6th CLASS

MATHEMATICS

IIT FOUNDATION

STUDY MATERIAL

1. MULTIPLES AND FACTORS

TEACHING TASK:

JEE MAIN LEVEL QUESTIONS

(1) The first 5 multiples of 8 are

8, 16, 24, 32, 40

Sum: 8+16+24+32+40 = 120

(2) The greatest factor of 13 is 13 itself.

→ Since 13 is a prime number, its only factor

1 and 13. Ans: A

(3) 7 is a prime number.

It has factors are 1 and 7 only

Ans: C

(4) Even number divided by even number only.

So 5 is not a factor of 16.

Ans: A

(5) Prime factorization of $16 = 2^4$

Number of factors $4+1 = 5$

16 number has exactly 5 factors.

Ans: A

(6) The factors of 36 are

1, 2, 3, 4, 6, 9, 12, 18, 36.

Number of factors of 36 is 9.

(7) three consecutive numbers of 5 are

$5x, 5(x+1), 5(x+2)$

Sum: $5x + 5(x+1) + 5(x+2) = 180$

$\Rightarrow 5x + 5x + 5 + 5x + 10 = 180$

$\Rightarrow 15x + 15 = 180$

$\Rightarrow 15x = 180 - 15$

$\Rightarrow 15x = 165$

$\Rightarrow x = \frac{165}{15} = 11$

$\boxed{x = 11}$

$5x = 5 \times 11 = 55$

∴ The smallest multiple of 5 is 55.

Ans:

(8) The 9^{th} multiple of 14 is

$14 \times 9 = 126$

$\therefore$ The 9^{th} multiple of 14 is 126

Ans: B

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(9) Two consecutive odd numbers are $x, x+2$.

The sum is 44

$$x + (x+2) = 44$$

$$2x + 2 = 44$$

$$2x = 44 - 2$$

$$x = \frac{42}{2}$$

$$x = 21$$

$\therefore$ The largest number $x+2$

$= 21 + 2$

$= 23$

Ans: B

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(10) From option A: $a+b$

$1+6 = 7$ is a prime number

From option B: $a+b$

$3+4 = 7$ is a prime number.

Ans: D

(11) While $c = a \times b$.

a and b are not necessary factors of c

Ans: B

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(12) The Largest multiple of 19 less than 2000

$$= \frac{2000}{19} = 105.26$$

The Largest Integer less than 105.26 is 105

$19 \times 105 = 1995$

So, 1995 is odd

Ans: B

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(13) We identify the next even number after 1000 is by adding 2 to 1000

$1000 + 2 = 1002$

∴ The Smallest even integer greater than 1000 is 1002.

Ans: A

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(14) If a is a factor of b

The common factor of a and b are the factor of a itself.

∴ the number of common factors of a and b is equal to the number of factors of a.

Ans: B

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(15) The factors of 192 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96 and 192.

Ans: D

JEE ADVANCED LEVEL QUESTIONS

MULTI CORRECT ANSWER TYPE:

(16) If a is a natural number, the multiples of a are obtained by multiplying a by other natural number.

$a = a, 2a, 3a, \ldots na.$

Ans: A, B, C, and D

(17) Option B: It is a prime number.

→ This statement is not necessary true Not all numbers divisible by 9 are prime for example 27.

Ans: B

(18) This statement-I is false. A multiple of number is not necessary a factor of that number.

→ Statement-II is true. If 15 is a multiple of x, then x divides 15 exactly without leaving a remainder, which means x is a factor of 15.

Ans: D

(19) Statement-I : The number of multiples of a given number is infinite, not finite.

Statement-II : For given Integer, there is a Limited number of Integers that can divide it without leaving a remainder.

Ans : D

COMPREHENSION TYPE

COMPREHENSION-1

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(20) By defination every number is a multiple of 1, because any number (n) can be expressed as $n = 1 \times n$ indicating that n is a multiple of 1.

Ans : A

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(21) The multiples of a number extend infinitely in the positive and negative direction.

Ans : D

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(22) Every number is divisible by 1. This make 1 is a factor of every number, because any number n divided by 1 equals n itself.

Ans: A

INTEGER TYPE

(23) The factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

The number of factors of 24 are 8.

(24) The number of multiples of 8 is infinite.

(25) The number 31 is a prime number, which means its only factors are 1 and 31 itself.

(26) 143 can be divided by 11

$\Rightarrow \frac{143}{11} = 13$

Both 11 and 13 are prime numbers.

