M-m}{2}$ Ans: B

07. minimum value $= n - \sqrt{l^2+m^2}$ Ans: B

08 $(a\sin x)^2 + (b\csc x)^2$

$= (a\sin x - b\csc x)^2 + 2(a\sin x)(b\csc x)$

$= (a\sin x - b\csc x)^2 + 2ab. \geq 2ab$

minimum value $= 2ab$ Ans: A

09. period of $k \cdot f(x) \rightarrow$ period of $f(x)$ Ans: A

10 $\tan 3x + \tan x$ $\rightarrow$ period $\tan 3x \rightarrow \frac{\pi}{3}$

$= \frac{\cancel{3\tan x - 4\tan^3 x}}{}$ period $\tan x \rightarrow \pi$

period L.C.M $\{\frac{\pi}{3}, \pi\} = \frac{\pi}{1} = \pi$.

Ans: D.

JEE MAINS LEVEL

01. $\sin\frac{\pi}{2}x + \cos\frac{\pi}{3}x$

period $\sin\frac{\pi}{2}x \rightarrow \frac{2\pi}{\left(\frac{\pi}{2}\right)} = 4.$

period $\cos\frac{\pi}{3}x \rightarrow \frac{2\pi}{\left(\frac{\pi}{3}\right)} = 6$

period L.C.M $\{4, 6\} = 12$

Ans: C

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02. $\frac{1}{2}(2\sin x\cos x) = \frac{1}{2}\sin 2x.$

max. of $\sin 2x \rightarrow 1$

max. of $\frac{1}{2}\sin 2x \rightarrow \frac{1}{2}$

Ans: C

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03 $\cos\theta + \cos 2\theta$

$= \cos\theta + 2\cos^2\theta - 1$

$= 2\cos^2\theta + \cos\theta - 1$

$= 2\left[\cos^2\theta + \frac{1}{2}\cos\theta - \frac{1}{2}\right) - 1$

$= 2\left(\cos^2\theta + 2\cdot\cos\theta\cdot\frac{1}{4} + \frac{1}{16} - \frac{1}{16} - \frac{1}{2}\right) - 1$

$= 2\left(\left(\cos\theta + \frac{1}{4}\right)^2 - \frac{9}{8}\right.$

minimum value $= -\frac{9}{8}$

Ans: A

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04 period of $\sin 2x = \frac{2\pi}{2} \rightarrow \pi$

period of $\cot 4x = \frac{\pi}{4}$

period L.C.M. of $\left(\pi, \frac{\pi}{4}\right) = \frac{\pi}{1} = \pi$

Ans. B

5. $\tan(x+2x+3x+\dots+nx)$

$= \tan(1+2+3+\dots+n)x$

$= \tan\left[\frac{n(n+2)}{2}\right]x$

period $\rightarrow \frac{\pi}{\left(\frac{n(n+1)}{2}\right)} = \frac{2\pi}{n(n+1)}$ Ans: B

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06 $f(x) = \frac{1}{5\sin x - 6}$

$-1 \le \sin x \le 1$

$-5 \le 5\sin x \le 5$

$-5-6 \le 5\sin x - 6 \le 5-6$

$-11 \le 5\sin x - 6 \le -1$

$-\frac{1}{11} \ge \frac{1}{5\sin x - 6} \ge -1$

$\therefore -1 \le \frac{1}{5\sin x - 6} \le -\frac{1}{11}$

Hence Range $= \left[-1, -\frac{1}{11}\right]$ Ans: B

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07 $\tan(x+2x+3x+\dots+nx)$

$= \tan(1+2+3+\dots+n)x$

$= \tan\frac{n(n+1)}{2}x.$

period $= \frac{\pi}{\left|\frac{n(n+1)}{2}\right|} = \frac{2\pi}{n(n+1)}$

$= \frac{\pi}{\Sigma n}$ Ans: B, C

08 **Statement I:**

$2\cos^2\theta + \sqrt{5}\sin\theta\cos\theta + 4\sin^2\theta$

$a\cos^2\theta + b\sin\theta\cos\theta + c\sin^2\theta$

$\therefore$ maximum value $= \frac{1}{2}(a+c) + \frac{1}{2}\sqrt{(a-c)^2+b^2}$

$= \frac{1}{2}(2+4) + \frac{1}{2}\sqrt{(2-4)^2+\cancel{5}}$

$= 3 + \frac{9}{2}$

$= \frac{9}{2}$ (True)

Statement II: The maximum value of

$a\cos^2\theta + b\sin\theta.\cos\theta + c\sin^2\theta$ is

$\frac{1}{2}(a+c) + \frac{1}{2}\sqrt{(a-c)^2+b^2}$ (False)

c

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09. $p\cos\frac{x}{4} + q\sin\frac{x}{4}$

maximum value $= c + \sqrt{a^2+b^2}$

$= 0 + \sqrt{p^2+q^2}$

$= (p^2+q^2)^{1/2}$

Ans: D

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10 $f(x) = 3\sin^2x + 5\sin x\cos x + 4\cos^2x$

