Real Numbers

Exercise 1.1

Question 1: Use Euclid's division algorithm to find the HCF of:

(i)135 and 225 (ii) 196 and 38220 (iii) 867 and 255

Answer 1:

(i) 135 and 225

Since 225 > 135, we apply the division lemma **'a=bq+r'** to a= 225 and b = 135 to obtain

$225 = 135 \times 1 + 90$

Since remainder $90 \neq 0$,

Now we apply the division lemma to divisor of previous step, that is 135 and remainder of previous step that is, 90 to obtain

$135 = 90 \times 1 + 45$

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain

$90 = 2 \times 45 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45, Therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

Since 38220 > 196,

we apply the division lemma to 38220 and 196 to obtain $38220 = 196 \times 195 + 0$.

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196, Therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

Since 867 > 255,

we apply the division lemma to 867 and 255 to obtain $867 = 255 \times 3 + 102$

Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain

$255 = 102 \times 2 + 51$, Here also remainder is not 0.

Now We consider the new divisor 102 and new remainder 51,

and apply the division lemma on 102 and 51 to obtain $102 = 51 \times 2 + 0$.

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51, Therefore, HCF of 867 and 255 is 51.

Question 2: Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.

Answer 2: Let a be any positive integer and b = 6.

Then, by Euclid's division lemma, a = 6q + r for some integer q ≥ 0, and r = 0, 1, 2, 3, 4, 5 because 0 ≤ r < 6.

Therefore, possible values of a = 6q or 6q + 1 or 6q + 2 or 6q + 3 or 6q + 4 or 6q + 5.

Also 6q+1 can be written as, 6q + 1 = 2 × 3q + 1 = 2k1 + 1, where k1 is a positive integer.

Similarly,

6q + 3 = (6q + 2) + 1 = 2 (3q + 1) + 1 = 2k2 + 1, where k2 is an integer.

6q + 5 = (6q + 4) + 1 = 2 (3q + 2) + 1 = 2k3 + 1, where k3 is an integer.

Clearly, if you look at the previous steps, 6q + 1, 6q + 3, 6q + 5 are of the form 2k + 1, where k is an integer.

Therefore, 6q + 1, 6q + 3, 6q + 5 are not exactly divisible by 2, because they are leaving a remainder behind.

Hence, these expressions of numbers, 6q+1, 6q+3, 6q+5 are odd numbers.

And therefore, any odd integer can be expressed in the form 6q + 1, or 6q + 3, or 6q + 5.

Question 3: An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer 3:

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

Apply Euclid's algorithm on 616 and 32.

616 = 32 × 19 + 8 …. Here, remainder is not zero, So apply Euclid's algorithm on 32 and 8 now.

32 = 8 × 4 + 0. Here, remainder is zero.

The HCF (616, 32) is divisor at this stage, which is nothing but 8.

Therefore, they can march in 8 columns each.

Question 4: Use Euclid's division lemma to show that the square of any positive integer is either of form 3m or 3m + 1 for some integer m. [Hint: Let x be any positive integer then it is of the form 3q, 3q + 1 or 3q + 2. Now square each of these and show that they can be rewritten in the form 3m or 3m + 1.]

Answer 4: Let a be any positive integer and b = 3. Then a = 3q + r for some integer q ≥ 0 And r = 0, 1, 2 because 0 ≤ r < 3 .

Therefore, a = 3q or 3q + 1 or 3q + 2. Then,

$a^2 = (3q)^2$ or $(3q + 1)^2$ or $(3q + 2)^2$

So, a = $\mathbf{9q^2}$ or $\mathbf{9q^2 + 6q + 1}$ or $\mathbf{9q^2 + 12q + 4}$

If a = $9q^2$ = 3 × $3q^2$. This can be written as **a = 3k1. Where k1 = $3q^2$**

If a = $9q^2$ + 6q +1 = 3 × ($3q^2$ + 2q) + 1. This can be written as a = 3k2 + 1. Where k2 = $3q^2$ + 2q.

a = $9q^2$ +12q +4 = 3 × ($3q^2$ + 4q + 1) + 1. This can be written as a = 3k3 + 1. Where k3 = $3q^2$ + 4q + 1.

Where k1, k2, and k3 are some positive integers.

Hence, it can be said that the square of any positive integer is either of the form 3m or 3m + 1.

Where m is some positive integer.

Question 5: Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Answer 5:

Let a be any positive integer and b = 3.

So, a = 3q + r, where q ≥ 0 and 0 ≤ r < 3. r can be 0, 1, 2.

So, a = 3q or 3q + 1 or 3q + 2.

Therefore, every number can be represented as these three forms. There are three cases.

Case 1: When a = 3q, $a^3 = (3q)^3 = 27q^3 = 9(3q^3)$= 9m. Where m is an integer such that m = $3q^3$.

Case 2: When a = 3q + 1, $a^3 = (3q +1)^3$ => $a^3 = 27q^3 + 27q^2 + 9q + 1$.

a^3 = 9(3q 3 + 3q 2 + q) + 1 => a^3 = 9m + 1 Where m is an integer such that m = $(3q^3 + 3q^2 + q)$

Case 3: When a = 3q + 2, $a^3 = (3q +2)^3$ => $a^3 = 27q^3 + 54q^2 + 36q + 8$ => a^3 = 9(3q 3 + 6q 2 + 4q) + 8

⇨ a^3 = 9m + 8 Where m is an integer such that m = (3q 3 + 6q 2 + 4q).

Therefore, the cube of any positive integer is of the form 9m, 9m + 1, or 9m + 8.