So, the prime factors of 143 are 11 and 13,

the sum : $11+13 = 24$

MATRIX MATCHING TYPE

(27)

(a) Prime factors of 99 : 3, 3, and 11 is an odd prime factors.

Ans : P

(b) Prime factor of 100 : 2, 2, and 5, 2 is an even prime factor.

Ans : Q

(c) The single digit perfect number is 6 since the sum of its divisors excluding itself, 1, 2, and 3 are equals to 6.

Ans : S

(d) The double digit perfect number is 28 since the sum of its divisors excluding itself, 1, 2, 4, 7 and 14 are equals to 28.

Ans : t.

(28) MATRIX MA

(a) Ans: S

(b) Ans: P

(c) Ans: Q

(d) Ans: R

LEARNERS TASK

Conceptual Understanding Questions (CUQ's)

(1) The first five natural numbers are 1, 2, 3, 4 and 5.

Product of the first five natural numbers

$= 1 \times 2 \times 3 \times 4 \times 5$

$= 120.$

Ans: A

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(2) The factors of 18 are 1, 2, 3, 6, 9 and 18.

The number 18 has a total of 6 factors.

Ans: B

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(3) Every Integer is divisible by 1.

$\therefore$ 1 is a factor of every number.

Ans: A

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(4) Every number is a factor of itself.

Because any number (n) divides n without leaving a remainder.

Ans: C

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(5) The greatest factor of any number n is n Itself. Because a number divides itself without leaving a remainder, and no other number greater than n, can be a factor of n.

Ans: D

(6) The number 0 is a multiple of every number. because any number multiplied by 0 results in 0.

Ans: D

(7) The odd multiple of 10 does not exist. Multiples of 10 are always even because 10 is an even number.

Ans: D

(8)

⇒ 6 is a perfect number because the sum of its proper divisors (1, 2, 3) equals the number itself (1+2+3=6).

⇒ 28 is a perfect number because the sum of its proper divisors (1, 2, 4, 7, 14) equals the number itself (1+2+4+7+14 = 28).

Ans: D

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(9) The concept of a greatest multiple of 10 doesn't exist because multiples of 10 continue infinitely.

Ans: D

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(10) 32 is not a multiple of 32 because 32 is not divisible by 6.

Ans: C

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(11) The factors of 100 are 1, 2, 4, 5, 10, 20, 25, 50 and 100... Hence, the sum is

$= 1+2+4+5+10+20+25+50+100 = 217.$

Ans: D

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(12) Multiples of 9 include numbers like 9, 18, 27, 36, ... etc.

There are infinitely multiples of 9.

Ans: B

JEE MAINS LEVEL QUESTIONS

(1) The LCM of 8 and 12

$LCM = 2^3 \times 3^1 = 8 \times 3 = 24$

So that the smallest number is 24.

Ans: A

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(2) The first 7 multiples of 9 are

9, 18, 27, 36, 45, 54, 63.

And the sum of total 7 multiples of 9 are

$9+18+27+36+45+54+63 = 252$

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(3) The factors of 100 are, 1, 2, 4, 5, 10, 20, 25, 50 and 100.

So, there are 9 factors of 100.

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(4)

(1)

$= \frac{1533}{73}$

$= 21$

$\therefore$ The 73 is a factor of 1533.

Ans: B

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(2) The factors of 1024 are 1, 2, 4, 8, 16, 32, 64, 128, 256, 512 and 1024.

So, number of factors of 1024 is 11.

Ans: A

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(3) The 100th multiple of 10 is calculated by multiplying 10 by 100 [$100 \times 10 = 1000$].

So, the 100th multiple of 10 is 1000.

=> The factors of 10 are 1, 2, 5 and 10. The least of these factors is 1.

=> Now add the 100th multiple of 10 and least factor of 10 is = $1000 + 1 = 1001$.

Ans: A

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(4) x is a factor of y.

y is a multiple of z

=> x and z are factors of y

Since x is a factor of y

and y is a multiple of z

both x and z are factors of y.

Ans: C

(5)

It directly matches the given information that $(a-b)$ and $(a+b)$ are factors of c.

$\Rightarrow (a-b) \times (a+b) = c$.

Ans: C

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(6)

Since 'c' is a common factor, it divides both 'a' and 'b'.

Because $a > b$, it follows that a is a longer multiple of c compared to b.

Ans: D

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(7)

$\Rightarrow \frac{x}{y} = z$

$\Rightarrow x = y \times z$

we can see that x is product of y and z hence x is a multiple of both y and z.