$f(x) = a\sin^2x + b\sin x\cos x + c\cos^2x$

max. value $= \frac{1}{2}(a+c) + \frac{1}{2}\sqrt{(a-c)^2+b^2}$

$= \frac{1}{2}(3+4) + \frac{1}{2}\sqrt{(3-4)^2+5^2}$

$= \frac{7}{2} + \frac{\sqrt{26}}{2}$

$= \frac{7+\sqrt{26}}{2}$

Ans: D

11. $f = 7 + 10 \cdot \cancel{\sin 2x + 8\sin^2 x}$

$f = \cancel{7 + 20\sin x \cos x} + 8\sin^2 x$

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11. $f = 7 + 10\sin 2x + 8\sin^2 x$ (14)

$f = 7 + 10\sin 2x - 4(1 - 2\sin^2 x) + 4$

$f = 11 + 10\sin 2x - 4\cos 2x$

minimum value $= 11 + \sqrt{100+16}$

$= 11 + \sqrt{116}$

$= 11 + 2\sqrt{29}$

Ans: B

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12 $f(x) = 11 + 5\sqrt{1 - \cos^2 2x} + 10\cos^2 x$

$f(x) = 11 + 5 \cdot \sqrt{\sin^2 2x} + 10\cos^2 x$

$f(x) = 11 + 5|\sin 2x| + 10\cos^2 x$

$= 11 + 5|\sin 2x| + 5(2\cos^2 x - 1) + 5$

$= 16 + 5|\sin 2x| + 5\cos 2x$

minimum value $= 16 - \sqrt{5^2 + 5^2}$

$16 - \sqrt{50}$

$= 16 - 5\sqrt{2}$

Ans: A

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13 $f(x) = 1 - 8\sin^2 x \cdot \cos^2 x$

$\cancel{= 1 - 2(2\sin x \cos x)^2}$

$= 1 - 2.$

$$f(x) = 1 - 8\sin^2 x \cdot \cos^2 x$$
$$= 1 - 2(2\sin x \cos x)^2$$
$$= 1 - 2\sin^2 2x$$

$$0 \le \sin^2 2x \le 1$$
$$\ge -2\sin^2 2x \ge -2$$
$$1 \ge 1 - 2\sin^2 2x \ge 1 - 2$$
$$-1 \le 1 - 2\sin^2 2x \le 0$$

maxi value $= 0$.

Ans : 0

7. $f(x) = \sin^2 x$

We know $0 \le \sin^2 x \le 1$

minimum value $= 0$

Ans: 0

5 a) $2\sin^2 x + \cos^2 x$

$\Rightarrow 2\sin^2 x + 1 - \sin^2 x$

15 a) $2\sin^2 x + \cos^2 x$

$= 2\sin^2 x + 1 - \sin^2 x$

$= \sin^2 x + 1$

$0 \le \sin^2 x \le 1$

$1 \le 1 + \sin^2 x \le 2 \rightarrow$ Range $[1,2]$

b) $1 + 8\sin^2 x \cdot \cos^2 x$

$= 1 + 2(2\sin x \cos x)^2$

$= 1 + 2 \cdot \sin^2 2x$

$0 \le \sin^2 2x \le 1$

$0 \le 2\sin^2 2x \le 2$

$1 \le 1 + 2\sin^2 2x \le 3 \rightarrow$ Range $[1,3]$

c) $\cos^2 x + \sin^4 x$

$= 1 - \sin^2 x + \sin^4 x$

$= 1 - \sin^2 x(1 - \sin^2 x)$

$= 1 - \sin^2 x \cdot \cos^2 x$

$= 1 - \frac{1}{4}(2\sin x \cos x)^2$

$= 1 - \frac{1}{4}\sin^2 2x$

$0 \le \sin^2 2x \le 1$

$0 \ge -\frac{1}{4}\sin^2 2x \ge -\frac{1}{4}$

$1 \ge 1 - \frac{1}{4}\sin^2 2x \ge 1 - \frac{1}{4}$

$\therefore \frac{3}{4} \le 1 - \frac{1}{4}\sin^2 x \le 1$

Range $= [\frac{3}{4}, 1]$

d) $3\sin^2 x + 4\cos^2 x$

$= 3\sin^2 x + 4 - 4\sin^2 x$

$= -\sin^2 x + 4$

$0 \le \sin^2 x \le 1$

$0 \ge -\sin^2 x \ge -1$

$4 \ge 4 - \sin^2 x \ge 3$

Range $= [3,4]$

THE END