Ans: C

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(8) First, we compute (xyz):

$$xyz = (ab)(bc)(ca) = a^2b^2c^2$$

Next, we compute fraction:

$$\frac{xyz}{abc} = \frac{a^2b^2c^2}{abc}$$

Simplifying the expression:

$$\frac{a^2b^2c^2}{abc} = abc.$$

Ans: D

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(9) The product of the Least common Multiple (LCM) and greatest common divisor of x denoted as x is x.

Ans: B.

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(10) n is a natural number. The consecutive multiples can be expressed as

$x = n,\ y = 2n,\ z = 3n,\ w = 4n,\ r = 5n$

The sum of x, y, z and r is

$x + y + z + w = n + 2n + 4n + 5n$

z is the sum

$z = 3n$

To find many time z is of $(x+y+w+r)$

$$\frac{x+y+w+r}{z} = \frac{12n}{3n} = 4$$

∴ The sum of x, y, w, and r is 4

Ans: C

(11) The common multiples by n, the consecutive multiples are given as

$a = n,\ b = 2n,\ c = 3n,\ d = 4n,\ e = 5n$

the sum, is 45.

$a + b + c + d + e = n + 2n + 3n + 4n + 5n$

$15n = 45$

$n = 3$

The next multiple after e,

$e = 5n \Rightarrow 5 \times 3 = 15$

The next multiple after e would be 6n

$6n = 6 \times 3 = 18.$ Ans: C

(12) From the given relationships,

d is a multiple of c, c is a multiple of b and b is a multiple of a.

∴ d is a factor of c.

Ans: D

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(13) ADVANCED LEVEL QUESTIONS

MULTI CORRECT ANSWER TYPE:

(13) If x is a composite number and its factors are a, b and c with one of these factors equal to x, the x must be a perfect number.

A perfect number is defined as a positive integer that is equal to the sum of its proper divisors (exactly itself).

Ans: A, C

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(14) Factors of 10 = 1, 2, 5, 10

∴ Sum of factors 1+2+5+10 = 18

So, 18 is a multiple of 3.

∴ The number x for which the sum of its factors is a multiple of 3 is 10.

Ans: D

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(15) a, b, c and d are 4 consecutive multiples of a number.

$b^2 = a \times d$.

option A ⇒ $4^2 = 2 \times 8$

$16 = 16$ It's satisfied the condition.

option B ⇒ $36^2 = 3 \times 12$

$36 = 36$ It's satisfied the condition.

option C ⇒ $10^2 = 5 \times 20$

$100 = 100$ It's also satisfied the condition.

Ans: All.

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(16) 9 is not a factor of 24.

Ans: C.

(17) Multiple of 3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

Multiple of 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50.

∴ Multiple of both 3 and 5 are 15, 30.

Ans: B and D

(18) If a, b, c, d, e, z are the even consecutive multiples of a number (x), the smallest multiple would be 2x, which is not 1.

⇒ If a, b, c, d, e, z are consecutive even factors of a number Y, a being 1 is not possible because 1 is not an even number.

Ans: D

(19) Statement I: The sum of the factors of a number is always greater than the number itself.

Statement II: The sum of the factors of 10 is 18

Ans: D

(20) Statement I.: The factors of 12 are 1, 2, 3, 4, 6, and 12.

Excluding 12, the sum of the factors (1+2+3+4+6 = 16, which is not 6.

Statement II: Examples of perfect numbers include 6 factors: 1, 2, 3; sum of factors = 6.

The number is defined as a positive integer that is equal to the sum of its proper divisors.

Ans: B

COMPREHENSION TYPE

COMPREHENSION - I

(21) The number of factors of 18 is

$18 = 2^1 \times 3^2$

∴ Number of factors = $(1+1)(2+1) = 2 \times 3 = 6$

Ans: C

(22) Number of factors of 180 is

$180 = 2^2 \times 3^2 \times 5^1$

Number of factors $= (2+1)(2+1)(1+1) = 3 \times 3 \times 2$

$= 18.$

Ans: A

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(23) option C: Number of factors of 102

$102 = 2 \times 3 \times 17$

factors $= 2 \times 2 \times 2 = 8.$

Ans: C

INTEGER TYPE

Question no (14)

If $y = 2^3$

$y = 8$

factors of 8 are 1, 2, 4, 8

factors of 8 is 4.

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(15) The number of factors of x is given by

$(2+1)(P+1) = 24$

Simplyfying $3(P+1) = 24$

$P+1 = 8$

$P = 8-1$

$P = 7_{11}$

MATRIX MATCHING TYPE

(16)

(a) Ans : t

(b) Ans : q

(c) Ans : p

(d) Ans : r